Proportional Reasoning

Fast Track GRASP Math Packet

Part 1

Version 1.2
Released 8/1/2024

http://www.collectedny.org/ftgmp

Table of Contents

Welcome!

Congratulations on deciding to continue your learning! We are happy to share this study packet on proportional reasoning. We hope that these materials are helpful in your efforts to earn your high school equivalency diploma. This group of math study packets will cover mathematics topics that we see on high school equivalency exams. If you study these topics carefully, while also practicing other math skills, you will increase your chances of passing the exam.

Please take your time as you go through the packet. You will find plenty of practice here, but it's useful to make extra notes for yourself to help you remember. You will probably want to have a separate notebook where you can recopy problems, write questions, and include information that you want to remember. Writing is thinking and will help you learn.

After each section, you will find an answer key. Try to answer all the questions and then look at the answer key. It's not cheating to look at the answer key, but do your best on your own first. If you find that you got the right answer, congratulations! If you didn't, it's okay. This is how we learn. Look back and try to understand the reason for the answer. Please read the answer key even if you feel confident. We added some extra explanations and examples that may be helpful. If you see a word that you don't understand, try looking at the *Vocabulary Review* at the end of the packet.

We hope you share what you learn with your friends and family. If you find something interesting here, tell someone about it! If you find a section challenging, look for support. If you are in a class, talk to your teacher and your classmates. If you are studying on your own, talk to people you know or try searching for a phrase online. Your local library should have information about adult education classes or other support. You can also find classes listed here: **https://www.acces.nysed.gov/aepp/find-adult-education-program**.

You are doing a wonderful thing by investing in your own education right now. You have our utmost respect for continuing to learn as an adult.

Please feel free to contact us with questions or suggestions.

Best of luck!

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Overview

Prerequisites	There are no prerequisites to *Proportional Reasoning, Part 1.* As long as you are able to read this packet independently, you don't have to study any other math packets first. Students should complete *Proportional Reasoning, Part 1* before working on *Proportional Reasoning, Part 2.*
Connections	Students can find more opportunity to practice using ratios and proportions in the Fast Track GRASP Math Packets, *Being Counted: Probability & Statistics, Parts 1 and 2*.

In this packet, you will explore concepts in proportional reasoning, including ratios, rates, and proportions.

<u>In Part 1, you will study the following topics:</u>

- Part-whole, part-to-part ratios, and rates
- Equivalent ratios
- Double number lines, ratio tables, and bar models
- The language of proportional reasoning

<u>In Part 2, you will build on what you learned in Part 1, and study the following topics:</u>

- Connections between ratios, fractions, percents, and decimals
- Making comparisons between ratios
- Solving proportions

<u>In addition to the learning the topics above, you will find the following materials to help you:</u>

- High School Equivalency Test Practice Questions. You will practice the concepts you have learned from this packet to work on these questions.
- A graphic organizer to study vocabulary is included, along with a vocabulary activity to review concepts. A glossary with important terms from this packet is also included for your study.
- Concept Circles can help you make connections between the concepts you have learned and help you remember those connections.

Assessment Questions

Calculator allowed

The following questions will help to see if this packet is right for you. Do your best to answer each question. If you can't answer, don't worry—this packet will help you answer questions like these and more. When you are finished with the questions, read our recommendations.

Question 1

Tickets for a baseball game cost $60 for a family of 5. Adult and youth tickets cost the same amount. Based on this information, decide whether each statement is TRUE or FALSE.

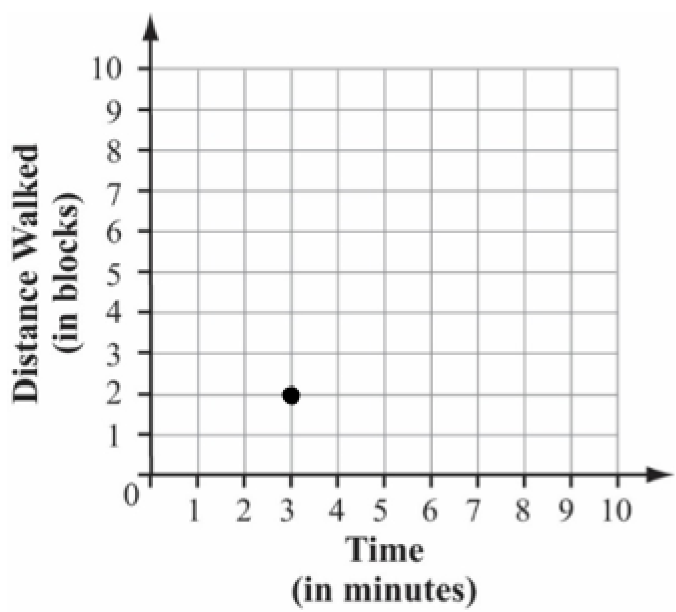
	TRUE	FALSE
2 tickets cost $24.		
For $40, you can buy 4 tickets.		
The cost is $12 per ticket.		
The cost for 10 tickets is $65.		

Question 2

It takes Mildred about 3 minutes to walk 2 blocks. A point has been plotted in the coordinate plane to represent this situation.

A. Plot a second point that represents an equivalent ratio.

B. Explain what the coordinates of the point you plotted represent.



Question 3

Julia uses a photocopier to enlarge a business logo. The original dimensions of the logo were 2” by 3”. Which of the following could ***not*** be the dimensions of the enlargement?

A. 4” by 6”

B. 4” by 5”

C. 6” by 9”

D. 8” by 12”

Question 4

The price of Gina’s dinner before tax and tip was $20.00. The restaurant added 10% tax to the bill, then Gina left a $3.00 tip. How much did Gina pay for her dinner in total?

A. $22.00

B. $24.00

C. $25.00

D. $33.00

Answer Key

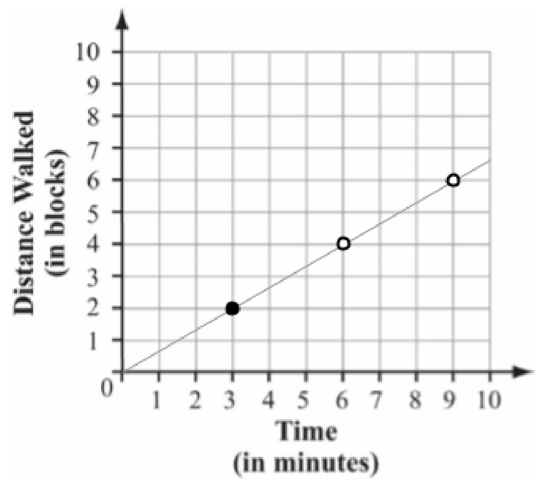
Question 1.

	TRUE	FALSE
2 tickets cost $24.	x	
For $40, you can buy 4 tickets.		x
The cost is $12 per ticket.	x	
The cost for 10 tickets is $65.		x

Question 2.

A. (6, 4) and (9, 6) should be the most common correct answers, but any points along the line would also be correct.

B. The student's explanation should show that they understand that the x value represents the time that it takes Mildred to walk y blocks. For example: It takes Mildred 6 minutes to walk 4 blocks.



Question 3. B

Question 4. C

Recommendations

Consider the following when making a decision about working through this packet:

- Student has some difficulty with Question 1 or 2: The student may choose to work through *Proportional Reasoning, Part 1.*
- Student has some difficulty with Question 3 and 4: If a student comfortably answers Questions 1 & 2, but has some difficulty with Questions 3 and 4, the student may feel confident enough to skip *Proportional Reasoning, Part 1* and go directly to *Proportional Reasoning, Part 2.*
- Student comfortably answers all four questions: The student may choose to work on a different packet. However, it is recommended that students complete the Test Practice Questions in *Proportional Reasoning, Part 2* for questions that require students to interpret a variety of proportional situations before they take the HSE exam.

This assessment asks students to demonstrate understanding of:
Question 1 *(from Proportional Reasoning, Part 1):* Compute unit rates, solve real-world problems using ratios or proportions (GED Quantitative Problem Solving Assessment Targets Content Indicator: Q.3.a, Q.3.c).

Question 2 *(from Proportional Reasoning, Part 1 and 2):* Compute unit rates, locate points in the coordinate plane, interpret unit rate as slope (GED Algebraic Problem Solving Assessment Targets Content Indicators: A.5.a, A.5.c, Q.3.a)

Question 2 *(from Proportional Reasoning, Part 1):* Compute unit rates, solve real-world problems using ratios or proportions (GED Quantitative Problem Solving Assessment Targets Content Indicator: Q.3.b).

Question 3 *(from Proportional Reasoning, Part 1):* Compute unit rates, solve real-world problems using ratios or proportions (GED Quantitative Problem Solving Assessment Targets Content Indicator: Q.3.d).

Vocabulary

It is important to understand mathematical words when you are learning new topics. The following vocabulary will be used a lot in this study packet:

convert, decimal, equivalent, proportion, quantity, rate, ratio, relationship

When we learn new vocabulary, it is good to think about your experience with the word. Asking questions like, "Have I heard this word before?", "When have I heard this word?", "What do I think this word means?" can help you build on what you already know.

Here's how it works. On the next page, you'll find a chart with each of the vocabulary words above. For each word, ask yourself how familiar you are with the word. For example, think about the word "*area.*" Which of these statements is true for you and your experience with the word "*area*"?

- I know the word "*area*" and use it in conversation or writing.
- I know the word "*area*," but I don't use it.
- I have heard the word "*area*" but I'm not sure what it means.
- I have never heard the word "*area*" at all.

In the chart on the next page, read each word and then choose one of the four categories and mark your answer with a ✔ (checkmark). Then write your best guess at the meaning of the word in the right column. If it's easier, you can also just use the word in a sentence.

Here's an example of how the row for "area" might look when you're done:

Word	I know the word and use the word	I know the word but don't use it	I have heard the word, but I'm not sure what it means	I have never heard the word	My best guess at the meaning of the word (or use the word in a sentence)
area	✓				A place or location, like a neighborhood or town

This activity is designed to help you start thinking about some of the important words you will find in this packet. As you go through the activities in this packet, you will learn more about these words, what they mean, and how to use them. You will learn more precise definitions that may come up during your high school equivalency exam.

There is a glossary with the definitions of useful vocabulary at the end of the packet.

Word	I know this word and use the word	I know this word but don't use it	I have heard this word, but I'm not sure what it means	I have never heard this word	My best guess at what this word means...
convert					
decimal					
equivalent					
proportion					
quantity					
rate					
ratio					
relationship					

Multiplication Warm-up

Practicing multiplication will help you understand proportions, but it is also true that you can use proportions to answer multiplication questions.

1) Fill in the missing numbers in the multiplication table below.

×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2		4		8		12		16		20
3	3		9		15		21		27	
4		8		16		24		32		40
5	5		15		25		35		45	
6				24				48		
7										
8		16				48				80
9										
10				40				80		

2) How did you find out the missing numbers?

3) If $3 \times 8 = 24$, what is 6×8?
How do you know?

Multiplication By Heart at Mathigon.com (https://mathigon.org/multiply) is a great way to practice multiplication.

Multiplication Warm-up - Answer Key

1)

×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	36	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

2) There are different strategies we can use to find the missing numbers. You might add the same number as you go down or across. Or you might see patterns in the numbers that help you fill in the table. Or you might have some of these multiplication facts memorized. You don't have to have them memorized, but it can be helpful.

3) $6 \times 8 = 48$. If three 8's is 24, then six 8's is twice as much. This is an example of proportional thinking.

Introduction to Proportional Reasoning

A typical day is full of ratios and proportions.

Imagine waking up and pouring yourself a bowl of granola. One serving of granola is ¼ cup and has 150 calories. Some of us keep track of how many calories and nutrients we take in. How many calories would be in one cup of granola? How much sugar? How many grams of protein? The Nutrition Fact labels on our food are full of ratios and proportions that can help us keep track of the food we eat.

nutrients in our food

Maybe after breakfast, we take a shower. If your shower uses about two gallons of water per minute, how much water do you use in 10 minutes? If you take a 10-minute shower every day, how much water do you use in a week?

Ratios help us get where we are going. After getting dressed, some people drive to school or work. The speedometer measures your speed in a ratio (miles per hour). The price of gas is also a ratio. On the sign below, gas costs $3.43 per gallon. This ratio is very important in household economics!

speedometer in a car

the price of gas

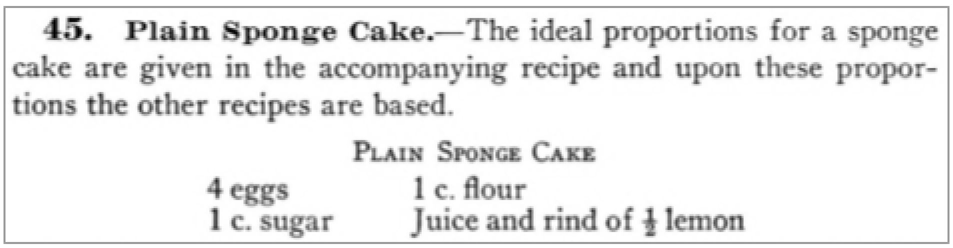
Some people ride a bike to work. If you ride a bike at a speed of 12 miles per hour for a half hour, how far did you travel? We can use proportional reasoning to find out.

Your heart rate is how many times your heart beats per minute. If you count your heart beats for 30 seconds, you can figure out how many times it would beat in one minute. When resting, my heart rate is about 70 beats per minute. A friend wears a Fitbit in yoga classes so that he can look at his heart rate data after class. His heart rate went up to 150 beats per minute in a difficult class!

lemons for sale

After exercising, we might stop at a grocery store to buy ingredients for a cake. The price of produce is often shown as a ratio. According to this sign, you can buy 4 lemons for $2.

Our recipe for sponge cake calls for eggs, sugar, flour, and lemons. The ingredients are all in proportion to each other. The recipe below is for one cake. If we need to make three cakes, how much will we need of each ingredient?


45. Plain Sponge Cake.—The ideal proportions for a sponge cake are given in the accompanying recipe and upon these proportions the other recipes are based.

Plain Sponge Cake

4 eggs	1 c. flour
1 c. sugar	Juice and rind of ½ lemon

part of a recipe from a cookbook

These are just a few examples of ratios and proportions we see in our daily lives. Ratios and proportions are some of the most important math that we study in adult basic education, since we use this kind of math every day. When we think about rates, fractions, percents, and decimals, we are using proportional reasoning. Improving your proportional reasoning will make you more powerful when you use math. They are also an important part of the high school equivalency math exam.

In this packet, we will explore how to make sense of ratios and proportions in the world. We will practice thinking about ratios so that you do <u>not</u> have to memorize any tricks to solve proportions. All you have to do is use your good thinking brain. You will practice *reasoning*, which is the process of using logic to arrive at a conclusion. We will practice reasoning with ratios and proportions, using the information we have and applying it to different situations.

In Part 1 of this packet, you will practice creating different kinds of ratios, making *equivalent ratios*, or ratios that represent the same value. You will use double number lines, ratio tables, and bar models to understand equivalent ratios. You will also learn what ratios look like on a graph.

In Part 2 of this packet, you will make connections between ratios, fractions, percents, and decimals and make comparisons between ratios. Then you will solve proportions using what you have learned about equivalent ratios.

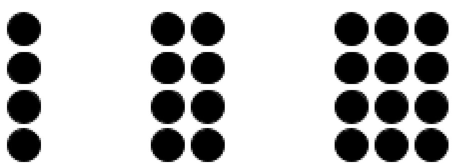
Sequences

A *sequence* is a list of things in order. Sometimes, a sequence can be a list of numbers, but it can also be a series of pictures.

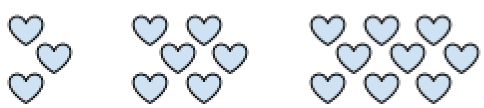
The pictures below show the first three steps in a sequence. Look for a pattern, then draw steps 4 and 5 of the sequence.

Example:

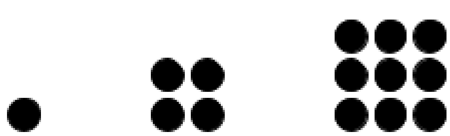
1 2 3 4 5



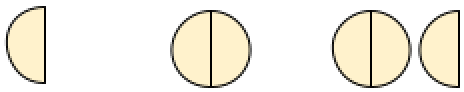
4) 1 2 3 4 5



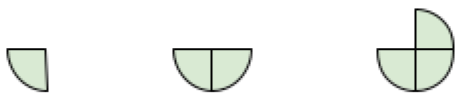
5) 1 2 3 4 5



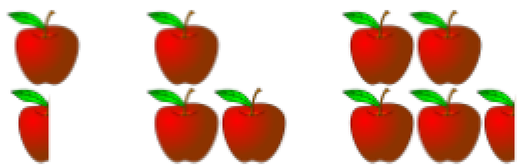
6) 1 2 3 4 5



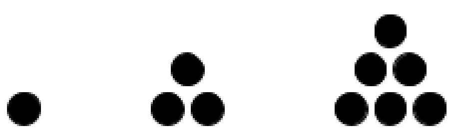
7) 1 2 3 4 5



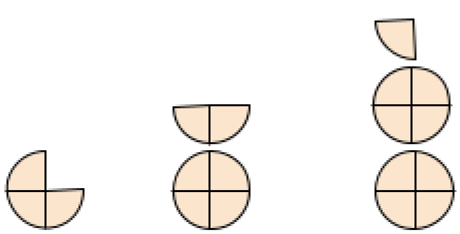
8) 1 2 3 4 5



9) 1 2 3 4 5



10) 1 2 3 4 5



11) 1 2 3 4 5

These are dollar bills ($1.00).

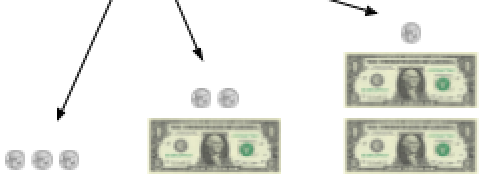
12) 1 2 3 4 5

This is a nickel ($0.05).

This is a dime ($0.10).

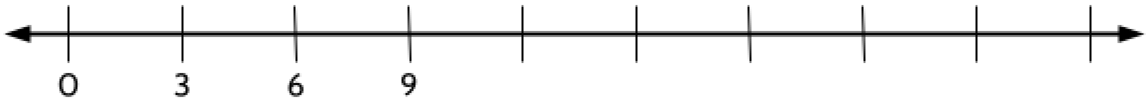
13) 1 2 3 4 5

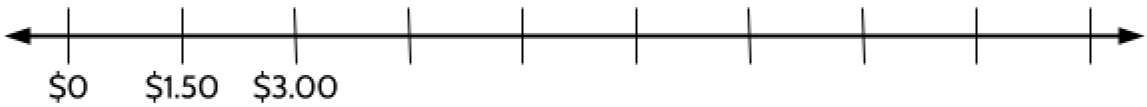
These coins are quarters ($0.25 each)

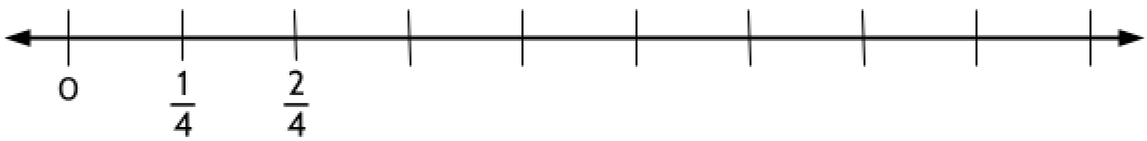


14) 1 2 3 4 5

Write in the missing numbers on the number lines below.

15) 

16) 

17) 

Complete these sequences by writing in the missing numbers.

18) 2, 4, 6, __________, __________, __________, __________, __________, __________, 20, 22, ...

19) $0.75, $1.50, $2.25, __________, __________, __________, __________, $6.00, $6.75, ...

20) 1.5, 3, 4.5, __________, __________, __________, __________, __________, 13.5, 15, ...

21) $1\frac{1}{4}$, $2\frac{2}{4}$, $3\frac{3}{4}$, __________, __________, __________, __________, 10, $11\frac{1}{4}$, ...

22) 24 is the second *term* in this sequence: 12, 24, 36, ...

What is the sixth term in the sequence?

23) $\frac{3}{8}$ is the third *term* in this sequence: $\frac{1}{8}$, $\frac{2}{8}$, $\frac{3}{8}$, $\frac{4}{8}$...

What is the ninth term in the sequence?

To play a game with more sequences, try **What Comes Next?** at https://bit.ly/whatcomesnextgame.

Introducing Ratios

A *ratio* is a comparison between two or more numbers. It is a relationship between two quantities. Here are some examples of ratios that we might see in the world around us:

Shopping: 2 apples for 1 dollar

Nutrition: 140 calories for 2 cookies

Wages: $16 per hour

Measurement: 12 inches per foot

Photography: 4 inches by 6 inches

Currency: 17.5 Mexican pesos for 1 U.S. dollar

Speed: 60 miles per hour

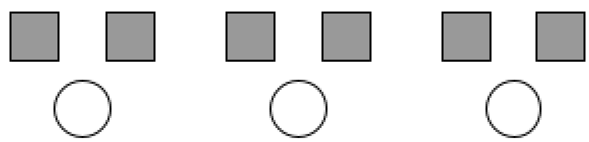
Fuel efficiency: 30 miles per gallon

Health: 70 heartbeats per minute

Sports: 221 free throws out of 237 attempts

As you can see, ratios are used in many different situations. Each of the ratios above compare one thing to another.

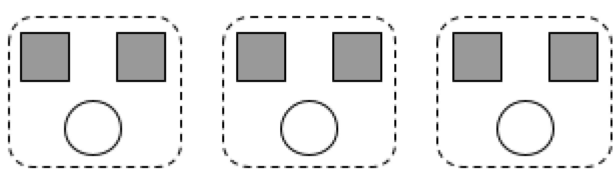
In this section, we will practice using different ratios to describe situations. For example, look at this collection of shapes:



Here are a few ways we could describe the collection:

- For every 6 squares, there are 3 circles.
- The ratio of squares to circles is 6 to 3, which can also be written as 6:3.
- The ratio of circles to squares is 3 to 6, which can also be written as 3:6.

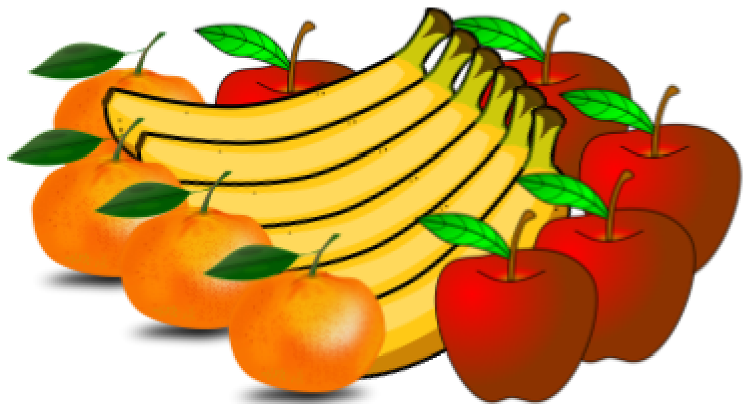
You might notice that the shapes are repeated in groups:



This allows us to compare the shapes in new ways:

- There are 2 squares for every 1 circle.
- There is 1 circle for every 2 squares.

Look at the collection of fruit and complete the sentences below.

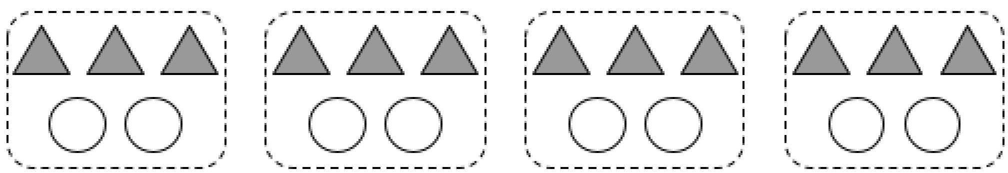


24) The ratio of apples to bananas is ________:________ .

25) The ratio of oranges to apples is ________:________ .

26) For every ________ bananas, there are ________ oranges.

Below, each of the groups of shapes show the ratio of 3:2, and the whole set of groups together also shows the ratio of 3:2.



Complete the sentences below to write more ratios based on the collection of shapes above.

27) The ratio of triangles to circles is 12:________ .

28) For every 12 ________________ , there are 8 ________________ .

29) For every 3 triangles, there are ________ circles.

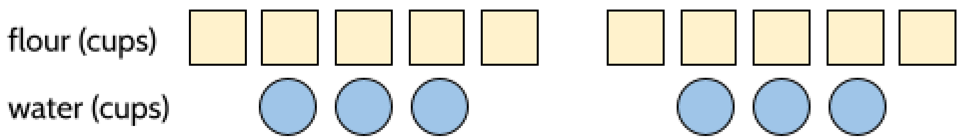
30) There are ________ circles for every ________ triangles.

The ratio 3:2 could be used to describe the relationship between other things. For example, it could mean 3 cups of flour for every 2 cups of butter in a recipe for cookies, or a soccer team that wins 3 games for every 2 games it loses, or a ramp that goes over 3 feet for every 2 feet it goes up.

The ratio 3:2 means a relationship between any two quantities where there are 3 of one quantity for every 2 of the other quantity.

There are many ratios in cooking and baking. Here is an example of a ratio used when baking.

31) Emil is using a recipe to make 2 loaves of bread for his family. The recipe calls for 10 cups of flour and 6 cups of water.



Read the statements below and circle all that correctly describe this situation.

A. The ratio of cups of flour to cups of water is 5:3.

B. For every cup of water, there are 2 cups of flour.

C. There is 1 cup of flour for every 3 cups of water.

D. For every 10 cups of flour, there are 6 cups of water.

E. There are 3 cups of flour for every cup of water.

F. The ratio of water to flour is 3:5.

32) The recipe above makes 2 loaves of bread. How many cups of flour would Emil need if he wants to make 4 loaves of bread?

A. 4

B. 10

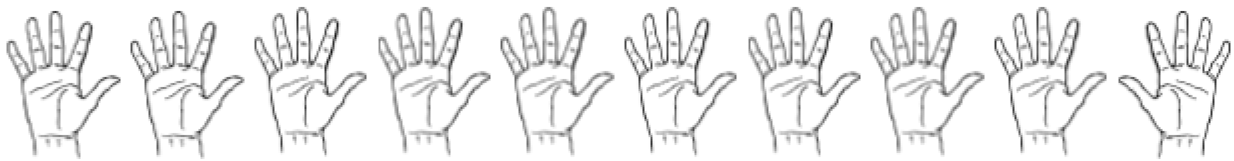
C. 12

D. 20

33) Draw the cups of flour and cups of water needed for 5 loaves of bread.

Part-Whole and Part-to-Part Ratios

In humans, about 9 people out of every 10 people are right-handed. This is called a ***part-whole ratio***, since <u>part</u> of the population (the number of right-handed people) is compared with the <u>whole</u> population (the total number of people).



Part-whole ratios are written as a fraction ($\frac{9}{10}$) or with a colon (9:10).

$$\frac{9\ right-handed\ people}{10\ total\ people} = \text{9 right-handed people out of every 10 people} = 9{:}10$$

For every part-whole ratio, there is a different part-whole ratio that can be made with the other part. For example, if 9 out of every 10 people are right-handed, then 1 out of every 10 people is left-handed.

Note: A part-whole ratio is a fraction since it compares part of a quantity to the whole quantity. A part-whole ratio is a ratio <u>and</u> a fraction.

34) Which of the following shows the part-whole ratio of 1 left-handed person out of every 10 people? Select all that apply.

   A. $\frac{1\ left-handed\ person}{10\ total\ people}$

   B. $\frac{1\ left-handed\ person}{9\ right-handed\ people}$

   C. 1 left-handed person:10 people

   D. 1 left-handed person:9 right-handed people

35) If 20 people were randomly chosen and asked which of their hands is dominant, how many would you expect to be left-handed?

36) If 30 random people were asked, how many would you expect to be left-handed?

Imagine we flipped 20 pennies and laid out the results on the right.

With a part-whole ratio, we can compare the number of "heads" to the total number of flips:

11 heads out of 20 flips

11 heads:20 total flips

$\frac{11\ "heads"}{20\ total\ flips}$

37) Fill in the blanks. Each example shows the ratio of the number of "tails" to the total number of flips.

9 tails out of _______ total flips

_______ tails:20 total flips

$\frac{\_\_\_\_\_\ tails}{20\ total\ flips}$

38) Which choice shows the part-whole ratio below?

Out of 20 M&Ms, 3 are yellow.

A. $\frac{3}{17}$

B. $\frac{3}{20}$

C. $\frac{17}{20}$

D. $\frac{17}{3}$

39) Which choices show the part-whole ratio below? Choose all that apply.

Out of 20 M&Ms, 4 are green.

A. 4 green:20 total

B. 4 green:16 total

C. 1 green:5 total

D. 20 total:4 green

If you want to compare the number of left-handed people to the number of right-handed people, you would use a ***part-to-part ratio***. Here are two examples of part-to-part ratios:

For every 9 right-handed people, there is 1 left-handed person.	or	For every 1 left-handed person, there are 9 right-handed people.

A part-to-part ratio compares part of the population (left-handed people) to another part of the population (right-handed people). Part-to-part ratios are often written with a colon (9:1) but can also be written as a fraction ($\frac{9\ right-handed\ people}{1\ left-handed\ person}$).

Note: Even though it can sometimes look like one, a part-to-part ratio is not a fraction. A fraction is a part-whole relationship. Comparing part of a quantity to another part of the quantity is a ratio, but is not a fraction.

Here are some ways to compare the number of left-handed people and the number of right-handed people:

$\frac{9\ right-handed\ people}{1\ left-handed\ person}$ = 9 right-handed people for every 1 left-handed person = 9:1

40) Which of the following shows the part-to-part ratio of 1 left-handed person for every 9 right-handed people? Select all that apply.

A. $\frac{1\ left-handed\ person}{10\ total\ people}$

B. $\frac{1\ left-handed\ person}{9\ right-handed\ people}$

C. 1 left-handed person:10 people

D. 1 left-handed person:9 right-handed people

41) In a survey of randomly chosen people, 5 of the people surveyed said they were left-handed. In the same survey, how many people would you expect to be right-handed?

42) If 36 people in a random survey were right-handed, how many would you expect to be left-handed?

Look at the results of our coin flip again. With a part-to-part ratio, we can compare the number of "heads" to the number of "tails."

43) How can we write this part-to-part ratio for coins? Choose all that apply.

A. $\frac{11 \text{ "}heads\text{"}}{9 \text{ "}tails\text{"}}$

B. 11:9

C. 9:11

D. $\frac{11 \text{ "}tails\text{"}}{9 \text{ "}heads\text{"}}$

44) Fill in the blanks to show three ways of comparing the number of "tails" to the number of "heads."

9 tails for every _______ heads

_______ tails:_______ heads

$\frac{\_\_\_\_\_ \ tails}{11 \ heads}$

45) Which choice shows the part-to-part ratio below? Choose all that apply.

In a pack of 56 M&Ms, 9 are yellow and 11 are green.

A. $\frac{9}{11}$

B. $\frac{9}{56}$

C. $\frac{11}{56}$

D. $\frac{11}{9}$

46) Which choice shows the part-to-part ratio below? Choose all that apply.

In a pack of 56 M&Ms, 9 are yellow and 11 are green.

A. 11 green:9 yellow

B. 9 yellow:11 green

C. 11 green:56 total

D. 9 green:56 total

47) Think about 10 friends, family members, or classmates. Write down their names.

In this group of 10 people...

...what is the ratio of left-handed people to total people?

...what is the ratio of right-handed people to total people?

...what is the ratio of left-handed people to right-handed people?

...what is the ratio of left-handed people to right-handed people?

48) For each of the following ratios, decide whether it is a part-to-part ratio (PTP) or a part-whole ratio (PW).

Ratios	part-to-part or part-whole
11 “heads” for every 9 “tails”	PTP
11 “heads” out of 20 coin flips	
Out of a class of 20 students, 12 students speak more than one language	
12 bilingual students for every 8 students who speak only one language	
4 blue M&Ms for every 3 yellow M&Ms	
42 correct answers for every 8 incorrect answers	
42 correct answers out of 50 answers on a test	
Out of 20 M&Ms, 3 are yellow	
5 M&Ms are brown and 4 M&Ms are green	

Practice your ratio skills with the situation below.

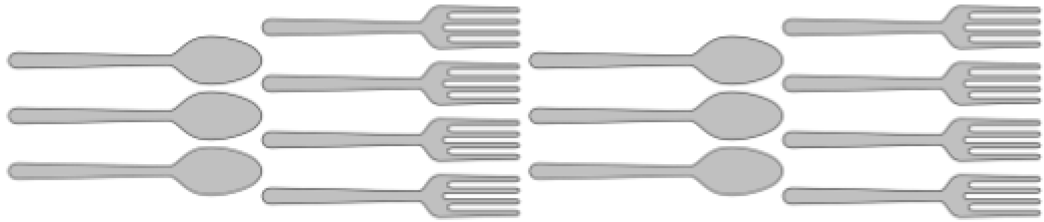
> Polly bought a bag of Polygon Pieces and emptied the candies onto a table. They were in the form of triangles (*t*), squares (*s*), pentagons (*p*), and hexagons (*h*).

49) Mark the following statements true or false.

Statements	T or F
There are 2 triangles for every 1 square.	T
The ratio of pentagons to triangles is 7:6.	
The ratio of hexagons to total candies is $\frac{4}{16}$.	
There are 7 pentagons out of 20 total pieces of candy.	
For every 2 hexagons, there are 3 triangles.	
The ratio of squares to total candies is 3:17.	
The ratio of triangles to total candies is $\frac{3}{10}$.	

50) Write your own true statement using ratios from Polygon Pieces diagram above.

51) Look at the image below and put a checkmark next to each true statement.



- ☐ There are 6 spoons for every 8 forks.
- ☐ The ratio of spoons to forks is 3:4.
- ☐ The ratio of forks to spoons is 8:6.
- ☐ There are 3 spoons for every 4 forks.
- ☐ There are 6 spoons for every 14 utensils.
- ☐ There are 8 forks for every 14 utensils.
- ☐ The ratio of forks to utensils is 8:14.
- ☐ The ratio of spoons to utensils is 6:14.

52) The ratio 3 spoons:4 forks is a…

A. part-to-part ratio

B. part-whole ratio

53) The ratio $\frac{6\ spoons}{14\ utensils}$ is a…

A. part-to-part ratio

B. part-whole ratio

54) The ratio 5 apples:15 pieces of fruits is a…

A. part-to-part ratio

B. part-whole ratio

55) The ratio $\frac{3\ bananas}{2\ oranges}$ is a…

A. part-to-part ratio

B. part-whole ratio

56) There are 16 students total in an adult education class. 12 of these students live in the Bronx. The remaining students live in Manhattan.

Write as many ratios as you can about the students in this class.

Rates

A *rate* is a ratio that shows the relationship between different kinds of quantities. To see the cost of bananas, we might look at the relationship between pounds and dollars. A price of $0.50 per pound is an example of a rate which compares dollars and pounds.

Here are some other examples of rates:

$16 per hour	80 heartbeats per minute	sales tax rate of 8.875 percent
50 miles per hour	10,000 steps per day	100 megabits per second (Mbps)

We often use the word per when we are working with rates. The word *per* means "for each" or "for every." So, being paid "$16 *per* hour" means 16 dollars *for each* hour of work.

57) What two quantities are compared in the rate shown on this sign?

A. number of apples and pounds

B. number of apples and dollars

C. quarters and dollars

D. pounds and dollars

58) What two quantities are compared in the rate shown on this gauge?

A. miles and gallons of gas

B. gallons of gas and hours

C. miles and hours

D. minutes and hours

59) What two quantities are compared in the rate shown on this sign?

A. miles and gallons of gas

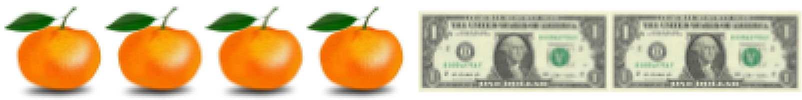
B. dollars and gallons of gas

C. gallons of gas and hours

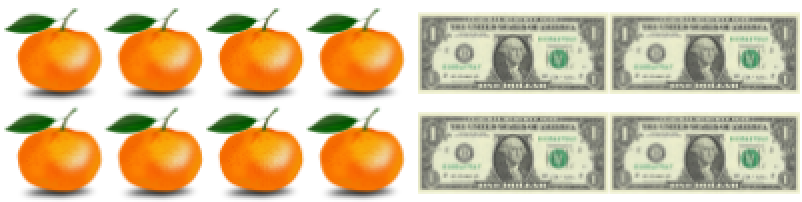
D. miles and hours

Look at the price of oranges shown on the right. This rate shows the relationship between the number of oranges and the price in dollars.

We could use a drawing to show that 4 oranges costs $2:

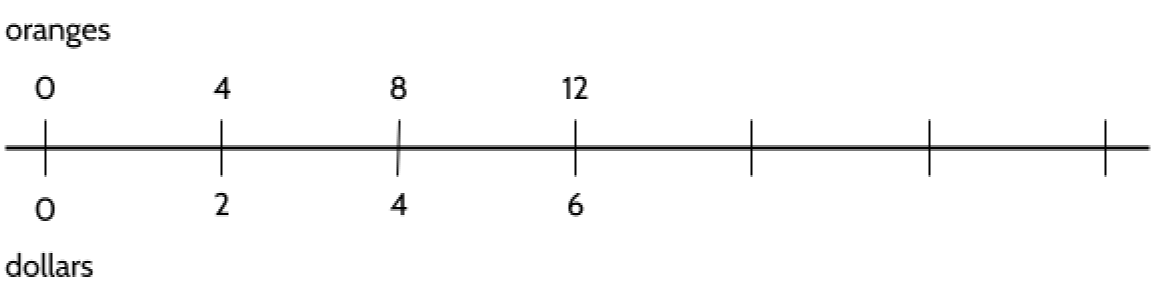


60) How much would 8 oranges cost?



61) Make a drawing below to show the cost of 12 oranges.

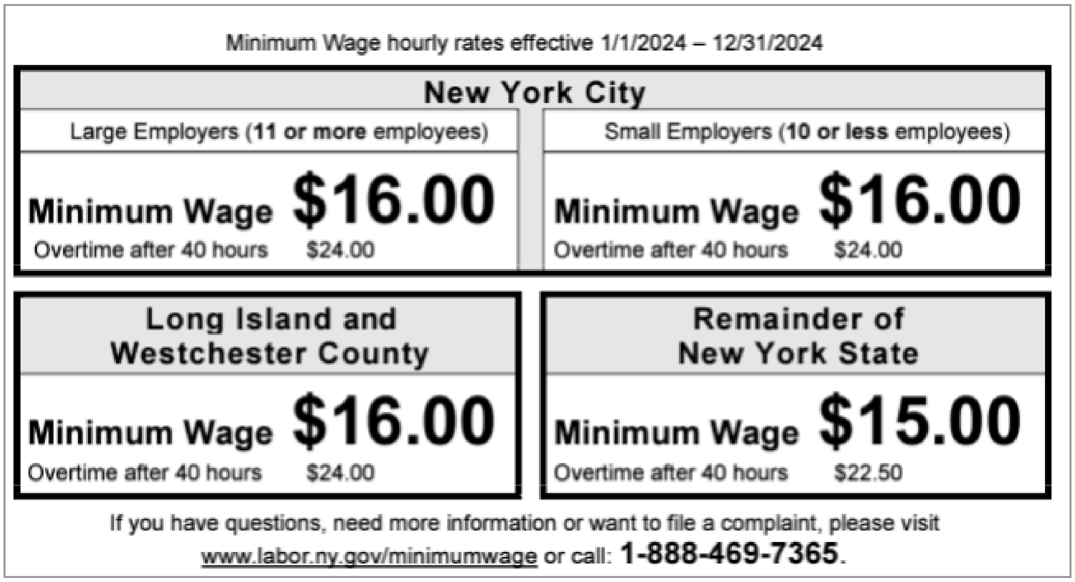
We can also use a double number line to see the prices of different quantities of oranges. This is an example of a double number line:



It is called a *double* number line because it shows two different quantities at the same time. The top of the number line shows numbers of oranges. The bottom of the number line shows the price.

62) What do you notice when you look at the oranges and dollars number line?

The following information is from the New York State Department of Labor.



Minimum Wage hourly rates effective 1/1/2024 – 12/31/2024

New York City

Large Employers (**11 or more** employees)	Small Employers (**10 or less** employees)
Minimum Wage $16.00 Overtime after 40 hours $24.00	**Minimum Wage $16.00** Overtime after 40 hours $24.00

Long Island and Westchester County	**Remainder of New York State**
Minimum Wage $16.00 Overtime after 40 hours $24.00	**Minimum Wage $15.00** Overtime after 40 hours $22.50

If you have questions, need more information or want to file a complaint, please visit www.labor.ny.gov/minimumwage or call: **1-888-469-7365**.

63) What two quantities are compared in the rates shown in this image?

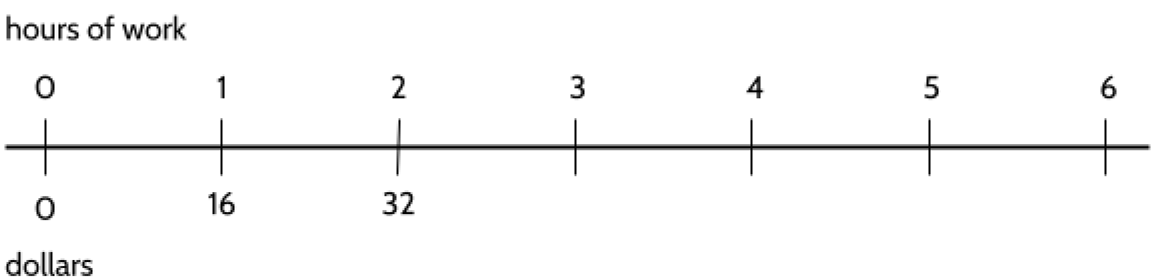
A. hours of work and number of employees

B. dollars and cents

C. large employers and small employers

D. dollars and hours of work

64) Write in the missing dollar amounts on the number line.



These quantities can be written as ratios: $\frac{\$16}{1\ hour}$, $\frac{\$32}{2\ hours}$, and $\frac{\$48}{4\ hours}$. Each of these ratios are *equivalent*, meaning they have the same value. Though they show different numbers of hours and amounts of money, they all follow the same hourly pay rate: 16 dollars for each hour of work.

65) Which of the following ratios is <u>not</u> equivalent to $16/hour?

A. 32 dollars:2 hours

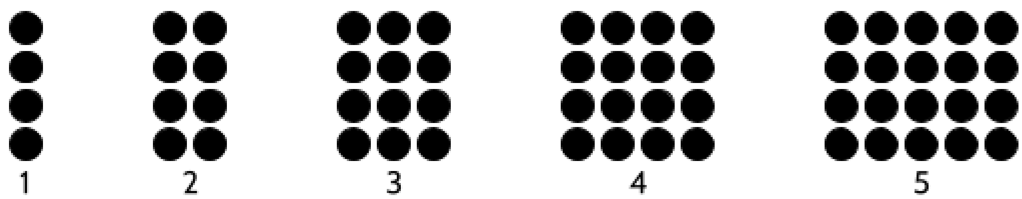
B. 45 dollars:3 hours

C. 80 dollars:5 hours

D. 160 dollars:10 hours

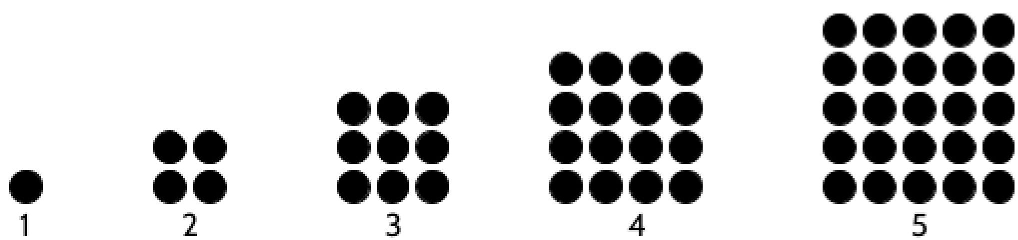
Introduction to Proportional Reasoning - Answer Key

Sequences



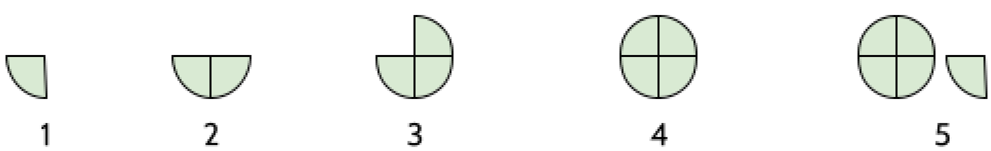
4)

5)



6)

7)



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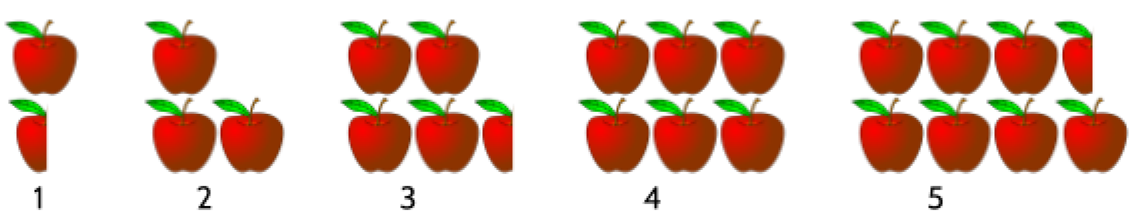

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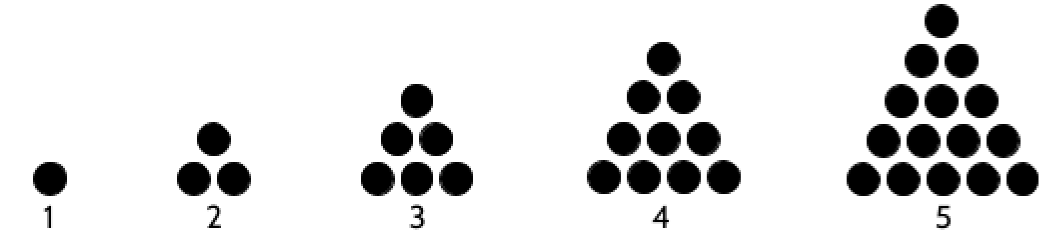
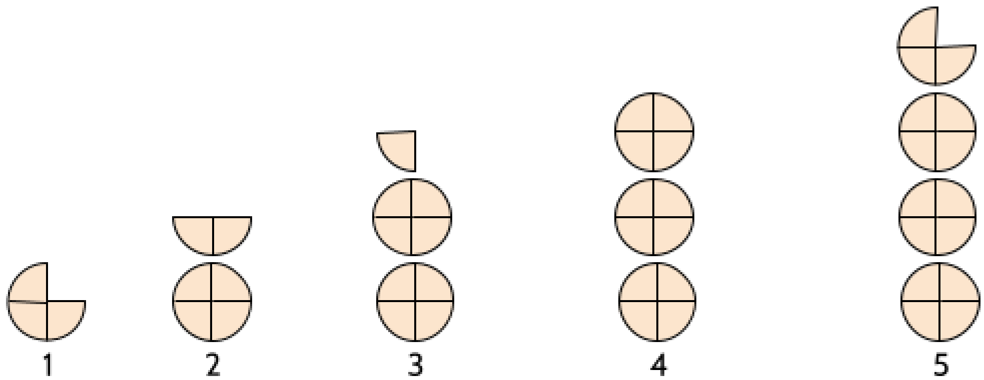
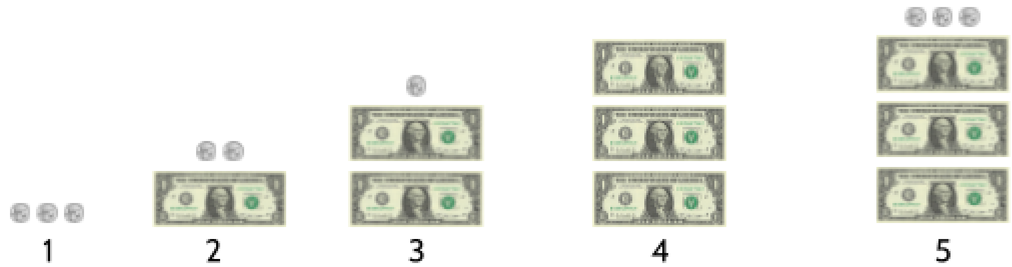
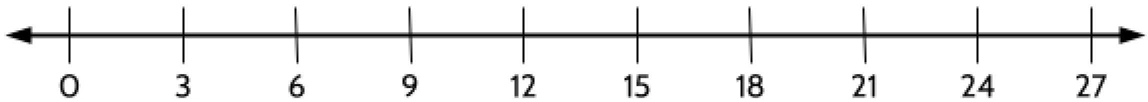
10) 

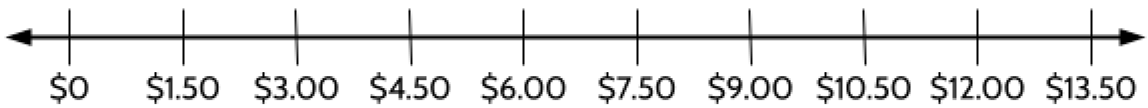
11) 

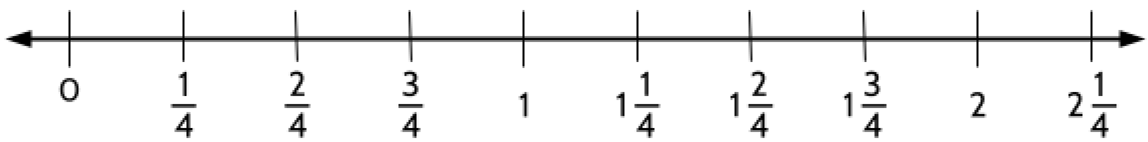
12) 

13) 

14) 

15) 

16) 

17) 

This series of fractions could also be written as $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \frac{5}{4}, \frac{6}{4}, \frac{7}{4}, \frac{8}{4}, \frac{9}{4}, \ldots$

18) 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, ...

19) $0.75, $1.50, $2.25, $3.00, $3.75, $4.50, $5.25, $6.00, $6.75, ...

20) 1.5, 3, 4.5, 6, 7.5, 9, 10.5, 11, 13.5, 15, ...

21) $1\frac{1}{4}$, $2\frac{2}{4}$, $3\frac{3}{4}$, 5, $6\frac{1}{4}$, $7\frac{2}{4}$, $8\frac{3}{4}$, 10, $11\frac{1}{4}$, ...

22) 72

23) $\frac{9}{8}$ or $1\frac{1}{8}$

Introducing Ratios

24) 5:6

25) 4:5

26) There are two right answers:
For every 6 bananas, there are 4 oranges.
For every 3 bananas, there are 2 oranges.

27) 8

28) triangles, circles

29) 2

30) There are a few right answers:
There are 8 circles for every 12 triangles.
There are 4 circles for every 6 triangles.
There are 2 circles for every 3 triangles.

31) A, D, F

32) D

33) Your drawing should show 25 cups of flour and 15 cups of water.

Part-Whole and Part-to-Part Ratios

34) A, C

35) 2

36) 3

37) 9 tails out of 20 total flips

9 tails:20 total flips

$$\frac{9\ \ tails}{20\ total\ flips}$$

38) B

39) A, C, D

40) B, D

41) 45

42) 4

43) All are correct ways of writing the part-to-part ratio

44) 9 tails for every 11 heads

9 tails:11 heads

$$\frac{9\ tails}{11\ heads}$$

45) A, D

46) A, B

47) Answers will vary.

48)

Ratios	part-to-part or part-whole
11 “heads” for every 9 “tails”	PTP
11 “heads” out of 20 coin flips	PW
Out of a class of 20 students, 12 students speak more than one language	PW
12 bilingual students for every 8 students who speak only one language	PTP
4 blue M&Ms for every 3 yellow M&Ms	PTP
42 correct answers for every 8 incorrect answers	PTP
42 correct answers out of 50 answers on a test	PTW
Out of 20 M&Ms, 3 are yellow	PTW
5 M&Ms are brown and 4 M&Ms are green	PTP

49)

Statements	T or F
There are 2 triangles for every 1 square.	T
The ratio of pentagons to triangles is 7:6.	T
The ratio of hexagons to total candies is $\frac{4}{16}$.	F
There are 7 pentagons out of 20 total pieces of candy.	T
For every 2 hexagons, there are 3 triangles.	T
The ratio of squares to total candies is 3:17.	F
The ratio of triangles to total candies is $\frac{3}{10}$.	T

50) Answers will vary.

51) All of these statements are true.

52) A

53) B

54) B

55) A

56) Answers will vary.

Rates

57) D

58) C

59) B

60) $4

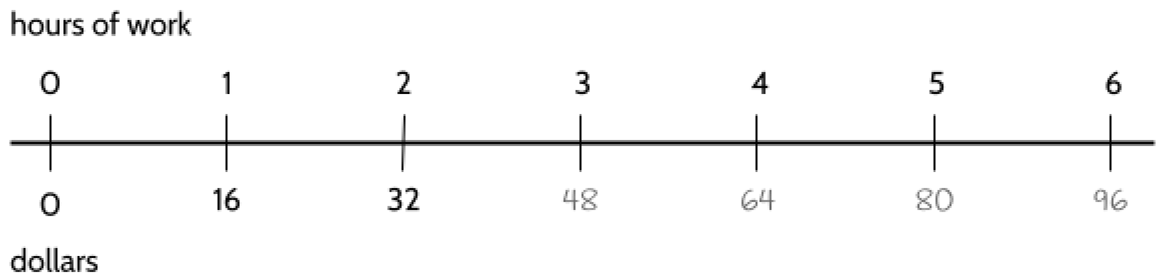
61) You should have six dollar bills in your drawing.

62) There are many things you might notice. A couple things we noticed:
The numbers on the top go up by 4 and the numbers on the bottom go up by 2.

There are some missing numbers.
Each number on top is twice as much as the number below.

63) D

64)



65) B

Equivalent Ratios: Making Copies and Splitting Evenly

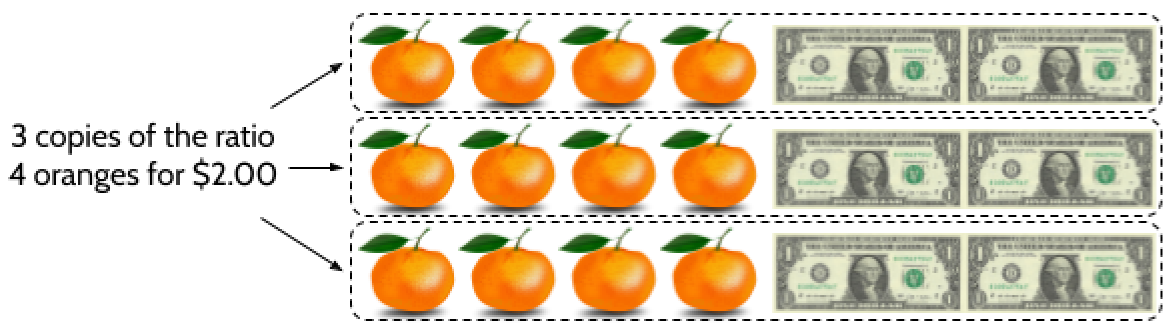
Equivalent ratios are ratios that have the same value, even though they may look different. For example, consider the following equivalent ratios:

- A wage of $16 per hour
- A wage of $32 for two hours of work
- A wage of $64 for four hours of work

$16 is not equivalent to $64, but a ratio of $16 for 1 hour of work is an equivalent ratio to $64 for 4 hours of work.

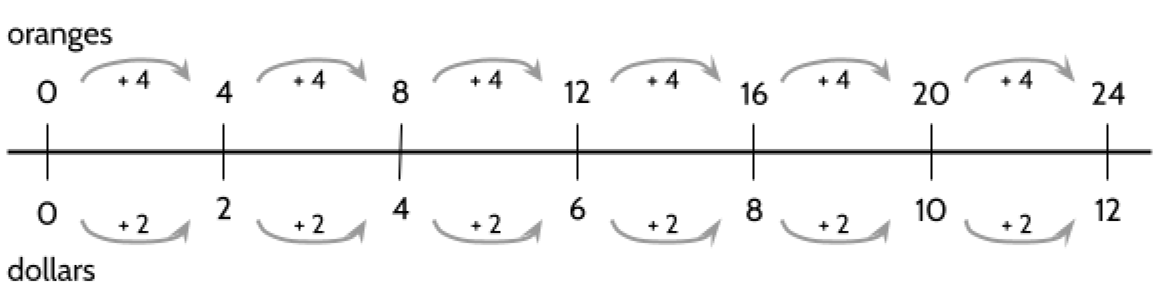
Equivalent ratios are helpful for solving all kinds of problems and can help us with math in our daily life.

One way to create equivalent ratios is to make copies of the original ratio. For example, the image below shows three copies of the ratio, 4 oranges for $2.00.



The new ratio of 12 oranges for $6.00 is equivalent to 4 oranges for $2.00.

A double number line helps us make more ratios that are equivalent to the original price. It is very similar to a regular number line.[1] Quantities (numbers) get bigger as you move to the right and smaller as you move to the left.



[1] For more practice with number lines, see [Number Lines and the Coordinate Grid](#).

A *ratio table* is another way to make equivalent ratios. Do you see how 4 oranges and 2 dollars are added for each new column?

oranges	4	8	12	16	20	24	28	32
dollars	2	4	6	8	10	12	14	16

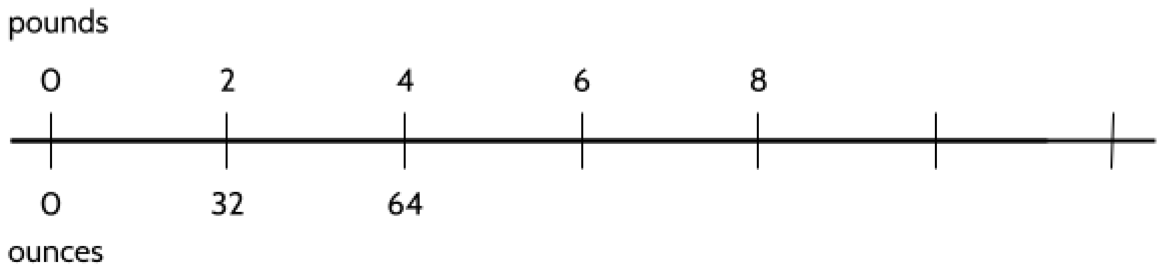
Each of these tools can show that $\frac{4\ oranges}{2\ dollars}$ is equivalent to $\frac{12\ oranges}{6\ dollars}$.

In this section, we will use drawings, double number lines, and ratio tables to practice making equivalent ratios.

Measurement Ratios with a Double Number Line

For each double number line below, fill in the missing numbers, then answer the questions.

1) Ounces and pounds are measurements of weight. There are 16 ounces in 1 pound.



2) How many ounces are in 14 pounds?

3) Which of these statements are true? Choose all that are true.

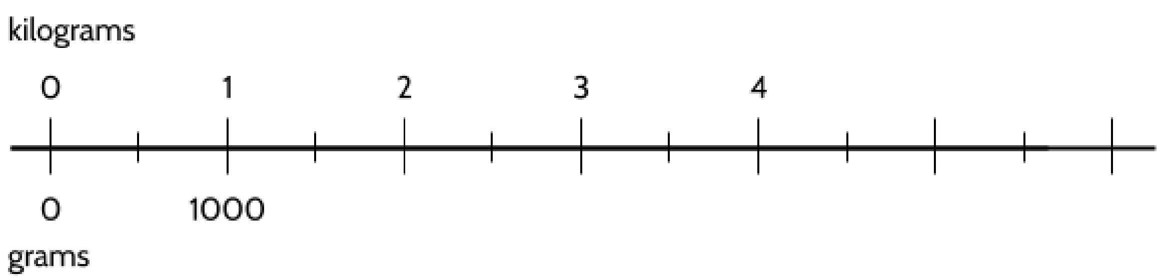
A. There are 16 ounces for every pound.

B. There are 16 pounds for every ounce.

C. There are 64 ounces for every 4 pounds.

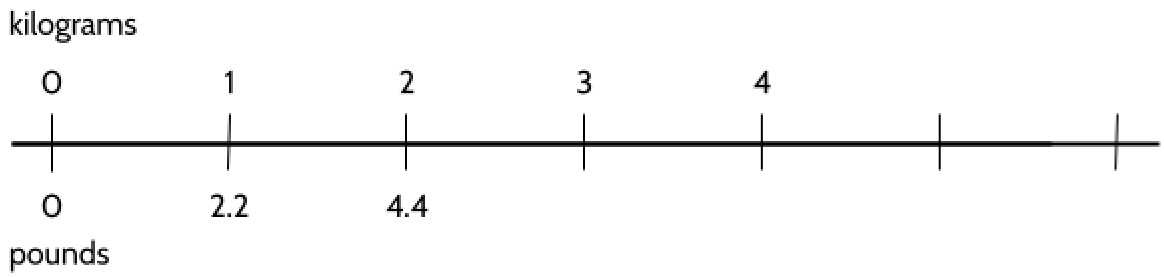
D. There are 5 pounds for every 80 ounces.

4) Grams and kilograms are measurements of weight. There are 1000 grams in 1 kilogram.



5) How many grams are in 5 ½ kilograms?

6) Kilograms are part of the metric system of measurement and pounds are part of the U.S. system of measurement. We can use ratios to *convert* (change) from one to the other. There are approximately 2.2 pounds in 1 kilogram.



7) Which of these ratios are correct? Select all that are true.

A. $\frac{1\ kilogram}{2.2\ pounds}$

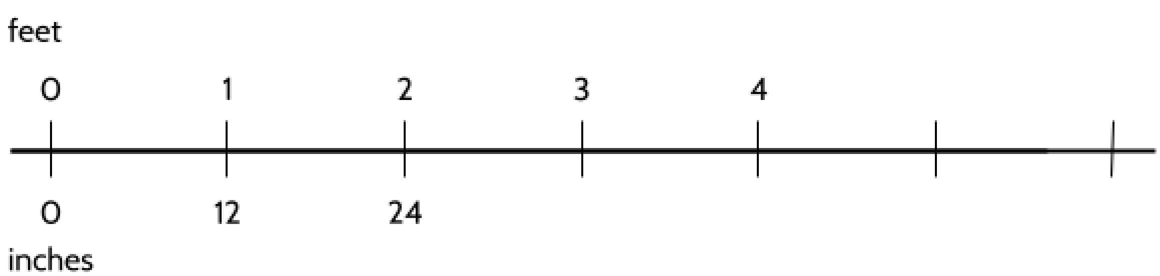
B. $\frac{1\ pound}{2.2\ kilograms}$

C. $\frac{4\ kilograms}{8.8\ pounds}$

D. $\frac{5\ kilograms}{5.5\ pounds}$

8) How many pounds would be equal to 5 kilograms?

9) Feet and inches are measurements of length used in the United States. There are 12 inches in 1 foot. (These are not drawn to size.)



10) Which of these ratios are correct? Select all that are true.

A. 1 foot:12 inches

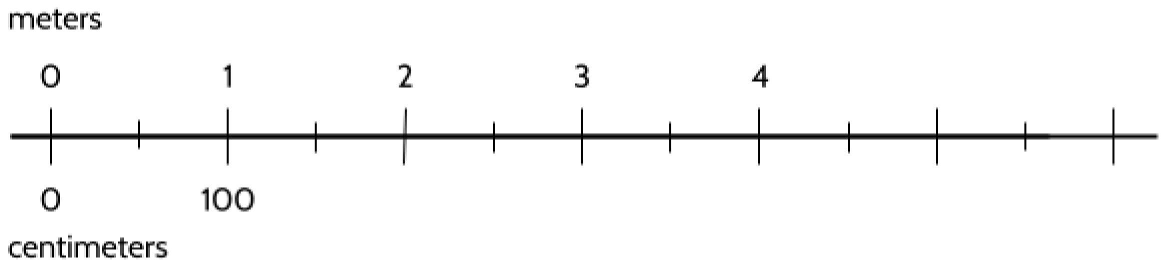
B. 24 inches:2 feet

C. 36 feet:3 inches

D. 5 feet:60 inches

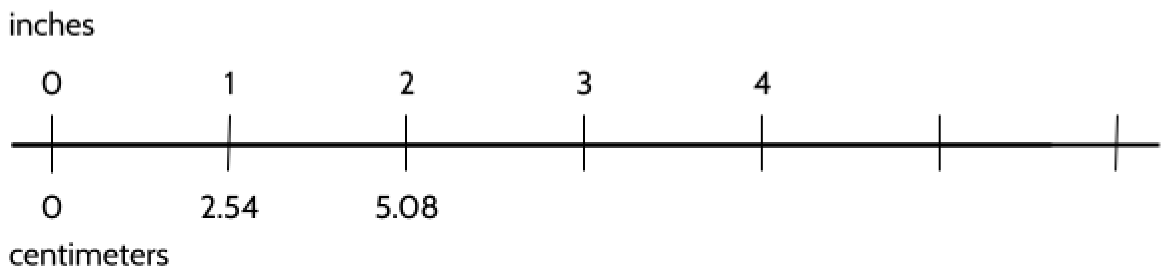
11) If someone is 6 feet tall, how tall are they if you measure in inches?

12) Meters and centimeters are measurements of length used in most countries.



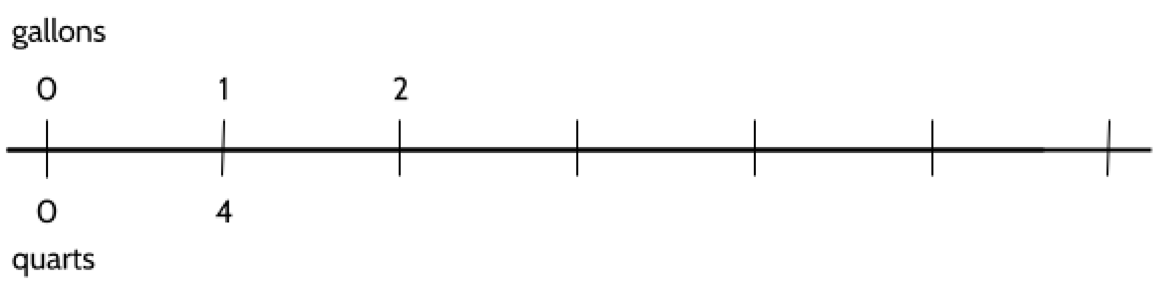
13) 350 centimeters is equal to how many meters?

14) Centimeters are part of the metric system of measurement and inches are part of the U.S. system of measurement, but we can use ratios to convert them. 1 inch equals approximately 2.54 centimeters.



15) 8 inches is equal to how many centimeters?

16) Gallons and quarts are measurements of volume. We often use them to measure liquids such as milk.



17) How many quarts are in 5 gallons?

A. $1\frac{1}{4}$

B. 20

C. 24

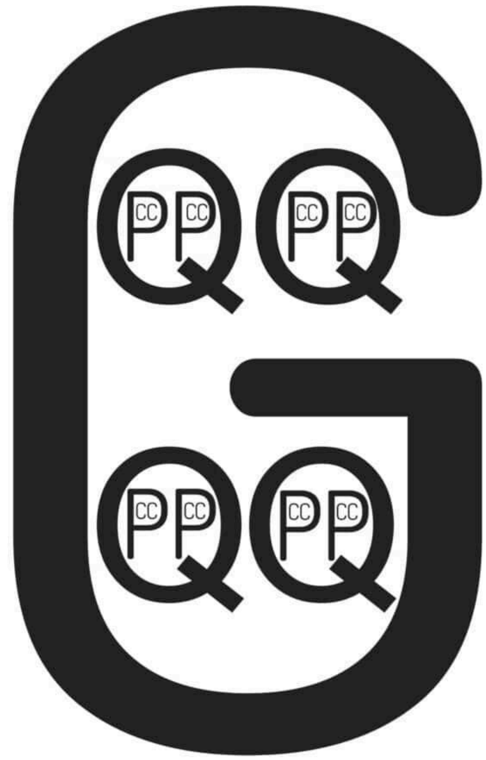
D. 100

18) The following diagram is used to remember the relationships between gallons (G), quarts (Q), pints (P), and cups (C), which are all measurements of volume.

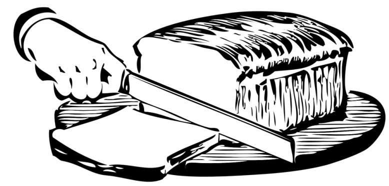
Some ratios in the diagram:

- 4 quarts in 1 gallon (4 Qs in 1G)
- 2 pints in 1 quart (2Ps in 1Q)
- 2 cups in 1 pint (2Cs in 1 P)

What other ratios do you see in this diagram? Write as many as you can below.



Making Equivalent Ratios with Ratio Tables



Baking requires attention to detail. It is important to measure carefully and use the correct amounts of ingredients. Understanding ratios can help you become an expert baker.

When making bread, many bakers use the following ratio: three cups of water and five cups of flour to make one loaf of bread.

Imagine that you own a bakery. Since you sell a lot of bread, you have to increase the amounts in the recipe so that you can make more than one loaf of bread at a time.

19) Your bakers use the ratio table below to calculate your quantities of flour and water. Unfortunately, one of the bakers spilled coffee on the paper. Can you replace the missing numbers?

cups of water	3	6	9	12		
cups of flour	5	10	15			
loaves of bread	1	2				

20) How many loaves of bread can you make with 10 cups of flour?

21) How much flour do you need for 12 cups of water?

22) How much water do you need for 30 cups of flour?

23) One of your regular customer calls with a special order for 10 loaves of bread. Complete the table below to show how much water and flour you will need.

cups of water	3	
cups of flour	5	
loaves of bread	1	10

As we increase the amount of bread we make for the bakery, we can use ratio tables to make sure that we follow the original ratio of 3 cups of water for every 5 cups of flour. All of the ratios in our table are equivalent to the original ratio of 3:5. We create equivalent ratios by multiplying the amount of each ingredient by the same number. If we use 10 times the amount of water, we need 10 times the amount of flour.

Here is the flour and water ratio table again, starting with the ratio $\frac{3\ cups\ of\ water}{5\ cups\ of\ flour}$.

$$3 \xrightarrow{\times 10} 30$$

cups of water	**3**	30
cups of flour	**5**	50

$$5 \xrightarrow{\times 10} 50$$

Both parts of the ratio $\frac{3}{5}$ can be multiplied by 10 to make ten loaves of bread.

$$\frac{3 \times 10}{5 \times 10} = \frac{30}{50}$$

If both parts of a ratio are multiplied by the same number, the new ratio is equivalent to the original ratio. The equivalent ratios of $\frac{6\ cups\ of\ water}{10\ cups\ of\ flour}$ and $\frac{30\ cups\ of\ water}{50\ cups\ of\ flour}$ were created with the ratio table.

24) Which of the following ratios would follow the 3:5 bread recipe for 20 loaves of bread?

A. 23 cups of water and 25 cups of flour

B. 30 cups of water and 50 cups of flour

C. 60 cups of water and 90 cups of flour

D. 60 cups of water and 100 cups of flour

25) If you charge $4.00 per loaf of bread, how much money would you make after you sell bread made from 20 cups of flour? Explain your answer below.

26) Which of the following ratios does <u>not</u> follow the 3:5 bread recipe?

A. 6 cups of water and 10 cups of flour

B. 12 cups of water and 20 cups of flour

C. 15 cups of water and 30 cups of flour

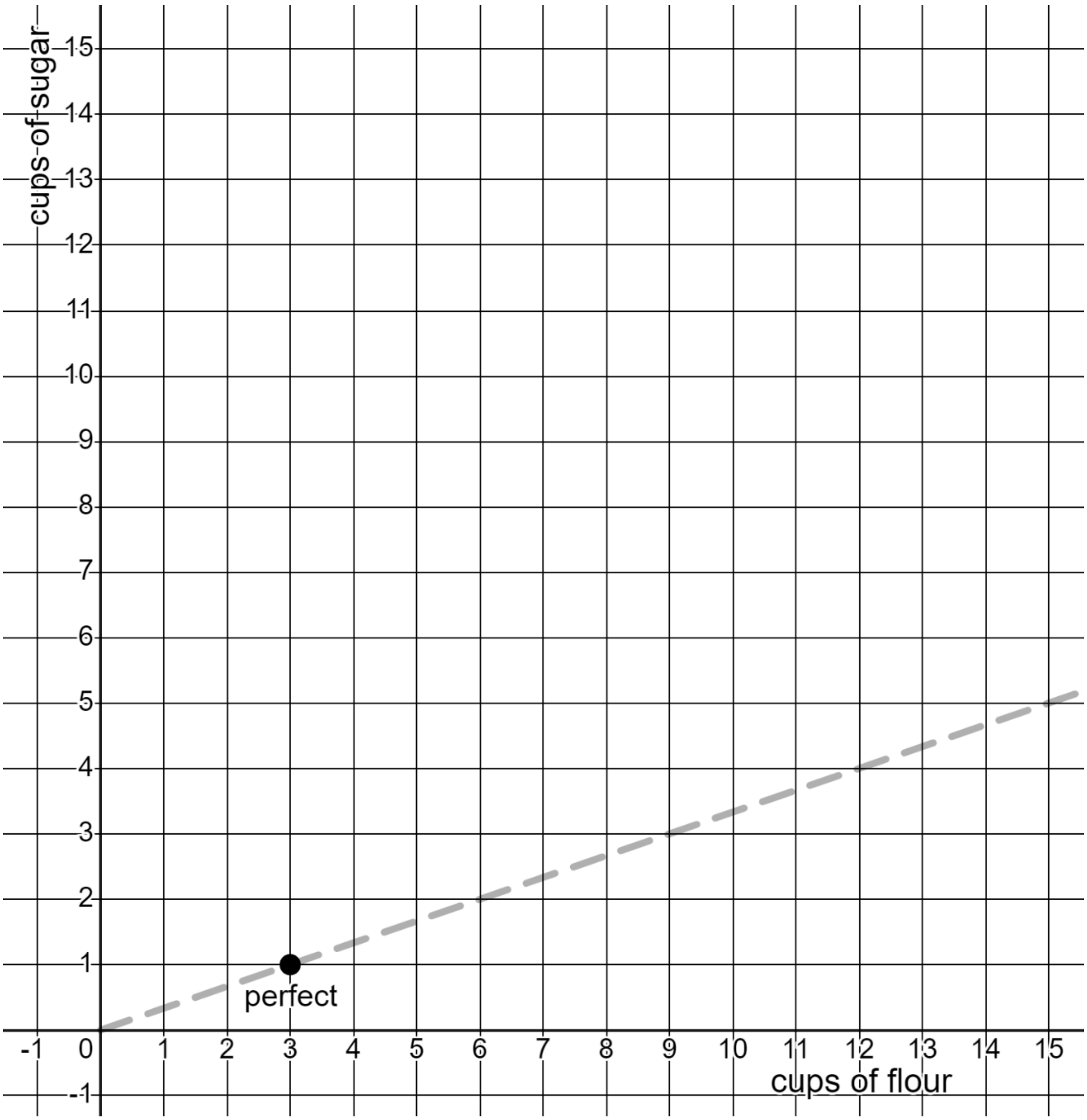
D. 21 cups of water and 35 cups of flour

A cookie recipe calls for three cups of flour for every one cup of sugar. Use this ratio of 3:1 to guide your reasoning below.

27) Complete the table below by determining which of the recipes are correct. Then predict how the cookies will turn out. (TOO SWEET, NOT SWEET ENOUGH, or PERFECT)

Cups of Flour	Cups of Sugar	Does the recipe follow the correct ratio? YES or NO	How will the cookies taste?
3	1	Yes	Perfect
4	2		
4.5	1.5		
6	2		
7	2		
8	4		
10	3		
12	4		
13	6		
15	5		

28) Now graph all the points from the previous page. The first recipe has been placed on the chart.



29) Look at the graph after you draw all the points. What do you notice?

Rates: Making Copies with Ratio Tables

Ratio tables allow us to add, subtract, multiply, and divide to make equivalent ratios. Complete the following tables to make different equivalent ratios.

30) A store is selling 4 apples for $3.00. Fill in the blanks in the ratio table below.

add multiply by 2

apples	4	8	12	20	40	60
dollars	3	6	9			

add multiply by 2

Hint: Try adding columns or multiplying columns by the same number.

31) After driving for 2 hours, you have traveled 100 miles.

hours	2	6	8		9	
miles	100			50		250

32) Currency exchange rates allow people to trade money from one country for money from another country. In December 2023, you could get about 11 U.S. dollars for 10 Euros.

Euros	10	20	40		90	100
U.S. dollars	11			55		

33) If you can get 11 U.S. dollars for 10 Euros, how many U.S. dollars could you get for 30 Euros?

34) In basketball, you get one point when you successfully shoot a free throw into the net. Stephen Curry from the Golden State Warriors is currently the best free throw shooter in the NBA. He completes about 90 out of every 100 free throws.

free throws completed	90					270
free throws attempted	100	200	400	50	450	

35) The U.S. Department of Health and Human Services recommends that Americans eat at least 2 ½ cups of vegetables per day.

cups of vegetables	$2\frac{1}{2}$					40
days	1	2	4	5	8	

36) A nurse measures a patients' heart rate as 20 heartbeats in 15 seconds.

Seconds	15	30	60	75		120
Heartbeats	20				120	

37) Spotify pays music artists about $0.003 per stream[2].

Streams	1	2	10	20	100	200
Dollars	$0.003	$0.006				

Use the ratio table to find the amount of money for larger numbers of streams.

Streams						
Dollars						

[2] A *stream* is when someone plays the song over the Internet. Spotify pays less than one penny to music artists each time their song is played. Some musicians have argued that they should receive 1 penny per stream.

Equivalent Fractions

The top number in a fraction is called the *numerator*. It is the value of the "part." The bottom number in a fraction is called the *denominator*. It is the value of the "whole" or the number of parts the whole is divided into.

numerator (part) → $\frac{1}{10}$ ← *denominator* (whole)

$\frac{1 \text{ left-handed people}}{10 \text{ all people}}$

When you multiply or divide the numerator and denominator by the same number, the value of the fraction doesn't change. The new fraction will be equivalent to the original fraction. In the example below, $\frac{2}{20}$ is equivalent to $\frac{1}{10}$.

38) Fill in the missing values in the ratio table.

× 2 × 5 ÷ 2 × 3

1	2			
10	20			

× 2 × 5 ÷ 2 × 3

39) Which of these equations is <u>not</u> true?

A. $\frac{1}{10} = \frac{2}{20}$

B. $\frac{10}{100} = \frac{5}{50}$

C. $\frac{2}{20} = \frac{15}{150}$

D. $\frac{1}{10} = \frac{10}{20}$

Multiply the top number and bottom number by any number to create equivalent fractions. Make sure you multiply the top and bottom number by the same number.

40) Create five equivalent fractions for $\frac{3}{4}$. (One has been done for you.)

3	6				
4	8				

41) Create five equivalent fractions for $\frac{7}{10}$.

7					
10					

42) Multiply or divide the numerator and denominator by any number to create five equivalent fractions for $\frac{1}{2}$.

$$\frac{1}{2} = \frac{2}{4} = \frac{\quad}{\quad} = \frac{\quad}{\quad} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$$

43) Create five equivalent fractions for $\frac{15}{6}$.

$$\frac{15}{6} = \frac{5}{\quad} = \frac{\quad}{\quad} = \frac{\quad}{\quad} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$$

44) Which of the following ratios is <u>not</u> equivalent to $\frac{1}{4}$?

A. $\frac{4}{16}$

B. $\frac{6}{24}$

C. $\frac{7}{35}$

D. $\frac{9}{36}$

How do you know?

45) Fill in the missing parts of the fractions to create equivalent ratios.

$$\frac{2}{10} = \frac{4}{\quad} = \frac{\quad}{45} = \frac{5}{\quad} = \frac{\quad}{55} = \frac{20}{\quad}$$

46) Which ratio is equivalent to $\frac{2}{10}$ and the other fractions above?

A. 1:10

B. 1:5

C. 2:5

D. 5:10

47) Using the digits 0 to 9 at most one time each, place a digit in each box so that you have two equivalent ratios.

$$\frac{\boxed{}}{\boxed{}} = \frac{\boxed{}\,\boxed{}}{\boxed{}}$$

Here is one solution:

$$\frac{\boxed{3}}{\boxed{1}} = \frac{\boxed{2}\,\boxed{4}}{\boxed{8}}$$

How many solutions can you find?

This is based on an activity from openmiddle.com, where similar challenges can be found.

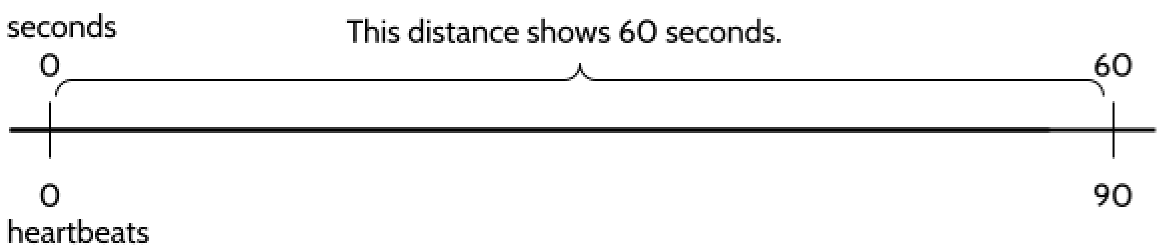
Splitting Ratios Evenly

So far, we have used addition and multiplication to create equivalent ratios. We can also use division to create equivalent ratios. In this section, we will create equivalent ratios by splitting the original ratio into equal pieces. To see how this works, we will use double number lines to represent ratios.

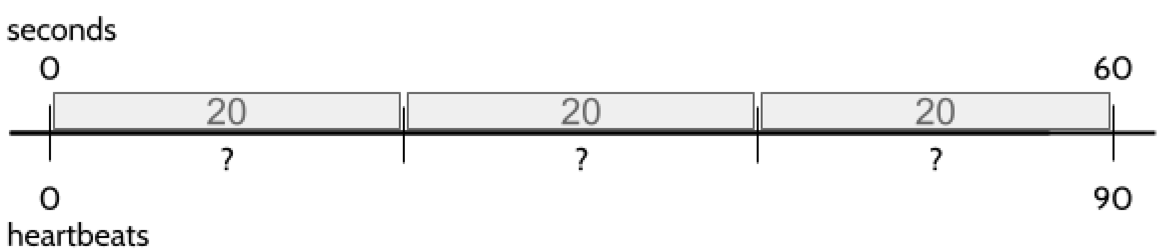
48) Try the following task:

A nurse measures a heart rate of 90 heartbeats in 60 seconds. Use the number line below to show how many heartbeats there would be in 20 seconds.

We can use the strategy of splitting evenly to find 20 seconds on this number line. The first step is to measure the distance between 0 and 60.

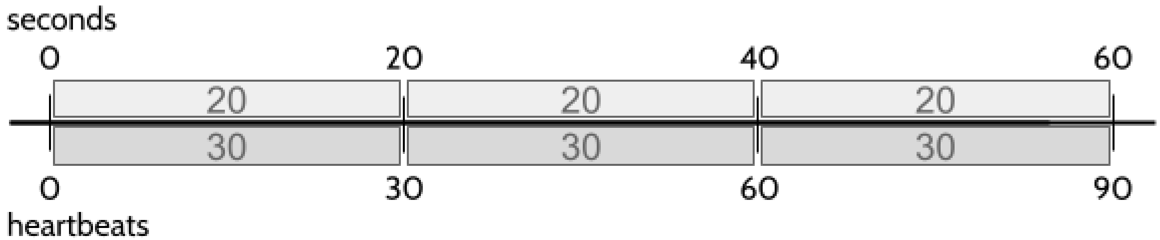


We can ask ourselves, “How many pieces 20 seconds long will fit in a distance of 60 seconds?” If we split 60 into three equal pieces, each piece will be 20 seconds long.



How many heartbeats happen in each 20 seconds?

To find the number of heartbeats in 20 seconds, we need to cut 90 seconds into three equal pieces as well.



We can see that 30 heartbeats would happen in 20 seconds. The ratio $\frac{30\ heartbeats}{20\ seconds}$ is equivalent to $\frac{90\ heartbeats}{60\ seconds}$. Our final answer might look like the number line below.

Use the number lines below to split the ratios evenly to find new equivalent ratios.

49) A nurse measures a heart rate of 100 heartbeats in 60 seconds. Use the number line below to show how many heartbeats there would be in 15 seconds.

50) Which of these ratios is equivalent to 100 heartbeats/60 seconds?

A. 50 heartbeats/20 seconds

B. 20 heartbeats/15 seconds

C. 30 heartbeats/20 seconds

D. 200 heartbeats/120 seconds

51) A computer printer prints 240 pages in 12 minutes. Use the number line below to show how many pages it would print in 4 minutes.

52) In December 2023, you could get about 60 Dominican pesos for 1 U.S. dollar (100 pennies). Use the number line to show how many pesos you could get for 20 U.S. cents.

53) We can use your work on the number line above to answer different questions. For example, how many Dominican pesos would you get for 40 U.S. cents?

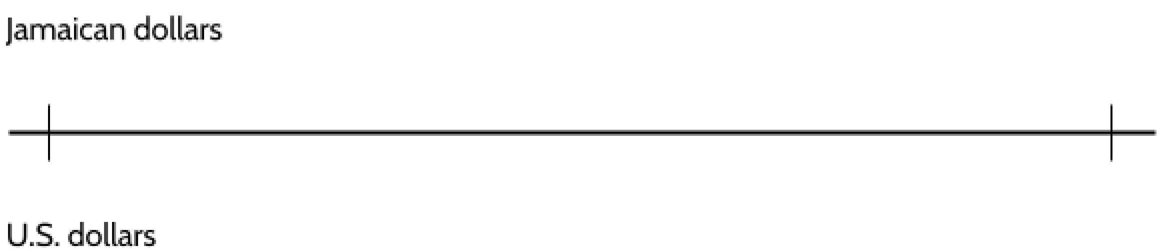
54) A grocery store sells 8 tangerines for $3.00. Use the number line below to show the price of 2 tangerines.

55) How much would 6 tangerines cost?

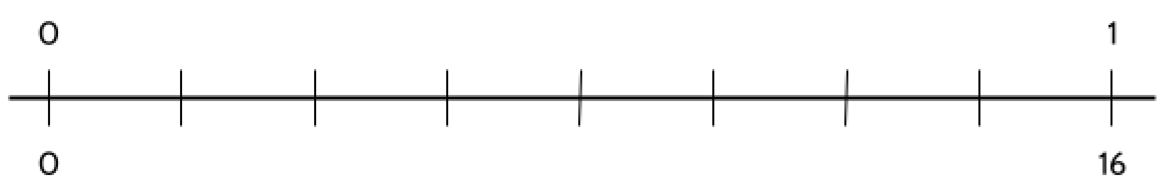
56) A power company charges $75 for 200 kilowatt-hours of electricity. Use the number line below to show the cost of 40 kilowatt-hours of electricity.

57) Every 30 days, a family pays $2000 rent for a 1-bedroom apartment. Use the number line below to show the amount of rent for 6 days.

58) In December 2023, you could get about 160 Jamaican dollars for 1 U.S. dollar. Use the number line to show how many Jamaican dollars you could get for 25 U.S. cents.



59) There are 16 cups in 1 gallon. How many cups are in $\frac{1}{8}$ of a gallon? Add labels and missing numbers to the number line below.



60) At a grocery store, cheddar cheese costs \$6.00 per pound. How much would $\frac{1}{4}$ of a pound cost?

61) How much would $\frac{3}{4}$ of a pound of cheddar cheese cost?

62) A Mitsubishi Mirage can travel about 400 miles with 10 gallons of gasoline. Add labels and missing numbers to the number line to show how many miles you could drive with 2 gallons of gasoline.

We can also use ratio tables to split ratios into equivalent ratios with smaller quantities.

63) Try the following task:

A nurse measures a heart rate of 90 heartbeats in 60 seconds. Use the ratio table below to show how many heartbeats there would be in 2 seconds. (You don't have to use all of the columns in the table.)

heartbeats	90				
seconds	60				

A heart rate of 90 heartbeats in 60 seconds is equivalent to 3 heartbeats in 2 seconds. There are different ways to show the two ratios are equivalent. Here is one way:

÷ 2 ÷ 5 ÷ 3

heartbeats	**90**	45	9	3
seconds	**60**	30	6	2

÷ 2 ÷ 5 ÷ 3

Each of these ratios ($\frac{90}{60}$, $\frac{45}{30}$, $\frac{9}{6}$, and $\frac{3}{2}$) are equivalent. If you multiply or divide each number in a ratio, the value of the ratio doesn't change.

64) Show a different way to change $\frac{90}{60}$ to $\frac{3}{2}$.

65) There are 960 calories in 12 slices of a brand of cheddar cheese. Show how many calories are in 3 slices.

slices	12				
calories	960				

66) A worker makes $1000 in 40 hours. Show how many hours they work to make $200.

hours					
dollars					

67) A grocery store charges $5.00 for a gallon of milk. At this price, how much money would 2 quarts cost? (Hint: There are four quarts in a gallon.)

quarts					
dollars					

68) At the same price above, how much would 3 quarts cost?

69) In a 5-minute shower, 10 gallons of water flows down the drain. How much water would a 2-minute shower use?

70) New York City charges about $1\frac{1}{2}$ cents per gallon for water usage. How much money does a 15-minute shower cost in NYC? Explain your thinking below.

71) A grocery store charges $6.00 for 5 apples. At this price, how much money would 1 apple cost?

72) At the same price above, how much would 4 apples cost?

Unit Rates

Unit rates answer the question, “How much (or how many) for one?” The unit rate of 25 miles per gallon means a car can travel 25 miles on 1 gallon of gas. In a unit rate, one of the quantities in the ratio is 1. The ratio shows how much of something per 1 unit of something else.

73) If a car travels 25 miles with 1 gallon of gas, how far would it travel with 3 gallons?

miles	25	
gallons	1	3

The *unit price* is the cost for one item or for one unit of measure. The price per pound is a common unit price in the United States. Unit price is a kind of unit rate.

74) If 1 pound of apples costs $3.00, how much would 5 pounds of apples cost?

pounds	1	
dollars	3.00	

You can make a unit rate from other kinds of rates. For example, if you were told oranges cost $2 for 4 oranges, you could figure out that the unit rate is $0.50 for 1 orange, which can be written as $0.50/orange. This answers the question, “How much money for 1 orange?”

Figure out the unit rate in the examples below.

75) If 4 pounds of broccoli cost $10.00, how much would 1 pound cost?

÷ 4

pounds	4	
dollars	10.00	

÷ 4

76) If 10 feet of chain link fencing cost $150.00, what is the unit price per foot?

feet		
dollars		

The unit price can be used to find the price of different quantities, even if the unit price isn't given. You can find the unit price, then find the cost of different quantities.

Question:
If 4 pounds of broccoli costs $10.00, how much would 5 pounds cost?

Answer:
Unit price: 1 pound for $2.50
Price for 5 pounds: $12.50

÷ 4 × 5

pounds	4	1	5
dollars	10.00	2.50	12.50

÷ 4 × 5

77) If 5 bananas cost $1.00, how much would 3 bananas cost?

bananas			
dollars			

78) If 6 pieces of candy cost $1.50, how much would 7 pieces of candy cost?

79) If you earn 150 dollars in 5 hours, how much money would you make in 12 hours?

There are always two different unit rates when comparing two quantities. For example, look at the following price:

Chocolate chip cookies

8 cookies for \$2.00

In the price, two quantities are being compared: the number of cookies and the number of dollars. There are two possible unit rates:

The price in dollars of 1 cookie

÷ 8

cookies	8	1
dollars	2	

÷ 8

The number of cookies for 1 dollar

÷ 2

cookies	8	
dollars	2	1

÷ 2

80) What is the price of 1 cookie?

81) How many cookies for 1 dollar?

82) Buying 2 pounds of tomatoes for \$3.00:

How much does 1 pound cost?

How many pounds do you get for 1 dollar?

83) Walking 20 miles in 5 hours:

How many miles in 1 hour?

How many hours to walk 1 mile?

84) Angela can run 10 laps in 30 minutes.

a. How many minutes does it take Angela to run one lap?

b. How many laps does she run per minute?

laps	10	1	
minutes	30		1

85) Lin rode a bike 20 miles in 120 minutes. Assuming they rode at a constant speed...

a. How fast did Lin ride in miles per hour?

b. How long did it take Lin to ride 5 miles?

c. How far did Lin ride in 15 minutes?

d. How fast did Lin ride in minutes per mile?

miles	20				
minutes	120				
hours					

86) Using the digits 0 to 9 at most one time each, place a digit in each box to make two equivalent ratios where one of the ratios is a unit rate.

$$\frac{\boxed{}\boxed{}}{\boxed{}\boxed{}} = \frac{\boxed{}}{\boxed{}}$$

Here is one solution:

$$\frac{\boxed{4}\boxed{5}}{\boxed{9}\boxed{0}} = \frac{\boxed{1}}{\boxed{2}}$$

How many solutions can you find?

This is based on an activity from openmiddle.com, where similar challenges can be found.

Is It Proportional?

A *proportion* is a statement that two ratios or fractions are equivalent. Each of the examples below are proportions:

$\frac{12\ inches}{1\ foot} = \frac{36\ inches}{3\ feet}$ 2:3 = 10:15 $\frac{100\ dollars}{92\ Euros} = \frac{25\ dollars}{23\ Euros}$

$16 pay for 1 hour or $32 pay for 2 hours

If 8 oranges cost $2.00, then 2 oranges would cost $0.50.

1 inch

1.5 inches

2 inches

3 inches

If one ratio is equivalent to another ratio, they are *proportional* to each other.

87) For each of the following, decide whether the two ratios are proportional to each other. Place a checkmark ✓ next to the ratios that are in proportion to each other.

- ☐ 1 pound of celery for $1.50 & 4 pounds for $6.00
- ☐ A batch of cookies with 3 cups of flour and 1 cup of sugar & a batch of cookies with 2 cups of flour and $\frac{1}{2}$ cup of sugar
- ☐ rent of $2000/month for 500 square feet & $3000/month for 750 square feet
- ☐ 20 heartbeats in 15 seconds & 60 heartbeats in 60 seconds
- ☐ $75 for 210 kilowatt-hours of electricity and $15 for 42 kilowatt-hours

Solve each of the following problems.

88) Theo and Emil are brothers who were born exactly three years apart. When Theo was 2 years old, Emil was 5. When Theo was 6, how old was Emil?

89) Emil and Theo are saving money to buy soccer jerseys. They decide that they will each put their full allowance of $5.00 per week into their savings. When they start saving money, Emil already has $20.00 saved and Theo has $10.00. How much does Theo have when Emil has $60.00?

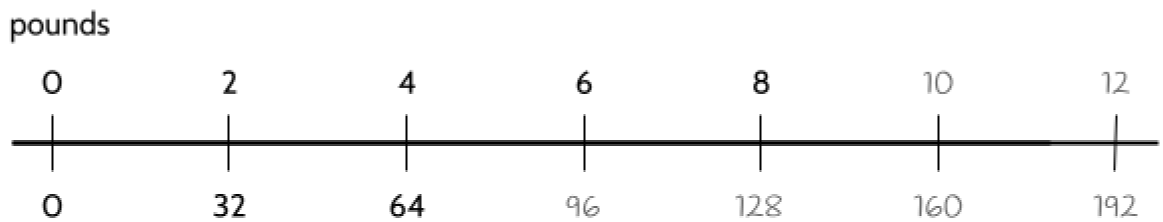
90) Dan and Lawrence are baking bread using the same recipe. Dan is making a loaf of bread using 5 cups of flour and 3 cups of water. Since he is making more bread, Lawrence uses 15 cups of flour. How many cups of water does Lawrence need?

91) Andrea and Casey are each planting peas in their gardens. Andrea plants 2 rows and Casey plants 4 rows. If Andrea's peas are ready to pick in 8 weeks, how long will it take for Casey's peas to be ready?

92) Only one of the four situations above is proportional. Circle the proportional situation.

Equivalent Ratios: Making Copies and Splitting Evenly - Answer Key

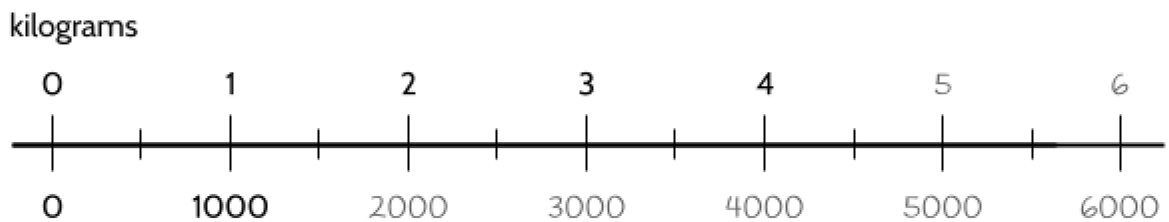
Measurement Ratios with a Double Number Line



1) ounces

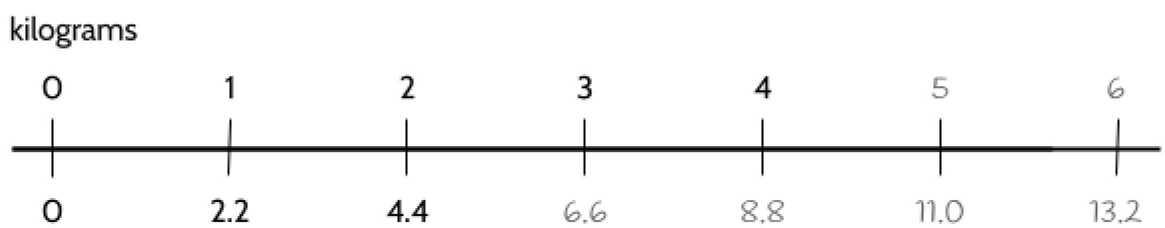
2) 224 ounces

3) A, C, D



4) grams

5) 5,500 grams



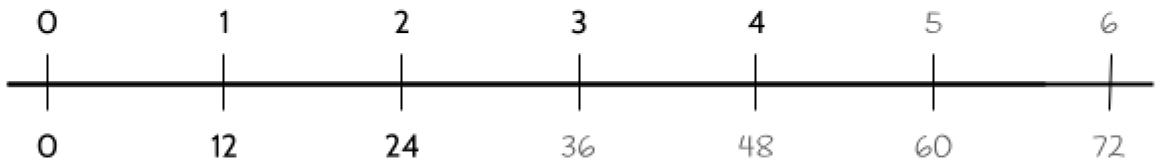
6) pounds

7) $\frac{1\ kilogram}{2.2\ pounds}$

$\frac{4\ kilograms}{8.8\ pounds}$

8) 11 pounds

feet

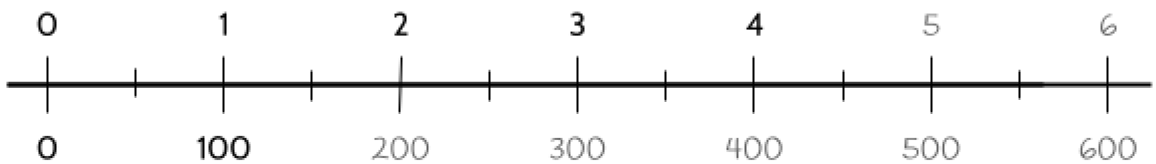


9) inches

10) All of these ratios are true.

11) 72 inches

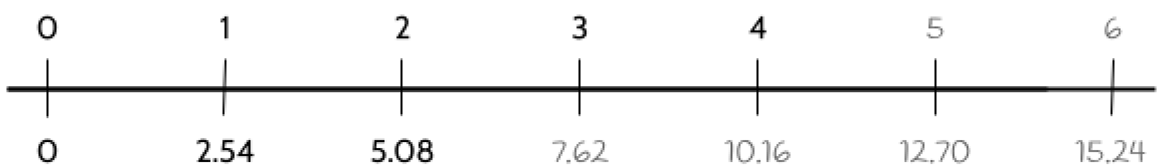
meters



12) centimeters

13) 3.5 or 3 ½ meters

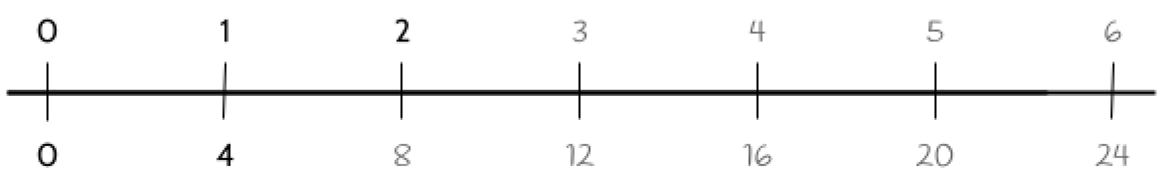
inches



14) centimeters

15) 20.32 centimeters

gallons



16) quarts

17) B

18) Answers will vary.

Making Equivalent Ratios with Ratio Tables

19)

cups of water	3	6	9	12	15	18
cups of flour	5	10	15	20	25	30
loaves of bread	1	2	3	4	5	6

20) 2 loaves

21) 20 cups of flour

22) 18 cups of water

23)

cups of water	3	30
cups of flour	5	50
loaves of bread	1	10

24) D

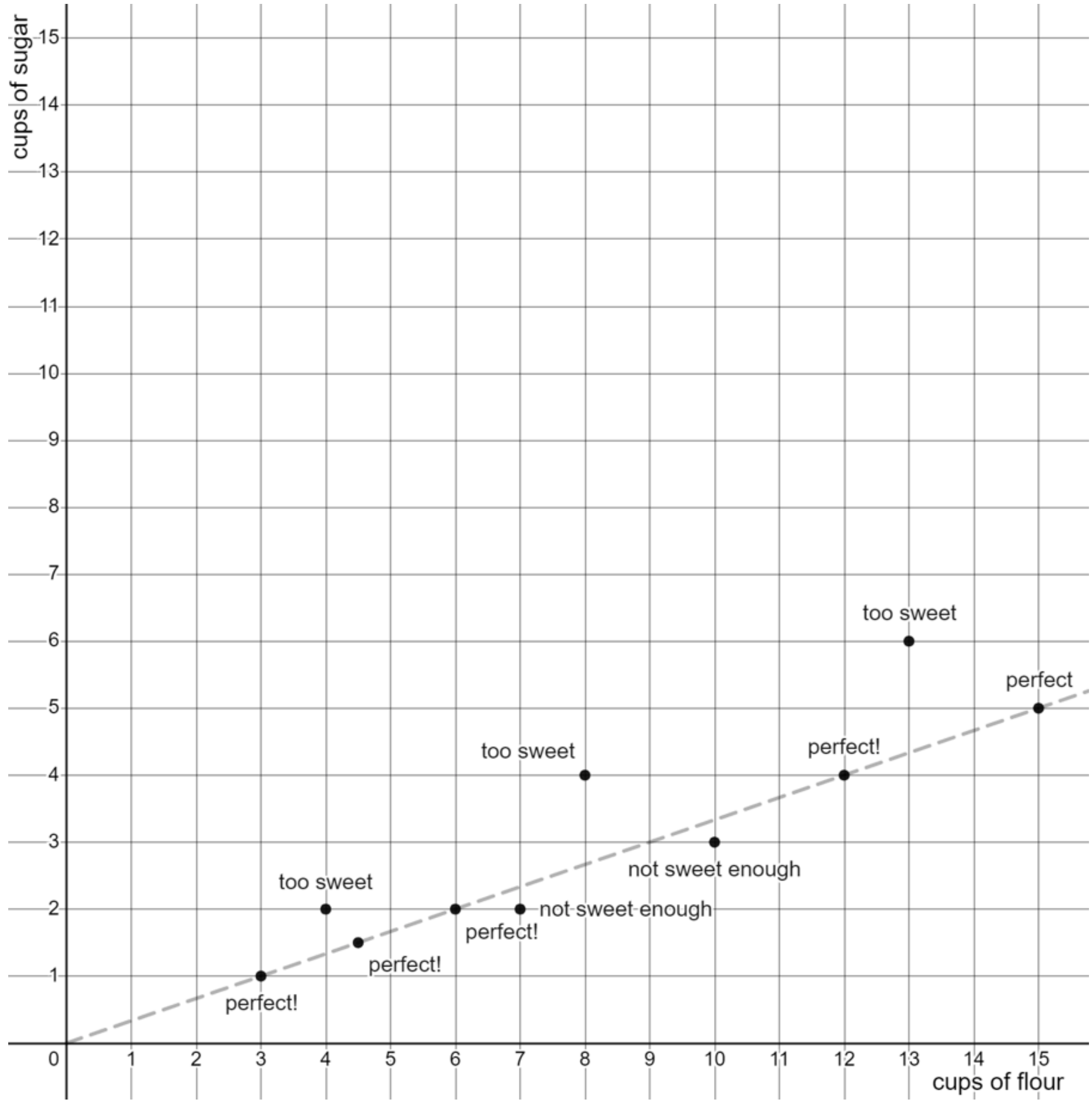
25) $16.00

26) C

27)

Cups of Flour	Cups of Sugar	Does the recipe follow the correct ratio? YES or NO	How will the cookies taste?
3	1	Yes	Perfect
4	2	No	Too sweet
4.5	1.5	Yes	Perfect
6	2	Yes	Perfect
7	2	No	Not sweet enough
8	4	No	Too sweet

Cups of Flour	Cups of Sugar	Does the recipe follow the correct ratio? YES or NO	How will the cookies taste?
10	3	No	Not sweet enough
12	4	Yes	Perfect
13	6	No	Too sweet
15	5	Yes	Perfect

28) 

29) There are many things you might notice. Here are a few things we noticed:
All of the perfect ratios of flour to sugar are on the dotted line.
All of the too sweet ratios are above the line.
All of the not sweet enough ratios are below the line.

Rates: Making Copies with Ratio Tables

add | multiply by 2

apples	4	8	12	20	40	60
dollars	3	6	9	15	30	45

add | multiply by 2

30)

31)

hours	2	6	8	1	9	5
miles	100	300	400	50	450	250

32)

Euros	10	20	40	50	90	100
U.S. dollars	11	22	44	55	99	110

33) 33 U.S. dollars

34)

free throws completed	90	180	360	45	405	270
free throws attempted	100	200	400	50	450	300

35)

cups of vegetables	$2\frac{1}{2}$	5	10	$12\frac{1}{2}$	20	40
days	1	2	4	5	8	16

36)

Seconds	15	30	60	75	90	120
Heartbeats	20	40	80	100	120	160

37)

Streams	1	2	10	20	100	200
Dollars	\$0.003	\$0.006	\$0.03	\$0.06	\$0.30	\$0.60

Answers will vary for the second table. Try large numbers of streams to see how much money the music artists make.

Equivalent Fractions

38)

× 2 × 5 ÷ 2 × 3

1	2	10	5	15
10	20	100	50	150

× 2 × 5 ÷ 2 × 3

39) D

40) Answers will vary.

41) Answers will vary.

42) Answers will vary.

43) Answers will vary.

44) C

45) The bottom number is always 5 times the top number.

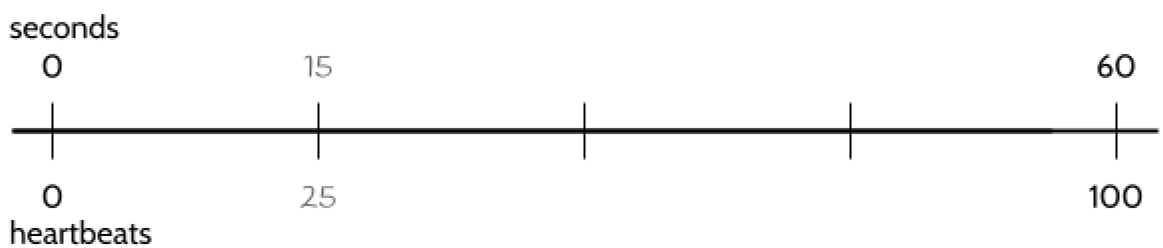
$$\frac{2}{10} = \frac{4}{20} = \frac{9}{45} = \frac{5}{25} = \frac{11}{55} = \frac{20}{100}$$

46) B

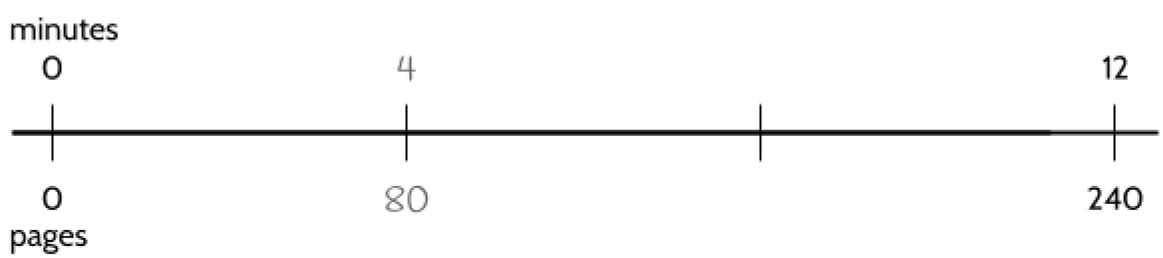
47) There are many correct answers. Here are a few more correct answers:
$\frac{3}{1} = \frac{27}{9}$, $\frac{4}{1} = \frac{32}{8}$, $\frac{5}{1} = \frac{20}{4}$, $\frac{7}{1} = \frac{42}{6}$, and more….

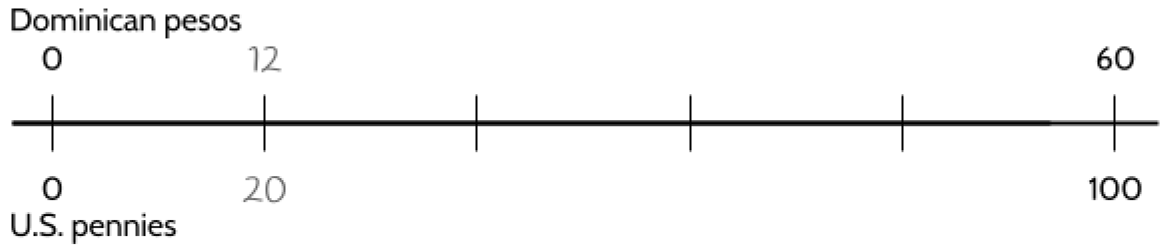
Splitting Ratios Evenly

48) 

49) 

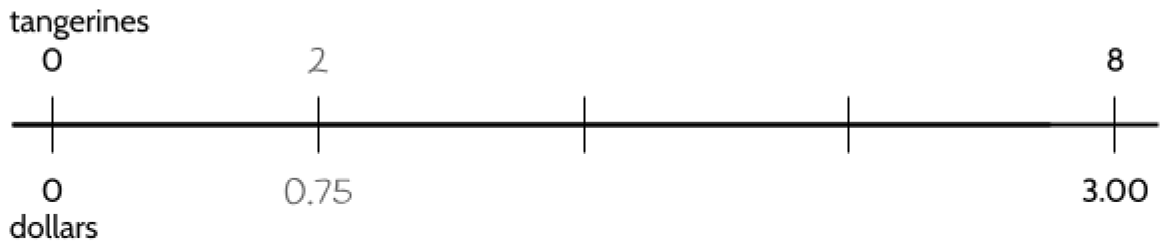
50) D

51) 



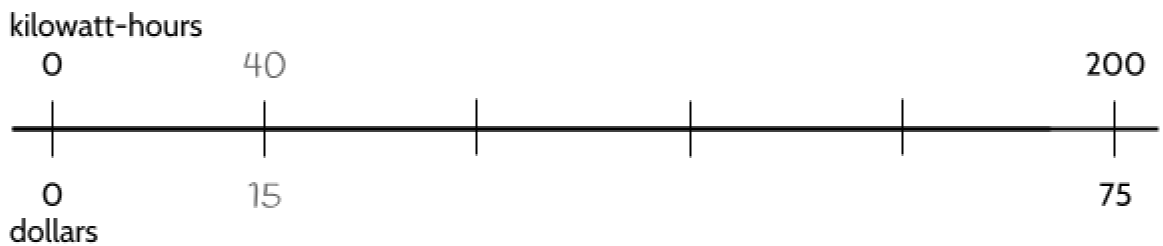
52)

53) 24 Dominican pesos



54)

55) $2.25



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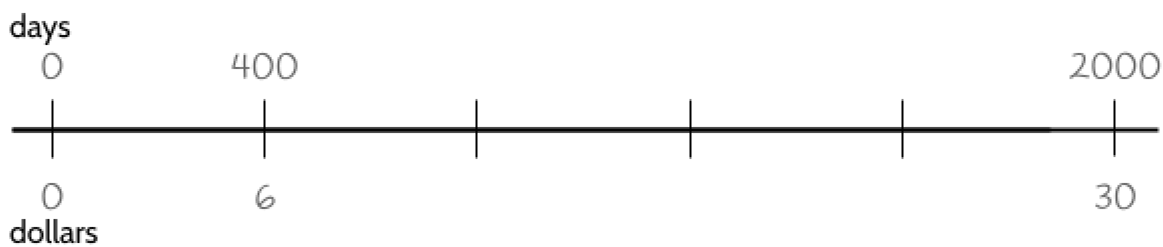

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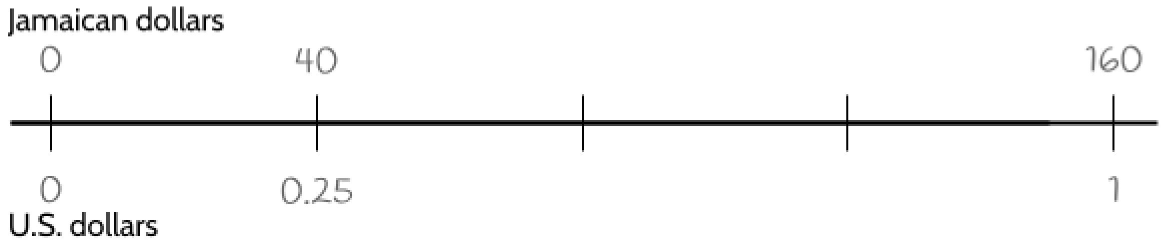

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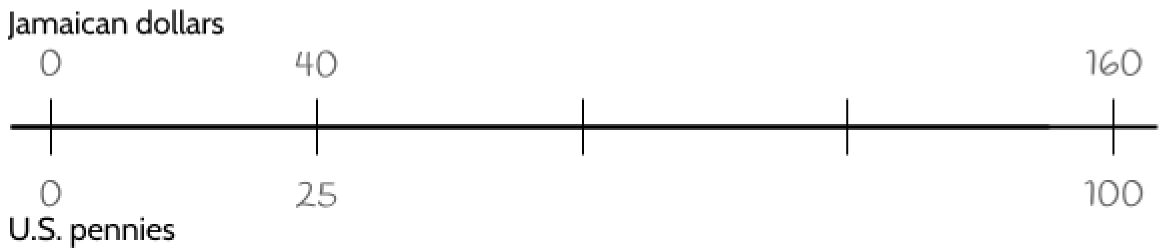

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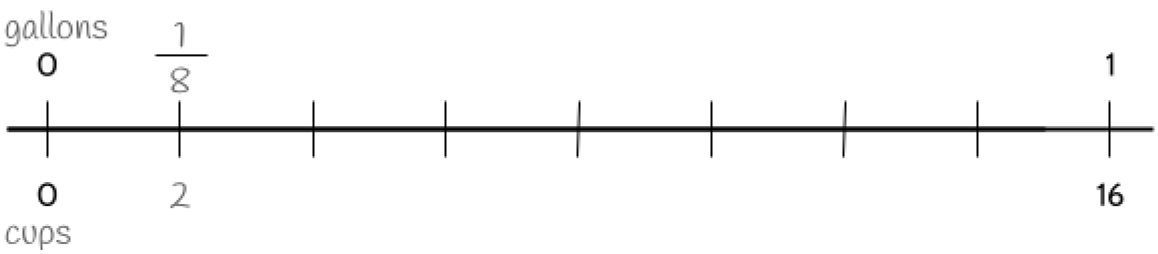
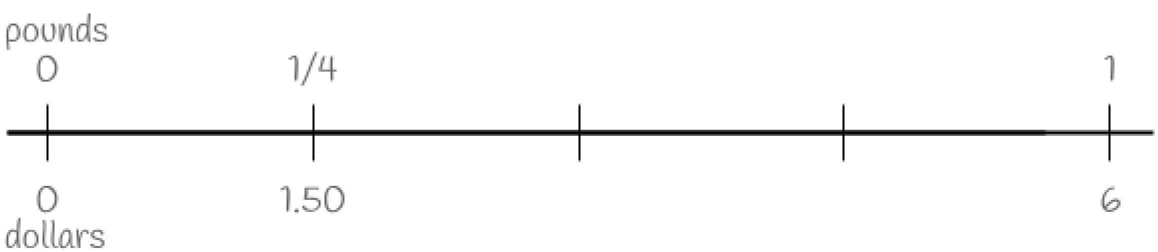
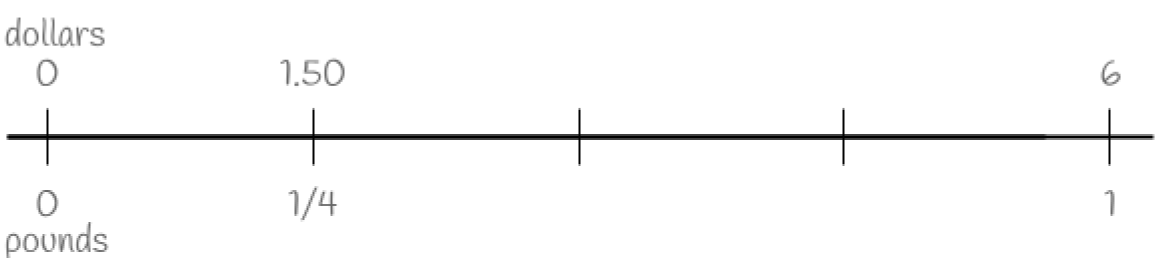


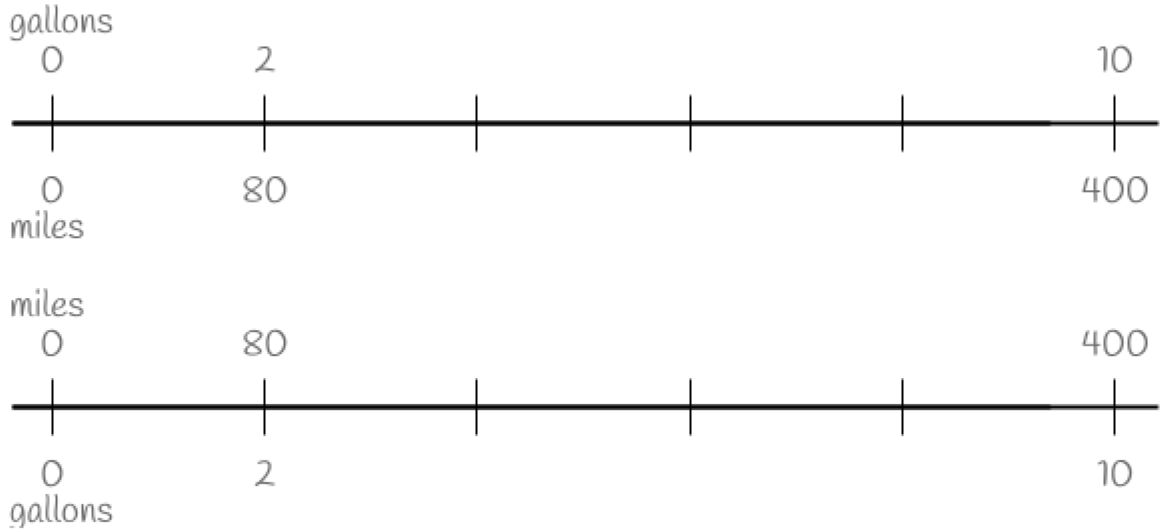
60) Either of the following would be correct. Either pounds or dollars can go on top of the number line.





61) $4.50

62) Either of the following would be correct. Either gallons or miles can go on top of the number line.



63) The following page has a possible solution.

64) You might use different strategies.

65) You might use different strategies. There are 240 calories in 3 slices of cheddar cheese.

66) You might use different strategies. The worker would work 8 hours to make $200.

67) You might use different strategies. 2 quarts would cost $2.50.

68) $3.75

69) You might use different strategies. A 2-minute shower would use 4 gallons.

70) $0.45. A 15-minute shower used 30 gallons of water. 30 gallons × 1 ½ cents = 45 cents.

71) You might use different strategies. Each apple would cost $1.20.

72) $4.80

Unit Rates

73)

miles	25	**75**
gallons	1	3

74)

pounds	1	**5**
dollars	3.00	**15.00**

75)

pounds	4	**1**
dollars	10.00	**2.50**

76)

feet	**10**	**1**
dollars	**150.00**	**15.00**

77)

bananas	**5**	**1**	**3**
dollars	**1.00**	**0.20**	**0.60**

78)

candy	**6**	**1**	**7**
dollars	**1.50**	**0.25**	**1.75**

79)

hours	**5**	**1**	**12**
dollars	**150**	**30**	**360**

80) $0.25 for 1 cookie

81) 4 cookies for 1 dollar

82) 1 pound for \$1.50
$\frac{2}{3}$ pounds for 1 dollar

83) 4 miles in 1 hour
1 mile in $\frac{1}{4}$ hour

84) a. 3 minutes for 1 lap
b. $\frac{1}{3}$ lap per minute

laps	10	1	$\frac{1}{3}$
minutes	30	**3**	1

85) a. 10 miles per hour (10 miles in 1 hour)
b. 30 minutes
c. 2.5 miles
d. 6 minutes per mile (1 mile in 6 minutes)

This is one way to use the ratio table. You may do it in a different way.

miles	20	**10**	**5**	**2.5**	**1**
minutes	120	**60**	**30**	**15**	**6**
hours	2	**1**	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{10}$

86) There are many correct answers. Here are a few more correct answers:
$\frac{15}{20} = \frac{3}{4}$, $\frac{96}{48} = \frac{2}{1}$, $\frac{34}{68} = \frac{1}{2}$, $\frac{72}{96} = \frac{3}{4}$, and more....

Is It Proportional?

87) 1 pound of celery for \$1.50 & 4 pounds for \$6.00
rent of \$2000/month for 500 square feet & \$3000/month for 750 square feet
\$75 for 210 kilowatt-hours of electricity and \$15 for 42 kilowatt-hours

88) 9 years old

89) $50.00

90) 9 cups

91) 8 weeks (It doesn't take twice as long for twice as many plants to grow. It takes the same amount of time.)

92) The bread baking situation is proportional. The other three situations are not proportional.

The Language of Proportional Reasoning

What Does the Word "Per" Mean?

The word *per* is often used when talking about rates. Here are some examples:

15 dollars *per* hour   70 heartbeats *per* minute   100 *per*cent

Per means *for each* or *for every*. We use *per* a lot when we talk about driving.

Question: How fast was the car moving?
Answer: 65 miles *per* hour. This means the car went 65 miles *for every* 1 hour of driving.

Question: How much did the gas cost?
Answer: $3.00 *per* gallon. This means you have to pay $3.00 *for each* gallon of gas you put in your car.

Question: What is your car's gas mileage?
Answer: 25 miles *per* gallon. This means your car travels 25 miles *for each* gallon in the tank.

The rates for driving speed, cost, and gas mileage above can be written in many ways:

speed	cost	gas mileage
65 miles per hour	*$3.00 per gallon*	*25 miles per gallon*
65 miles/hour	*3 dollars/gallon*	*25 miles/gallon*
65 mph	*$3.00/gallon*	*25 mpg*

1) Complete the following table for the rate of 65 miles per hour.

Hours Driving	Distance Traveled
1	65 miles/1 hour
2	130 miles/2 hours
3	
5	
10	

2) Complete the following table for the rate of 25 miles per gallon. This is a measure of how far a car will travel with a certain amount of gasoline.

Number of Gallons	Gas Mileage
1	25 miles/1 gallon
2	
3	
5	
10	

3) Complete the following table for the rate of $3.00/gallon.

Number of Gallons	Cost
1	$3.00/1 gallon
2	$6.00/2 gallons
3	
5	
10	

4) Look up the terms below in an online dictionary. What do they mean?

per annum:

per diem:

per capita:

percent:

Complete the following matching activity. Connect the quantity on the left with the quantity on the right. Then use the combined phrase to fill in the sentences below.

heartbeats		hour
kilometers		person
dollars		container
calories	per	mile
eggs		class
servings		day
students		minute

5) The doctor measured the patient's heart rate at 190 heartbeats per minute .

6) Kindergarten classes in Albany have an average of 19 ____________________________ .

7) As of 2024, Employers in New York City are required to pay a minimum wage of 16 ____________________________ .

8) Americans consume about 270 ____________________________ each year.

9) To understand driving distances in other countries, you can use the conversion rate of 1.6 ________________________________ .

10) Moderately active women between 31 and 50 years old should consume a maximum of 2,000 ________________________________, according to the Dietary Guidelines for Americans.

11) Packaged snacks often have three or more ________________________________ .

Ratios Written in Different Ways

Once you start looking for numbers written as ratios, you will see them everywhere. Here are some examples from recent news reports:

- More than 1 in 4 high school students are vaping (using e-cigarettes). About 1 in every 10 middle school students currently vape. (Dec. 2019, U.S. News and World Report)
- 7 in 10 Americans surveyed say they are still concerned about rising prices for everyday purchases, down from 83% in the summer of 2022. (Feb. 2024, PR Newswire)
- About $\frac{3}{5}$ of the New York City high school graduating class of 2018 enrolled in higher education, such as college or a vocational program. (Nov. 2019, Chalkbeat.org)
- Playing for the Houston Rockets basketball team, James Harden has made 87% of his free throws this season. (Dec. 2019, SportingNews.com)
- With a batting average of .335, Tim Anderson of the Chicago White Sox had the best hitting percentage in major league baseball in 2019. (Dec. 2019, ESPN.com)

Each of these values are written in different ways, but they are all ratios:

1 in 4   1 in every 10 $\frac{3}{5}$ 87%   0.335

12) Use a newspaper, advertisements, television, or the Internet to find three examples of numbers written as ratios.

13) Write one of the ratios in other ways. For example, if it is a percent, change it to a fraction. Or if it is a fraction, change it to an equivalent fraction.

The Language of Proportional Reasoning - Answer Key

What Does the Word "Per" Mean?

1)

Hours Driving	Distance Traveled
1	65 miles/1 hour
2	130 miles/2 hours
3	195 miles/3 hours
5	325 miles/5 hours
10	650/10 hours

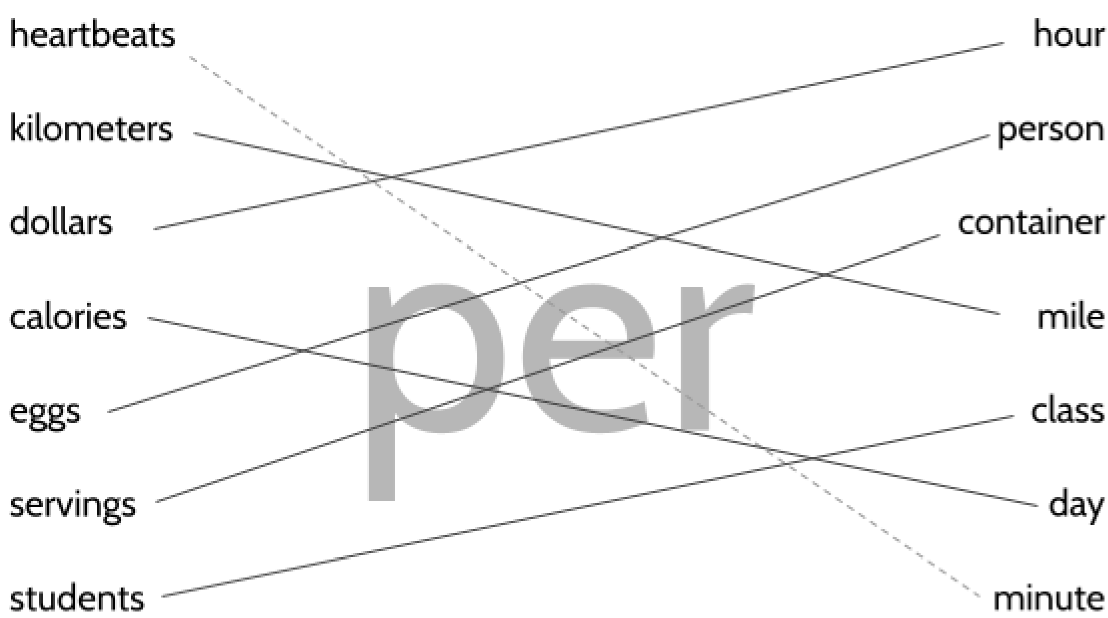
2)

Number of Gallons	Gas Mileage
1	25 miles/1 gallon
2	50 miles/2 gallons
3	75 miles/3 gallons
5	125 miles/5 gallons
10	250 miles/10 gallons

3)

Number of Gallons	Cost
1	$3.00/1 gallon
2	$6.00/2 gallons
3	$9.00/3 gallons
5	$15.00/5 gallons
10	$30.00/10 gallons

4) We recommend Merriam-Webster.com, since it gives examples of words used in sentences. You could also use Dictionary.com. What did you find out?



5) heartbeats per minute

6) students per class

7) dollars per hour

8) eggs per person

9) kilometers per mile

10) calories per day

11) serving per container

Ratios Written in Different Ways

1) Answers will vary. Please share the ratios you found with us. Email: Eric (eric.appleton@cuny.edu) and Mark (mark.trushkowsky@cuny.edu)

2) Answers will vary.

Vocabulary Review

You can use this section to look up words used in this math packet.

area (noun): The size of a flat surface, measured by counting squares.

equivalent (adjective): Equal in value. Example: $\frac{1}{2}$ and 0.5 are *equivalent*.

multiple (noun): A number that can be divided by another number evenly, with no remainder. 25 is a *multiple* of 5.

number line (noun): A picture of a straight line on which all numbers can be placed. Numbers get bigger as you move to the right and smaller as you move to the left.

fraction (noun): A part of a whole amount. $\frac{1}{2}$ or "one half" is an example of a fraction.

> The top number (**numerator**) shows how many parts we have. It is the value of the "part" in a part-whole relationship.
>
> The bottom number (**denominator**) says how many equal pieces the whole is divided into. It is the value of the "whole" in a part-whole relationship.

per (preposition): for each, for every

- The car was traveling 40 miles *per* hour.
- Most showers use 2.1 gallons of water *per* minute.
- The taxi charges $2.75 *per* mile.
- The air pressure in your car tires should be 32-35 pounds *per* square inch.

percent (noun): A percent is a part-whole ratio which is "out of" 100. *Per*- means "for every" and -*cent* means "100," so the word *percent* literally means "for every 100." For example, "5 percent" means "5 for every 100." We use the symbol % to show percent.

product (noun): The result of multiplication. 4 times 5 gives a *product* of 20.

proportion (noun): a statement that two ratios or fractions are equivalent. This is an example of a proportion: $\frac{12\ inches}{1\ foot} = \frac{36\ inches}{3\ feet}$. It can also mean a proper or equal share or the size of something compared to something else.

> disproportionate (adjective): Larger or smaller size, share, or cost than we expect

rate (noun): A ratio that compares two different quantities, such as miles and hours (speed) or dollars and pounds (price).

- The sink was leaking at a *rate* of ½ an ounce of water per minute.
- The train traveled at a *rate* of 80 miles per hour.
- The hybrid car gets a gas mileage *rate* of 50 miles per gallon.

ratio (noun): A comparison between two or more numbers using multiplication or division. It is a relationship between two quantities.

> A **part-whole ratio** compares part of a quantity to the whole quantity. Example: 9 right-handed people out of every 10 people
>
> A **part-to-part ratio** compares part of a quantity to another part of the quantity. Example: 1 left-handed person for every 9 right-handed people.

sequence (noun): A sequence is a list of things in order. Often, a sequence is a list of numbers in order.

series (noun): A group of things that come one after another

set (noun): A collection of things

term (noun): A number (or other quantity) that is part of a series

unit rate (noun): A unit rate shows how much of something per 1 unit of something else. Examples: $0.50 per orange, 65 miles per hour, and 25 students for every teacher.

unit price (noun): The cost for one item or for one unit of measure. A unit price is a unit rate that shows the price of something.

Sources

Abrahamson, D., & Cigan, C. (2003). A design for ratio and proportion instruction. *Mathematics Teaching in the Middle School*, 8(9), 493-501. [Section: Rates]

Braaten, M. (2018). Rigorous Proportional Reasoning: So Much More than the Cross Product. *The Math Practitioner* (Spring 2018). The Adult Numeracy Network. [Section: Which is a Better Deal?]

Cai, J., & Wang, T. (2006). US and Chinese teachers' conceptions and constructions of representations: A case of teaching ratio concept. International Journal of Science and Mathematics Education, 4, 145-186. [Section: Practice Test Questions]

Chapin, S. H. & Johnson, A. (2006). *Math Matters: Understanding the Math You Teach*. Grades K-6. Math Solutions Publications. [Section: Unit Rates]

Common Core Standards Writing Team. (2011). Progressions for the common core state standards in mathematics. Tucson, AZ: University of Arizona Institute for Mathematics and Education. Downloaded from https://mathematicalmusings.org/ on August 23, 2023.

de la Cruz, J. A. (2013). Selecting proportional reasoning tasks. Australian Mathematics Teacher, The, 69(2), 14-18.

Ercole, L. K., Frantz, M., & Ashline, G. (2011). Multiple ways to solve proportions. Mathematics Teaching in the Middle School, 16(8), 482-490. [Section: Solving Proportions]

GED.com. (2016). Assessment guide for educators: mathematical reasoning. [Section: Overview]

GED.com. (2016). Math study guide. [Section: Practice Test Questions]

Hinds, S. (2006). Multiplication Matters: Grouping, Decimals, Fractions, and Negative Numbers in the Basic Education, pre-GED and GED Math Classroom. CUNY Adult Literacy/HSE/ESL Program. [Section: Skip Counting]

Illustrative Mathematics. Molly's Run. Retrieved on December 20, 2023 from https://tasks.illustrativemathematics.org/content-standards/7/RP/A/1/tasks/828 [Section: Practice Test Questions]

Illustrative Mathematics. The Price of Bread. Retrieved on December 20, 2023 from https://tasks.illustrativemathematics.org/content-standards/7/RP/A/3/tasks/1330 [Section: Bread and Wages]

Illustrative Mathematics. Riding at a Constant Speed, Assessment Variation. Retrieved on December 20, 2023 from https://tasks.illustrativemathematics.org/content-standards/tasks/1175. [Section: Finding Unit Rates]

Forrest, K., Schnabel, D., & Williams, M. (2006). Math by the Month: January 2006. Teaching Children Mathematics, 12(5), 248-249. Retrieved Nov 30, 2023, from https://doi.org/10.5951/TCM.12.5.0248

Forrest, K., Schnabel, D., & Williams, M. (2006). Math by the Month: February 2006. Teaching Children Mathematics, 12(6), 304-305. Retrieved Nov 30, 2023, from https://doi.org/10.5951/TCM.12.6.0304 [Section: Rates]

Frykholm, J. (2013). Learning to think mathematically with ratio tables: A resource for teachers, a tool for young children. Math Learning Center. [Sections: Equivalent Ratios: Making Copies and Splitting Evenly]

Lawrence, A. & Hennessy, C. (2007). Sizing Up Measurement: Activities for Grades 6-8 Classrooms. Math Solutions. Scholastic Inc.

Lobato, J., Ellis, A., & Zbiek, R. M. (2010). *Developing Essential Understanding of Ratios, Proportions, and Proportional Reasoning for Teaching Mathematics: Grades 6-8*. National Council of Teachers of Mathematics. 1906 Association Drive, Reston, VA 20191-1502.

Lonberg-Lew, S. (2019). Reasoning about multiplication facts with the area model. The Adult Numeracy Network Math Practitioner, Volume 25, Issue 1, Fall/Winter 2019.

Lonberg-Lew, S. (2022). Proportional Reasoning: Strategies that Make Sense. Math/Numeracy PD (ATLAS). MN Adult Education Professional Development. Retrieved from https://www.youtube.com/watch?v=iDH8hvk2-Mg [Section: Solving Proportions]

Lonberg-Lew, S. (2023). Will This Be on the Test? SABES Mathematics and Adult Numeracy Curriculum & Instruction Center. Full resource available at https://sabes.org/content/will-be-test-inspiration-tackling-standardized-test-questions-conceptually-and-creatively [Section: Practice Test Questions]

Manly, M., & Ginsburg, L. (2010). Algebraic Thinking in Adult Education. National Institute for Literacy. [Sections: Skills Practice]

Math Is Figure-Outable with Pam Harris (podcast). (2021). Ep 48: Fractions, A Money Model. Retrieved on December 6, 2023 from https://podcast.mathisfigureoutable.com [Section: Understanding Ratios]

Math Is Figure-Outable with Pam Harris (podcast). (2022). Ep 112: The Double Open Number Line Model. Retrieved on December 18, 2023 from https://podcast.mathisfigureoutable.com [Section: Understanding Ratios]

New York's Minimum Wage. New York State Department of Labor. Retrieved on December 29, 2023 from https://labor.ny.gov/minimumwage [Section: Understanding Ratios]

OpenMiddle.com. Source problems available for free at web site. [Sections: Equivalent Fractions, Unit Rates, Comparing Ratios Using Decimals and Percents]

Open Up Resources. (2023). 6–8 Mathematics 3rd Edition Curriculum. Retrieved on December 6, 2023 from https://access.openupresources.org/curricula [Sections: Introduction to Ratios]

Rabbit Coat Color Genetics. Oregon State University. Retrieved on December 14, 2023 from https://extension.oregonstate.edu/collection/rabbit-coat-color-genetics. [Section: Understanding Ratios]

Student Achievement Partners. (2015). Ratio and Rates Mini-Assessment. Achieve the Core. Retrieved from https://achievethecore.org/page/1051/ratios-and-rates-mini-assessment on December 22, 2024. [Section: Assessment Questions]

Small, M. (2019). Understanding the Math We Teach and how to Teach it: K-8. Stenhouse Publishers.

U.S. Department of Agriculture and U.S. Department of Health and Human Services. Dietary Guidelines for Americans, 2020-2025. 9th Edition. December 2020. Available at DietaryGuidelines.gov.

Van de Walle, J. A. (2013). *Elementary and Middle School Mathematics: Teaching Developmentally*. New York. [Sections: Introduction to Ratios]

What Comes Next? (game). (2013). The Adult Numeracy Network. Retrieved from https://adultnumeracynetwork.org/What-Comes-Next.

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