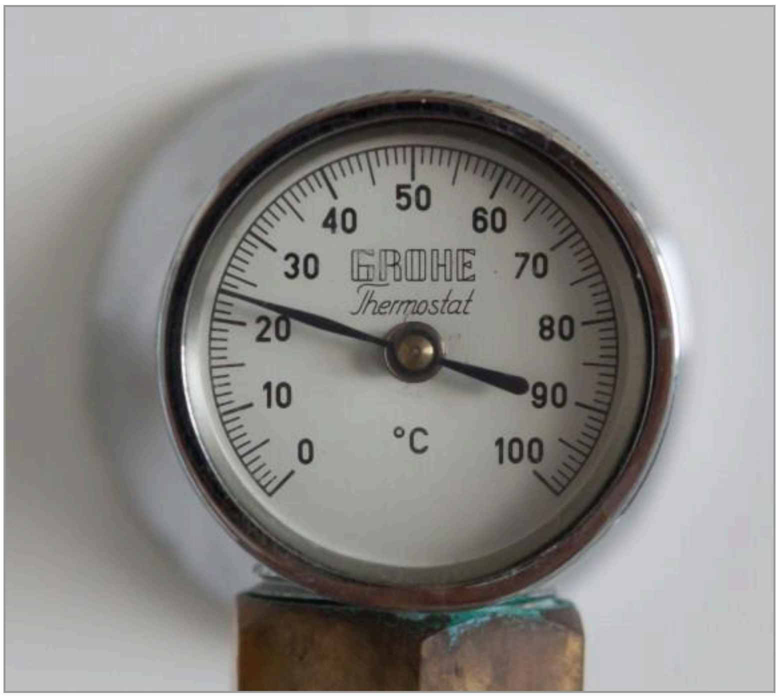
Number Lines and the Coordinate Grid

Fast Track GRASP Math Packet

Part 1



Version 1.2
Updated 01/07/2025

This Fast Track GRASP Math Packet was made possible through support from the New York State Education Department, Office of Adult Career and Continuing Education Services.

http://www.collectedny.org/ftgmp

Table of Contents

Overview

Prerequisites	There are no prerequisites to *Number Lines and Coordinate Planes, Part 1.* As long as you are able to read this packet independently, you don't have to study any other math packets first. Students should complete *Number Lines and Coordinate Planes, Part 1* before working on *Number Lines and Coordinate Planes, Part 2.*

In this packet, you will explore concepts in

In Part 1, you will study the following topics:

- Plotting points on a number line, including fractions, decimals, and signed numbers
- Measurement and distance on a number line
- Absolute value

In Part 2, you will build on what you learned in Part 1, and study the following topics:

- Plotting points and interpreting points on the coordinate grid
- Drawing lines and shapes on the coordinate grid
- Data on the coordinate grid (including scatter plots) and correlation

In addition to the learning the topics above, you will find the following materials to help you:

- High School Equivalency Test Practice Questions. You will practice the concepts you have learned from this packet to work on these questions. The answer key for this section explains the correct answers, and also some of the wrong answers.

- A graphic organizer to study vocabulary is included, along with a vocabulary activity to review concepts. A glossary with important terms from this packet is also included for your study.

- Concept Circles can help you make connections between the concepts you have learned and help you remember those connections.

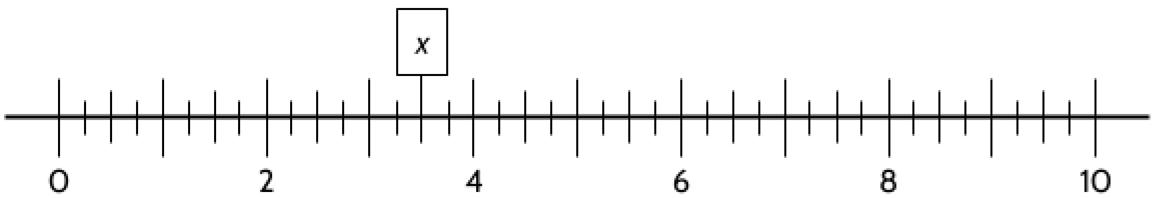
Assessment Questions

Calculator allowed

The following questions will help to see if this packet is right for you. Do your best to answer each question. If you can't answer, don't worry—this packet will help you answer questions like these and more. When you are finished with the questions, read our recommendations.

Question 1

Look at the number line below.



Which of the following numbers is not at position *x* on the number line above?

A. $2\frac{3}{4}$

B. $\frac{7}{2}$

C. $3\frac{1}{2}$

D. $\frac{14}{4}$

Question 2

Place the following numbers on the number line below: 0 $-2\frac{1}{2}$ $|-5|$

Question 3

Imagine the numbers $-4\frac{3}{4}$ and $1\frac{1}{4}$ plotted on a number line. What would the distance be between the two points?

A. $-4\frac{3}{16}$

B. $-3\frac{1}{2}$

C. $3\frac{1}{2}$

D. 6

Question 4

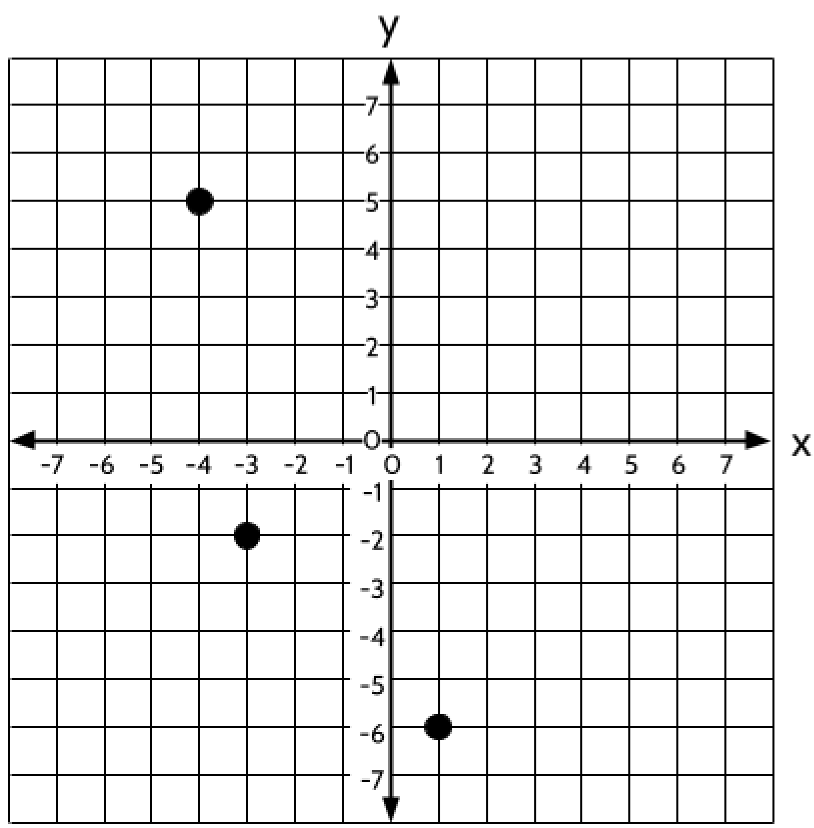
Look at the coordinate plane below. What are the coordinates of the three locations on the graph?

A. (-4, 5), (-2, 3), (-6, 1)

B. (-4, 5), (-3, -2), (1, -6)

C. (4, 5), (3, 2), (1, 6)

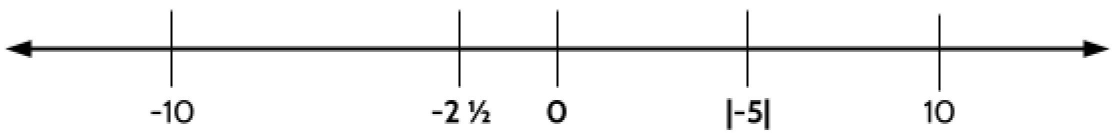
D. (5, -4), (-2, -3), (-6, 1)



Answer Key

Question 1. Choice A. $2\frac{3}{4}$ is at a different position on the number line.

Question 2. The exact positions of the numbers are shown on the number line below.



Question 3. Choice D. *6*.

Question 4. Choice B. *(-4, 5), (-3, -2), (1, -6)*

Recommendations

Consider the following when making a decision about working through this packet:

- Student has some difficulty with Question 1, 2, or 3: The student may choose to work through *Number Lines and Coordinate Planes, Part 1.*
- Student has some difficulty with Question 4: If a student comfortably answers Questions 1, 2 & 3, but has some difficulty with Question 4, the student may feel confident enough to skip *Number Lines and Coordinate Planes, Part 1* and go directly to *Number Lines and Coordinate Planes, Part 2.)*.
- Student comfortably answers all four questions: The student may choose to work on a different packet. However, it is recommended that students complete the Test Practice Questions in *Number Lines and Coordinate Planes, Part 2*, for questions that require students to interpret a variety of data representations before they take the HSE exam.

This assessment asks students to demonstrate understanding of:
Question 1 *(from Number Lines and Coordinate Planes, Part 1):* Ordering rational numbers (GED Quantitative Problem Solving Assessment Targets Content Indicator: Q.1.a).

Question 2 *(from Number Lines and Coordinate Planes, Part 1):* Ordering rational numbers, absolute value (GED Quantitative Problem Solving Assessment Targets Content Indicators: Q.1.a, Q.1.d).

Question 3 *(from Number Lines and Coordinate Planes, Part 1):* Determine the distance between two rational numbers on the number line, including using the absolute value of their difference (GED Quantitative Problem Solving Assessment Targets Content Indicators: Q.1.d).

Question 4 *(from Number Lines and Coordinate Planes, Part 2):* Locate points in the coordinate plane (GED Algebraic Problem Solving Assessment Targets Content Indicators: A.5.a.)

Vocabulary

It is important to understand mathematical words when you are learning new topics. The following vocabulary will be used a lot in this study packet:

absolute value, coordinate, difference, horizontal, increment, magnitude, signed number, vertical

When we learn new vocabulary, it is good to think about your experience with the word. Asking questions like, "Have I heard this word before?", "When have I heard this word?", "What do I think this word means?" can help you build on what you already know.

Here's how it works. On the next page, you'll find a chart with each of the vocabulary words above. For each word, ask yourself how familiar you are with the word. For example - the word "*area.*" Which of these statements is true for you and your experience with the word "*area*"?

- I know the word "*area*" and use it in conversation or writing.
- I know the word "*area*," but I don't use it.
- I have heard the word "*area*" but I'm not sure what it means.
- I have never heard the word "*area*" at all.

In the chart on the next page, read each word and then choose one of the four categories and mark your answer with a ✔ (checkmark). Then write your best guess at the meaning of the word in the right column. If it's easier, you can also just use the word in a sentence.

Here's an example of how the row for "area" might look when you're done:

Word	I know the word and use the word	I know the word but don't use it	I have heard the word, but I'm not sure what it means	I have never heard the word	My best guess at the meaning of the word (or use the word in a sentence)
area	✓				A place or location, like a neighborhood or town

This activity is designed to help you start thinking about some of the important words you will find in this packet. As you go through the activities in this packet, you will learn more about these words, what they mean, and how to use them. You will learn more precise definitions that may come up during your high school equivalency exam.

There is a glossary with the definitions of useful vocabulary at the end of the packet.

Word	I know this word and use the word	I know this word but don't use it	I have heard this word, but I'm not sure what it means	I have never heard this word	My best guess at what this word means...
absolute value					
coordinate					
difference					
horizontal					
increment					
magnitude					
signed number					
vertical					

Welcome!

Congratulations on deciding to continue your learning! We are happy to share this study packet on number lines and the coordinate grid. We hope that these materials are helpful in your efforts to earn your high school equivalency diploma. This group of math study packets will cover mathematics topics that we see on high school equivalency exams. If you study these topics carefully, while also practicing other math skills, you will increase your chances of passing the exam.

Please take your time as you go through the packet. You will find plenty of practice here, but it's useful to make extra notes for yourself to help you remember. You will probably want to have a separate notebook where you can recopy problems, write questions, and include information that you want to remember. Writing is thinking and will help you learn.

After each section, you will find an answer key. Try to answer all the questions and then look at the answer key. It's not cheating to look at the answer key, but do your best on your own first. If you find that you got the right answer, congratulations! If you didn't, it's okay. This is how we learn. Look back and try to understand the reason for the answer. Please read the answer key even if you feel confident. We added some extra explanations and examples that may be helpful. If you see a word that you don't understand, try looking at the *Vocabulary Review* at the end of the packet.

We hope you share what you learn with your friends and family. If you find something interesting here, tell someone about it! If you find a section challenging, look for support. If you are in a class, talk to your teacher and your classmates. If you are studying on your own, talk to people you know or try searching for a phrase online. Your local library should have information about adult education classes or other support. You can also find classes listed here: **https://www.acces.nysed.gov/aepp/find-adult-education-program**.

You are doing a wonderful thing by investing in your own education right now. You have our utmost respect for continuing to learn as an adult.

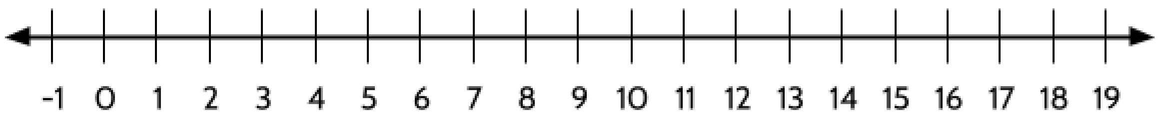
Please feel free to contact us with questions or suggestions.

Best of luck!

Eric Appleton (eric.appleton@cuny.edu) & Mark Trushkowsky (mark.trushkowsky@cuny.edu)
CUNY Adult Literacy and High School Equivalency Program

Introduction to Number Lines

Here is an example of a number line:

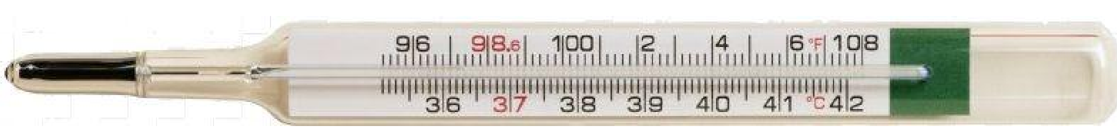


We see number lines on the job, at home, and as we move through our day.

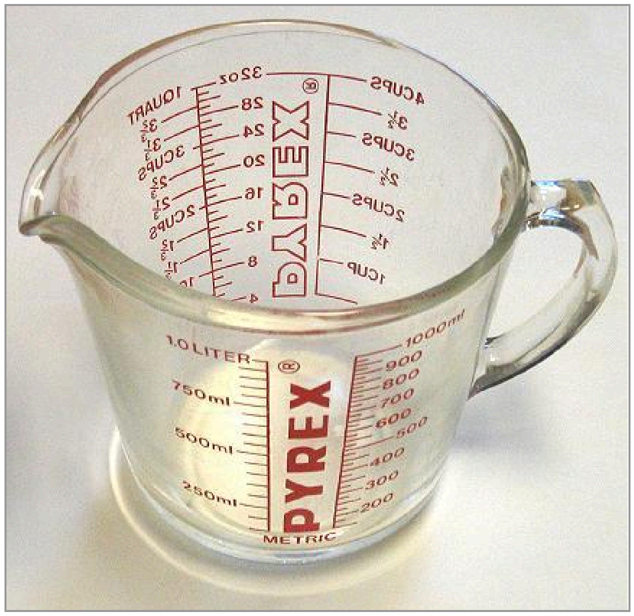
Construction workers use measuring tapes on the job.

Tailors and dressmakers use a similar kind of tape measure when making clothing.

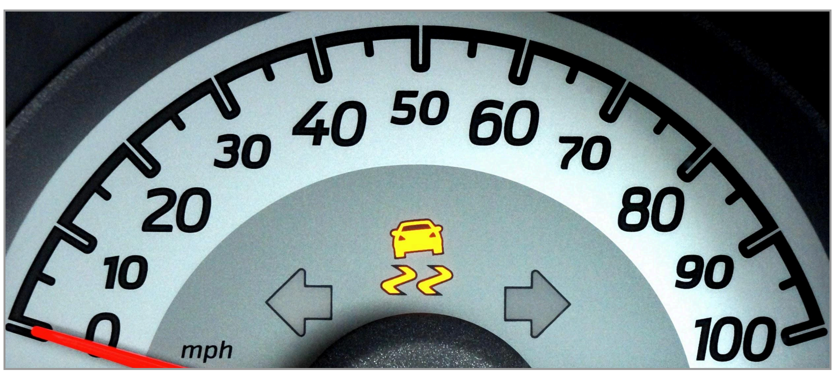
Parents might use a thermometer when a child is sick.



If you work as a cook in a restaurant, or cook at home, you probably use a measuring cup to measure ingredients.



When we drive a car, the speedometer tells us how fast we are going.



Each of these tools use number lines for measurement. There are many other examples of number lines in the world. We use number lines when we make measurements of things like distance, volume, weight, time, temperature, etc. If you pump air into a bike tire, the tire pump probably has a pressure gauge with a number line. When we turn on an oven, the knob has a number line to measure heat. Even maps on your mobile phone have number lines that measure distance. A clock is a circular number line. Number lines are everywhere.

Understanding how number lines work is important for measuring all kinds of things in our lives. They also help us compare numbers to other numbers. We can see how close or how far away numbers are from each other. We can also use number lines to make calculations.

Understanding the number line can help improve your mental math, when you do math in your head without writing anything down.

Every number can be thought of as a point on the number line. Because of this, number lines are important for all kinds of mathematics. Number lines are used in algebra, geometry, statistics, and many other kinds of mathematics.

In Part 1 of this packet, you will practice placing numbers on a number line, measuring with a number line, and comparing the size of numbers using a number line. We will use whole numbers, fractions, decimals, and signed numbers.

In Part 2 of this packet, you will practice using the coordinate grid with two number lines on a grid. The two number lines intersect to show the position of objects or to keep track of two different kinds of information. You will practice plotting points and reading points on the coordinate grid. This will help prepare you to use the coordinate grid for graphs in algebra, functions, and statistics.

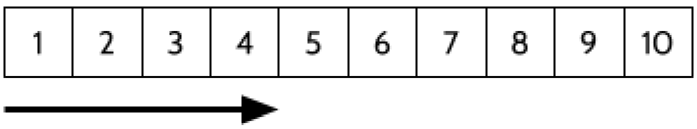
Both sections of this packet will help you with the math section of the high school equivalency exam along with reading charts and graphs on science and social studies high school equivalency exams.

Using Number Lines

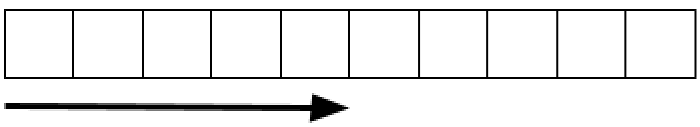
Before we look at number lines, we will start with a *number path*, shown below.

1	2	3	4	5	6	7	8	9	10

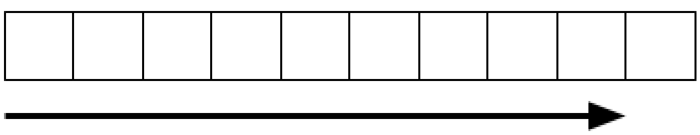
We can use a number path for measurement. The arrow below measures 4 spaces.



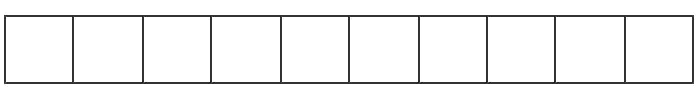
1) Write in the numbers below. How many spaces does the arrow measure? _______



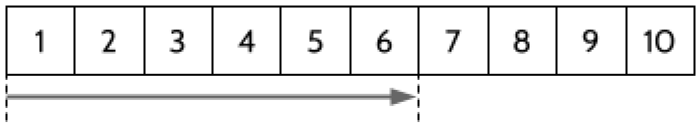
2) What is the measurement of the arrow below? _______



3) Fill in the number path, then draw an arrow with a measurement of 6 spaces.

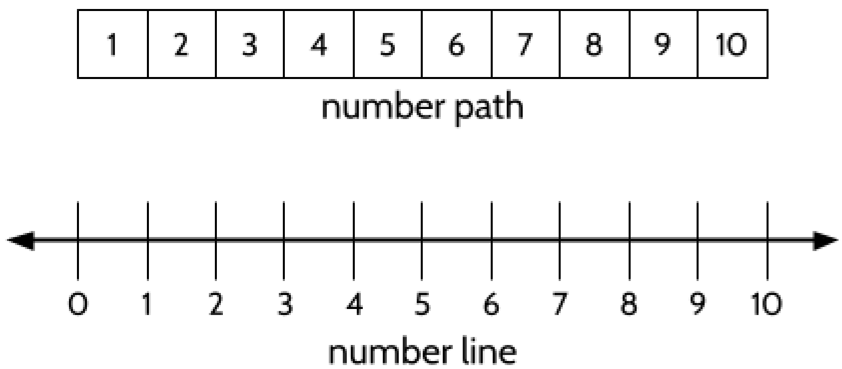


To draw an arrow 6 spaces long, we need to start on the left side of the 1 box and go all the way to the right side of the 6 box.



The arrow above is 6 spaces in length. We count each box, starting on the left and counting to the right.

Now, we will start to look at number lines. Look at the number path and number line below.

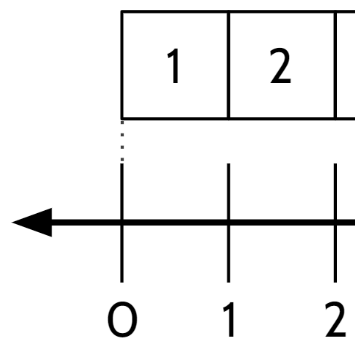


4) How is the number line different from the number path? How are they similar?

What is different?	What is the same?

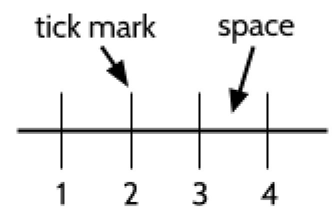
One thing that is different is that this number line starts with 0.

The number 0 is at the left side of the 1 box in the number line.



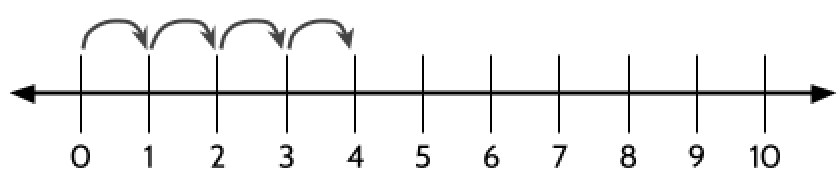
One thing that is the same about the number line and the number path is that we count spaces. In the number path, we count *boxes*. In the number line, we count *spaces* between the lines.

We can count the spaces between the numbers by using jumps. The jumps count the spaces between the lines. In this packet, we call these lines, tick marks. ***Tick marks*** are small lines used on a number line to help us find the location of numbers.

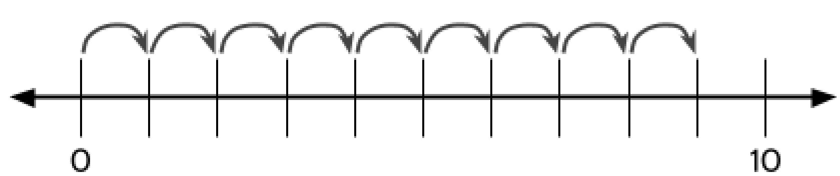


On a number line, we can use "jumps" to measure the number of spaces. Start with 0 and jump to the right.

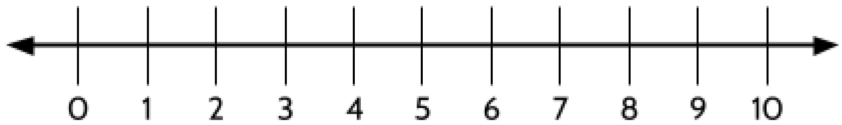
This number line shows a measurement of 4 spaces or jumps.



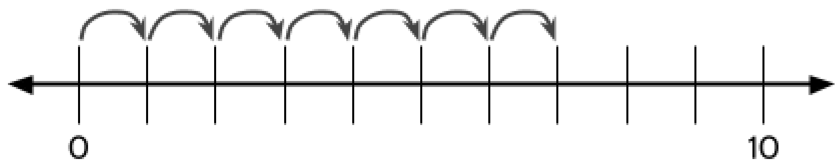
5) How many jumps are shown on the number line below? _____9_



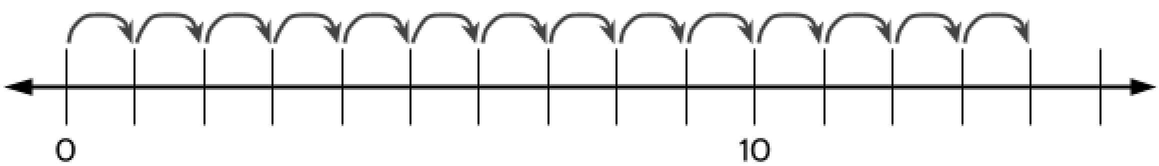
6) Draw jumps to show a measurement of 3 spaces.



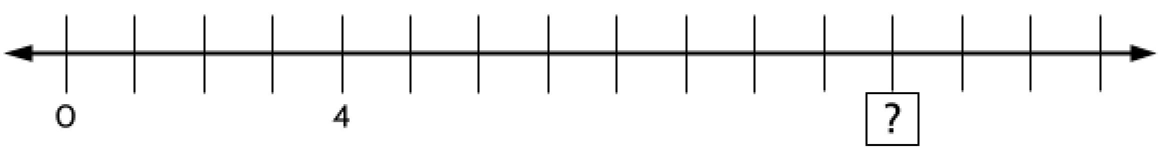
7) What measurement is shown by the jumps below?



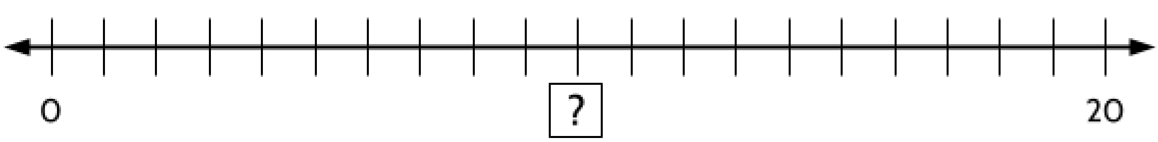
8) What measurement is shown by the jumps below?



9) What is the mystery measurement below? Draw jumps to find out.

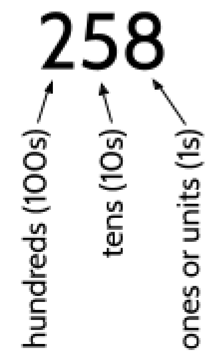


10) What is the mystery measurement below?

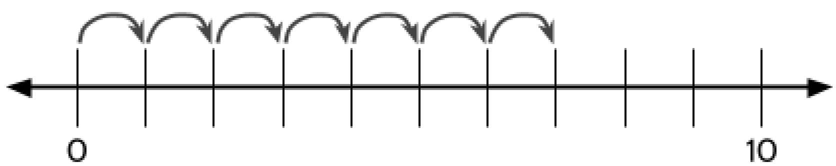


Units

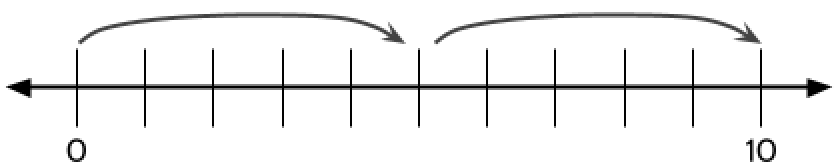
The word ***unit*** has a few different meanings. In mathematics, a unit can mean *one* (1). For example: The number 258 has two hundreds, five tens, and eight units.



This number line shows jumps of 1 unit at a time:



11) How many units are in each jump below? _______



The word *unit* is also used in the phrase ***unit of measurement***, which means the quantity of length, volume, weight, time, etc. that is being measured. For example, in the United States, we usually use inches, feet, yards, and miles as our units of measurement to measure distance. In other parts of the world, the units of measurement for distance are based on the meter. Millimeters, centimeters, and kilometers are units of measurement for distance.

12) What are some units of measurements we use to measure weight?

The word unit comes from Latin. The prefix *uni*- often means one. Here are some examples of words with *uni*-:

unicellular - having one cell, such as a bacteria

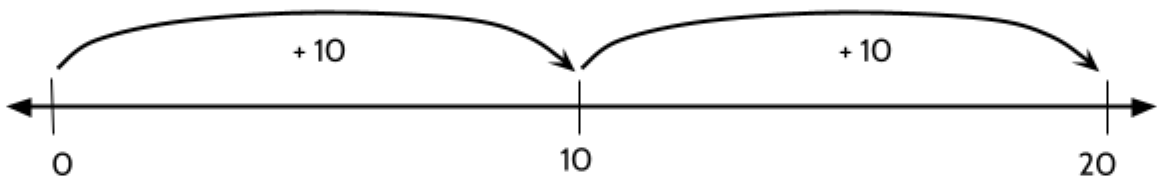
union - joining together two or more things

unique - one of a kind

unite - to bring together to form a single unit

Using Increments

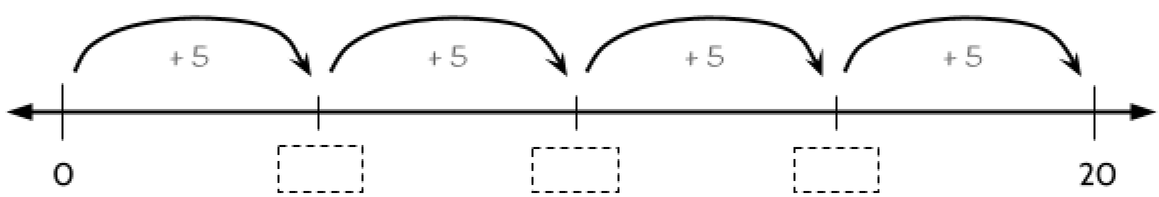
In these number lines, we will go from 0 to 20 with different sized jumps. For example, in the number line below, there are two jumps and each jump is +10.



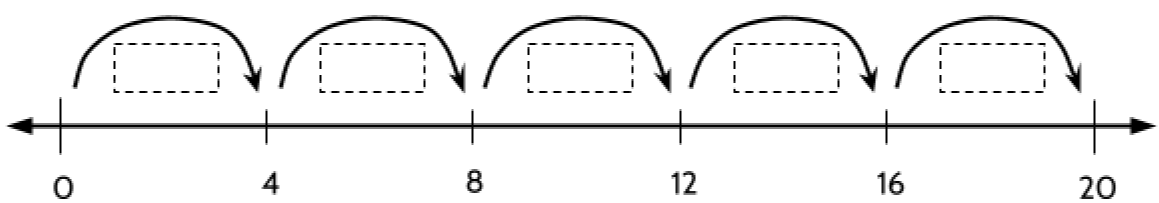
Each of the jumps are the same size. It takes two jumps of +10 to go from 0 to 20.

Add the missing numbers in the number lines below and answer the questions.

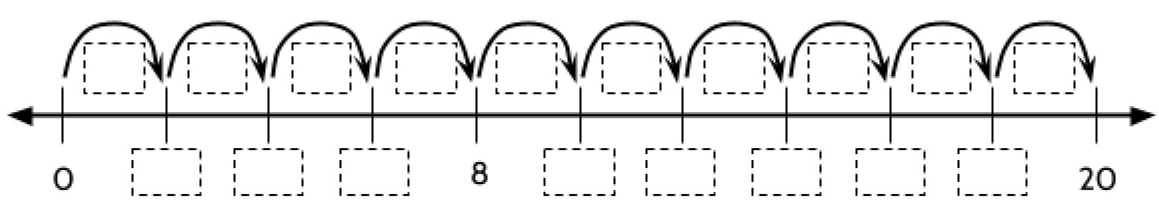
13) How big is each jump? __+5__



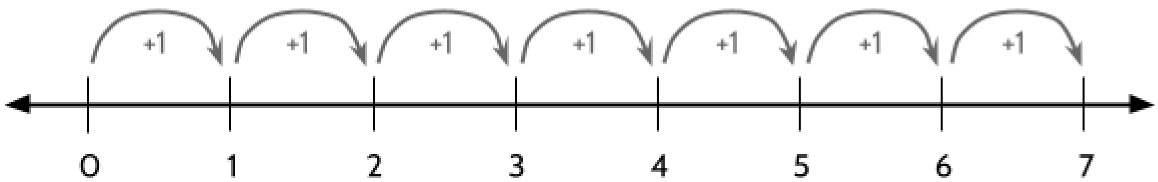
14) How big is each jump? ______4__



15) How big is each jump? ______2__



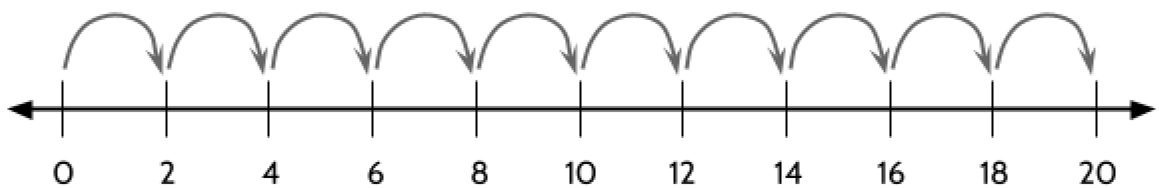
In the number line below, the numbers are growing by +1. This is the ***increment*** for this number line. The numbers 1, 2, 3, 4, 5, etc. are growing by an increment of +1.



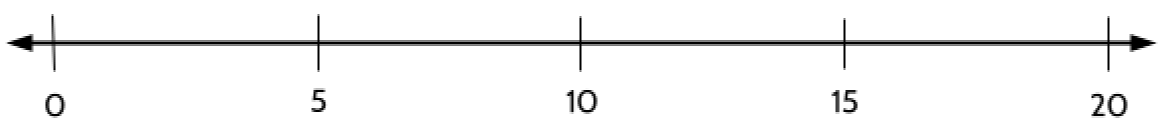
You can find the value of an increment on a graph by looking at the distance between tick marks. Ask yourself, what is the change in the numbers as they grow? The size of the jump is the increment.

Look at the following number lines and determine the value of the increment.

16) How big is each jump? ____2___ (Hint: In other words, how are the numbers 0, 2, 4, 6, etc. changing?

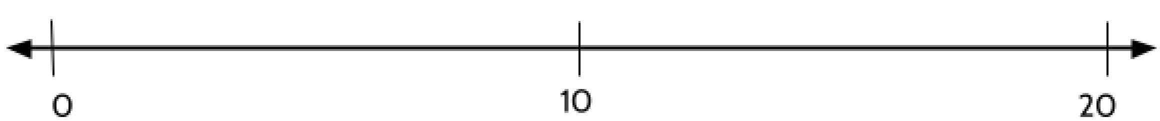


17) What is the increment between numbers below? _____5__



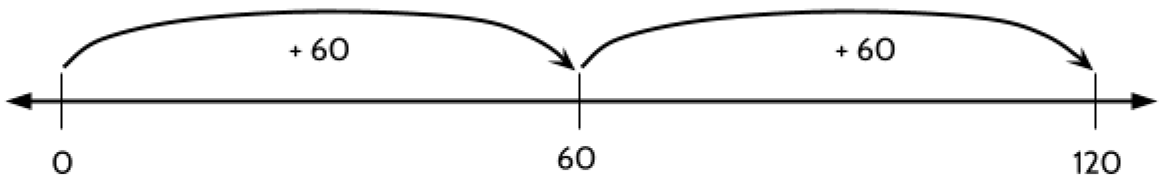
18) What is the increment between numbers below? ______4_

19) What is the increment between numbers below? _____10__



0 to 120 with Different Increments

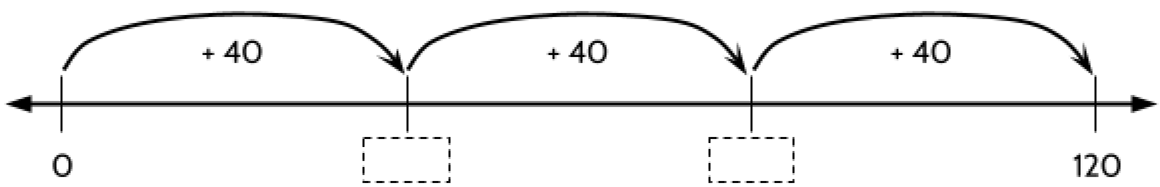
In these number lines, we will go from 0 to 120 with different numbers of jumps (with increments of different sizes). For example, in the number line below, each jump is +60 and we land on the number 60 between 0 and 120.



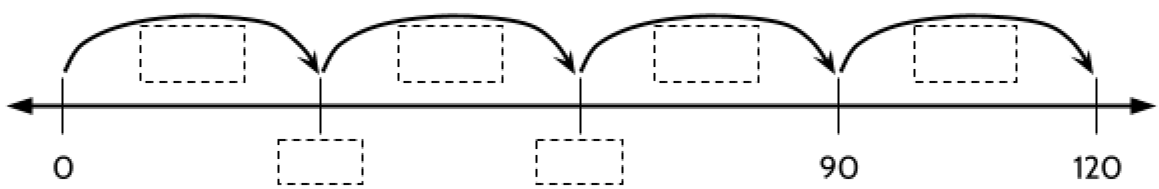
Each increment is the same size. It takes two increments of +60 to go from 0 to 120.

Add the missing numbers and answer the questions.

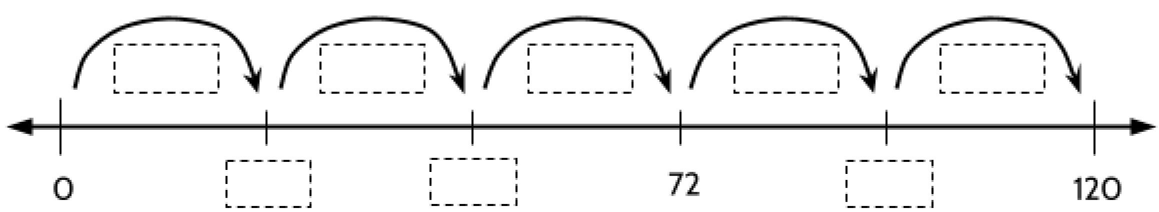
20) What is the size of the increment? (In other words, how big is each jump?) __+40__



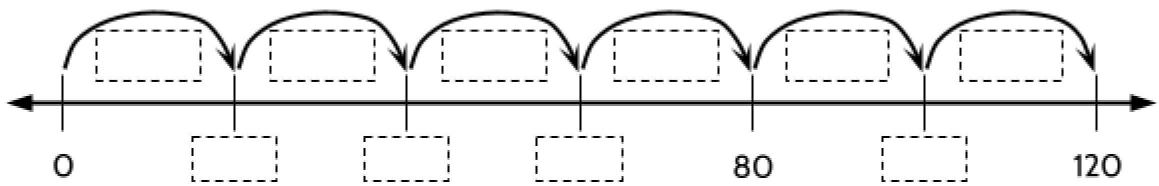
21) What is the size of the increment? ______30__



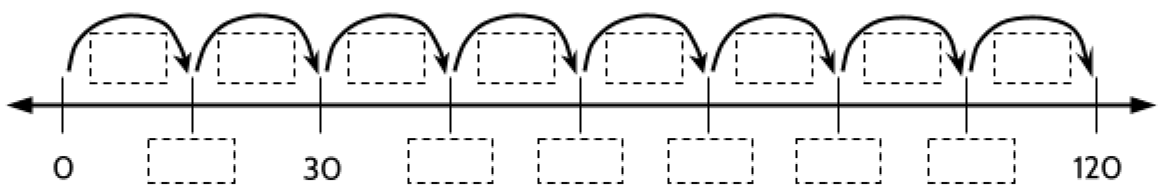
22) What is the size of the increment? _____24___



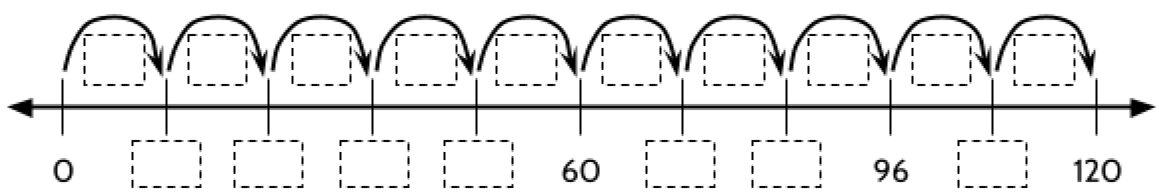
23) What is the size of the increment? _______20_



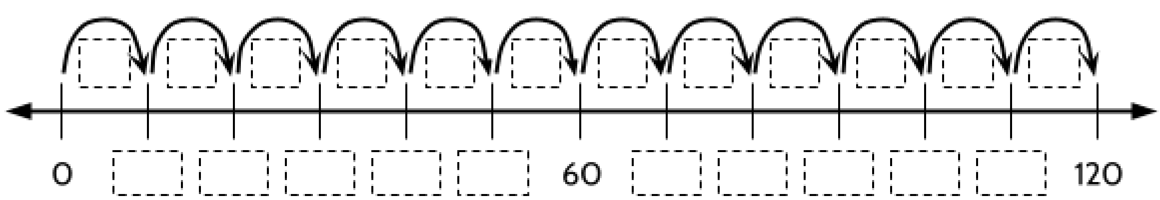
24) What is the size of the increment? _____15____



25) What is the size of the increment? _______12_



26) What is the size of the increment? _______10_



27) Imagine a number line that shows 24 jumps from 0 to 120. What is the size of the increment?

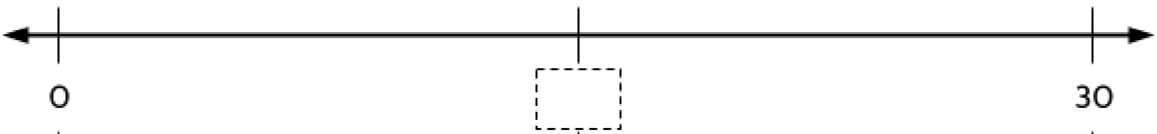
______5__

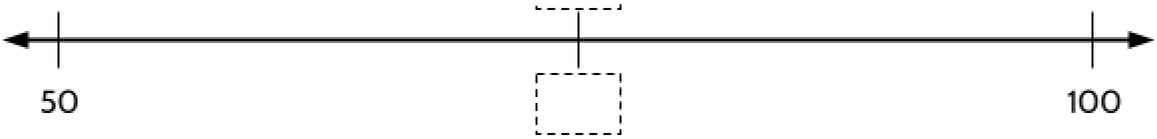
Find the Middle Number

In these questions, you will practice placing numbers in the correct position on a number line. What number is missing in the middle of the number line?

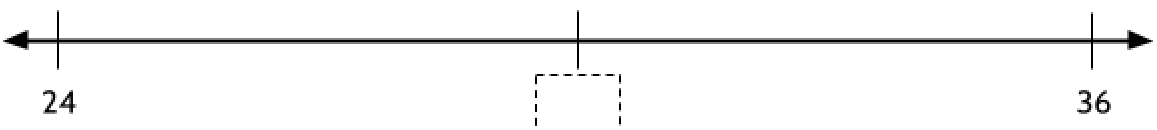
Add the missing numbers to each of the number lines below.

28)

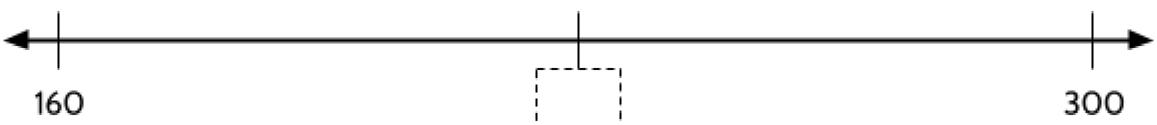




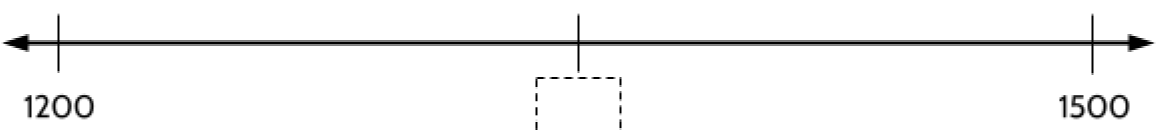
29)



30)



31)

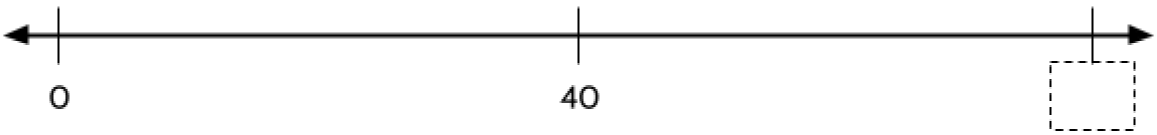


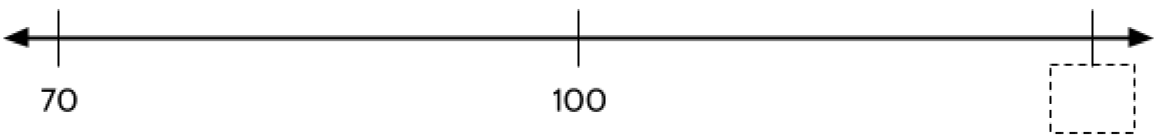
32)

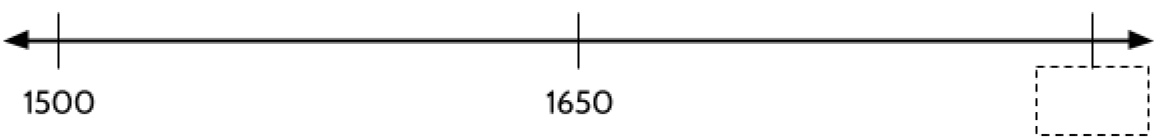
Find Outside Numbers

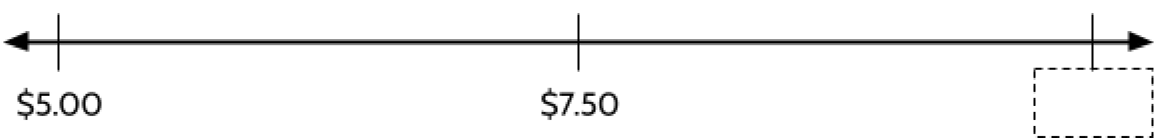
In these exercises, you will continue to practice placing numbers in the correct position on a number line. What number is missing on the right side of the number line?

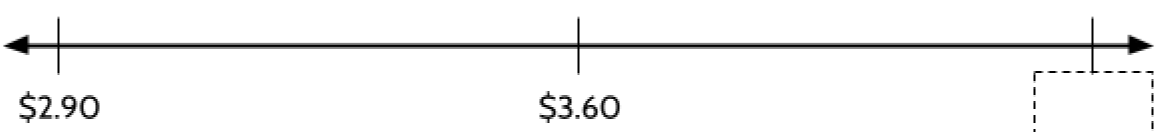
Add the missing numbers to each of the number lines below.

33) 

34) 

35) 

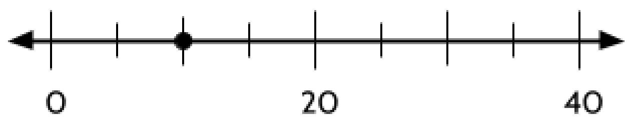
36) 

37) 

Reading Tick Marks on a Number Line

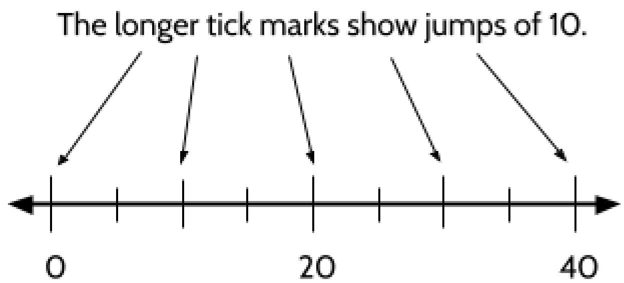
Tick marks help us find numbers on the number line. Tick marks are often longer or shorter to help us read number lines more easily. The numbers below the line help us know the meaning of the different tick marks.

In the number line below, what number do you think the point is on?

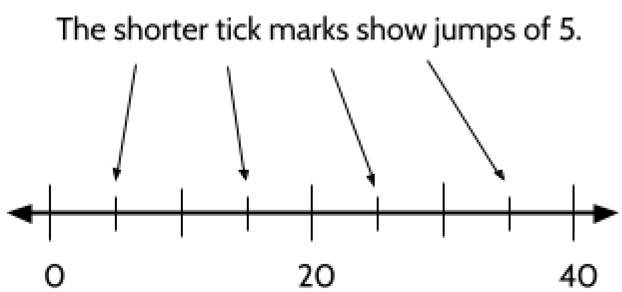


The longer tick mark is halfway between the 0 and 20, so we know it represents 10. The same is true for the tick mark halfway between 20 and 40, which represents 30.

The longer jumps of 10 help our eyes move along the number line.

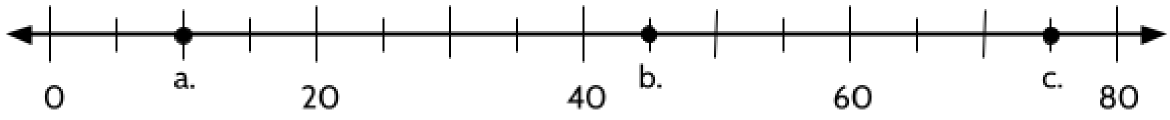


The longer tick marks also help us understand the shorter tick marks. On this number line, the shorter tick marks are halfway between the longer tick marks. If the longer tick marks are jumps of 10, then the shorter tick marks are jumps of 5.



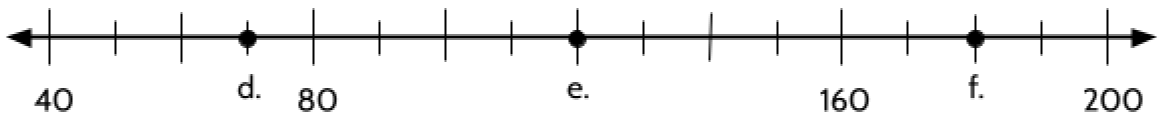
Find the values of the labeled points in the following number lines.

38)



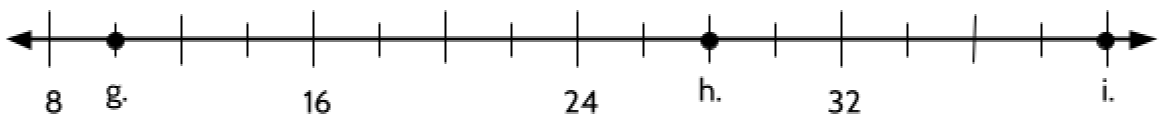
a. 10____ b. ________ c. ________

39)



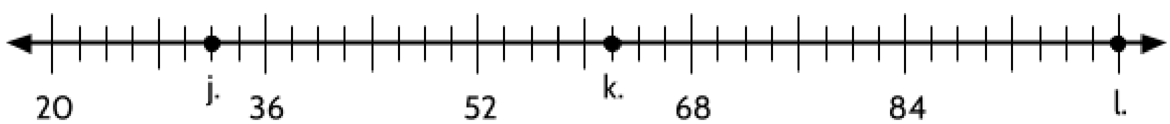
d. ________ e. ________ f. ________

40)



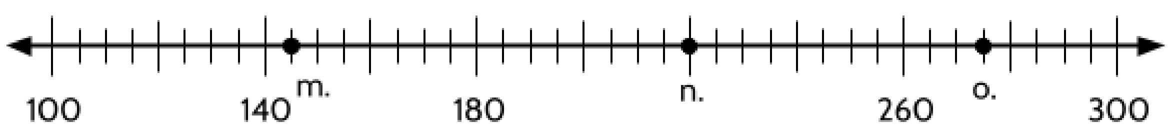
g. ________ h. ________ i. ________

41)



j. ________ k. 62________ l. ________

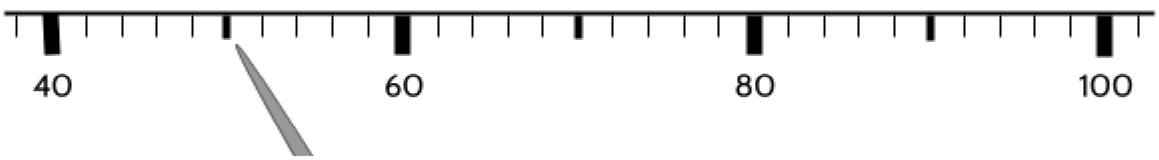
42)



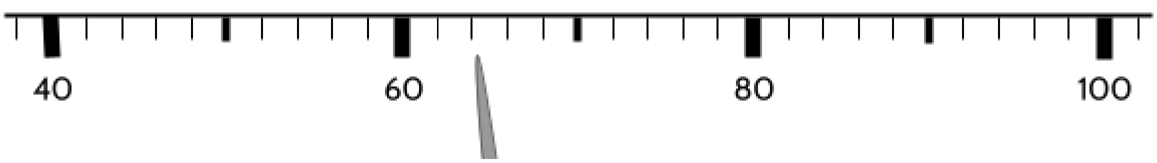
m. ________ n. ________ o. ________

Let's look at increments on some of the number lines from our everyday lives. This speedometer measures speed in kilometers per hour. The arrow points to the number that represents the speed of the car.

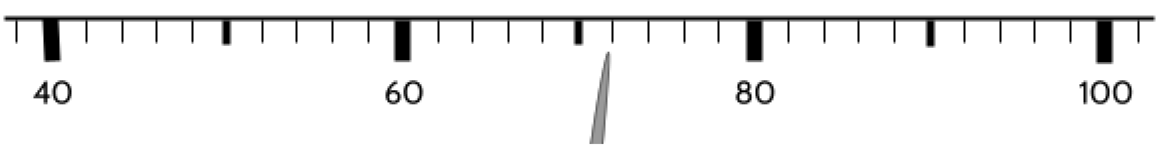
43) What is the speed of the car in the speedometer below? ________ km/h



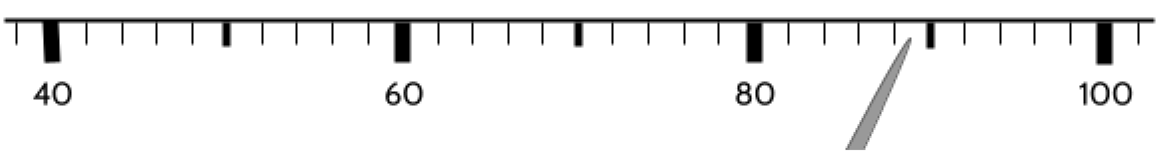
44) What is the speed of the car in the speedometer below? ________ km/h



45) What is the speed of the car in the speedometer below? ________ km/h

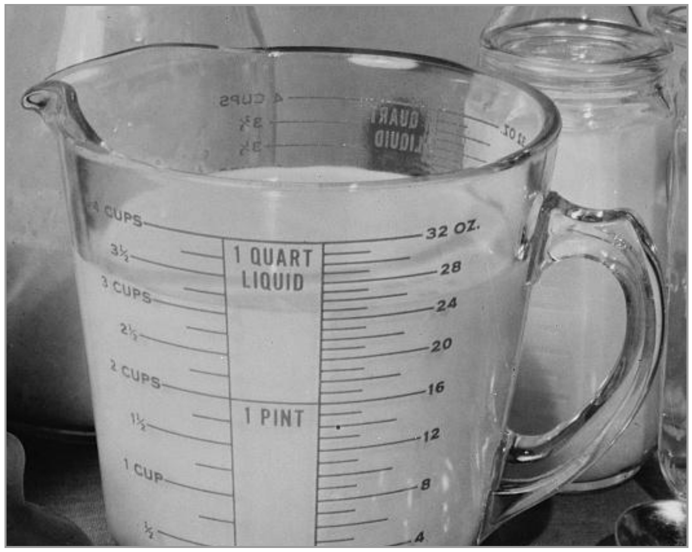


46) What is the speed of the car in the speedometer below? ________ km/h



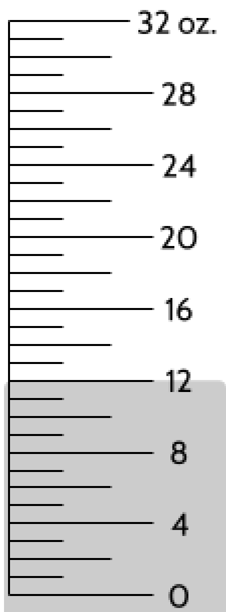
Number lines often have different types of tick marks. The tick marks of different sizes show the value of distances on the number line. We use tick marks so that we don't have to write all the numbers for every position.

This measuring cup uses cups, pints, quarts, and fluid ounces, which are different ways of measuring the volume of ingredients for cooking.



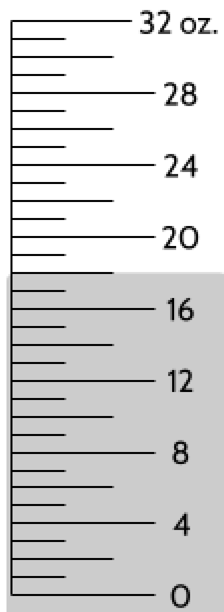
The number lines below show different amounts of milk in the measuring cup, measured in fluid ounces. How much milk is measured in each number line?

47)



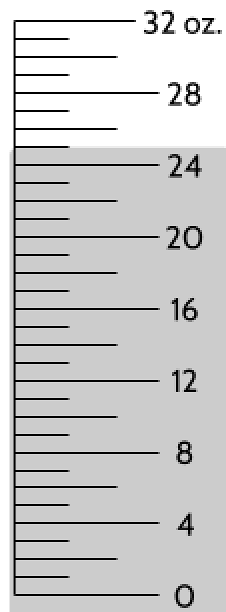
______ oz.

48)



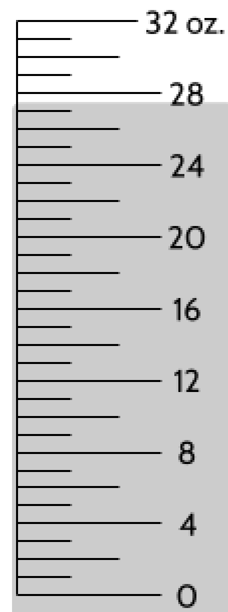
______ oz.

49)



______ oz.

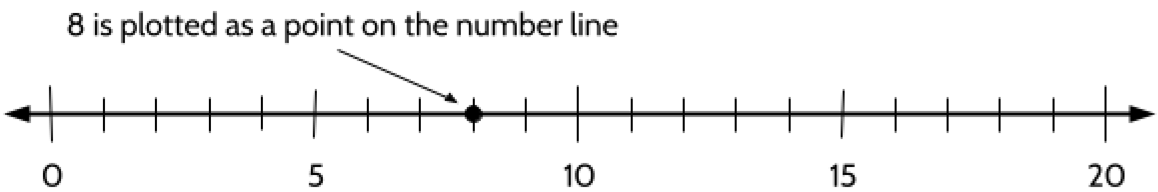
50)



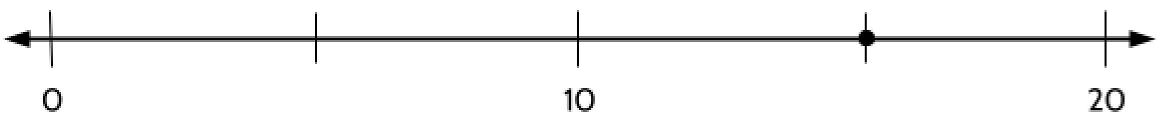
______ oz.

Points on a Number Line

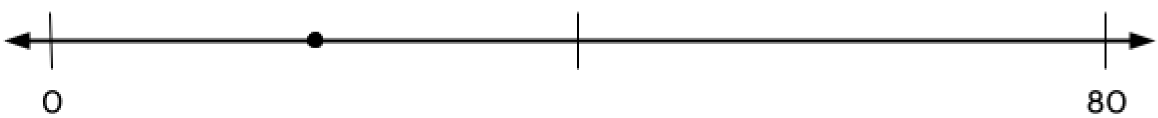
The word *plot* means to place a point on a graph, such as a number line or coordinate grid. On a number line, each point represents one number. You might be asked the value of the point. The point on the number line below represents the number 8.



51) What number does the point on the number line represent? ________

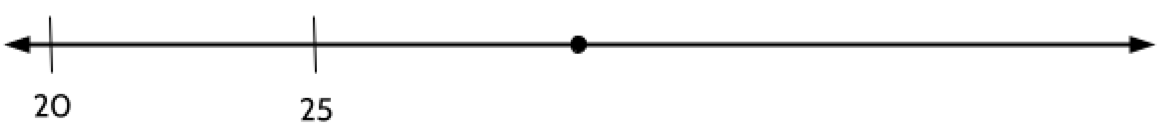


52) What number does the point on the number line represent? ________



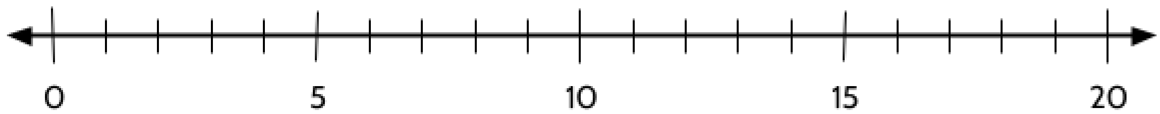
53) What number does the point on the number line represent? ________

54) What number does the point on the number line represent? ____30___

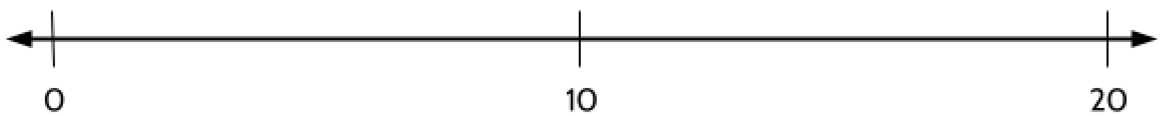


Now you will practice plotting points on number lines. You will have to use problem-solving skills to decide where the points should go. Add each number as a point to the number lines below.

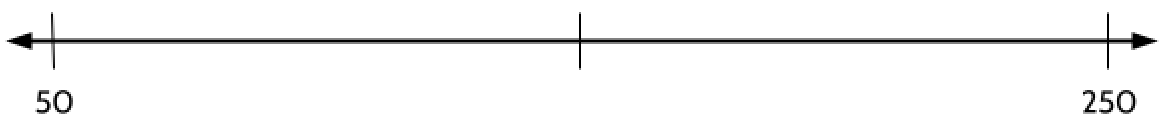
55) Plot 17 on the number line.



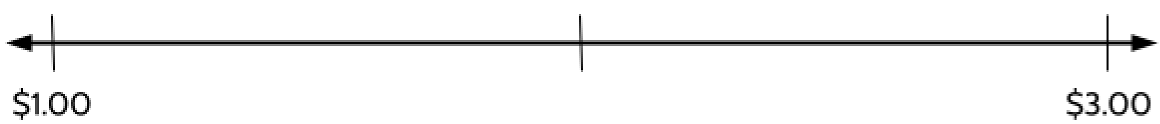
56) Plot 5 on the number line.



57) Plot 200 on the number line.



58) Plot $2.50 on the number line.

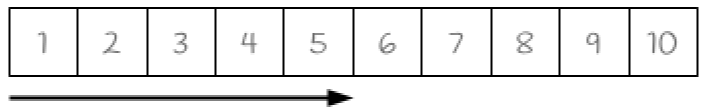


59) Plot 8 on the number line.

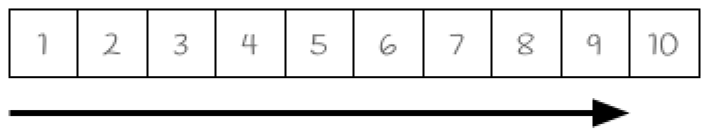
Introduction to Number Lines - Answer Key

Using Number Lines

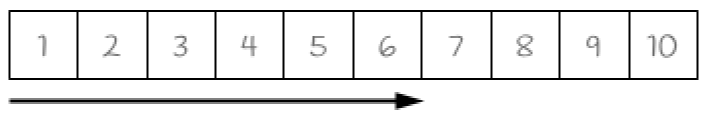
1) 5 spaces



2) 9 spaces



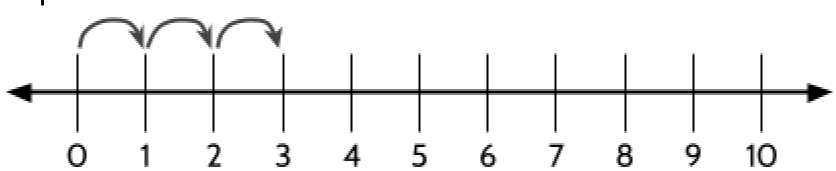
3)



4) The similarities and differences are explained after this question, in the packet.

5) 9 spaces

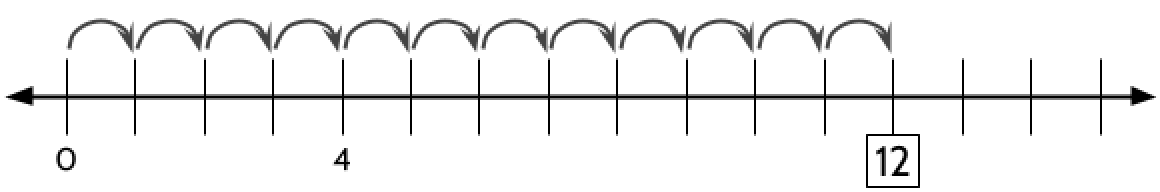
6)



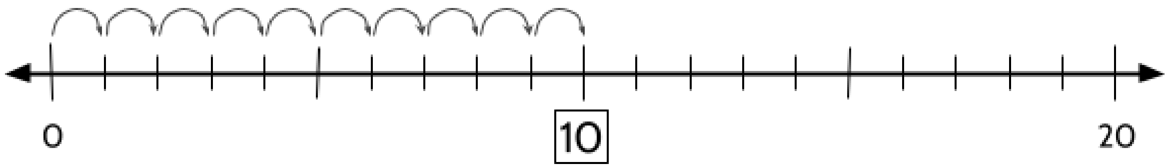
7) 7 spaces

8) 14 spaces

9) 12 spaces



10) 10 spaces

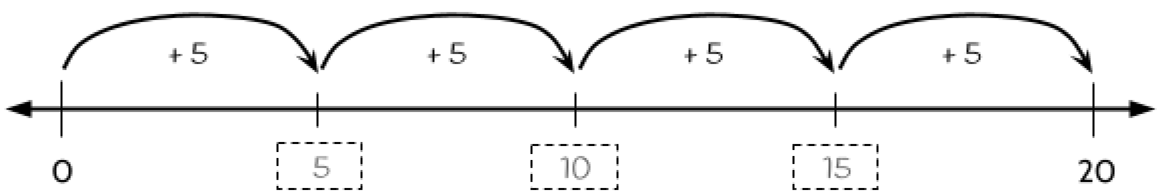


11) 5

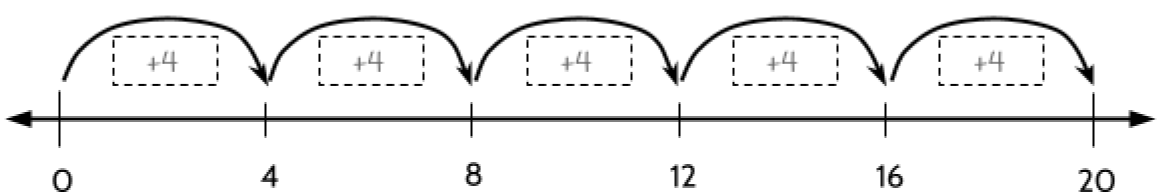
12) In the United States, we use ounces, pounds, and tons as units of measurement for weight. We also use units of measurement based on the gram, such as milligrams and kilograms.

Using Increments

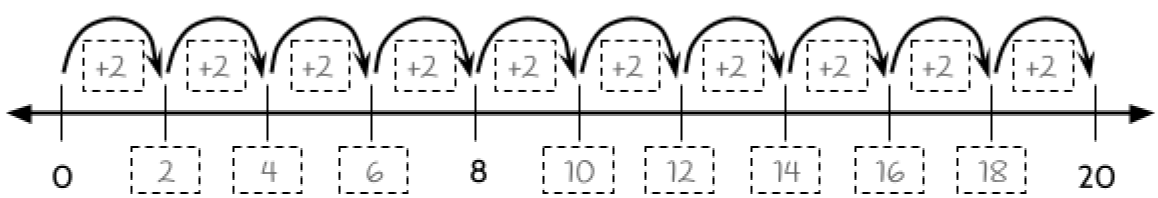
13) +5



14) +4



15) +2



16) 2

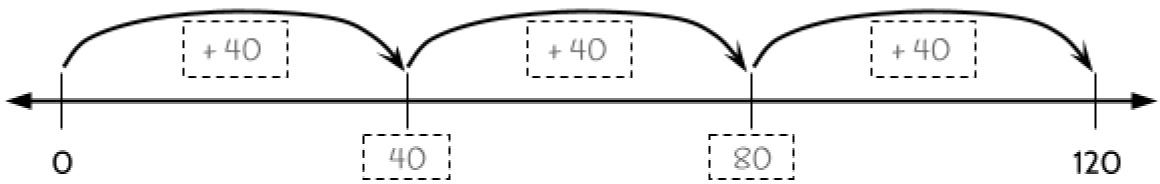
17) 5

18) 4

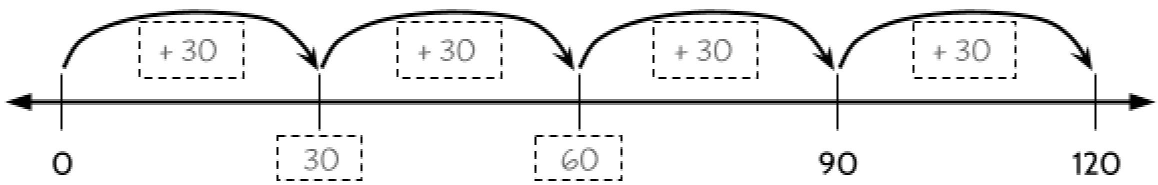
19) 10

0 to 120 with Different Increments

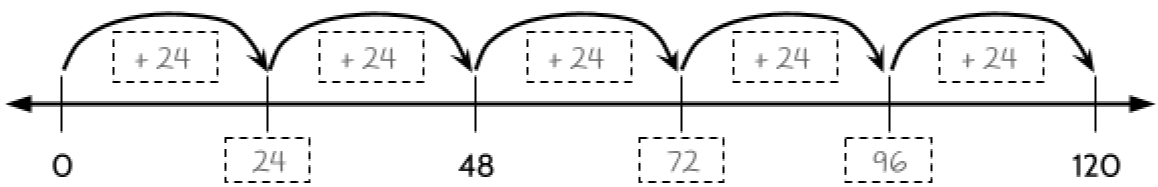
20) +40



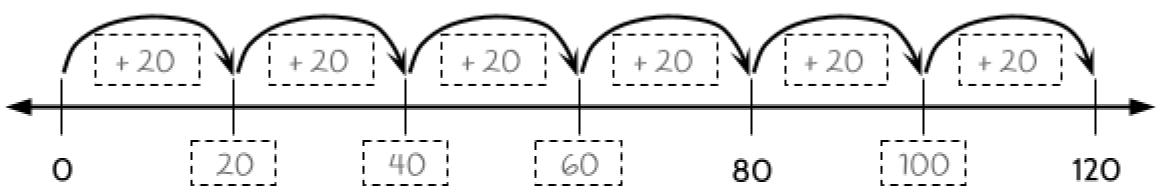
21) +30



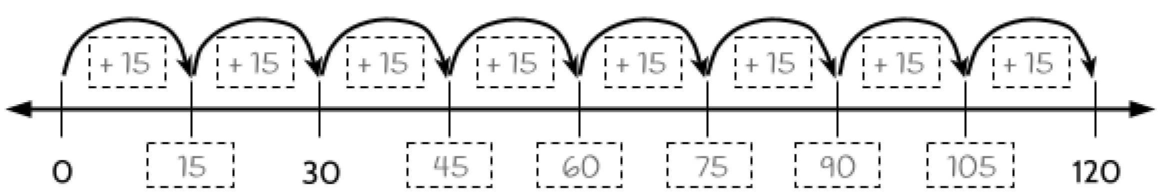
22) +24



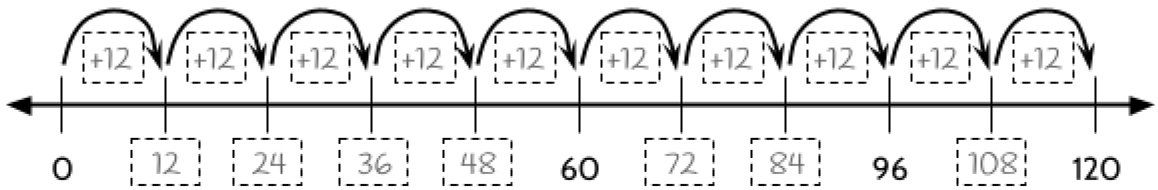
23) +20



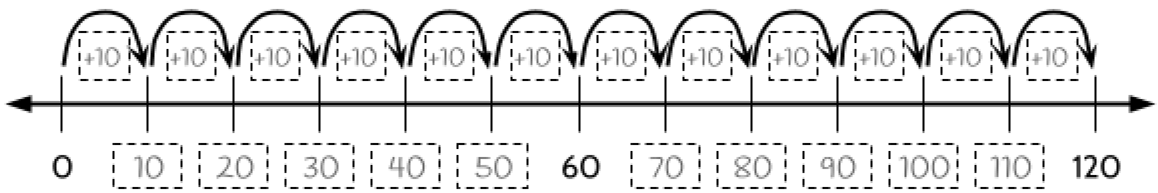
24) +15



25) +12

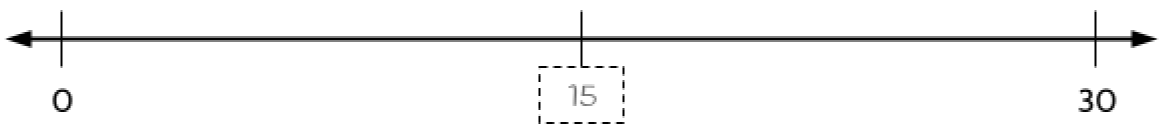


26) +10

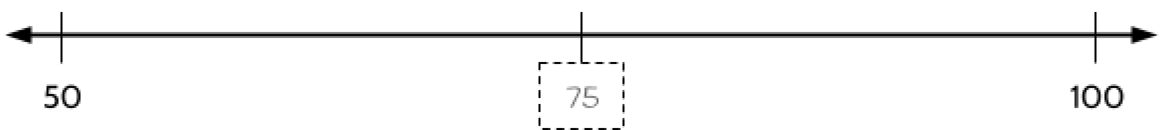


27) +5

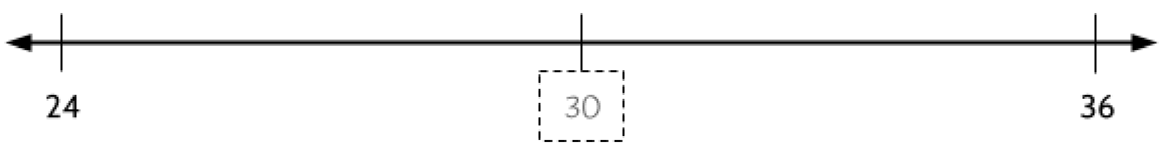
Find the Middle Number



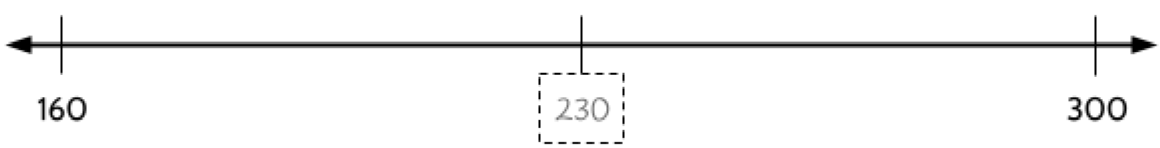
28)



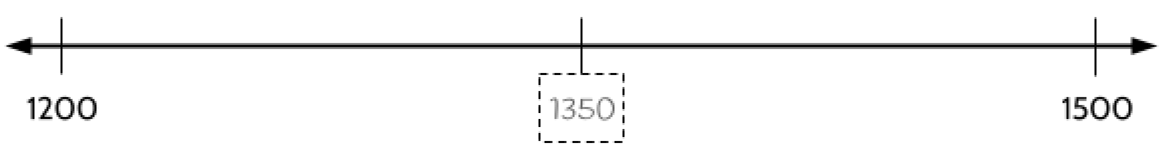
29)



30)

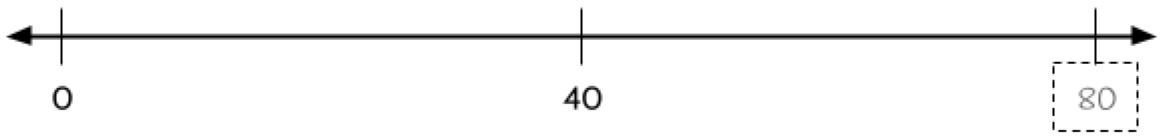


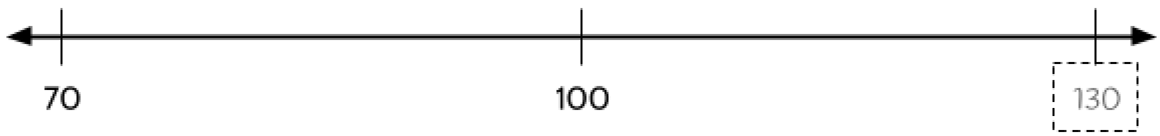
31)

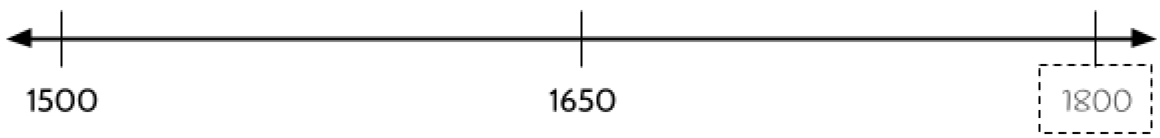


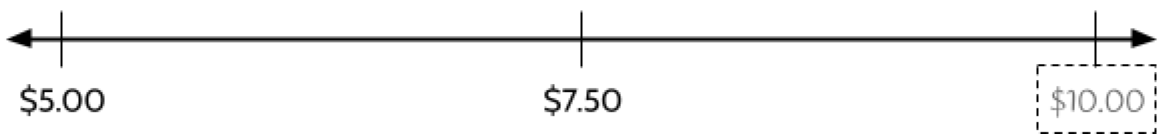
32)

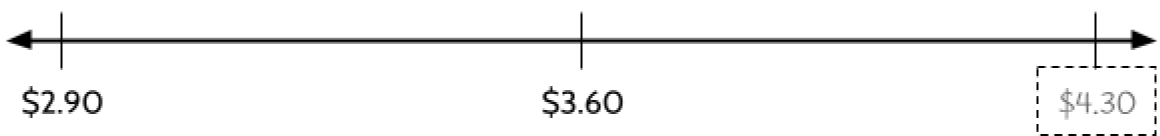
Find Outside Numbers

33) 

34) 

35) 

36) 

37) 

Reading Tick Marks on a Number Line

38) a. 10, b. 45, c. 75

39) d. 70, e. 120, f. 180

40) g. 10, h. 28, i. 40

41) j. 32, k. 62, l. 100

42) m. 145, n. 220, 275

43) 50 km/h

44) 64 km/h

45) 72 km/h

46) 89 km/h

47) 12 oz.

48) 18 oz.

49) 25 oz.

50) 27.5 oz. or 27 ½ oz.

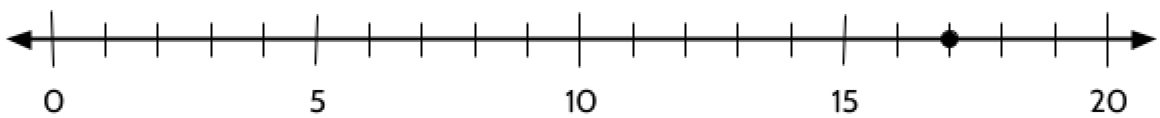
Points on a Number Line

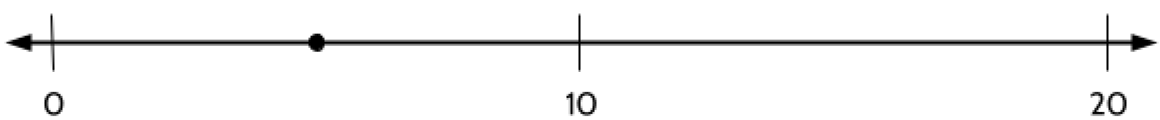
51) 15

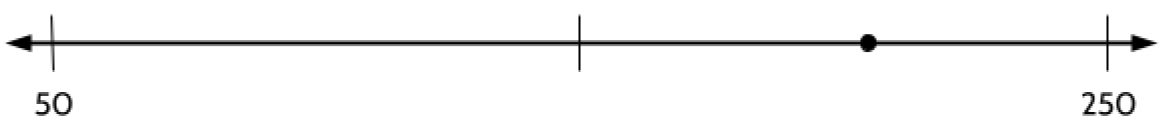
52) 20

53) 125

54) 30

55) 

56) 

57) 

58) 

59) 

Expanding the Number Line

Comparing Values: Equal, Greater Than, and Less Than

In this section, we will use number lines to compare different numbers and values. In mathematics, we have four primary ways of comparing values:

Comparison	Symbol	Example
is equal to	**=**	$4 \times 5 = 20$
is not equal to	**≠**	$3 \times 6 \neq 15$
is greater than	**>**	$1 > 0$
is less than	**<**	$3 < 2$

The symbol = or the ***equal sign*** is used in ***equations***. In an equation, the left side of the equal sign is equal to the right side of the equal sign. Here are a few examples of equations:

$3 + 5 = 6 + 2$ $7 \times 5 = 35$ $(6 \times 4) - 15 = 3 \times 3$

In the first equation, $3 + 5$ is equal to 8 and $6 + 2$ is also equal to 8. Both sides of the equation are in balance with each other.

The symbols ≠, >, and < are used in ***inequalities***. In an inequality, the two quantities are <u>not</u> necessarily equal to each other. Here are some examples of inequalities:

$8 \neq 12$	$16 > 15$	$0.5 < 1$
8 is not equal to 12	16 is greater than 15	0.5 is less than 1

The symbol ≠ can be used to say two quantities are not equal to each other. We can use > and < to compare the sizes of two quantities. The symbol > or the ***greater-than sign*** is used when the value on the left side of the equal sign is larger than the value on the right side. The symbol < or the ***less-than sign*** is used when the value on the left side of the equal sign is smaller than the value on the right side.

You might imagine a scale that compares the "weight" of the quantity on the left with the quantity on the right. Of course, numbers don't really have weight, but we can imagine that larger numbers are heavier and make the scale tilt to one side. If the values on the left and right are equal in value, the scale is balanced.

inequality

$4 > 1$

The left side is greater.

equation

$3 = 2 + 1$

The two sides are equal.

inequality

$2 < 10$

The left side is smaller.

Choose one of these symbols (=, ≠, >, or <) and write it between the quantity on the left and the right, so that the comparison between the two quantities is true. Find the value of each side. Then write a sentence comparing the two quantities. The first one is done for you.

1) $4 \times 8 \quad > \quad 100 \div 4$

32 is greater than 25.

2) $20 \qquad 17 + (2 \times 4)$

3) $2.5 \times 3 \qquad 1.5 \times 5$

4) $1.2 + 0.3 \qquad 0.9 + 0.08$

5) $(2 \times 7) + 3 \qquad 2 \times (7 + 3)$

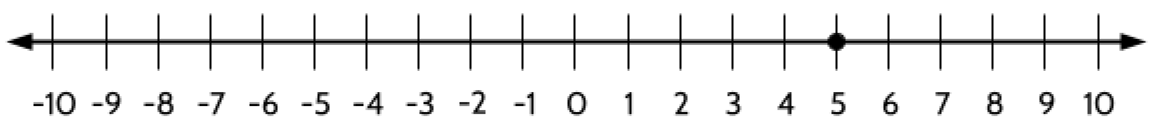
Now, we will graph equations and inequalities on a number line. We will start with an equation:

$$x = 5$$

How many solutions does this equation have? In other words, how many values for x make the left side equal to the right side?

Possible values of x	equation	Is the value a solution to $x = 5$?
4	$4 = 5$	No. 4 is not equal to 5.
5	$5 = 5$	**Yes.** 5 is equal to 5.
6	$6 = 5$	No. 6 is not equal to 5.

5 is the only number that makes $x = 5$ true. For that reason, the graph of $x = 5$ on a number line is a point.



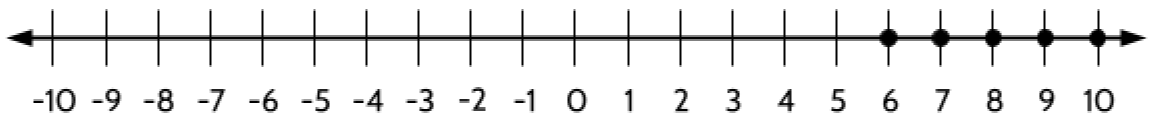
We can also use number lines to graph inequalities. What if we wanted to graph $w > 5$, which means w is greater than 5?

6) Complete the following table.

Possible values of x	inequality	Is the value a solution to $w > 5$?
4	$4 > 5$	No. 4 is not bigger than 5.
5	$5 > 5$	**5 is equal to 5**
6	> 5	6 is greater than 5
7	> 5	7 is greater than 5
8	> 5	8 is greater than 5
9	> 5	
10	> 5	
15	$15 > 5$	**Yes**. 15 is bigger than 5.

On the previous page, you should have found that 6, 7, 8, 9, 10, and 15 are solutions to $w > 5$. The number 5 is <u>not</u> a solution to $w > 5$ because 5 is not greater than 5.

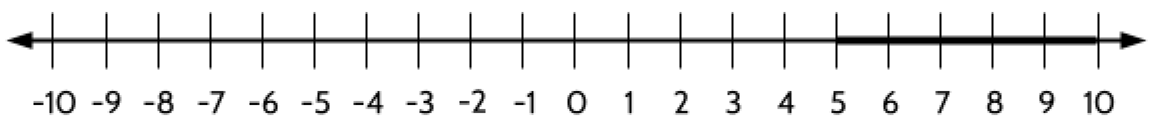
Maybe we could graph $w > 5$ like this? Does this seem right to you?



The graph above shows the solutions of 6, 7, 8, 9, and 10.

What about a solution between the numbers we have as solutions? 5 ½ is greater than 5, so it is a solution. Maybe you see other possible solutions between 6 and 7, or between 7 and 8.

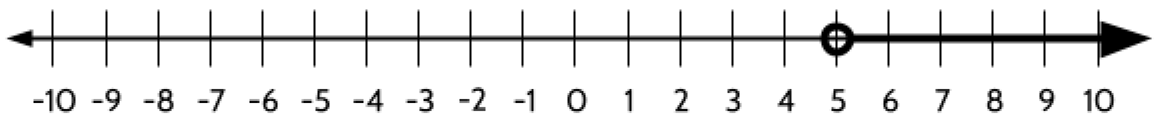
The solutions don't have to be whole numbers; they can be fractions or decimals. For this reason, we use a line to graph an inequality. The thick line between 5 and 10 on the number line shows that any position on the number line between 5 and 10 is a solution.

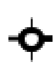


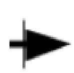
However, there are two things the graph above doesn't show. It doesn't show:

- 5 is <u>not</u> a solution to $w > 5$.
- Numbers bigger than 10 are solutions to $w > 5$. For example, 15 is a solution. So is 100 for that matter.

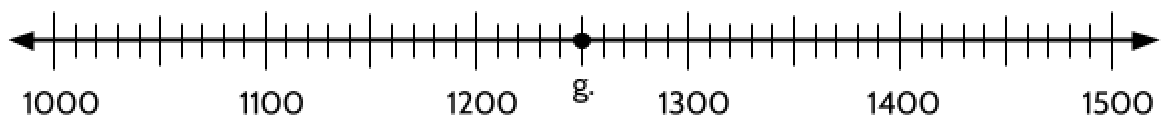
This is the correct graph for $w > 5$.



The open circle  means 5 is <u>not</u> a solution to $w > 5$.

The arrow to the right  means that all larger numbers are also solutions to $w > 5$.

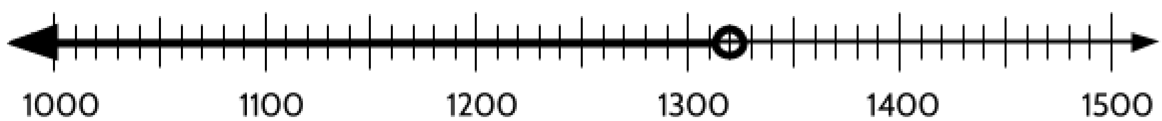
Let's think about a situation in the world. Gina knows exactly how much money she has in her bank account because she checked it yesterday. In the number line below, the point *g* shows the exact amount of money in her account.



7) Complete this equation: g = ________

We use an equal sign for the value of *g* because the amount of money in Gina's bank account is a specific amount.

Here is another situation: Mardgrina checked her bank account a few days ago and saw that the balance was $1320, but she has used her debit card since then and knows that the amount of money is lower now. The graph below shows the amount of money in Mardgrina's bank account.



8) Which of the following values are possible for Mardgrina's bank account? Choose all that apply.

A. 800

B. 1100

C. 1300

D. 1330

9) Which of the following matches the graph for Mardgrina's bank account (*m*)?

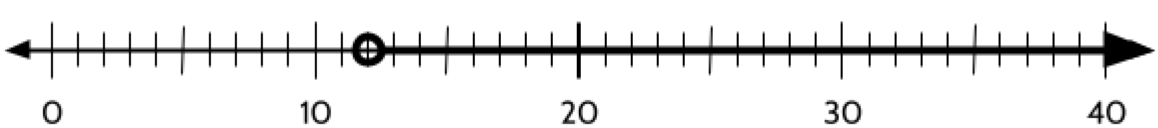
A. $m < 1300$

B. $m < 1320$

C. $m > 1300$

D. $m > 1320$

Ayantu is having a party for a friend's birthday. She invited a lot of people, but doesn't know how many people will actually come. The number line below shows how many people, *a*, are coming to Ayantu's party.



10) Based on the graph, how many people might come to Ayantu's party? Choose all that apply.

A. 10
B. 12
C. 15
D. 45

11) Which of the following matches the graph for Ayantu's party?

A. $a = 12$
B. $a < 12$
C. $a > 12$

There are also two more comparison symbols which are related to the symbols =, >, and <:

Symbol	Meaning	Example	
≥	is greater than or equal to	$x \geq 50$	*x* is greater than or equal to 50.
≤	is less than or equal to	$y \leq 8$	*y* is less than or equal to 8.

Here are a few examples of how we can use these symbols:

Cost of the party ≥ \$200 — The party costs ***greater than or equal*** **to** \$200. The party could cost \$200 or it could cost more.

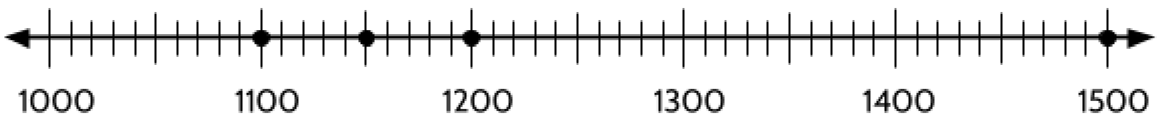
Class enrollment ≤ 30 students — The class is ***less than or equal to*** 30 students. The class might have 30 students or it might have fewer students.

Khom is leading a fundraiser for art supplies at his son's school. The school has a goal of raising at least \$1100, but if the fundraiser collects more money, the school will gladly use it for art classes and other activities. This inequality can be used to describe the amount of money to be collected in the school fundraiser (*f*): $f \geq 1100$

12) For the inequality, $f \geq 1100$, which of the following values of *f* are possible?

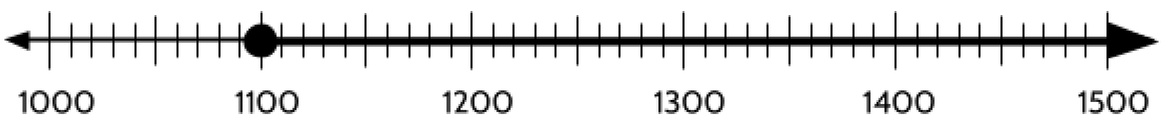
A. 1000
B. 1100
C. 1150
D. 1200
E. 1500
F. 3000

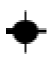
Let's try graphing the inequality, $f \geq 1100$. What if we put some of these values of *f* on a number line?



The number line above has points at 1100, 1150, 1200, and 1500, which are all possible amounts of money collected in the fundraiser. However, there are many other possible amounts.

Since inequalities have many possible solutions, we use a line instead of points. The number line below shows a graph of the school fundraiser and the inequality $f \geq 1100$.



The closed circle  at 1100 means that the school fundraiser could collect exactly 1100. The arrow points to the right to show that it could also be any larger number. Any point to the right of 1100 is a solution to $f \geq 1100$. For example, 3000 is a possible solution even though it isn't shown on the number line.

13) Select **all** the values of w from the list that make $w \geq 3$ true.

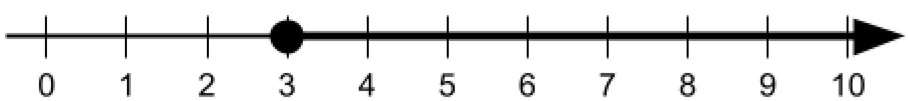
A. 0

B. 3

C. 8

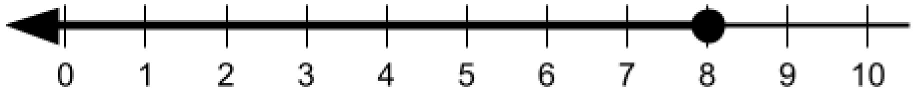
D. 11

This number line below shows a graph of an inequality.



14) What inequality describes the values shown on the number line above?

This number line below shows a graph of $w \leq 8$.



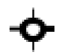
15) Select **all** the values of w from the list that make $w \leq 8$ true.

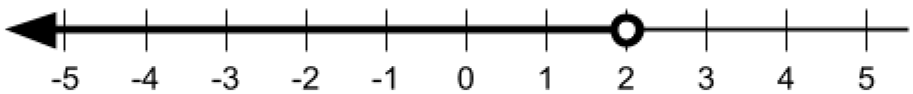
A. 0

B. 8

C. 9

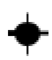
D. 11

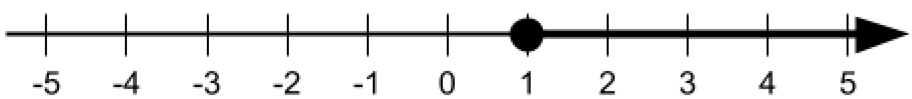
On a number line, the open circle  means the number where the open circle is placed **is not** included as a solution. For example, the following number line shows the inequality $x < 2$.



What if x was 2? Would 2 make the inequality true?

2 < 2 Is this a true number sentence? No, it isn't true. 2 is not bigger than 2. That's why the number line doesn't include 2. The open circle shows that 2 isn't a solution to this inequality.

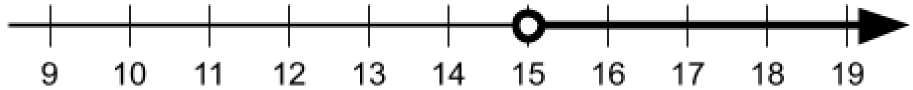
The closed circle  means the number where the closed circle is placed <u>is</u> included as a solution. For example, the following number line shows the inequality $y \geq 1$.



What if y was 1? Would this value make the inequality true?

1 ≥ 1 Is this a true number sentence? Is 1 greater than or equal to 1? Well, 1 isn't greater than 1, but they are equal, so the number sentence is true. That's why the number line includes 1. The closed circle shows that 1 is a solution to this inequality.

16) Which inequality does the following number line represent?



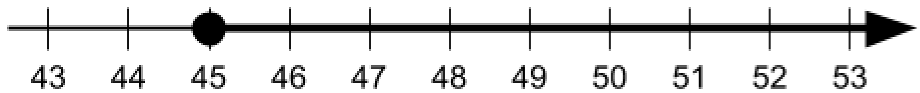
A. $z < 15$

B. $z \leq 15$

C. $z > 15$

D. $z \geq 15$

17) Which inequality does this number line represent?



A. $n < 45$

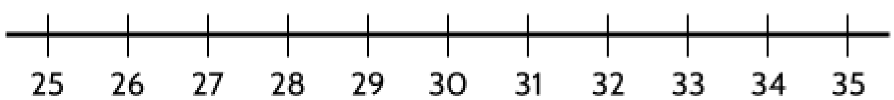
B. $n \leq 45$

C. $n > 45$

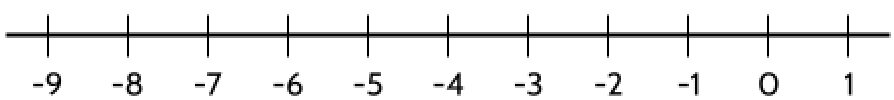
D. $n \geq 45$

18) Graph the solutions to each inequality onto the number lines by drawing arrows and circles.

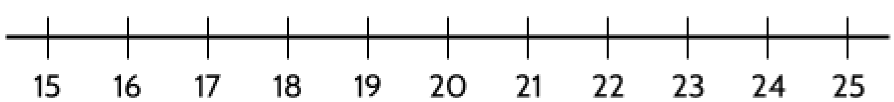
$p > 31$



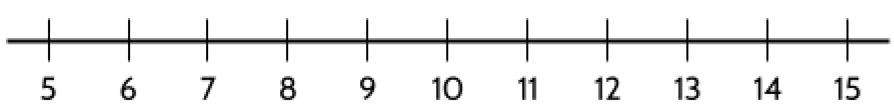
$q \leq 1$



$r \geq 15 + 6$



$s < 15 - 6$



Choose the correct inequality to represent the situations below.

19) The speed limit on a highway is 65 mph. The variable *s* represents the speeds that are allowed.

A. $s \geq 65$

B. $s \leq 65$

C. $s > 65$

D. $s = 65$

20) Certain movies have age restrictions. For example, if a movie is rated R, then only people who are 17 years old or older can watch it. The variable *a* represents the ages of people who can watch the movie.

A. $a \geq 17$

B. $a \leq 17$

C. $a < 17$

D. $a = 17$

21) A bakery has enough ingredients (flour, sugar, butter, etc.) to make up to 100 cakes each day. The variable *c* represents how many cakes they make.

A. $c \geq 100$

B. $c \leq 100$

C. $c > 100$

D. $c = 100$

22) Children must be at least 48 inches tall to ride the Cyclone roller coaster. The variable *h* represents the heights of children who can ride the roller coaster.

A. $h \geq 48$

B. $h \leq 48$

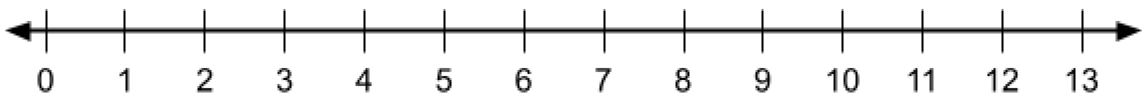
C. $h < 48$

D. $h = 48$

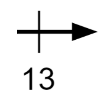
23) What could the inequality $x \geq 10$ represent? Think of a real world situation that would match this inequality.

Signed Numbers (Positive and Negative Numbers)

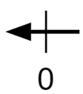
You may have noticed that number lines often have arrows on the right side and left side. The arrow on the right side of the number line means that we can include all larger numbers on the same number line.



13 is the largest number on the number line above. The arrow to the right means that 14, 15, 16, and ALL larger numbers are also included on the number line even if they aren't shown.



24) What do you think the arrow on the left side of the number line means?



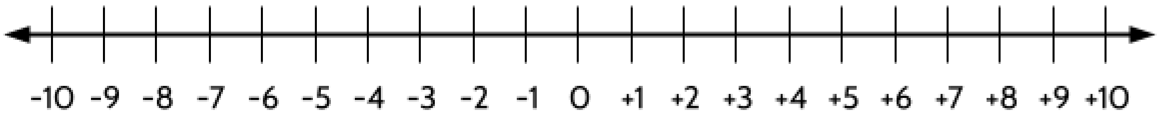
As we move to the right on a number line, the numbers get bigger. Example: $13 > 12$. We can imagine many, many other numbers larger as the number line grows to the right.

As we move to the left on a number line, the numbers get smaller. Example: $0 < 5$. To the left of 0 are numbers that are smaller than 0. You might ask, "How is that possible? How can a number be smaller than 0?"

Here are some examples of quantities smaller than 0:

- 10 degrees below zero
- $500 owed in credit card debt
- A bank fee of $40 after bouncing a check on an empty bank account
- 12,500 feet below the ocean surface (the location of the Titanic, on the floor of the ocean)
- 282 feet below sea level (the altitude of Death Valley, California)

This number line shows numbers greater than 0 and numbers less than 0:

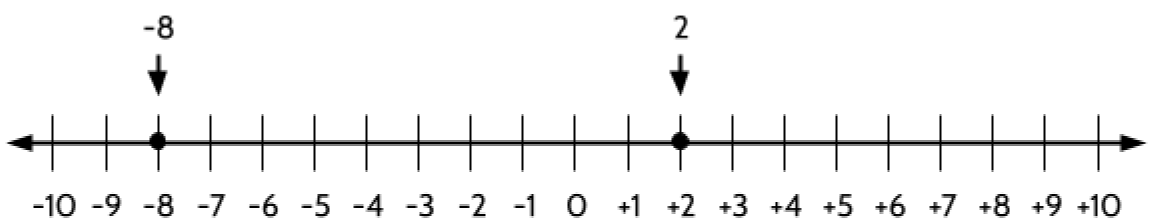


Positive whole numbers start at 1 and get bigger as we go to the right on the number line (1, 2, 3, 4, etc.). We can use a ***plus sign*** + to show that a number is positive.

Negative whole numbers start at -1 and get smaller as we go to the left on the number line (-1, -2, -3, -4, etc.). We use a ***negative sign*** - to show that a number is negative.

Let's compare the size of numbers on the number line using > (greater than) and < (less than). For example, which is larger, 2 or -8?

If you are not sure which number is larger, think about where the numbers are on the number line. The number line below shows -8 and 2.



2 is larger than -8 because numbers get bigger as we move to the right on the number line.

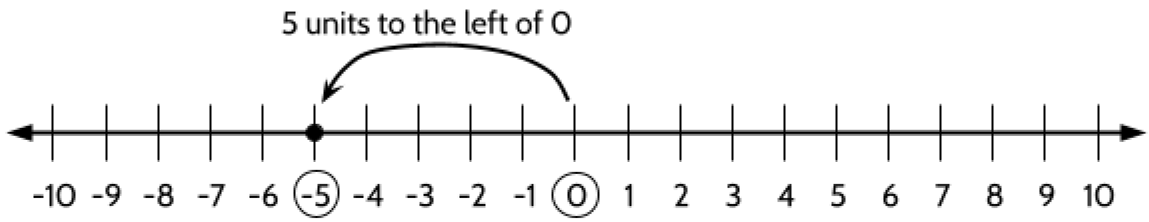
25) Use > or < to compare the pairs of numbers below. Note: If you don't see + or - before a number, you can assume that the number is positive.

	> or <			> or <	
+9	>	+5	-10		6
+6		+7	0		-3
-4		+3	0		3
-5		-7	1		-15

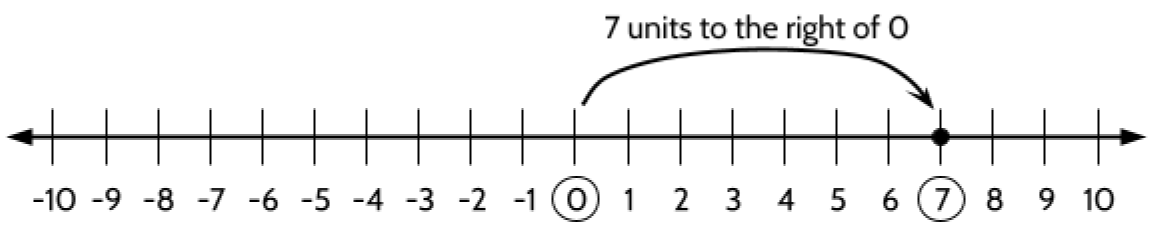
You may have wondered about 0. Is it positive or negative? 0 (zero) is larger than all negative numbers and smaller than all positive numbers. It is not positive or negative.

Finding the Opposite

To plot signed numbers on a number line, start at 0 and then count to the right or left, depending on the sign of the number. To plot a negative number on a number line, start at 0 and count to the left. For example, to plot -5 on a number line, start at 0 and then count 5 units to the left.

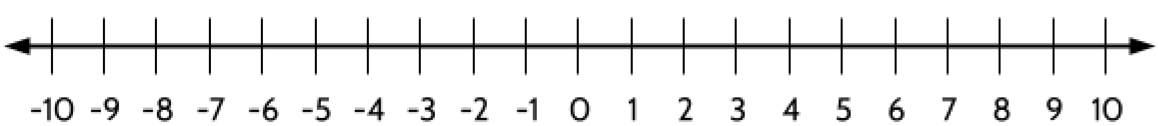


To plot a positive number on the number line, start at 0 and count to the right. For example, to plot +7 on a number line, start at 0 and then count 7 units to the right.

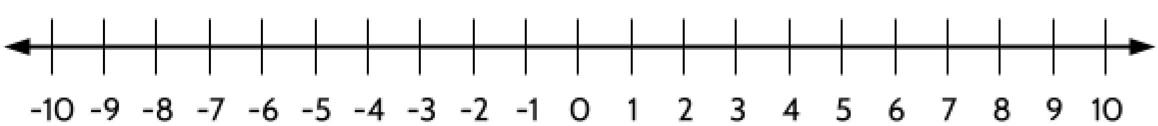


Plot these signed numbers on the number lines below.

26) +9



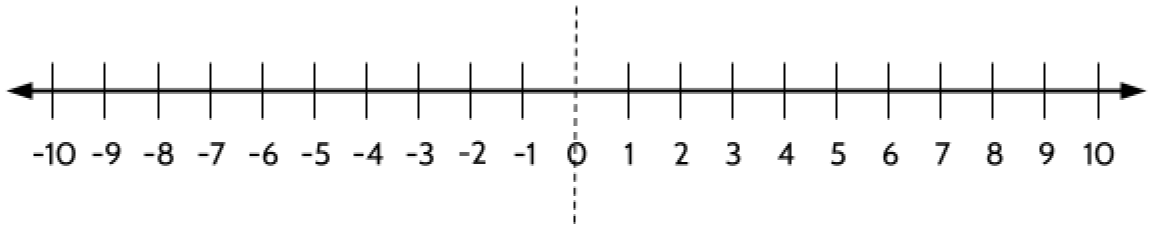
27) -3



When we move to the coordinate grid in Part 2, we will use the same rule for plotting points on the horizontal axis:

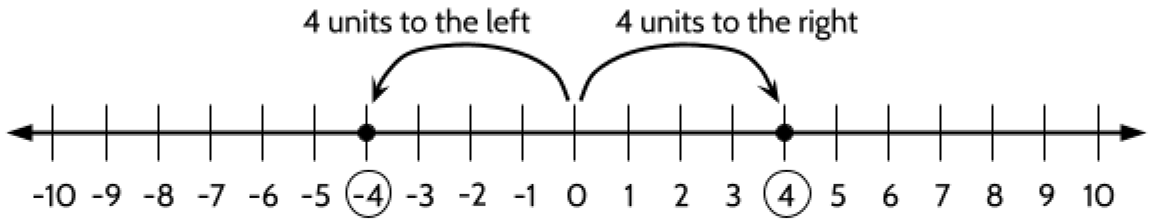
> Start at 0 and count to the right for positive numbers and start at 0 and count to the left for negative numbers.

Look at the number line below. You can see that 0 is the middle of this number line. Imagine folding the paper in half along the dotted line.



If you fold the paper on the dotted line, each positive number on the right would fold to meet its opposite on the negative side. The number 1 would meet -1, the number 2 would meet -2, and so on. This is because each number is the same distance from 0.

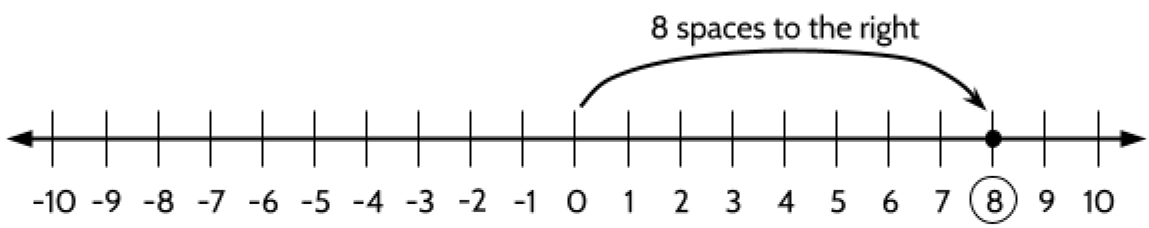
The numbers 4 and -4 are plotted on the number line below.



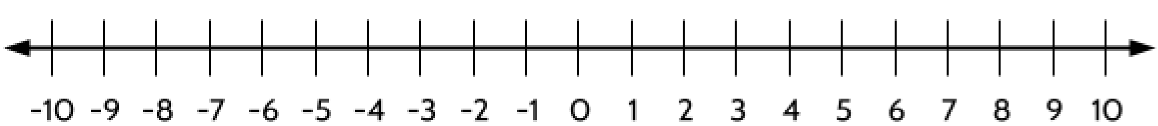
We can see that 4 and -4 are the same distance from 0, on opposite sides. For this reason, -4 is considered the opposite of 4.

> To find the opposite of a number, look for the number on the other side of 0 that is the same distance from 0.

28) What is the opposite of 8? ______



29) What is the opposite of -3?



The Meaning of – , the Minus Sign

The ***minus sign*** can have different meanings, depending on the situation.

<u>The minus sign can mean subtraction</u>. For example, in the equation 100 - 25 = 75, the minus sign means subtract 25 from 100.

<u>The minus sign can mean that a number is negative</u>. For example: -25 means that the number is "negative 25."

<u>The minus sign can mean "opposite</u>." For example: -25 means that the number is opposite 25, which means it is 25 units on the other side of the number line.

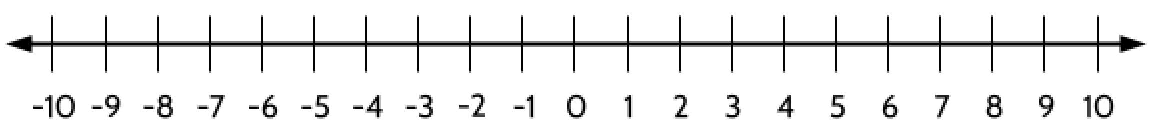
You can use each of these meanings to understand the minus sign. Try these questions.

30) Write 3 subtracted from 10, using a minus sign.

31) Write "negative 15."

32) Write the opposite of 0.5, using a minus sign.

Let's explore the mathematical meaning of the word, ***opposite***. Use a number line to think about these questions.



33) What is the opposite of 10?

34) What is the opposite of -7?

35) What is the opposite of 0? How do you know?

36) What do you think -(-5) means? (It could also be written - -5. The parentheses are added to make it easier to read.)

The expression -x means "the opposite of x."

37) If x equals 12, what is -x?

38) If x equals -12, what is -x?

39) Write the value of -x if:

$x = 2$ -x = _______

$x = -3$ -x = _______

40) True or false? Explain your answers.

-x is always negative.

-x can be positive.

41) For each expression, write an explanation of what the minus signs mean.

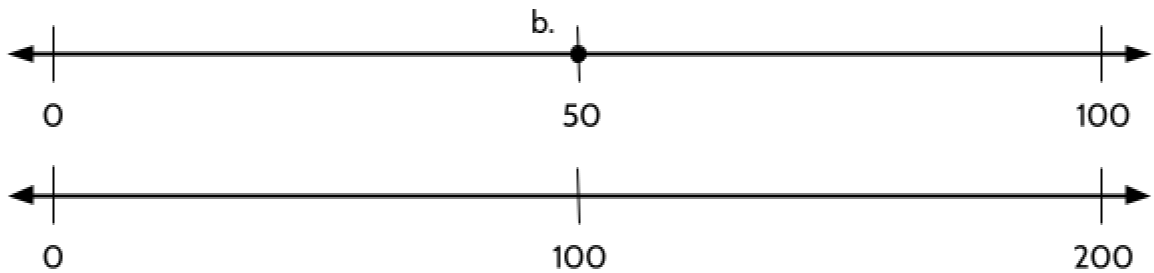
-3	This means "negative 3," which is the opposite of +3.
7 - 4	
- (-3)	
4 - 7	
3 - x	
- (-x)	
- (4 + 2)	

Plotting the Same Points on Two Number Lines

In this exercise, we will practice plotting points on a few number lines. We will see how the size of the number changes where we place the points.

42) Graph these numbers on the TWO number lines below. The point b) has been added.

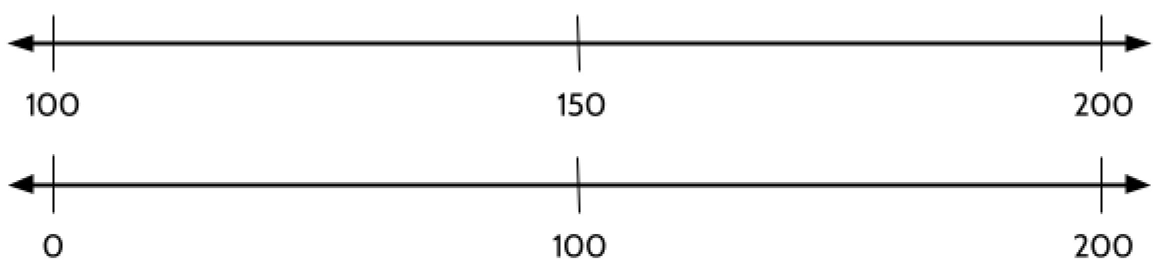
a) 25 b) 50 c) 75



43) What do you notice when you look at the points on the number lines above?

44) Graph these numbers on the TWO number lines below.

a) 125 b) 175 c) 110



45) What do you notice when you look at the points on the number lines above?

Making Copies and Splitting Equally

In this exercise, you will practice two important strategies for locating numbers on the number line. These two strategies can also help you become more comfortable with fractions and decimals. We will practice making copies first.

Making Copies

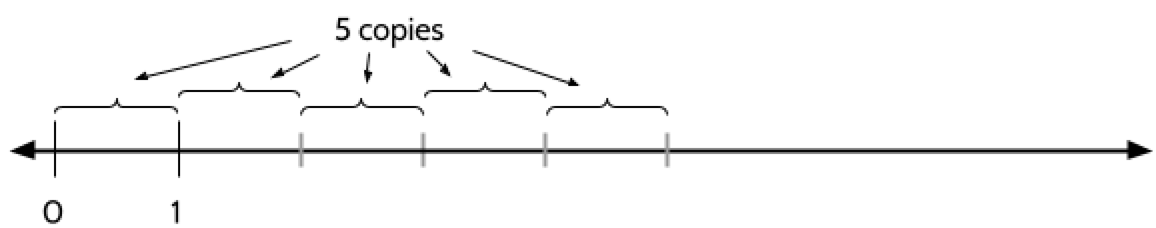
As an example, try the following task:

Plot 5 on this number line.

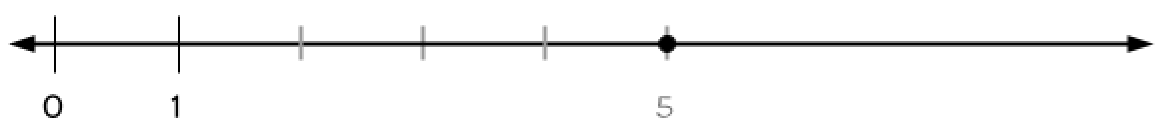
We can use the strategy of making copies to find the right position for the number 5 on this number line. The first step is to measure the distance between 0 and 1. The distance is 1 unit.

Next, we can ask ourselves, “How many copies of this distance do we need to make 5?”

We need five 1’s to make 5, so we can make 5 copies of the distance between 0 and 1.



Finally, we plot 5 on the number line.



Practice making copies as a strategy to plot the points below.

46) Plot 3 on the number line below.

47) Plot 6 on the number line below.

48) Plot 20 on the number line below.

49) Plot 50 on the number line below.

50) Plot 200 on the number line below.

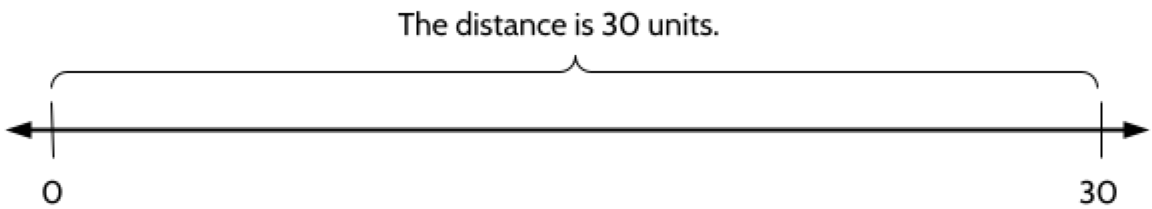
51) Plot $4.00 on the number line below.

Splitting Evenly

As an example, try the following task:

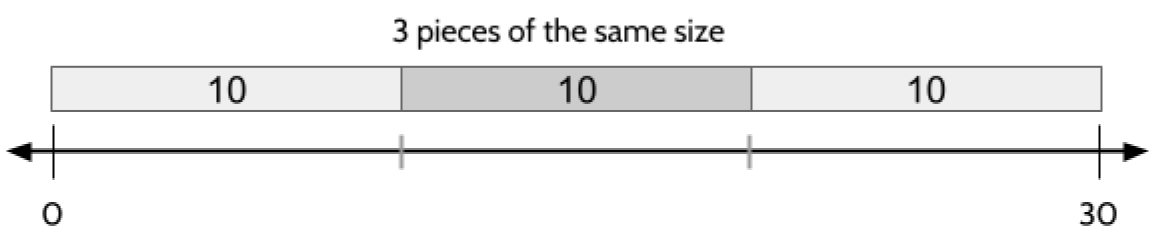
Plot 10 on this number line.

We can use the strategy of splitting evenly to find the right position for the number 10 on this number line. The first step is to measure the distance between 0 and 30.



Since we want to plot the number 10 on the number line, we can ask ourselves, “How many pieces 10 units long will fit in a distance of 30?”

If we split 30 into three equal pieces, each piece will be 10 units long.



Finally, we plot 10 on the number line.

Practice splitting evenly as a strategy to plot the points below.

52) Plot 25 on the number line below.

53) Plot 2 on the number line below.

54) Plot 30 on the number line below.

55) Plot 60 on the number line below.

56) Plot 400 on the number line below.

57) Plot $0.20 on the number line below.

You might use a few different strategies to plot the points below.

58) Plot 2 on the number line below.

59) Plot 60 on the number line below.

60) Plot 75 on the number line below.

61) Plot 15 on the number line below.

62) Plot 9 on the number line below.

63) Plot -8 on the number line below.

Expanding the Number Line - Answer Key

Comparing Values: Equal, Greater Than, and Less Than

1) $4 \times 8 > 100 \div 4$
   32 is greater than 25.

2) $20 < 17 + (2 \times 4)$
   20 is less than 17 added to 2 multiplied by 8.
   (This is one way to write the inequality as a sentence. You might have another way.)

3) $2.5 \times 3 = 1.5 \times 5$
   2.5 multiplied by 3 is equal to 1.5 multiplied by 5.
   (This is one way to write the inequality as a sentence. You might have another way.)

4) $1.2 + 0.3 > 0.9 + 0.08$
   1.2 plus 0.3 is greater than 0.9 added to 0.08.
   (This is one way to write the inequality as a sentence. You might have another way.)

5) $(2 \times 7) + 3 < 2 \times (7+3)$
   2 times 7 added to three is less than 2 multiplied by the sum of 7 and 3.
   (This is one way to write the inequality as a sentence. You might have another way.)

6)

Possible values of x	equation	Is the value a solution to $w > 5$?
4	$4 > 5$	No. 4 is not bigger than 5.
5	$5 > 5$	No. 5 is not bigger than 5.
6	$6 > 5$	Yes. 6 is bigger than 5.
7	$7 > 5$	Yes. 7 is bigger than 5.
8	$8 > 5$	Yes. 8 is bigger than 5.
9	$9 > 5$	Yes. 9 is bigger than 5.
10	$10 > 5$	Yes. 10 is bigger than 5.
15	$15 > 5$	Yes. 15 is bigger than 5.

7) 1250

8) A, B, C

9) B

10) C, D

11) C

12) B, C, D, E, F

13) B, C, D

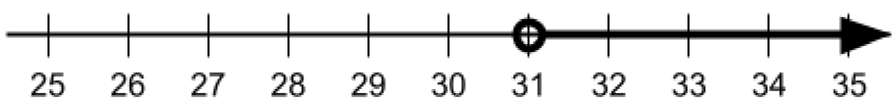
14) $w \geq 3$

15) A, B

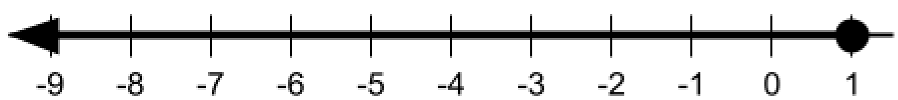
16) C

17) D

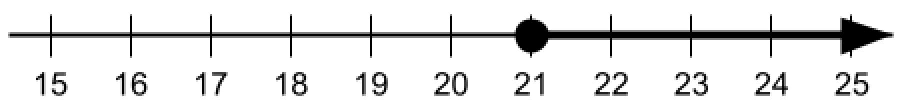
18)



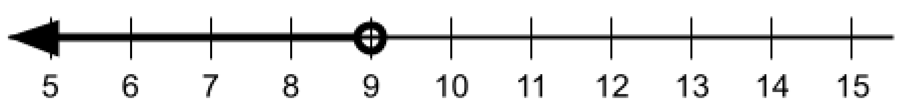
$p > 31$



$q \leq 1$



$r \geq 15 + 6$



$s < 15 - 6$

19) B

20) A

21) B

22) A

23) The inequality $x \geq 10$ could represent many different situations. Here are a few possibilities:

- Kids have to be at least 10 years old to swim in the deep end of a pool.
- Everyone will have to pay at least $10 to buy food for the party.
- You have to get at least 10 out of 15 questions right to pass a quiz.

What other examples can you think of?

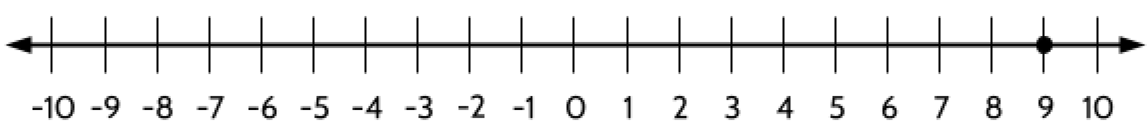
Signed Numbers (Positive and Negative Numbers)

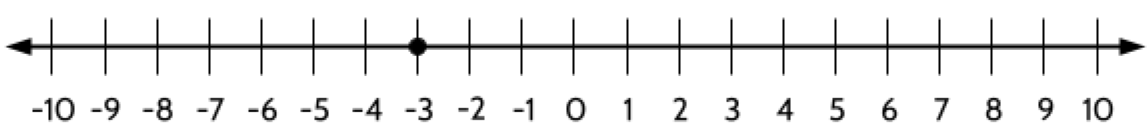
24) The arrow to the left means that numbers smaller than 0 are also included on the number line even if they aren't shown.

25) The answers are below.

	> or <				> or <	
+9	>	+5		-10	<	6
+6	<	+7		0	>	-3
-4	<	+3		0	<	3
-5	>	-7		1	>	-15

Finding the Opposite

26) 

27) 

28) -8

29) 3

The Meaning of – , the Minus Sign

30) 10 - 3

31) -15

32) -0.5

33) -10

34) 7

35) 0
Explanation: The opposite of 0 is 0. On a number line, an opposite is a number the same distance from 0, but on the other side of 0. The number 0 in the middle. It can't be reflected to the other side of the number line because it is where the reflection starts. (You might have another explanation.)

36) 5
Explanation: -(-5) means "the opposite of the opposite of 5." The opposite of 5 is -5. The opposite of -5 is 5. So, the opposite of the opposite of 5 is 5. As an equation:
$-(-5) = 5$

37) -12

38) 12

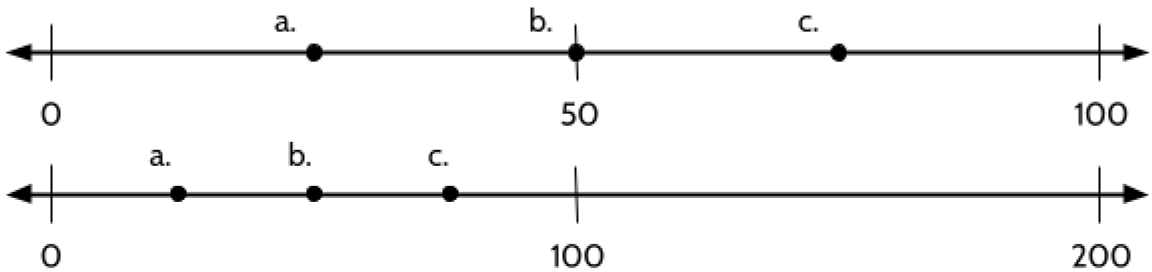
39) -2
3

40) False. Look at the second example in the previous question. In that example, -*x* is not negative.
True. Look at the second example in the previous question.

41)

-3	This means "negative 3," which is the opposite of +3.
7 - 4	This means 4 subtracted from 7.
- (-3)	This means the opposite of the opposite of 3.
4 - 7	This means 7 subtracted from 4.
3 - *x*	This means *x* subtracted from 3.
-(-*x*)	This means the opposite of the opposite of *x*.
-(4 + 2)	This means the opposite of the sum of 4 and 2. We add 4 and 2 to get 6, then find the opposite of 6, which is -6.

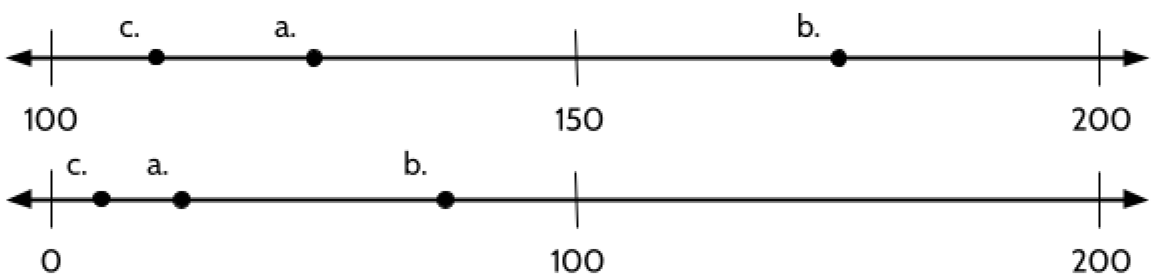
Plotting the Same Points on Two Number Lines



42)

43) You may notice different things. Here are a few things we noticed:

- The numbers are in the same order in both number lines (a, b, c)
- There is less space between the numbers on the second number line
- a) is halfway between 0 and b) on both number lines.



44)

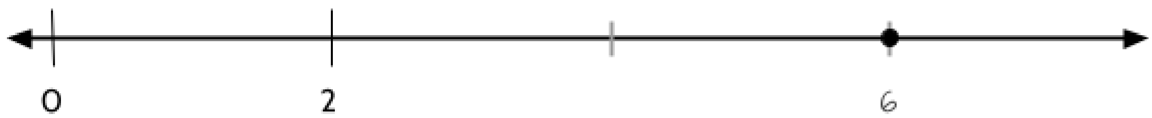
45) You may notice different things. Here are a few things we noticed:

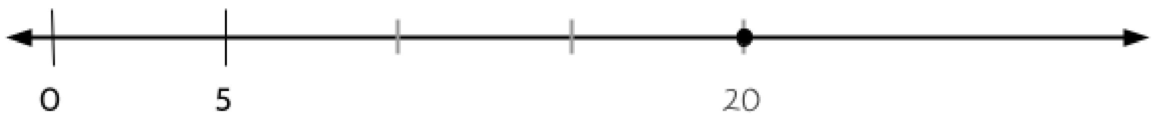
- The numbers are in the same order in both number lines (a, b, c)
- There is less space between the numbers on the second number line
- On both number lines, the distance between a) and 0 is the same as the distance between b) and 100

Making Copies and Splitting Equally

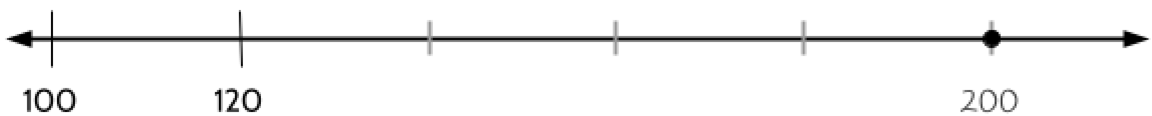
You might use different strategies than the examples below. There is no one right way to locate a point on a number line. There are many different ways to do it. These are just some examples.

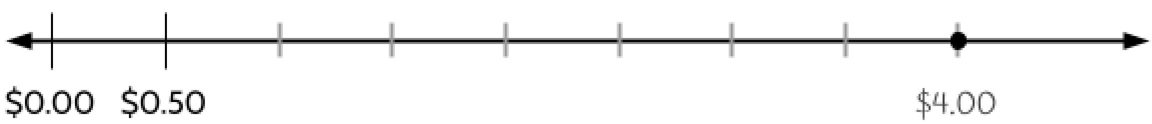
46)

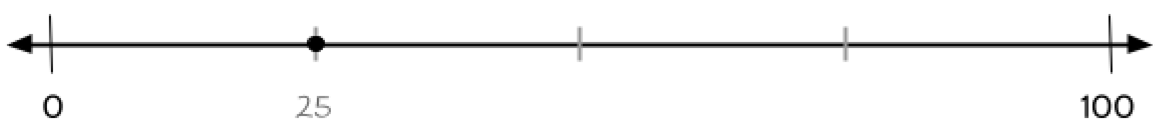
47) 

48) 

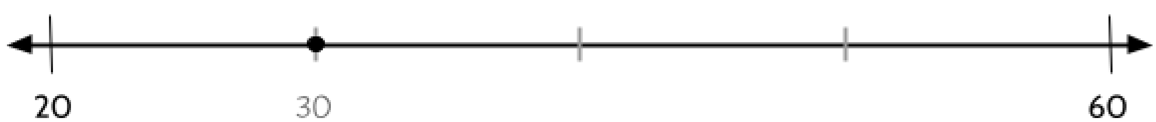
49) 

50) 

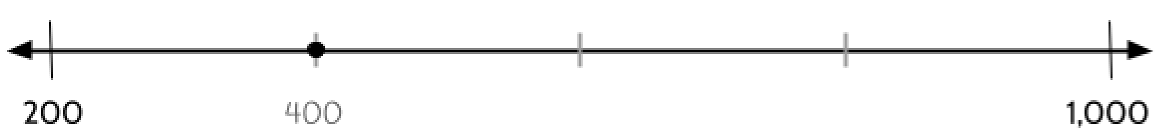
51) 

52) 

53) 

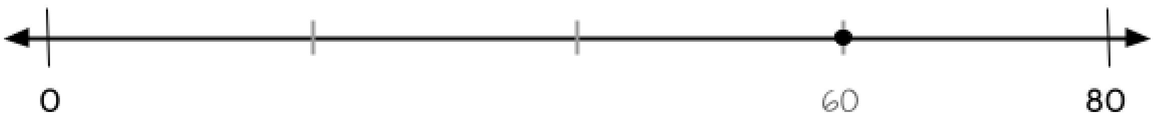
54) 

55) 

56) 

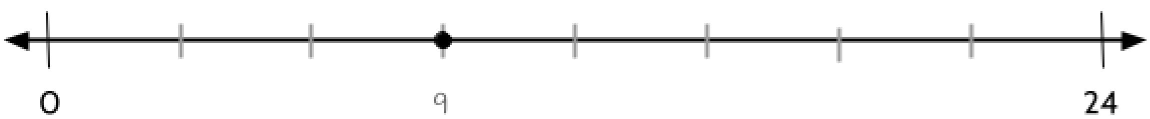
57) 

58) 

59) 

60) 

61) 

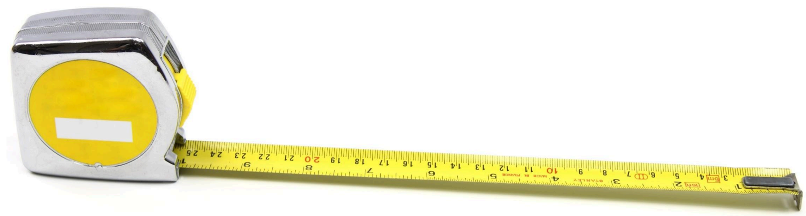
62) 

63) 

Measurement with a Number Line

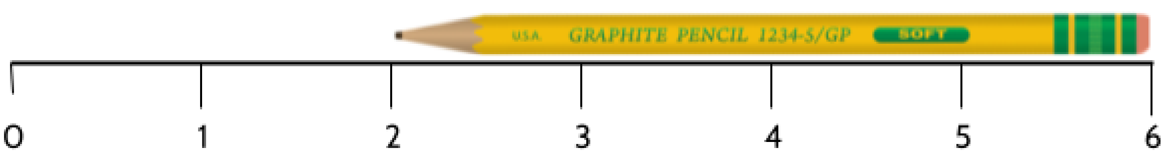
Magnitude and Absolute Value (or the Distance from 0)

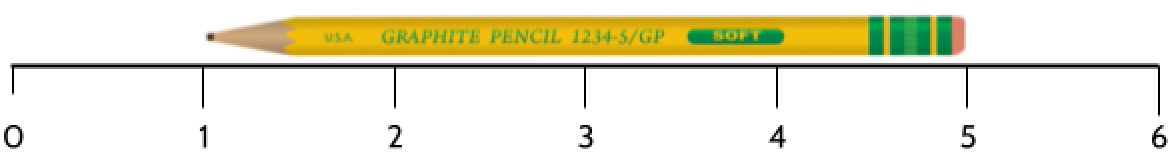
We can use number lines to understand mathematical ideas that are important for a high school diploma and success in college. In this section, we will use number lines to help us understand the idea of ***absolute value***.

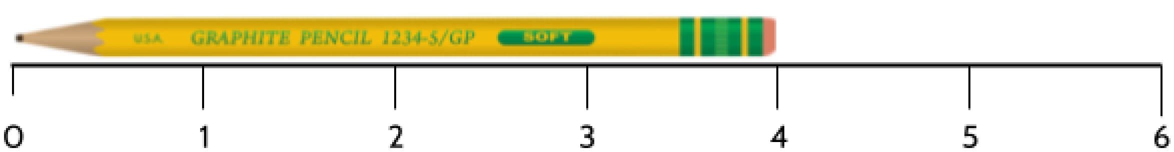


A measuring tape is an example of a number line commonly used in everyday life. We use measuring tapes and rulers to measure distance. The diagrams below show a pencil measured with a ruler.

1) Which of the following measurements show that the pencil is 4 inches long?

A.


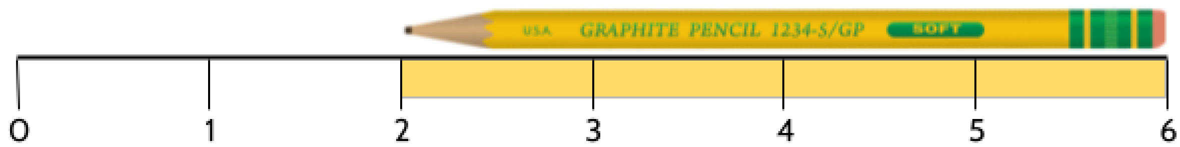
B.


C.


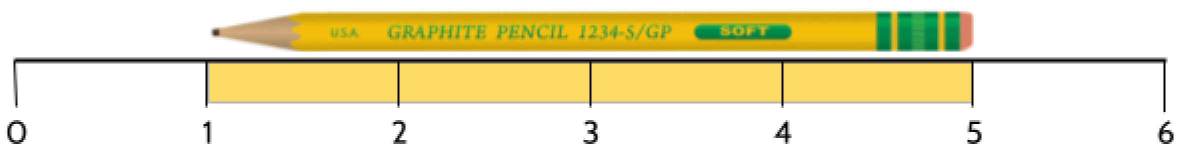
2) Explain your answer here.

Actually, all of these ways of using a ruler could show that the pencil is 4 inches long. If we shade the inches below the pencil, we can see that there are 4 shaded inches in each diagram.

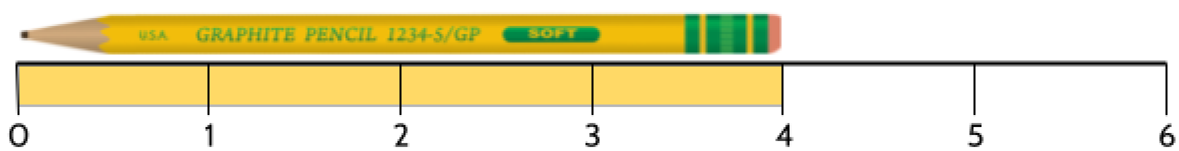
In this diagram, the pencil starts at 2 inches and ends at 6 inches. This is a distance of 4 inches, as shown by the shaded sections of the ruler.



In this diagram, the pencil starts at 1 inch and ends at 5 inches. This is a distance of 4 inches.

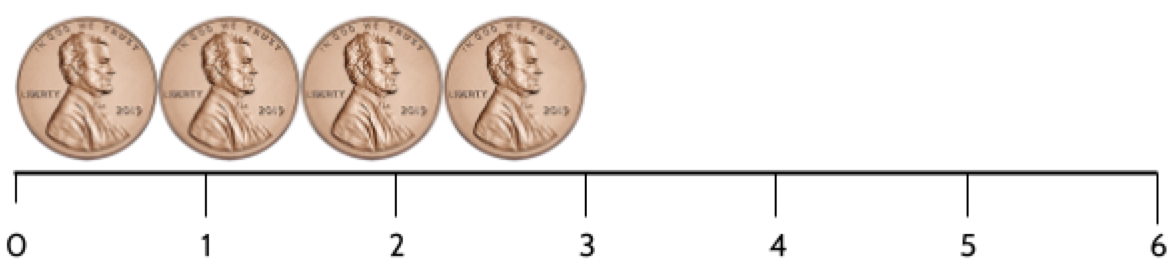


In this diagram, the pencil starts at the left side of the ruler, which is 0 inches, and ends at 4 inches. This is a distance of 4 inches.



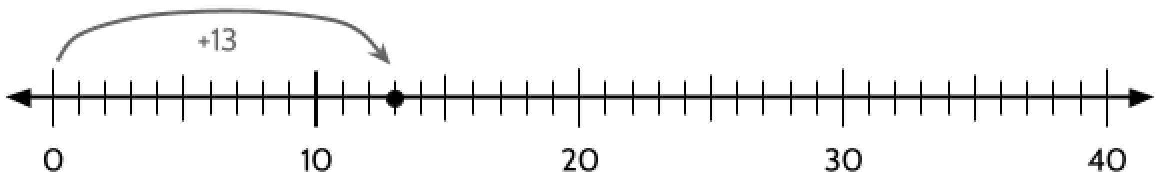
When learning to read a ruler, it is usually recommended that you start at 0, as shown in the diagram above. If you start at 0, you can read the ruler to find the measurement.

3) What is the length of the line of pennies below?



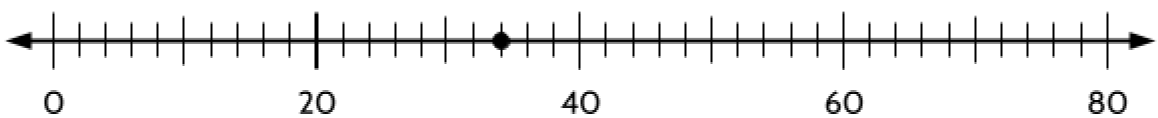
4) If you had 12 pennies lined up in a row, what would be the distance from the beginning to the end of the line of pennies?

A number's distance from 0 on a number line is its ***magnitude***. For example, the magnitude of the point on the number line is 13.

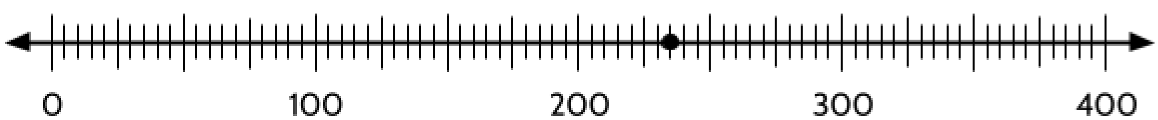


Let's practice using the word, magnitude.

5) What is the magnitude of the point on the number line below? (In other words, how far is the point from 0?)



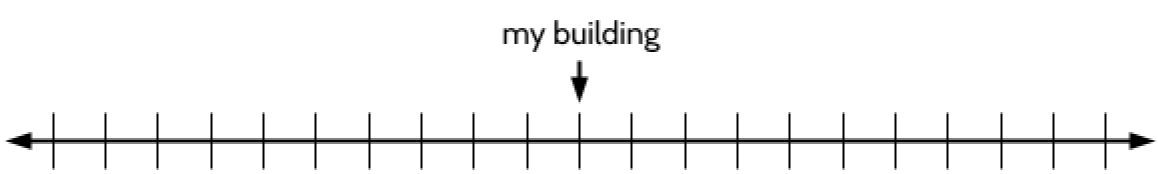
6) What is the magnitude of the point on the number line?



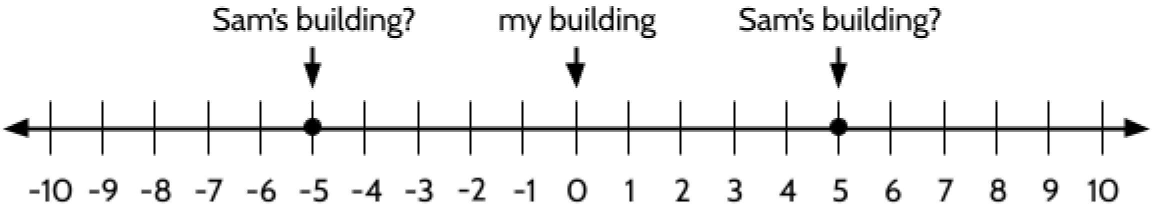
7) What is the magnitude of the number 23?

To understand magnitude, I want you to think about the apartment building where I live in Brooklyn (shown in the diagram below). I live in the middle of the block. My friend, Sam, lives 5 buildings away from me.

8) Mark all possible points where you think Sam might live.

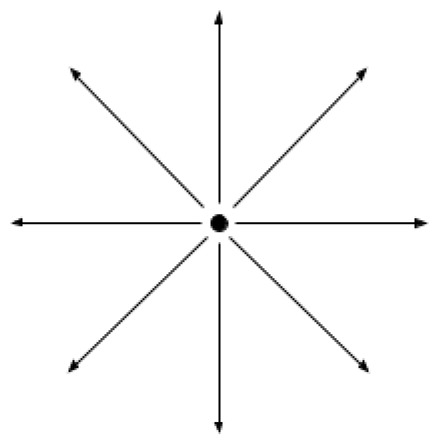


The distance from where I live to Sam's building is 5 buildings. It could be 5 buildings to the right. It could also be 5 buildings to the left. Here is the block on a number line.



If my building is located at 0, Sam's building could be located at 5 or -5. The distance from 0 to 5 is 5. The distance from 0 to -5 is also 5.

Imagine that you are in the middle of a large field, maybe a soccer or football field. You could turn and walk 20 feet in any direction. The distance would be 20 feet. The distance you walk is never negative, no matter the direction.

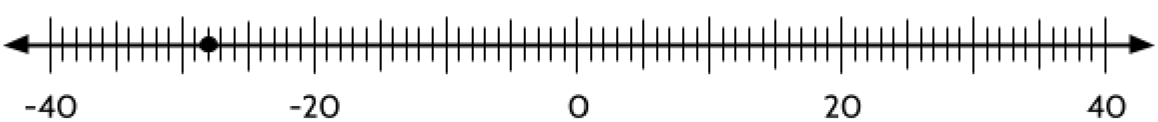


9) What is the distance between -8 and 0?

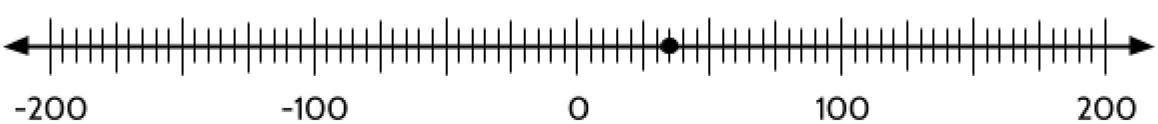
A. 8

B. -8

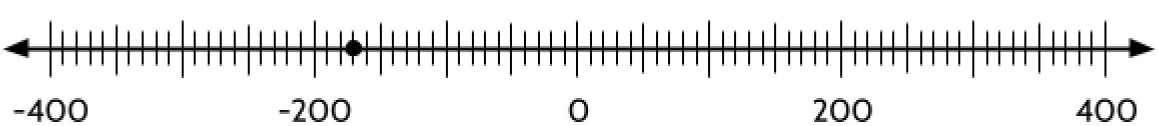
10) What is the magnitude of each of the following points? Remember that magnitude is the distance of a number from 0.



_____



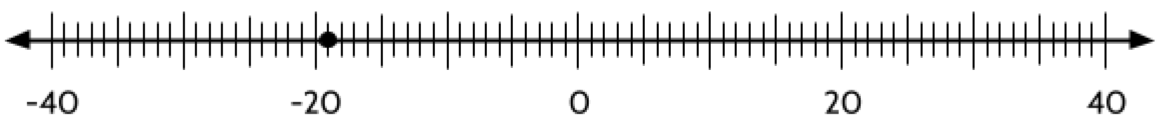
_____



_____

Absolute value is another way to say magnitude, or distance from 0. These two vertical lines | | are used to mean absolute value. When you see these two vertical lines around a number or expression, the symbols mean that you should find the absolute value of everything inside. For example, $|-5|$ means "the absolute value of -5."

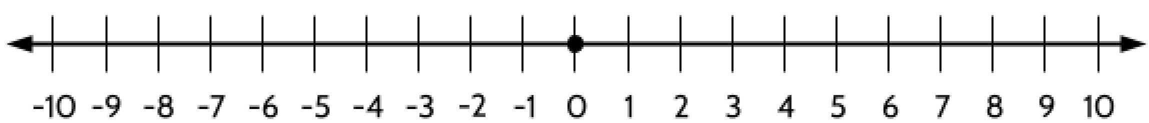
11) What is the absolute value of the point on the number line below?



12) What is the absolute value of -30?

13) What is $|-10|$ equal to?

14) What is the magnitude of the point on the number line below?



15) What is $|0|$?

On the last page, you read that distance is never a negative number. Does that mean that distance is always a positive number?

The answer is, distance is almost always a positive number. The one exception is when the distance is 0. The number 0 is not positive or negative.

16) Which of the following statements is true?

   A. The absolute value of a number is always positive.

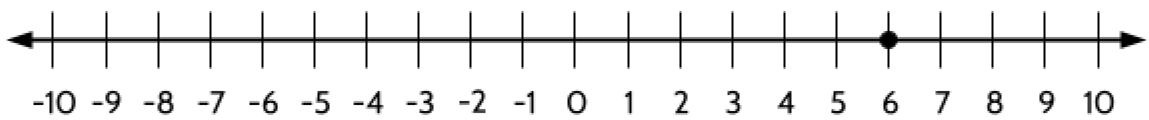
   B. The absolute value of a number is always negative.

   C. The absolute value of a number can be positive or negative.

   D. The absolute value of a number is either positive or equal to 0.

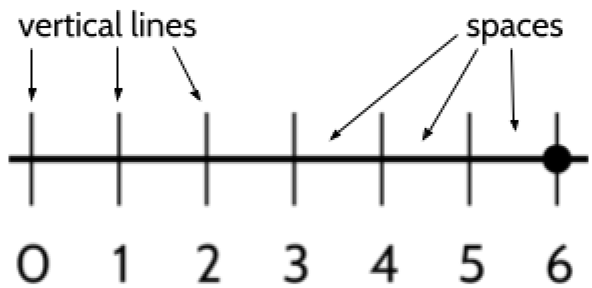
Explain the Mistake - Magnitude

Jazae and Helen are talking about the magnitude of the point on the number line below.



Jazae says the magnitude is 7, because she counts 7 vertical lines above the numbers 0 through 6.

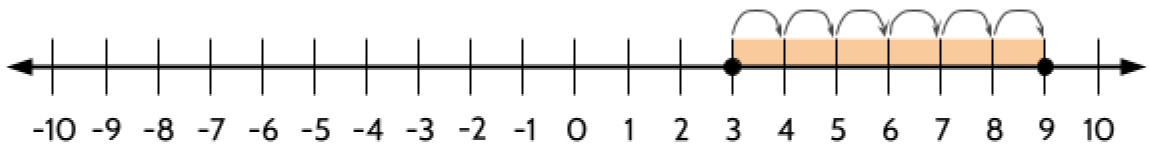
Helen says the magnitude is 6 since there are 6 spaces between the numbers 0 and 6.



17) Who is correct? Why?

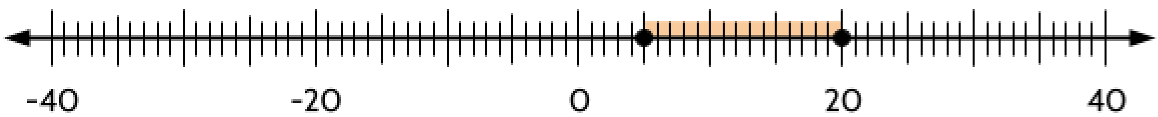
Distance Between Points on a Number Line

In this exercise, you will practice finding the distance between two points on a number line. As an example, let's look at the difference between the numbers 3 and 9. We added jumps and shaded the units between 3 and 9 to help us see the distance.

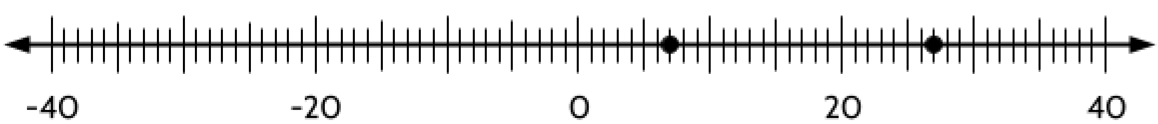


If we count the shaded units above, we can see that the distance between 3 and 9 is 6 units.

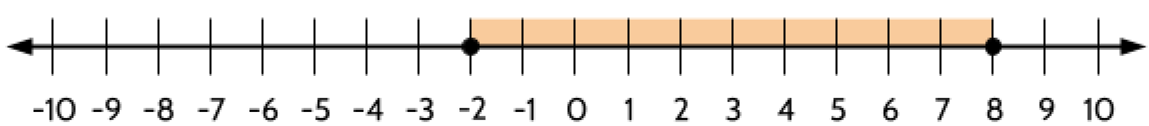
18) What is the distance between the two points on the number line below? ______



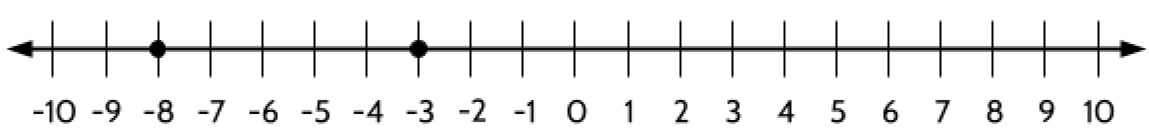
19) What is the distance between the two points on the number line below? ______



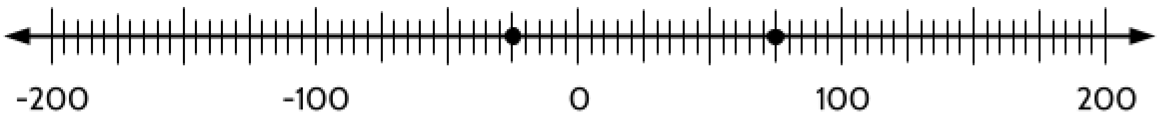
20) What is the distance between the two points on the number line below? ______



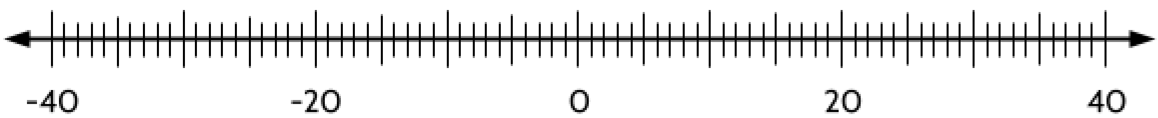
21) What is the distance between the two points on the number line below? ______



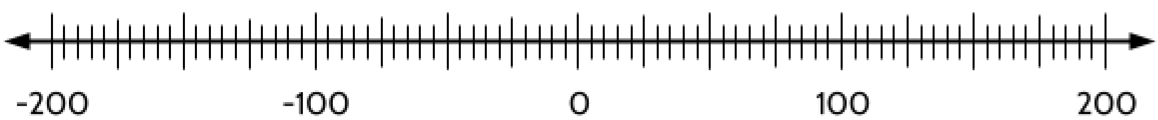
22) What is the distance between the two points on the number line below? ______



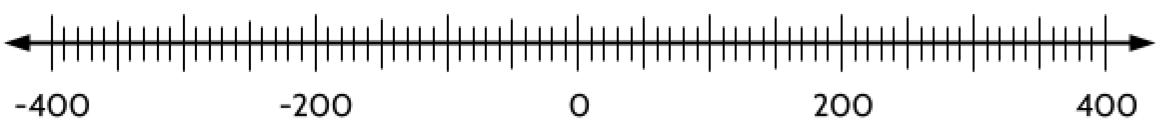
23) What is the distance between -8 and -22? ______ Use the number line below.



24) What is the distance between -45 and 15? ______ Use the number line below.



25) What is the distance between 70 and 350? ______ Use the number line below.



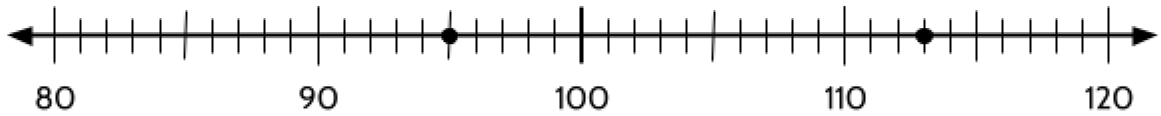
26) What is the distance between -10 and 25? ______

The ***difference*** between two numbers is similar to ***distance***. The difference between 9 and 3 is 6. The difference between 20 and 5 is 15. Subtraction is often used to find the distance or difference between two numbers.

There is one important distinction between *distance* and *difference*:

- The *distance* between two numbers cannot be negative. For example, the distance between 9 and 3 is 6, and the distance between 3 and 9 is also 6. It's the same distance.
- The *difference* between two numbers can be positive or negative. It depends on which number comes first in the subtraction. Example:
  - 9 - 3 = 6 (positive result)
  - 3 - 9 = -6 (negative result)

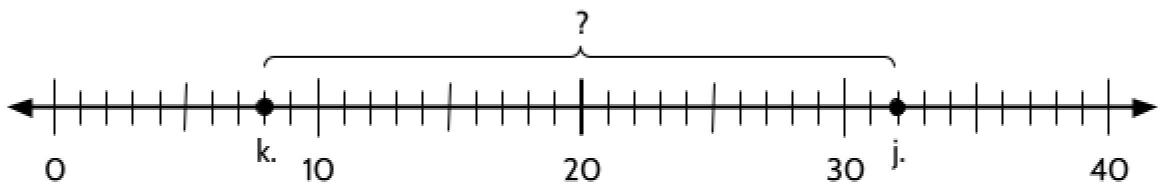
27) Look at the number line below to complete the next two questions.



The <u>distance</u> between the two numbers above is ______.

The <u>difference</u> between the two numbers above could be ______ or ______.

28) Jacob and his son, Khalil, put their ages on a number line.



How old is Jacob (j)? ______

How old is Khalil (k)? ______

What is the difference in their ages? ______

29) There is a 35 year age difference between Paulina and her son. If Paulina's son is 14 years old, how old is Paulina?

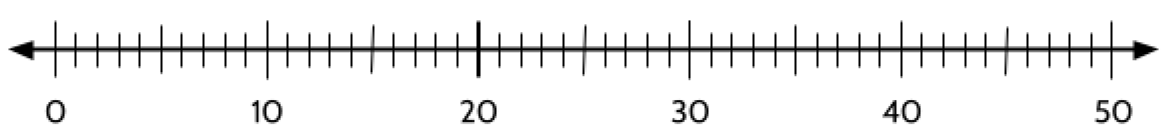
A. 21

B. 35

C. 41

D. 49

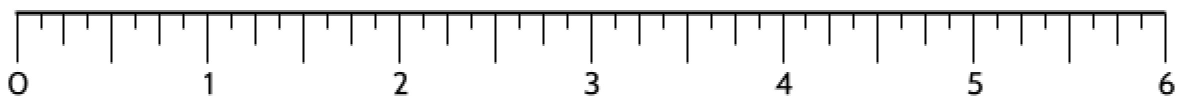
30) Use the number line below to explain your answer to the previous question.



Fractions of an Inch

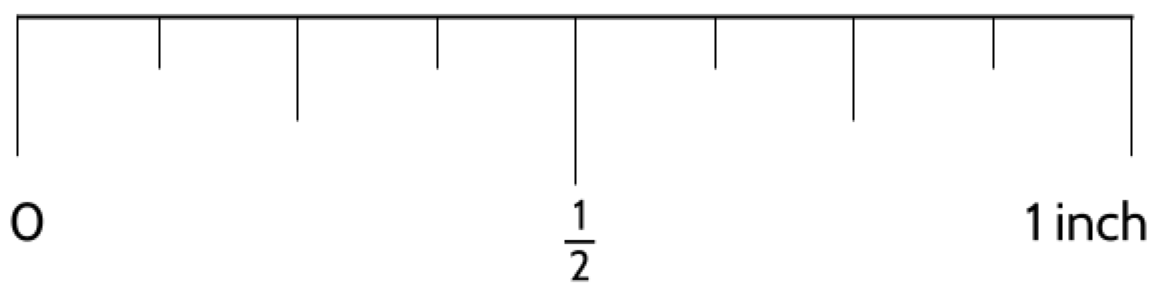
So far we have focused on whole numbers on the number line. There are also values between the whole numbers. In this section, we will focus on number lines with tick marks between whole numbers. Good examples to start with are rulers and measuring tapes, which use tick marks between whole numbers for more precise measurements.

Here are the first six inches on a standard U.S. ruler.



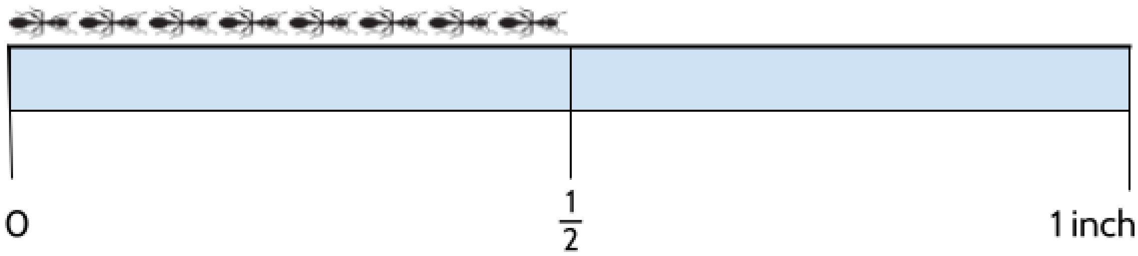
Let's look at the marks between 0 and 1 inch on a standard U.S. ruler.

31) Add fractions that you already know. The fraction $\frac{1}{2}$ has been added for you.



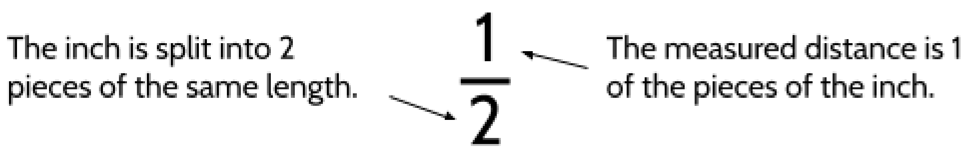
Let's look at a ½ inch. It is called a "half inch" because it is half the distance from 0 to 1. But why is it written in this way: $\frac{1}{2}$? We will try to answer that together.

In the diagram below, we are measuring the length of a line of ants. The inch and the ants have been magnified. The distance between 0 and 1 is split into two equal shaded pieces.



The 2 on the bottom of the fraction $\frac{1}{2}$ means that the inch is split into 2 pieces of the same size.

The 1 on the top of the fraction $\frac{1}{2}$ means we are measuring 1 of the 2 pieces.

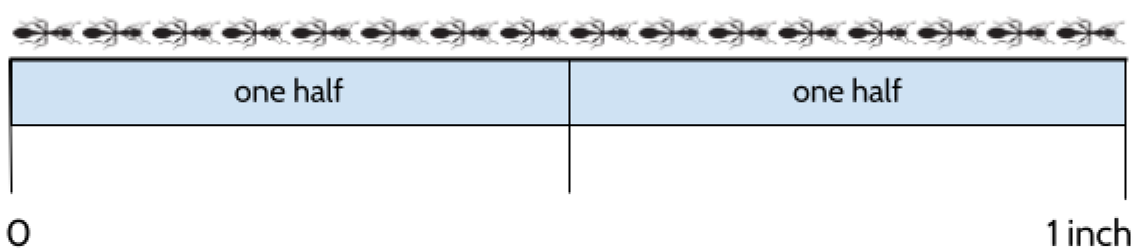


When we make a fraction, the first step is to split the whole into pieces of equal size. The bottom of the fraction tells us how many pieces. It is also the name of the fraction. For example, if there are two pieces, they are called "halves" (plural of "half").

32) Based on the diagram below, what does the fraction $\frac{2}{2}$ (two halves) mean?

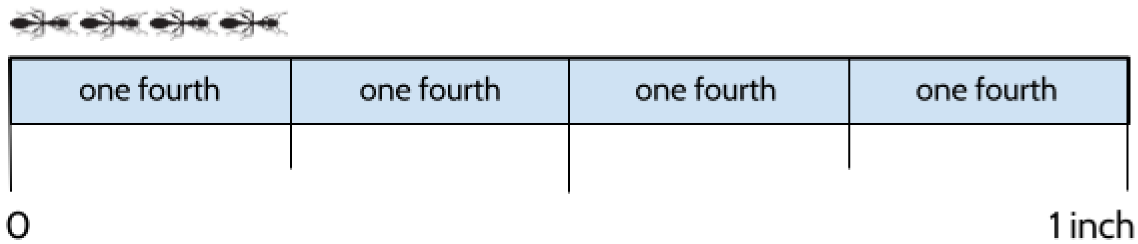
The inch is split into ______ pieces of the same length.

The measured distance is ______ of the pieces of the inch.



33) How long is the line of ants above, in inches? ______

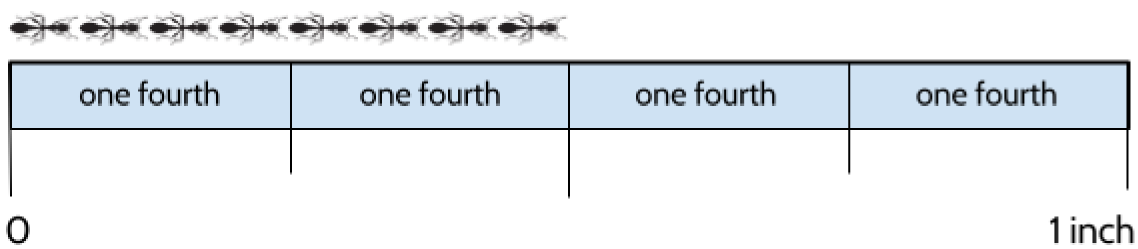
What if we split an inch into 4 equal pieces? How long is this line of ants?



34) Fill in the blanks, based on the diagram above.

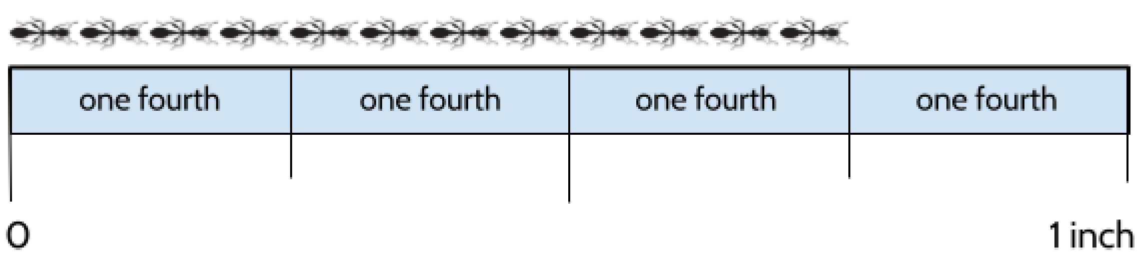
The inch is split into ______ pieces of the same length. → $\frac{1}{4}$ ← The measured distance is ______ of the pieces of the inch.

Once an inch is split into four pieces, you can make measurements using fourths (also known as quarters). The line of ants below measures "two fourths" of an inch long.



"Two fourths" can be written as $\frac{2}{4}$ inches. (You can also write the fraction as 2/4.)

35) How long is the line of ants below, in inches? Fill in the blanks below.



The inch is split into ______ pieces of the same length. → $\frac{\square}{4}$ ← The measured distance is ______ of the pieces of the inch.

36) What does “one fourth” + “one fourth” + “one fourth” equal?

37) Which of these is a correct way to say $\frac{2}{4}$?

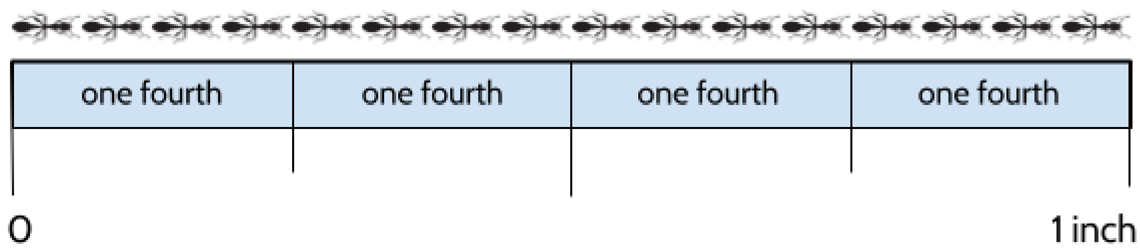
A. “two slash four”

B. “four halves”

C. “two fourths”

D. “two fours”

38) How long is the line of ants in the diagram below? Choose all that apply.



A. $\frac{1}{4}$ inch

B. $\frac{4}{4}$ inch

C. $\frac{4}{1}$ inches

D. 1 inch

39) What does “one fourth” + “one fourth” + “one fourth” + “one fourth” + “one fourth” equal? Choose all that apply.

A. $1\frac{1}{4}$

B. $1\frac{1}{2}$

C. $\frac{4}{5}$

D. $\frac{5}{4}$

40) Which of these values is equal to 12 fourths? Choose all that apply.

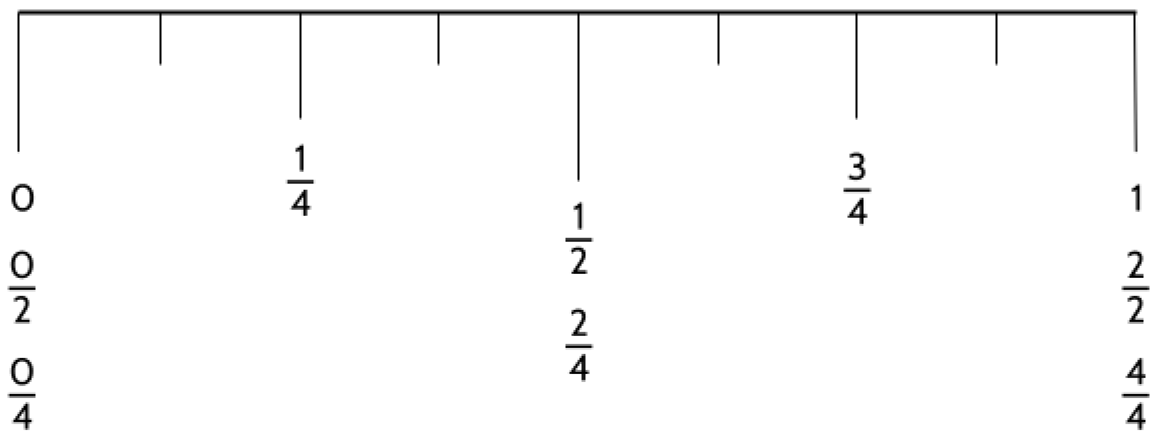
A. $1\frac{1}{2}$

B. $\frac{12}{4}$

C. 3

D. $\frac{4}{12}$

41) The following diagram shows halves and fourths labeled between 0 and 1 inch.



What do you notice?

What questions do you have?

At the beginning of this activity, we split one inch into two pieces in order to measure $\frac{1}{2}$ and $\frac{2}{2}$. Then, we split one inch into four pieces to make $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, and $\frac{4}{4}$.

Now, let's break an inch into eight pieces.

42) If you divide 1 into 8 pieces, what size is each piece?

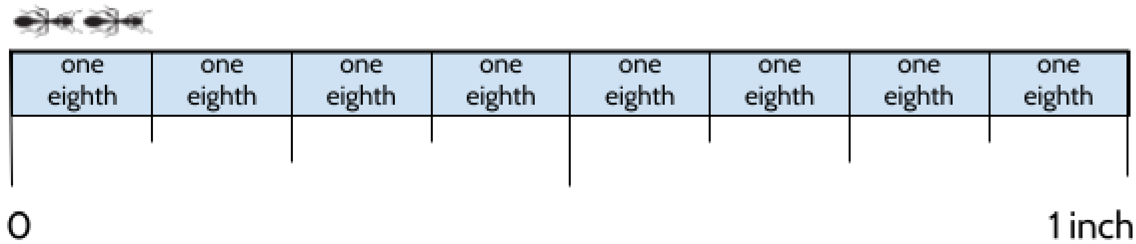
A. $\frac{1}{8}$

B. 8

C. $\frac{8}{1}$

D. 18

43) The diagram below shows one inch split into eight pieces. How long is the line of ants?



A. $\frac{1}{2}$

B. $\frac{1}{4}$

C. $\frac{1}{6}$

D. $\frac{1}{8}$

44) If a fraction breaks an inch into eight equal pieces, what is the name of the fraction?

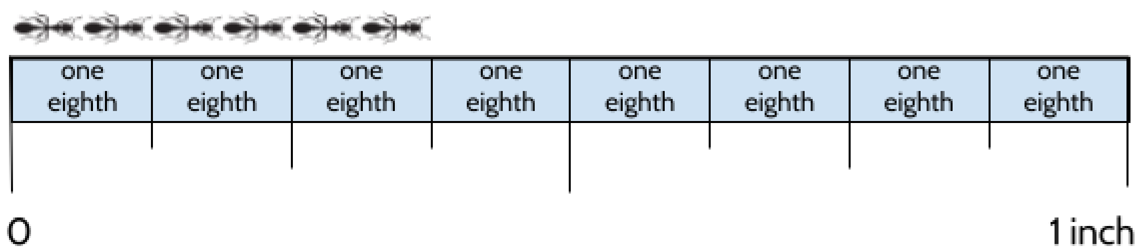
A. halves

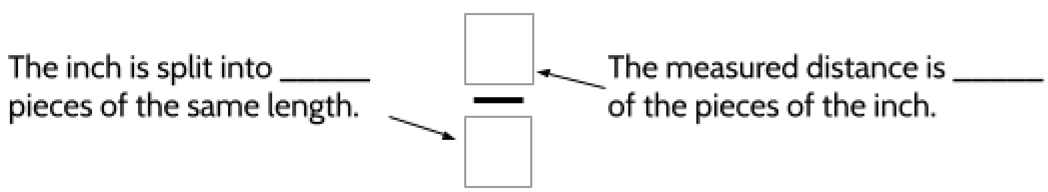
B. thirds

C. fourths (quarters)

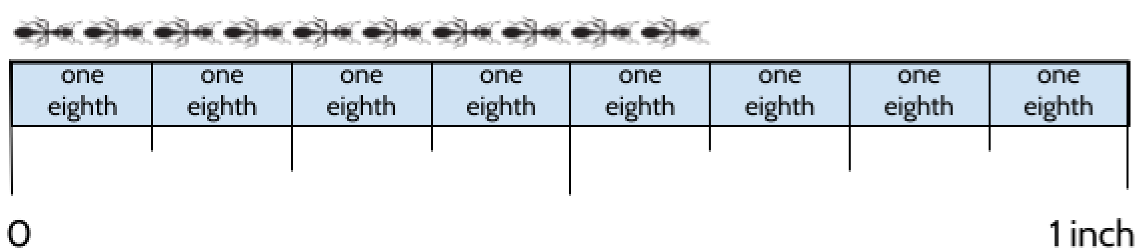
D. eighths

45) How long is the line of ants in the diagram below? Fill in the blanks below.

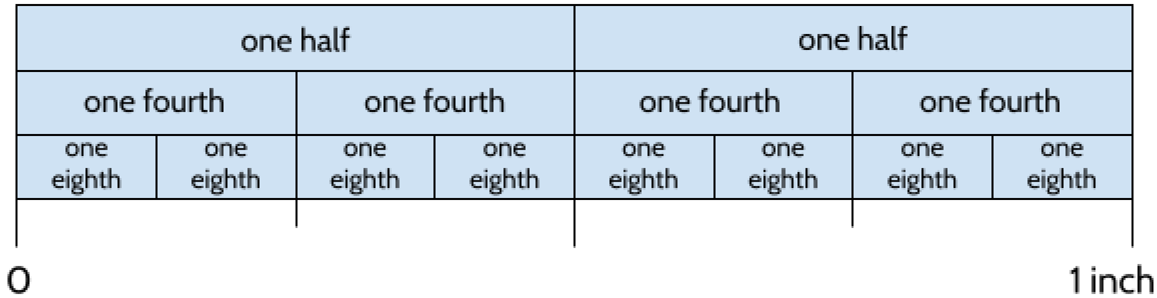




46) How long is the line of ants below, in inches? ______________________________



Look at the diagram and answer the questions below.



47) How many fourths are in one half?

48) How many eighths are in one fourth?

49) How many eighths are in one half?

50) Fill in the blanks.

two eighths = _______ fourth

four eighths = _______ fourths = _______ half

six eighths = _______ fourths

eight eighths = _______ fourths = _______ halves

51) True or False?

$\frac{1}{2} = \frac{2}{4}$	___T___	$\frac{3}{4} = \frac{6}{8}$	_______
$\frac{1}{2} = \frac{4}{8}$	_______	$\frac{1}{2} = \frac{3}{4}$	_______
$\frac{5}{8} = \frac{3}{4}$	_______	$\frac{10}{8} = \frac{5}{4}$	_______

52) FIll in the blanks.

five fourths = ______________ halves

three eighths = ______________ fourths

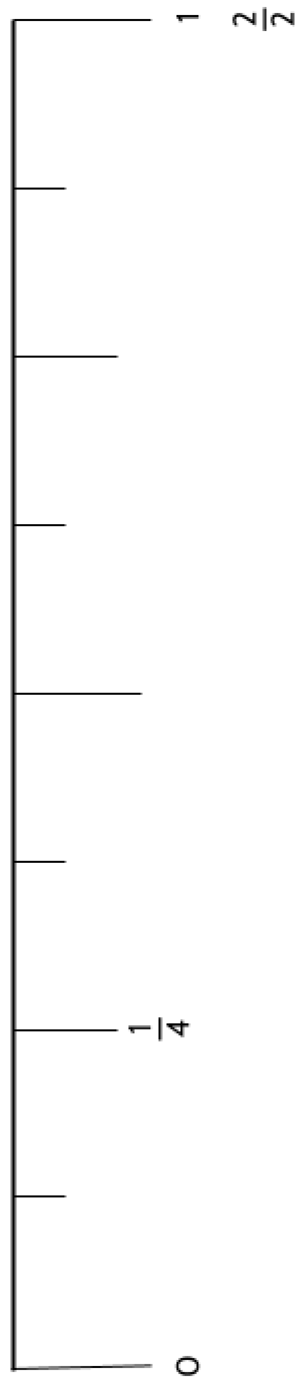
six fourths = ______________ halves

five halves = ______________ eighths

Inch Challenge

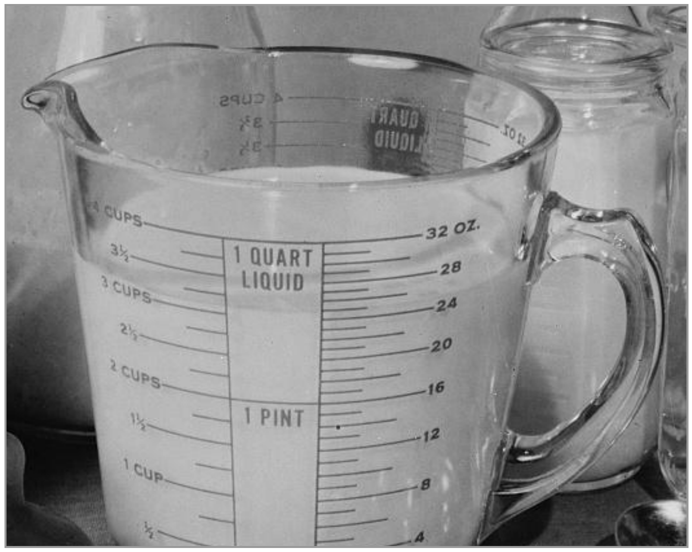
53) Can you add all the missing measurements between 0 and 1 inch?

A few measurements have been added for you.



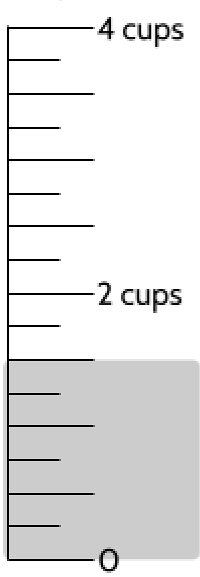
Measuring Cups

We looked at the fluid ounce measurements on this measuring cup earlier. The cup also shows measurements in cups, which is a way of measuring the volume of ingredients.



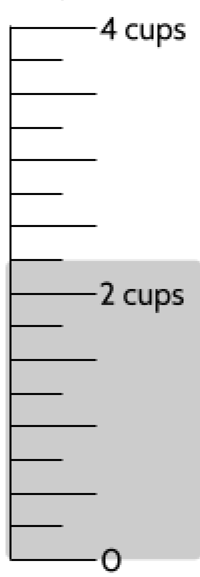
The number lines below show different amounts of milk in the measuring cup, measured in cups. How much milk is measured in each number line?

54)



________ cups

55)



________ cups

56)

________ cups

57)

________ cups

Making Copies and Splitting Equally (with Fractions)

In this exercise, you will practice making copies and splitting equally to locate fractions on the number line. We will practice making copies first.

Making Copies

As an example, try the following task:

Plot 1 on this number line.

We can use the strategy of making copies to find 1 on this number line. The first step is to measure the distance between 0 and $\frac{1}{3}$. The distance is $\frac{1}{3}$ units.

Next, we can ask ourselves, "How many copies of this distance do we need to make 1?"

We need $\frac{1}{3} \times\ 3$ to make 1, so we can make 3 copies of the distance between 0 and $\frac{1}{3}$.

Finally, we plot 1 on the number line.

Practice making copies as a strategy to plot the points below.

58) Plot 1 on the number line below.

59) Plot 1 on the number line below.

60) Plot 1 on the number line below.

61) Plot $\frac{5}{3}$ on the number line below.

62) Plot $\frac{7}{4}$ on the number line below.

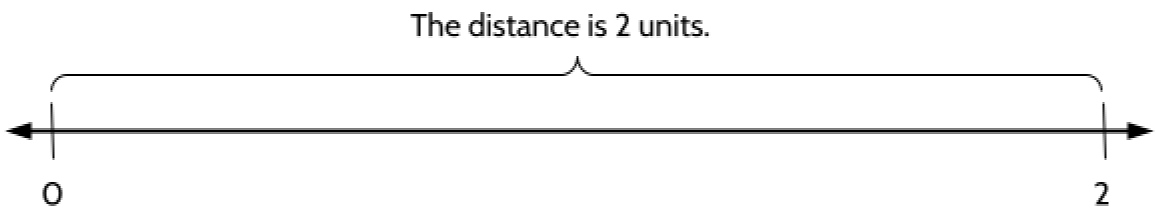
63) Plot $\frac{9}{8}$ on the number line below.

Splitting Evenly

As an example, try the following task:

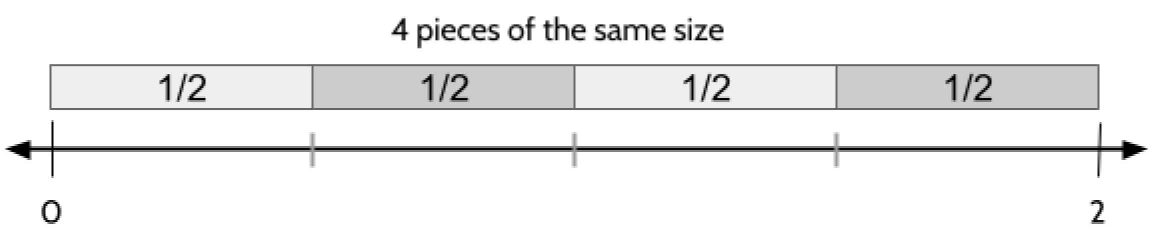
Plot $\frac{1}{2}$ on this number line.

We can use the strategy of splitting evenly to find the right position for the number $\frac{1}{2}$ on this number line. The first step is to measure the distance between 0 and 2.

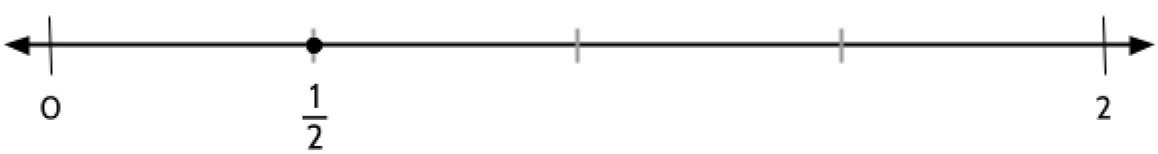


Since we want to plot the number $\frac{1}{2}$ on the number line, we can ask ourselves, "How many pieces $\frac{1}{2}$ units long will fit in a distance of 2?"

If we split 2 into four equal pieces, each piece will be $\frac{1}{2}$ units long.



Finally, we plot $\frac{1}{2}$ on the number line.



Practice splitting equally as a strategy to plot the points below.

64) Plot $\frac{1}{3}$ on the number line below.

65) Plot $\frac{1}{3}$ on the number line below.

66) Plot $\frac{1}{4}$ on the number line below.

67) Plot $\frac{1}{8}$ on the number line below.

68) Plot $\frac{1}{5}$ on the number line below.

69) Plot $\frac{1}{4}$ on the number line below.

You might use a few different strategies to plot the points below.

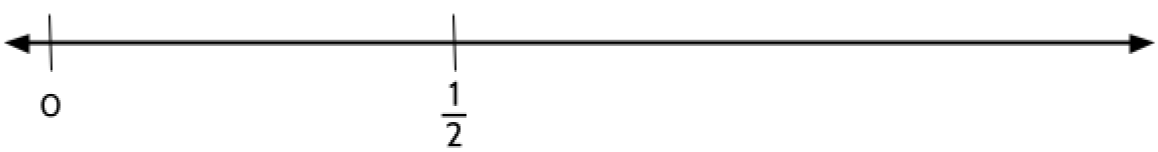
70) Plot $\frac{3}{5}$ on the number line below.

71) Plot $\frac{4}{3}$ on the number line below.

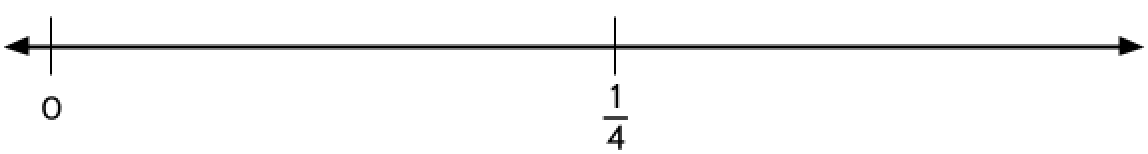
72) Plot 1 on the number line below.

73) Plot $\frac{1}{2}$ on the number line below.

74) Plot $\frac{1}{3}$ on the number line below.



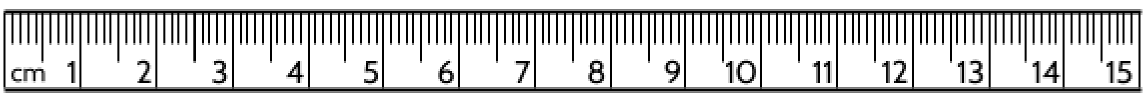
75) Plot $\frac{3}{8}$ on the number line below.



Fractions of a Centimeter

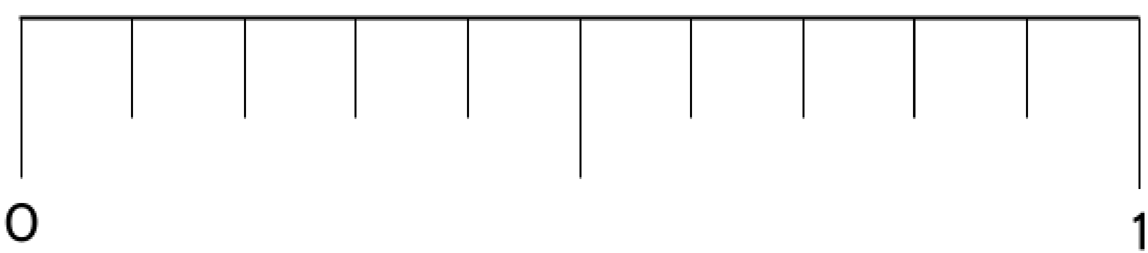
In this section, we will practice using marks on a ***metric*** ruler for precise measurements. A metric ruler uses centimeters, which are part of the metric system of measurement, used in most countries in the world. In the United States, the metric system is especially used in science and the medical field.

Here are the first 15 centimeters on a metric ruler.



We will start by looking at the marks between 0 and 1 centimeters. The diagram below shows a centimeter magnified so that we can see the marks between 0 and 1 easily.

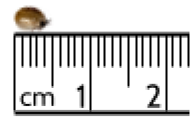
76) Add fractions of a centimeter that you already know.



A few things about centimeters:

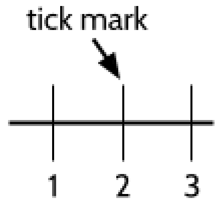
- Centimeter is often written as *cm*.
- There are 2.54 cm in an inch
- There are about 30 cm in a foot.
- There are 100 cm in a meter (*Centimeter* means "hundredth of a meter.")

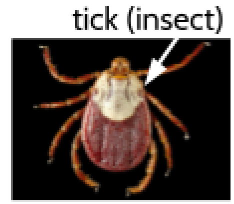
Fractions of a centimeter are useful when we want to measure something very small. For example, here is an image of a tick measured on a metric ruler.

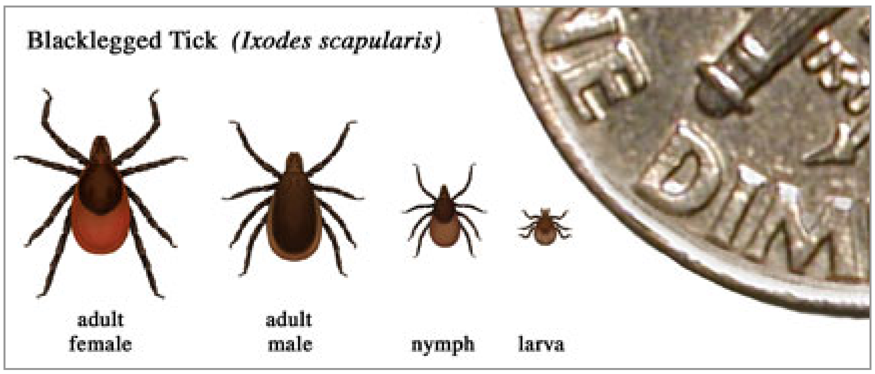


We have been using the word *tick* to mean the vertical mark between increments on a number line. *Tick* also refers to an insect which feeds on the blood of animals.

Scientists study ticks because many of them carry bacteria that cause Lyme disease in humans and pets, usually dogs.[1]







Ticks have four life stages:

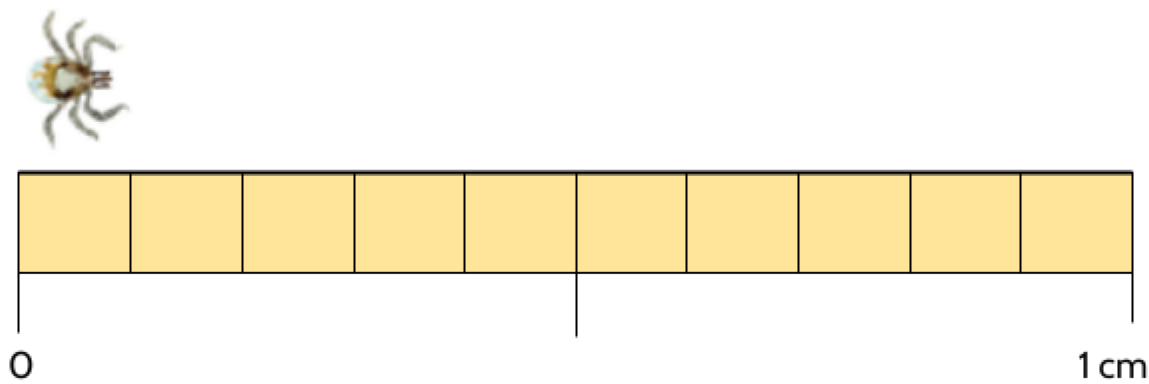
Stage 1: the egg stage
Stage 2: the larval stage
Stage 3: the nymph stage
Stage 4: the adult stage

As babies, ticks are tiny. As they get older, they grow larger. The tick in the ruler measurement above is really big for a tick. After they eat, ticks swell up and become larger.

77) Have you ever had an experience with a tick? Tell us about it.

[1] For information about ticks, including how to safely remove them from a person or animal, visit https://www.cdc.gov/ticks.

A scientist is collecting ticks at different stages of life and measuring their length, using a metric ruler. The diagram below shows a magnification of a tick larva.



78) Fill in the blanks, based on the diagram above.

$\frac{1}{10}$

1 ← The measured distance is ______ of the pieces of the centimeter.

10 ← The centimeter is split into ______ pieces of the same length.

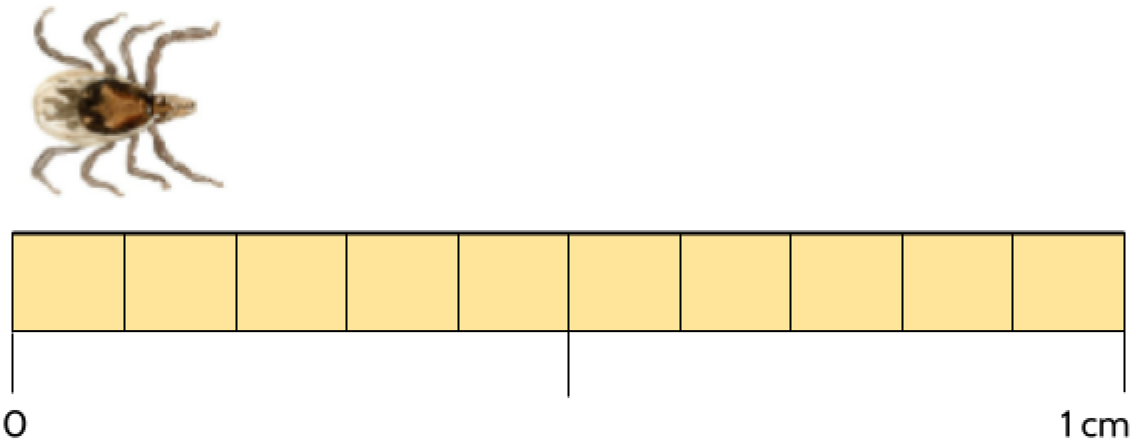
Once a centimeter is split into ten pieces, we can make measurements using tenths. The tick larva above measures "one tenth of a centimeter." This can be written as $\frac{1}{10}$ cm or 0.1 cm.

79) Complete the table.

Fraction	Decimal	Words
$\frac{1}{10}$	0.1	one tenth
$\frac{2}{10}$	0.2	two tenths
$\frac{3}{10}$		
$\frac{4}{10}$		
$\frac{5}{10}$		

Fraction	Decimal	Words
$\frac{6}{10}$		
$\frac{7}{10}$		
$\frac{8}{10}$		
$\frac{9}{10}$		
$\frac{10}{10}$		

80) The scientist also collected a tick in the nymph stage of life. How long is the nymph tick in centimeters, including the legs?



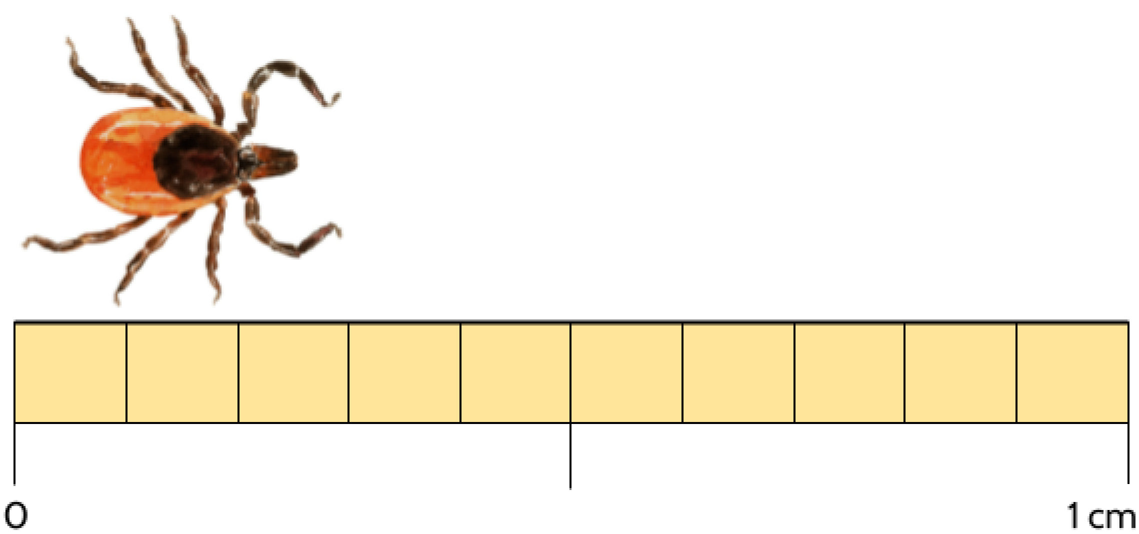
A. $\frac{1}{10}$ cm

B. $\frac{2}{10}$ cm

C. $\frac{1}{8}$ cm

D. $\frac{3}{10}$ cm

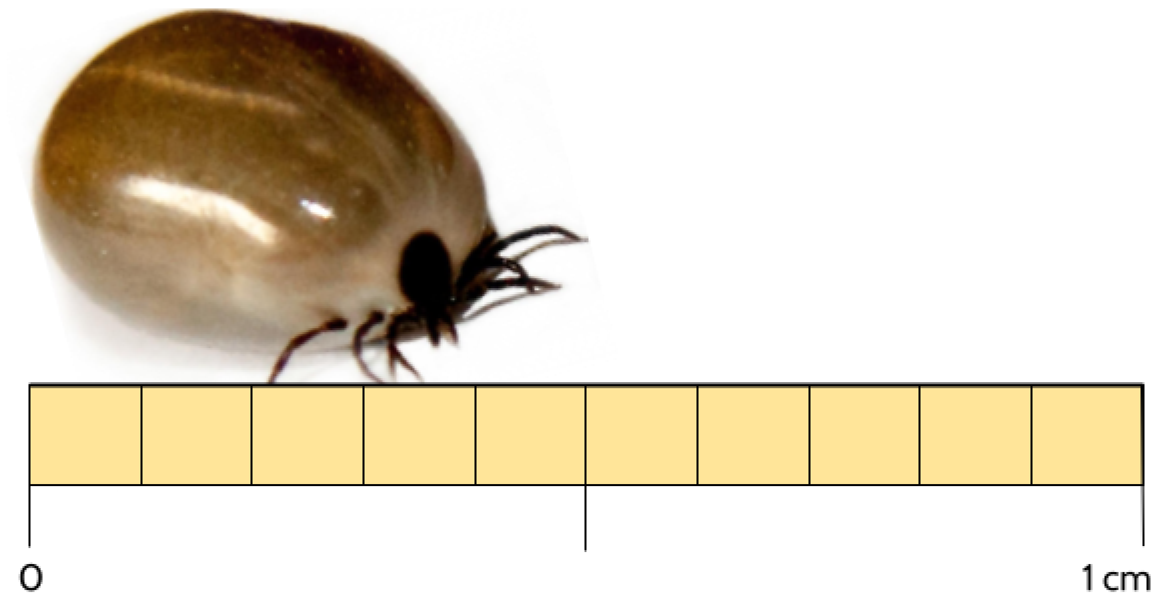
81) The scientist then measured an adult female tick, including the legs.



Write the length of the tick in three different ways:

Fraction: $\frac{\quad}{10}$ cm Decimal: 0.______ cm Words: _______ tenths of a centimeter

82) Finally, the scientist measured the length of an adult tick, just after it had finished feeding.



How long is the tick in the diagram above, including legs? Choose all that apply.

A. $\frac{1}{5}$ cm

B. $\frac{5}{10}$ cm

C. $\frac{1}{2}$ cm

D. $\frac{5}{5}$ cm

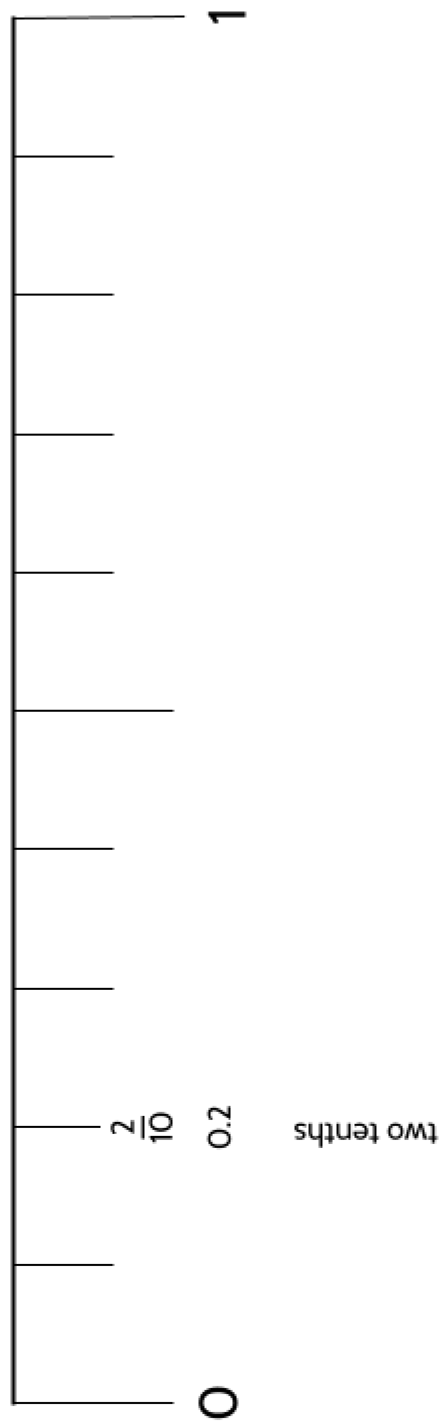
83) Another name for a tenth of a centimeter is a *millimeter*. There are 10 millimeters (mm) in a centimeter. Continue the table.

As a fraction	As a decimal	Number of millimeters
$\frac{1}{10}$ cm	0.1 cm	1 mm
$\frac{2}{10}$ cm	0.2 cm	
$\frac{3}{10}$ cm		
$\frac{4}{10}$ cm		
$\frac{5}{10}$ cm		

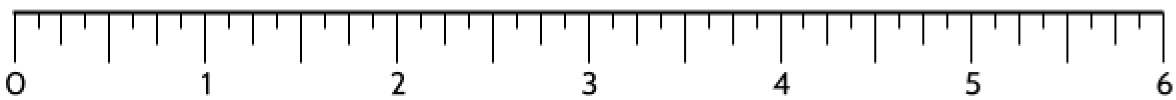
Centimeter Challenge

84) Can you add all the missing fractions and decimals between 0 and 1 centimeter?

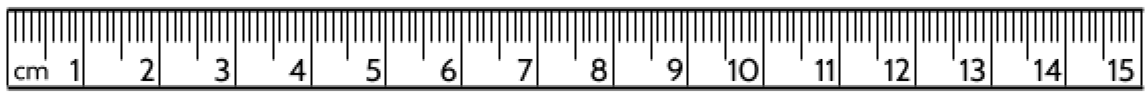
A few measurements have been added for you.



85) Here are the first six inches on a standard U.S. ruler.



Here are the first 15 centimeters on a metric ruler.



Compare the two rulers and tell us what you notice. Use fractions and decimals to explain what you see.

Measuring with a Number Line - Answer Key

Magnitude and Absolute Value (or the Distance from 0)

1) All of these ways of using a ruler could show that the pencil is 4 inches long. The question is explained in the packet.
2) Answers may vary.
3) 3 inches
4) 9 inches
5) 34
6) 235
7) 23
8) The answer is explained in the packet.
9) 8
10) 28, 35, 170
11) 19
12) 30
13) 10
14) 0. The magnitude of 0 is 0, because it is 0 units away from 0.
15) 0. The absolute value of 0 is 0, because it is 0 units away from 0.
16) D

Explain the Mistake - Magnitude

17) Helen is correct. When measuring the distance between numbers on a number line, we should measure spaces between the lines.

Distance Between Points on a Number Line

18) 15

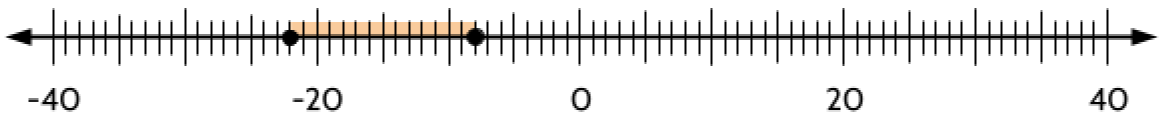
19) 20

20) 10

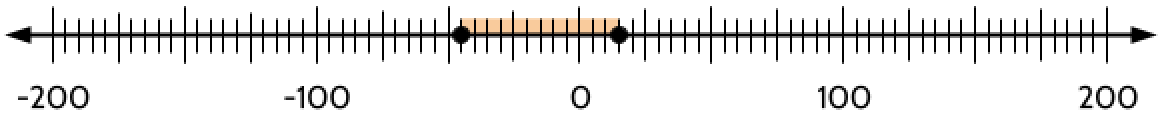
21) 5

22) 100

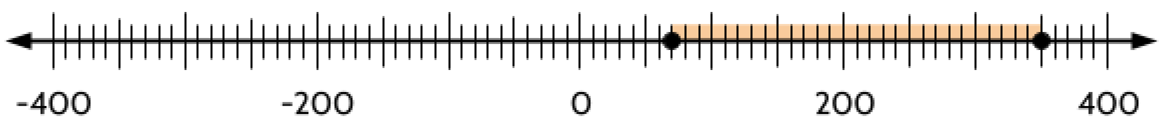
23) 14



24) 60



25) 280



26) 35

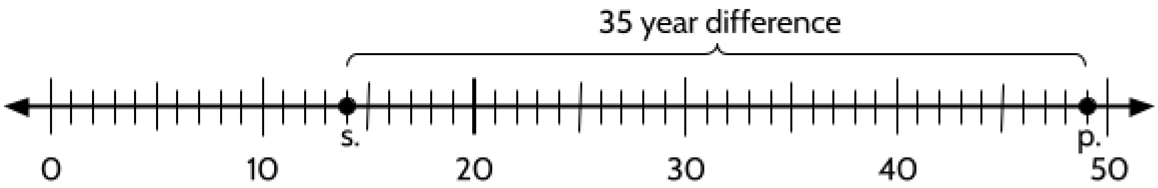
27) <u>18</u>. Distance is never a negative number. It is always positive or 0.

<u>18</u> or <u>-18</u>. The difference depends on which way you go on the number line. If you start at 113 and subtract 95, you will get +18. If you start at 95 and subtract 113, you will get -18.

28) 8
32
24 (or -24)

29) 49

One thing to notice about this problem is that we are given the difference (in ages) in the problem. In the earlier questions, we were given both ages and had to figure out the age difference. In this problem, we know the age difference is 35 years. In fact, we know that Paulina is 35 years older than her son (since she can't be younger than her son). If her son is 14 years old and Paulina is 35 years older, then she must be 49 years old, because 49-35 is 14.

30) 

We encourage you to use number lines to solve word problems. It can be a helpful tool for making sense of mathematical situations.

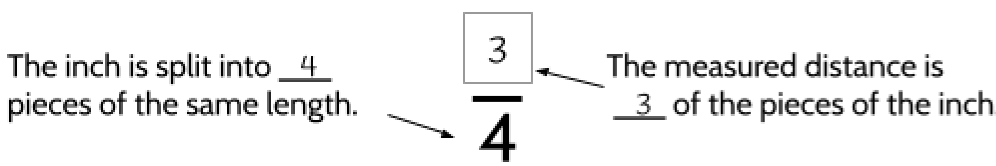
Fractions of an Inch

31) We will label each of these fractions in the next few pages of the packet.

32) The measured distance is 2 of the pieces of the inch.
The inch is split into 2 pieces of the same length.

33) 1 inch or 2/2 inches. These two measurements are equal.

34) The measured distance is 1 of the pieces of the inch.
The inch is split into 4 pieces of the same length.

35) 

36) "three fourths" or 3/4

37) C

38) B, D

39) A, D

40) B, C

41) There are many things you might notice. Here are some things we notice:

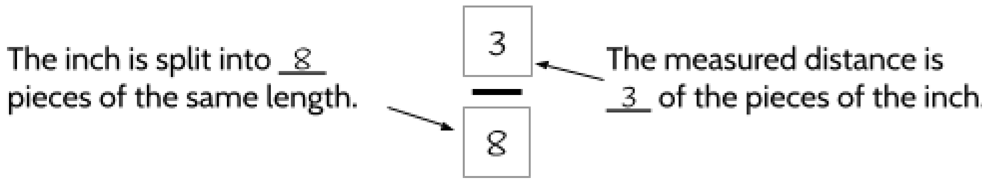
- 1/2 and 2/4 inches are the same length
- 2/2 and 4/4 are both equal to 1
- There are some lines that we haven't labeled yet

What questions do you have about this diagram?

42) A

43) D

44) D

45) 

46) five eighths, 5/8 (Both answers are correct)

47) 2

48) 2

49) 4

50) two eighths = one fourth
four eighths = two fourths = one half
six eighths = three fourths
eight eighths = four fourths = two halves

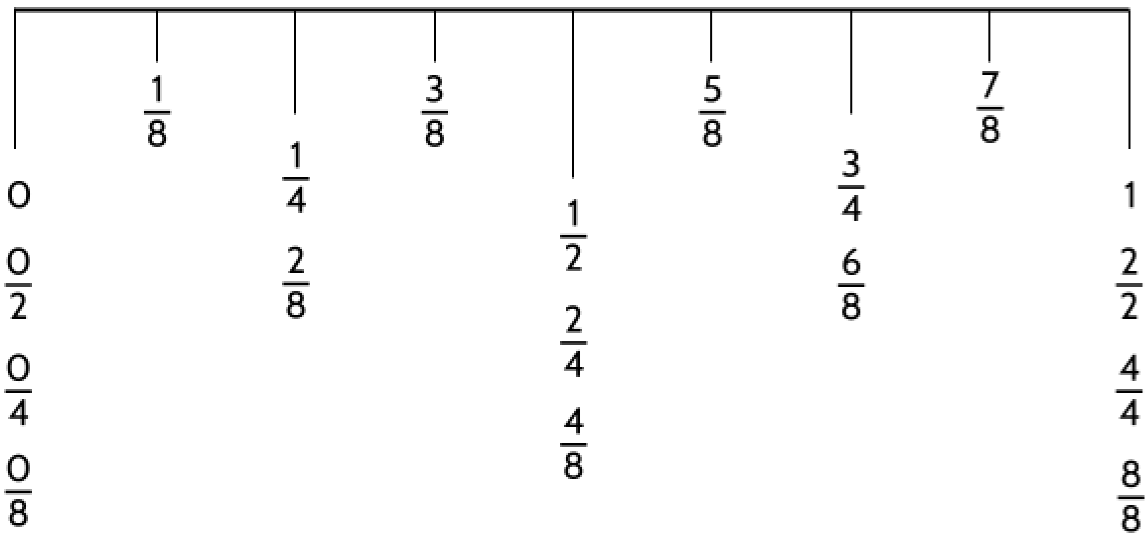
51) Answers are below.

$\frac{1}{2} = \frac{2}{4}$	T	$\frac{3}{4} = \frac{6}{8}$	T
$\frac{1}{2} = \frac{4}{8}$	T	$\frac{1}{2} = \frac{3}{4}$	F
$\frac{5}{8} = \frac{3}{4}$	F	$\frac{10}{8} = \frac{5}{4}$	T

52) five fourths = two and a half halves
six fourths = three halves
three eighths = one and a half fourths
five halves = twenty eighths

Inch Challenge

53)



Measuring Cups

54) 1 ½ cups

55) 2 ¼ cups

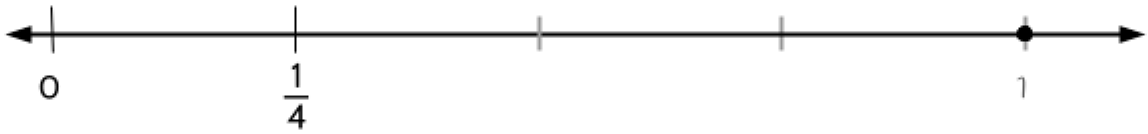
56) ¾ cups

57) 3 ⅛ cups

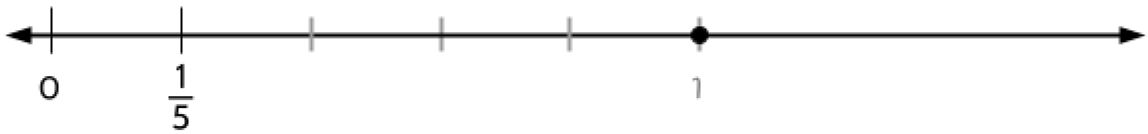
Making Copies and Splitting Equally (with Fractions)

You might use different strategies than the examples below. There is no one right way to locate a point on a number line. There are many different ways to do it. These are just some examples.

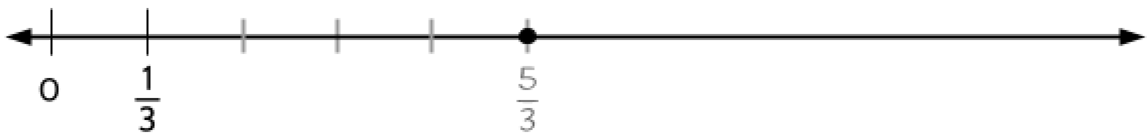
58)



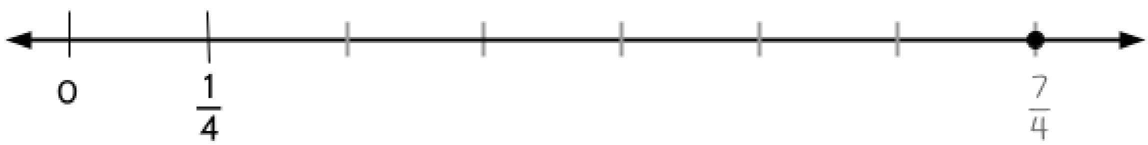
59)



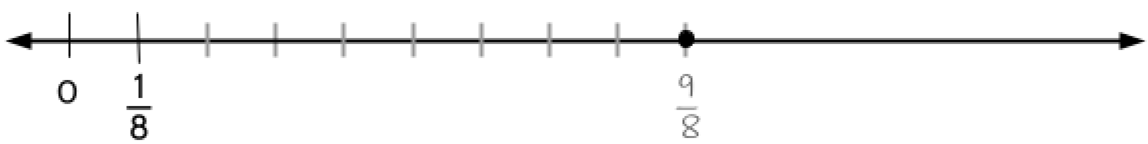
60)



61)

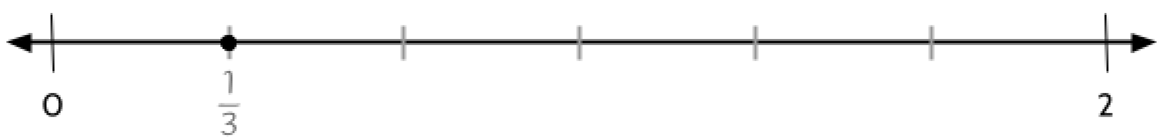


62)

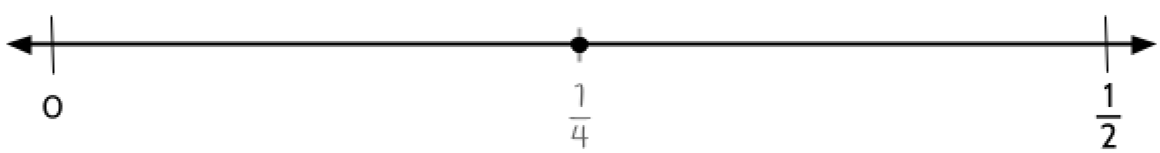


63)

64)



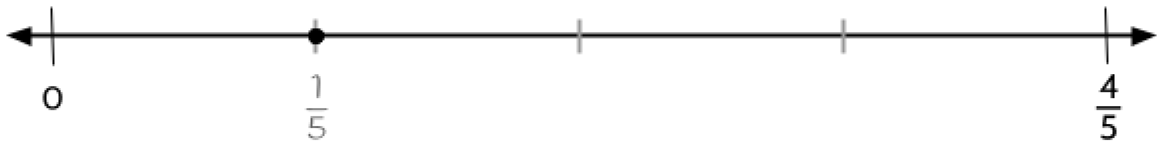
65)



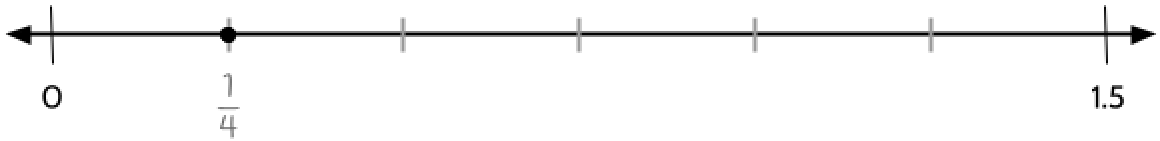
66)

67)

68)

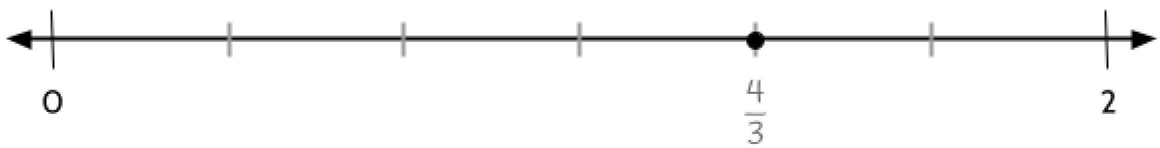


69)

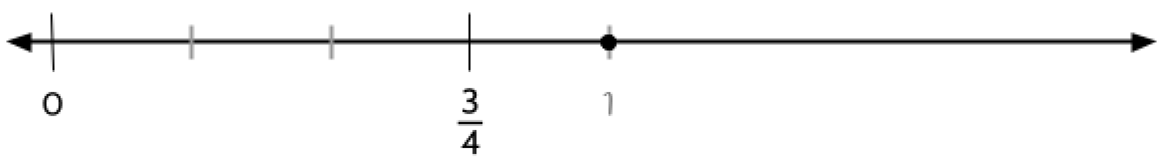


70)

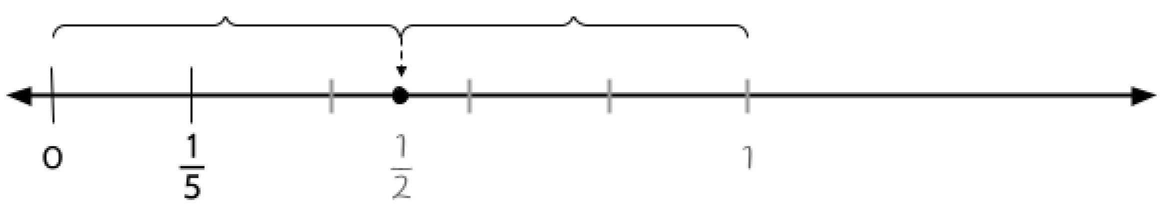
71)



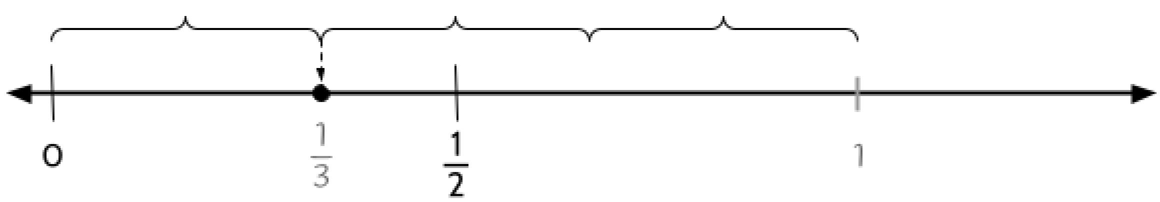
72)



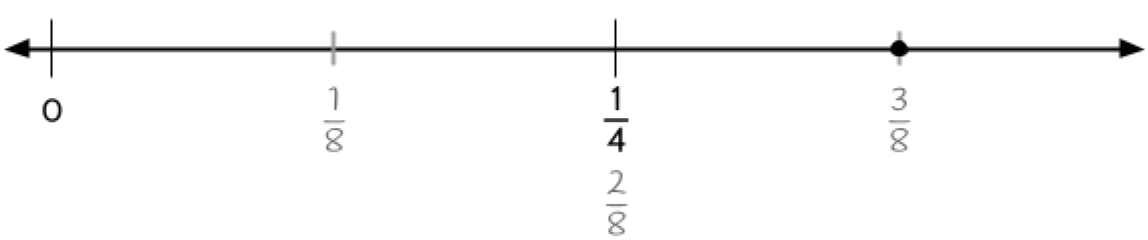
73)



74)



75)



Fractions of a Centimeter

76) We will label each of these fractions in the next few pages of the packet.

77) My family used to have dogs when I was a kid. I lived in Illinois and there were lots of ticks there. We had to check the dogs for ticks every night when they came inside. Sometimes, we found ticks that had swelled up. I remember finding a big one inside our dog's ear.

78) The measured distance is 1 of the pieces of the centimeter.
The centimeter is split into 10 pieces of the same length.

79) The answers are below.

Fraction	Decimal	Words
$\frac{1}{10}$	0.1	one tenth
$\frac{2}{10}$	0.2	two tenths
$\frac{3}{10}$	0.3	three tenths
$\frac{4}{10}$	0.4	four tenths
$\frac{5}{10}$	0.5	five tenths

Fraction	Decimal	Words
$\frac{6}{10}$	0.6	six tenths
$\frac{7}{10}$	0.7	seven tenths
$\frac{8}{10}$	0.8	eight tenths
$\frac{9}{10}$	0.9	nine tenths
$\frac{10}{10}$	1*	ten tenths

*Be careful with this one. 0.10 is not the same as $\frac{10}{10}$. The fraction ten-tenths is equal to 1.

80) B

81) Fraction: $\frac{3}{10}$ cm, Decimal: 0.3 cm, Words: three tenths of a centimeter

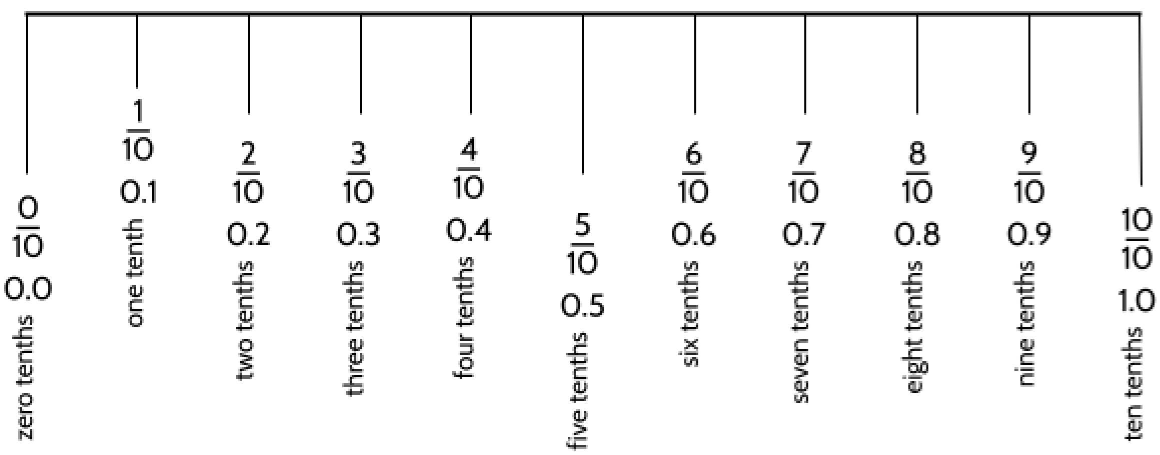
82) B, C

83) The answers are below.

As a fraction	As a decimal	Number of millimeters
$\frac{1}{10}$ cm	0.1 cm	1 mm
$\frac{2}{10}$ cm	0.2 cm	2 mm

$\frac{3}{10}$ cm	0.3 cm	3 mm
$\frac{4}{10}$ cm	0.4 cm	4 mm
$\frac{5}{10}$ cm	0.5 cm	5 mm

Centimeter Challenge



84)

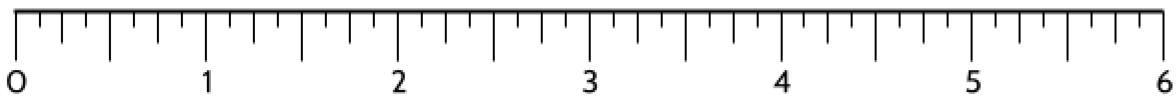
85) There are many things you might notice. Some things we notice:

- There are more lines between the numbers on the metric ruler.
- There are 8 spaces between the numbers on the standard U.S. ruler.
- The standard U.S. ruler shows halves, fourths (quarters), and eighths.
- There are 10 spaces between the numbers on the metric ruler.
- The metric ruler shows halves and tenths.
- A centimeter is smaller than an inch.
- 6 inches is just a bit longer than 15 centimeters.
- There are about 2 ½ centimeters in every inch.

Applying Knowledge of Number Lines

Explain the Mistake - Fractions

Angelina and Bibi are talking about the fractions $1\frac{1}{2}$ and $\frac{12}{8}$ on a ruler.



Angelina says that the two fractions represent the same distance. She says that $1\frac{1}{2}$ and $\frac{12}{8}$ are equal.

Bibi says that $1\frac{1}{2}$ and $\frac{12}{8}$ are different. She says that halves and eighths are different things, so they can't be equal.

1) Who is correct? Why?

Identify 5/8 on a Number Line

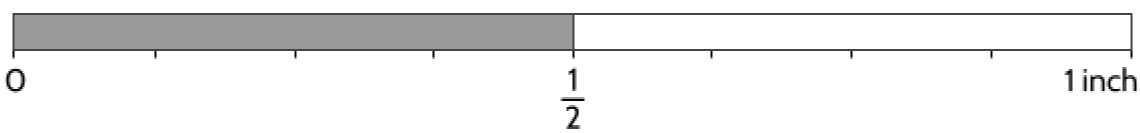
2) Directions: Label the point where $\frac{5}{8}$ belongs on the number line. Be as precise as possible.

This is based on an activity from openmiddle.com, where similar challenges can be found.

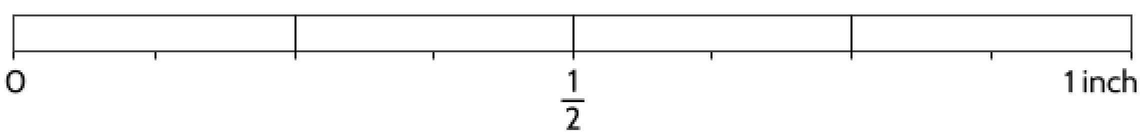
Comparing the Size of Fractions and Decimals

In this exercise, you will use your knowledge of fractions and decimals to compare their sizes. You can use the number line to help you make decisions.

We can show $\frac{1}{2}$ inch by shading in pieces of an inch.



3) Show $\frac{3}{4}$ inch by shading in pieces of the inch below. Then, label $\frac{3}{4}$ on the diagram.



4) Which of the following number sentences are true?

A. $\frac{3}{4} > \frac{1}{2}$

B. $\frac{3}{4} = \frac{1}{2}$

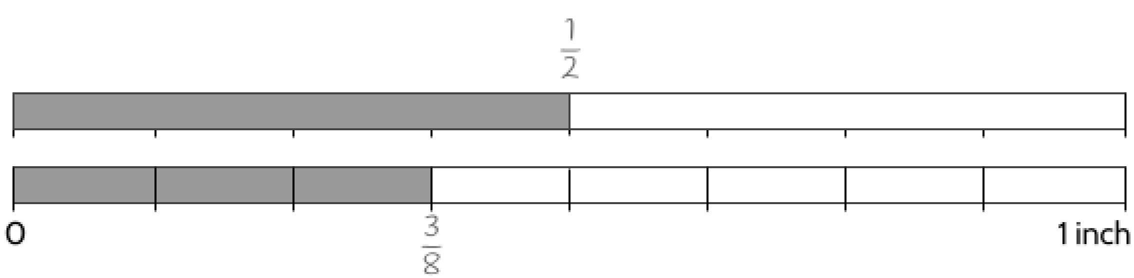
C. $\frac{1}{2} > \frac{3}{4}$

D. $\frac{1}{2} < \frac{3}{4}$

5) How do you know?

Compare the fractions below. Shade pieces on the two number lines, then add >, <, or = between the two fractions. (The first set of bars is shaded for you.)

6) $\frac{1}{2}$ $\frac{3}{8}$



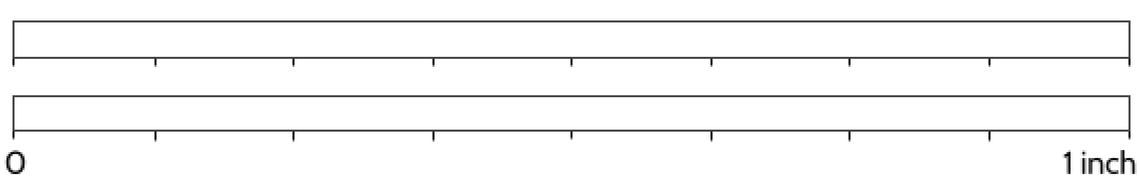
7) $\frac{1}{2}$ $\frac{5}{8}$

8) $\frac{1}{4}$ $\frac{2}{8}$

9) $\frac{1}{4}$ $\frac{3}{8}$

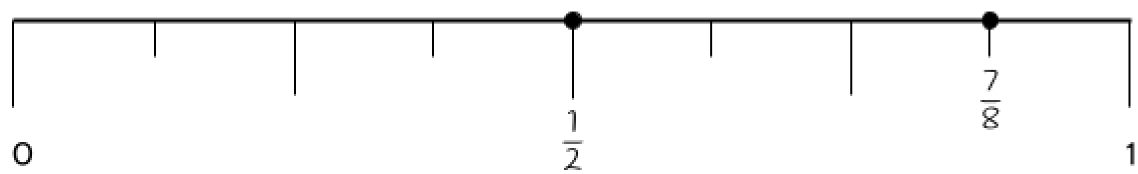
10) $\frac{3}{4}$ $\frac{5}{8}$

11) $\frac{3}{4}$ $\frac{6}{8}$



What if you wanted to compare the numbers $\frac{1}{2}$ and $\frac{7}{8}$?

For these two fractions, you might want to use the fractions of an inch number line since it shows halves and eighths. Looking at the number line below, do you see which number is bigger?



There are many ways to see that $\frac{7}{8}$ is bigger than $\frac{1}{2}$. Here are a few:

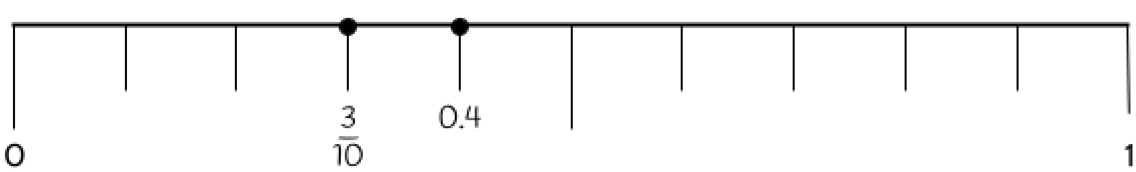
- $\frac{7}{8}$ is farther to the right on a number line.
- $\frac{7}{8}$ is closer to 1. It is only $\frac{1}{8}$ away.

We can compare these two numbers with either of these inequalities:

$\frac{7}{8} > \frac{1}{2}$ or $\frac{1}{2} < \frac{7}{8}$

What if you wanted to compare the numbers $\frac{3}{10}$ and 0.4?

For these two fractions, you might want to use the fractions of a centimeter number line since it shows tenths. Look at the number below and compare the size of the two numbers.



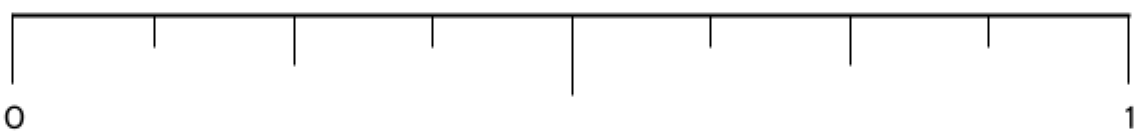
We can see that 0.4 is bigger. We can write a comparison of these two numbers with either of these inequalities:

$0.4 > \frac{3}{10}$ or $\frac{3}{10} < 0.4$

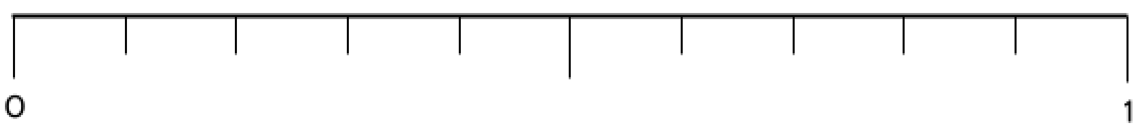
On the next page, try using the number lines to make comparisons. If you have another way of comparing the size of fractions or decimals, please show us.

You may want to use these number lines to compare the numbers below.

Fractions of an inch



Fractions of a centimeter

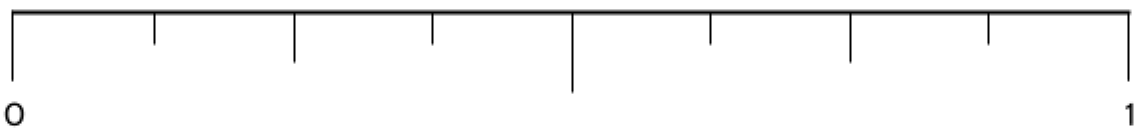


12) Complete the table.

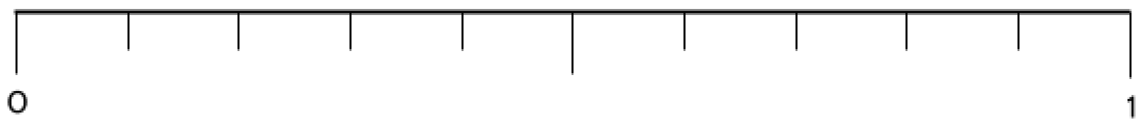
Compare the numbers with >, <, or =		
$\frac{1}{2}$	<	$\frac{7}{8}$
$\frac{3}{4}$		$\frac{1}{2}$
$\frac{1}{4}$		$\frac{1}{8}$
$\frac{5}{10}$		0.6
$\frac{5}{8}$		$\frac{2}{4}$
$\frac{7}{8}$		$\frac{4}{4}$
0.3		$\frac{3}{10}$
$\frac{2}{4}$		$\frac{5}{8}$
$\frac{1}{2}$		$\frac{4}{8}$

Compare the numbers with >, <, or =		
$\frac{1}{2}$		$\frac{4}{8}$
$\frac{3}{8}$		$\frac{1}{4}$
$\frac{2}{8}$		$\frac{1}{10}$
$\frac{1}{2}$		0.4
$\frac{7}{8}$		$\frac{3}{4}$
$\frac{7}{10}$		$\frac{3}{4}$
0.3		$\frac{1}{4}$
$\frac{5}{8}$		$\frac{3}{4}$
$\frac{7}{8}$		$\frac{9}{10}$

Fractions of an inch



Fractions of a centimeter



13) Use >, <, and = to compare the following numbers:

$\frac{1}{2}$ $\qquad$ $\frac{6}{8}$

$\frac{6}{8}$ $\qquad$ $\frac{1}{4}$

$\frac{1}{4}$ $\qquad$ $\frac{1}{2}$

14) Put these numbers in order from least to greatest:

$\frac{1}{2}, \frac{6}{8}, \frac{1}{4}$

15) Use >, <, and = to compare the following numbers:

$-\frac{1}{4}$ $\qquad$ -0.4

-0.4 $\qquad$ $\frac{3}{8}$

$\frac{3}{8}$ $\qquad$ $-\frac{1}{4}$

16) Put these numbers in order from least to greatest:

$\frac{3}{8}, -\frac{1}{4}, -0.4$

Identify 1/4 on a Number Line

17) Directions: Label the point where $\frac{1}{4}$ belongs on the number line. Be as precise as possible.

This is based on an activity from openmiddle.com, where similar challenges can be found.

Applying Knowledge of Number Lines - Answer Key

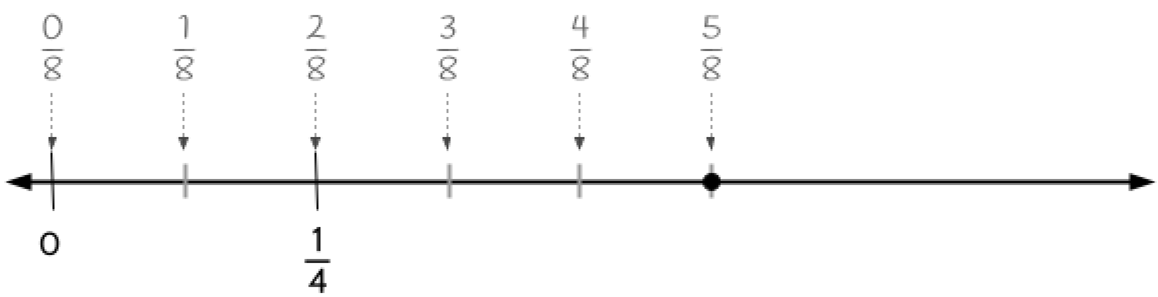
Explain the Mistake - Fractions

1) Angelina is correct. There are different ways to show that these two fractions are equivalent on the number line. 1 ⅓ is a point halfway between 1 and 2. The fraction 12/8 means 12 eighths. If you find the fraction ⅛ and then make 12 copies, you could be in the same position as 1 ½. You might have a different way to explain why these two fractions are the same.

Identify 5/8 on a Number Line

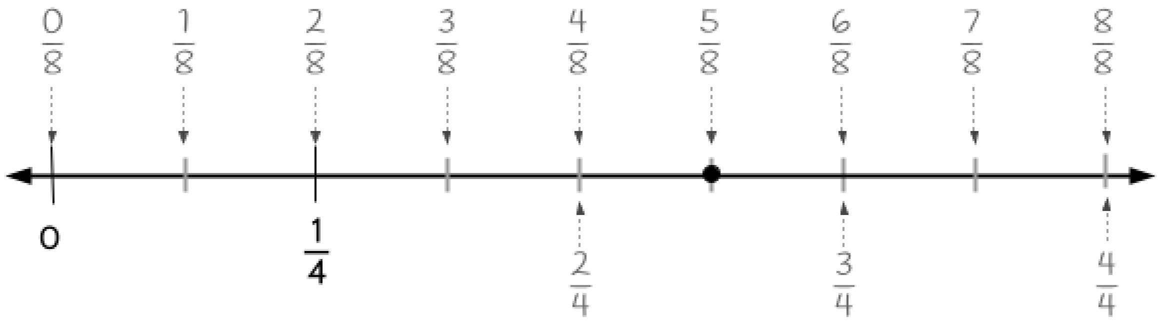
2) There are different ways to find the location of $\frac{5}{8}$ on the number line below. Here is one way:

- Split $\frac{1}{8}$ into two pieces of the same size. Each piece is $\frac{1}{4}$.
- Make 5 copies of $\frac{1}{8}$ to find $\frac{5}{8}$.

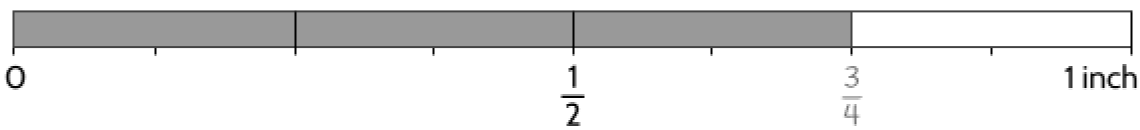


Here is another way:

- Make copies of $\frac{1}{4}$ to find $\frac{4}{4}$, which is the same as 1.
- Split 1 into eight pieces of the same size to find $\frac{1}{8}$.
- Make 5 copies of $\frac{1}{8}$ to find $\frac{5}{8}$.



Comparing the Size of Fractions and Decimals

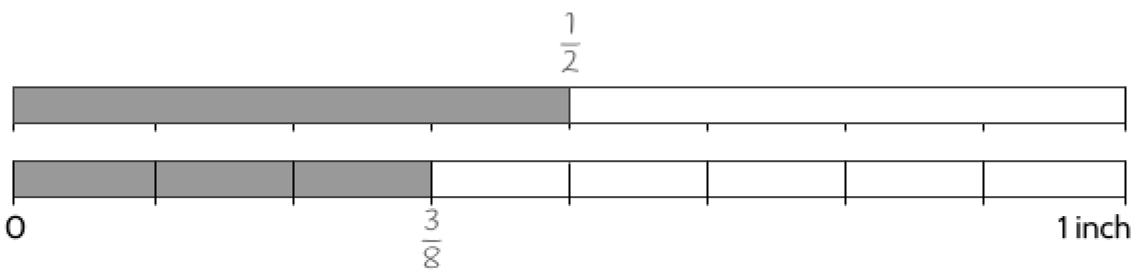
3) 

4) A, D

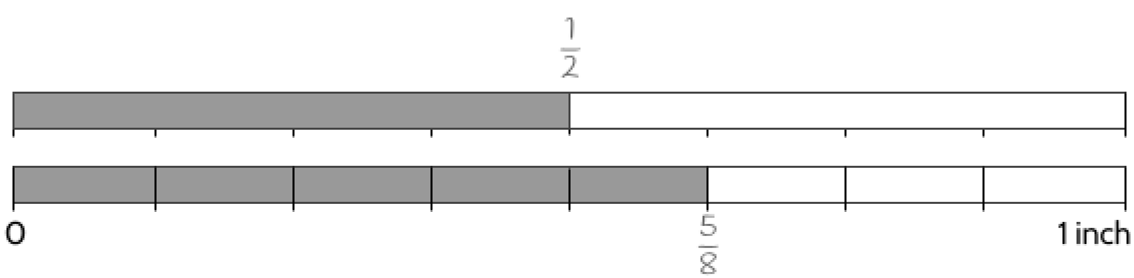
5) There are different ways to show that ¾ is greater than ½. Here are a few ways:

- ¾ is to the right of ½ on a number line, so it must be larger.
- 3 out of 4 pieces is longer than 1 out of 2 pieces.

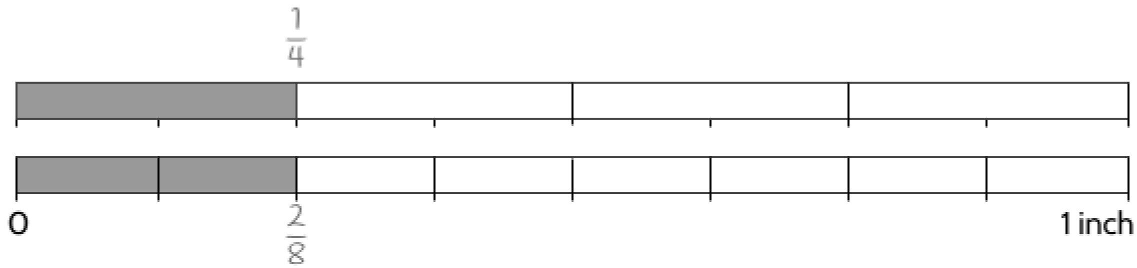
6) $\frac{1}{2} > \frac{3}{8}$



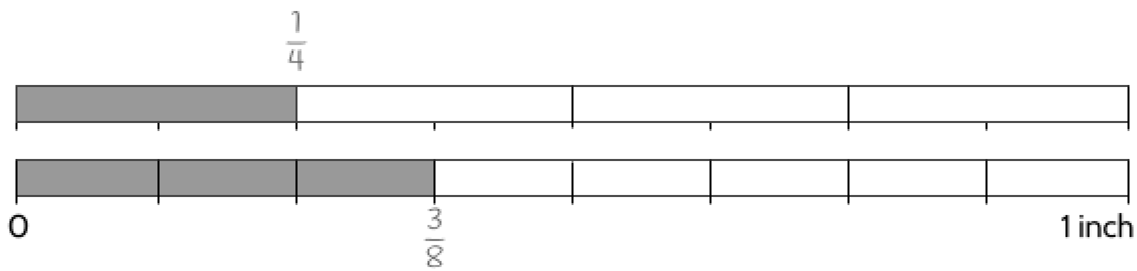
7) $\frac{1}{2} < \frac{5}{8}$



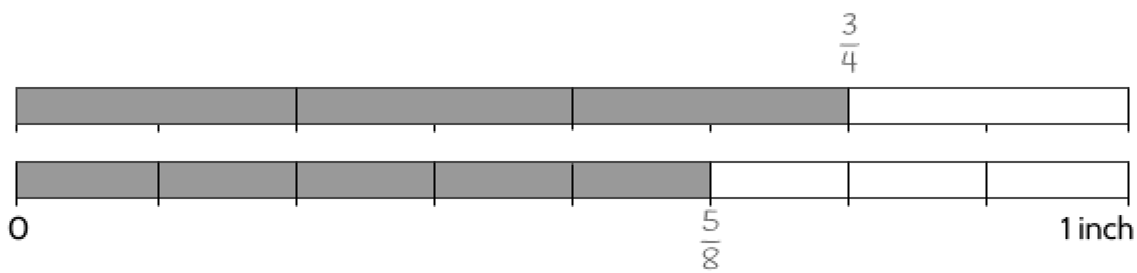
8) $\frac{1}{4} = \frac{2}{8}$



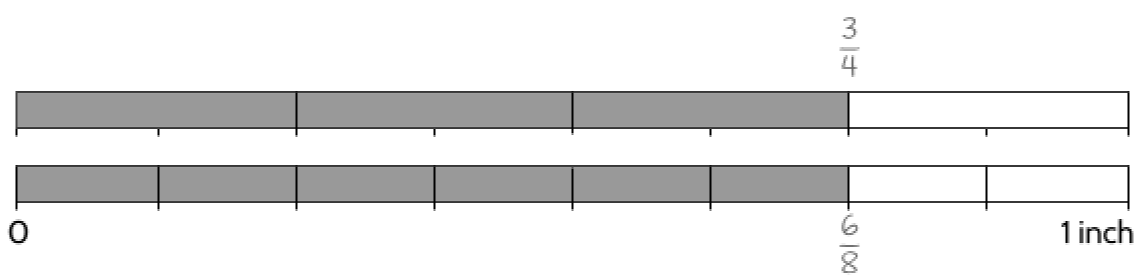
9) $\frac{1}{4} < \frac{3}{8}$



10) $\frac{3}{4} > \frac{5}{8}$



11) $\frac{3}{4} = \frac{6}{8}$



12) The answers are below.

Compare the numbers with >, <, or =		
$\frac{1}{2}$	<	$\frac{7}{8}$
$\frac{3}{4}$	>	$\frac{1}{2}$
$\frac{1}{4}$	>	$\frac{1}{8}$
$\frac{5}{10}$	<	0.6
$\frac{5}{8}$	>	$\frac{2}{4}$
$\frac{7}{8}$	<	$\frac{4}{4}$
0.3	=	$\frac{3}{10}$
$\frac{2}{4}$	<	$\frac{5}{8}$
$\frac{1}{2}$	=	$\frac{4}{8}$

Compare the numbers with >, <, or =		
$\frac{1}{2}$	=	$\frac{4}{8}$
$\frac{3}{8}$	>	$\frac{1}{4}$
$\frac{2}{8}$	>	$\frac{1}{10}$
$\frac{1}{2}$	<	0.4
$\frac{7}{8}$	>	$\frac{3}{4}$
$\frac{7}{10}$	<	$\frac{3}{4}$
0.3	>	$\frac{1}{4}$
$\frac{5}{8}$	<	$\frac{3}{4}$
$\frac{7}{8}$	<	$\frac{9}{10}$

13) $\frac{1}{2} < \frac{6}{8}$

$\frac{6}{8} > \frac{1}{4}$

$\frac{1}{4} < \frac{1}{2}$

14) $\frac{1}{4}, \frac{1}{2}, \frac{6}{8}$

15) $-\frac{1}{4} > -0.4$

$-0.4 < \frac{3}{8}$

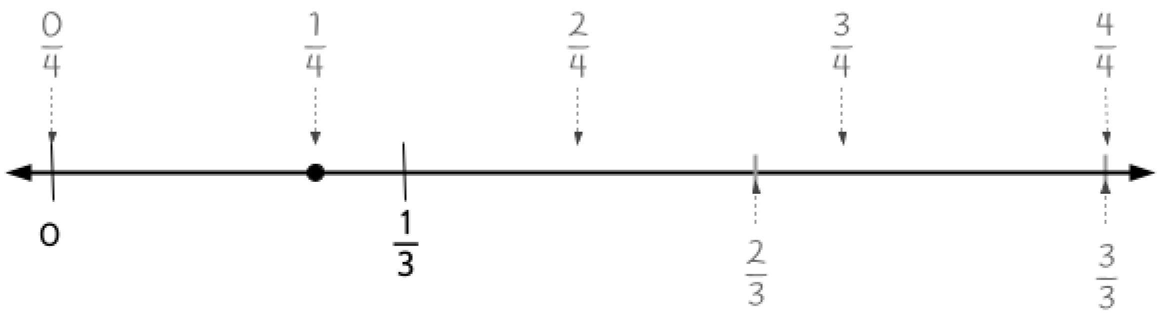
$\frac{3}{8} > -\frac{1}{4}$

16) $-\ 0.4, \quad -\frac{1}{4}, \frac{3}{8}$

Identify 1/4 on a Number Line

17) There are many ways to find the location of $\frac{1}{4}$ on the number line below. Here is one way:

- Make 3 copies of $\frac{1}{3}$ to find $\frac{3}{3}$, which is the same as 1.
- Split 1 into four even pieces to find $\frac{1}{4}$.



Vocabulary Review

Glossary

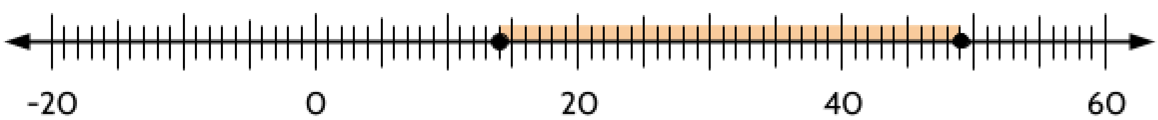
You can use this section to look up words used in this math packet.

absolute value (noun): A measurement of the size of a number, equal to the number's distance from zero on the number line. Synonym: magnitude

coordinate grid (noun): A flat surface formed when two straight number lines cross each other at right angles. An exact position on the grid can be described using coordinates. The *coordinate grid* is also called the coordinate plane.

denominator (noun): The bottom number in a fraction. Shows how many equal parts the item is divided into.

difference (noun): How much one number differs from another. One way to visualize the difference is how far one number is from another on a number line. For example, Paulina is 49 and her son is 14, so there is a 35 year age difference between them.



equal to (adjective): When two numbers or expressions have the same value. For example, 4 is equal to 2 + 2.

equation (noun): A number sentence that shows two quantities are equal by using the equal sign. $2 \times 3 = 6$ is an equation. 2×3 is equal to 6.

increment (noun): The amount by which something grows.

inequality (noun): A number sentence in which two expressions are not equal to each other. Example:

greater than (adjective): The symbol > (called the greater-than sign) is used in number sentences where the values on the left side of the equal sign are larger than the values on the right side.

less than (adjective): The symbol < (called the less-than sign) is used in number sentences where the values on the left side of the equal sign are smaller than the values on the right side.

magnitude (noun): A measurement of the size of a number, equal to the number's distance from zero on the number line. Synonym: absolute value

number line (noun): A picture of a straight line on which all numbers can be placed. Numbers get bigger as you move to the right and smaller as you move to the left.

numerator (noun): The top number in a fraction. It shows how many parts we have.

plot (verb): To place a point representing a number or pair of numbers on a graph, such as a number line or coordinate grid.

signed number (noun): A number shown as positive or negative. For example, +9 is a signed number that we read as "positive 9." –12 is a signed number that we read as "negative 12."

tick mark (noun): small lines used on a number line we use to help us find the location of numbers.

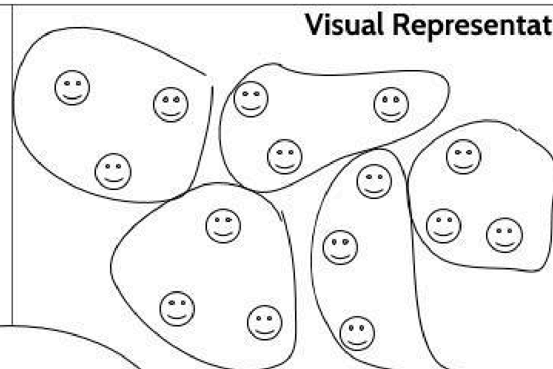
Using Graphic Organizers to Learn Vocabulary

In order to learn math vocabulary, we need to practice using it in different ways. In this activity, you will choose a few words from this packet that you want to practice, then you will complete a graphic organizer for each word. Look at the sample for the word *quotient* below.

To complete the graphic organizer, you will choose a word and then answer four questions:

- What is the definition of the word? You can look at the vocabulary review on page 82 for help. Try to write the definition in your own words to really make the word yours.
- Make a visual representation. You can make a drawing or diagram that will help you remember what the word means.
- What are some examples of the word you're studying? Below you can see that there are examples of *quotients*, which are the answers to division problems.
- What are some non-examples of this word? These are things that are **not** the words you're studying. For example, 24 is **not** the quotient of 4 divided by 6.

quotient

What is it?	Visual Representation
A quotient is the result of dividing one number by another. It is the answer to a division question.	
What are some examples?	**What are some non-examples?**
15 divided by 3 equals **5** 66 ÷ 6 = **11** 63/18 = **3.5** 5, 11 and 3.5 are quotients in these calculations. population ÷ area = **density**	4 times 6 equals 24 18 + 5 = 23 17 – 2.5 = 14.5 3.5 × 18 = 63

What is it?

Visual Representation

What are some examples?

What are some non-examples?

Sources

Ahrendt, S., Monson, D., & Cramer, K. (2021). Promoting discourse: Fractions on number lines. *Mathematics Teacher: Learning and Teaching PK-12*, 114(4), 284-289. [Section: Fractions on a number line]

Chang, L. (1985). Multiple methods of teaching the addition and subtraction of integers. *The Arithmetic Teacher*, 33(4), 14-19.

Chapin, S. H. & Johnson, A. (2006). *Math Matters: Understanding the Math You Teach*. Grades K-6. Math Solutions Publications. [Section: Overview]

GED.com. (2016). Assessment Guide for Educators: Mathematical Reasoning. [Section: Overview]

Ellis, M. W., & Bryson, J. L. (2011). A conceptual approach to absolute value equations and inequalities. The Mathematics Teacher, 104(8), 592-598. [Section: Measurement with a Number Line]

Frank, A. R. (1989). Counting skills–A foundation for early mathematics. The Arithmetic Teacher, 37(1), 14-17. [Introduction to Number Lines]

Frykholm, J. (2010). Learning to think mathematically with the number line: A resource for teachers, a tool for young children. Math Learning Center. [Sections: Introduction to the Number Line]

Lannin, J. K., van Garderen, D., & Kamuru, J. (2020). Building a strong conception of the number line. Mathematics Teacher: Learning and Teaching PK-12, 113(1), 18-24. [Section: Introduction to Number Lines]

McCabe, T., Sorto, M. A., & White, A. (2010). Activities for Students: Algebra on the Number Line. *The Mathematics Teacher*, 104(5), 379-386. [Sections: Placing Numbers on a Number Line & Absolute Value]

OpenMiddle.com. Source problems available for free at web site.

Peyser, E. E., & Bobo, J. (2022). Linking Number Sense to Linear Space. Mathematics Teacher: Learning and Teaching PK-12, 115(2), 113-121. [Section: Introduction to Number Lines]

Siebert, D., & Gaskin, N. (2006). Creating, naming, and justifying fractions. Teaching Children Mathematics, 12(8), 394-400. [Sections: Expanding the Number Line, Measurement with a Number Line]

Small, M. (2019). Understanding the Math We Teach and how to Teach it: K-8. Stenhouse Publishers. [Sections: Signed Numbers, Operations with Signed Numbers]

Picciotto, H. (2016). Algebra Lab Gear. Didax. [Section: Signed Numbers]

Van de Walle, J. A. (2003). *Elementary And Middle School Mathematics*. New York.

What is a Number Line? Bitesize. BBC. Retrieved from https://www.bbc.co.uk/bitesize/topics/zc3d7ty/articles/zdhdqhv on June 20, 2023.

Version

V. 1.1 10/5/2023: Corrected errors, added vocabulary practice and graphic organizer

V. 1.0 6/30/2023: First version released