# LINES & ANGLES

**NOTE**: These materials were originally made available in the NYSED/CUNY Fast Track GRASP Math Packet titled: Lines, Angles & Shapes: Measuring Our Word, which has been retired.

Other materials from that packet have been reorganized and expanded and can be found in the NYSED/CUNY Fast Track GRASP Math Packet titled: Two-Dimensional Geometry.

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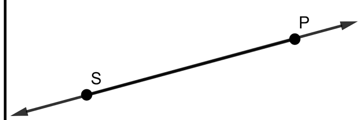
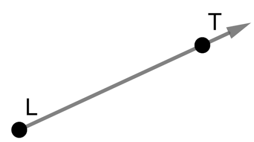
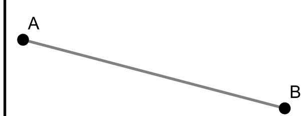
# Lines, Rays, and Segments

In the chart below, there are three figures that each look like lines. They are all straight. In geometry we have a specific name for each of them. It is helpful to understand the ways that they are different from each other.

1. Fill in the following chart

| **Name** | Line | Ray | Line Segment |
|---|---|---|---|
| **Figure** |   |   |   |
| **Notation**[1] | $\overleftrightarrow{SP}$ **or** $\overleftrightarrow{PS}$ | $\overrightarrow{LT}$ | $\overline{AB}$ **or** $\overline{BA}$ |
| **Characteristics** *(Briefly describe each figure. What makes it different from the other two?)* | | | |

[1] *Notation* is the way we write something in mathematics, using symbols, letters, or numbers.

Of these three figures, the most basic is the **line**, which you already read about. In the chart above, *Line SP* has arrows pointing in opposite directions. Lines in geometry are drawn this way to show that they continue in both directions forever.

Most geometric figures, like shapes and angles, are made up of *parts* of a line.

A **segment** (or line segment) is part of a line, defined by two endpoints and all the points between them. The rungs of a ladder are physical examples of line segments.


The name of this line segment is $\overline{AB}$ or $\overline{BA}$. For line segments, the order of the points doesn't matter. However, it is important that you use the end points in the name.

A **ray** is a part of a line, starting with one endpoint and made up of all the points on one side of that endpoint. A beam of light from a flashlight is a physical example of a ray.

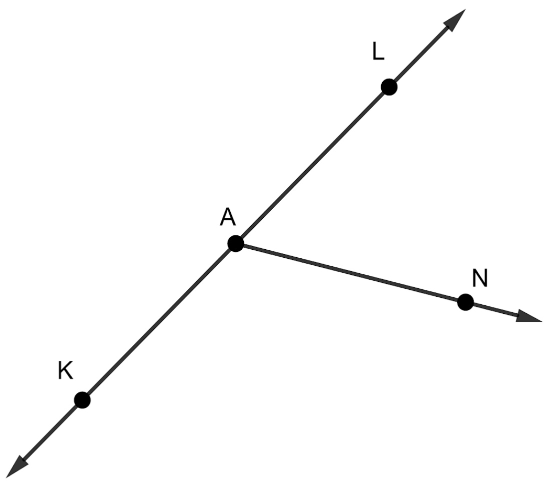
The name of this ray is $\overrightarrow{CE}$. When we name rays, the name <u>must</u> start with the endpoint (C, in this case). The first letter is the endpoint and the second letter is any point the ray goes through.


2. Write all possible notations for each drawing.

| a. | b. | c. |
|---|---|---|
|   |   |   |

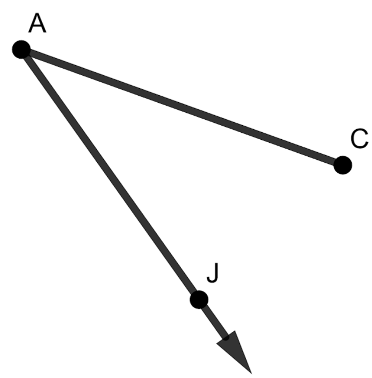
d. For this figure, use the notation that describes the <u>entire</u> ray.


e. *Fill in the blanks.*




___ intersecting with ___ at Point ___.

f. *Fill in the blanks.*




___ intersecting with ___ at Point ___.

3. Draw the following figures.

a. $\overrightarrow{EK}$

b. $\overline{SR}$

| | |
|---|---|
| c. $\overleftrightarrow{JL}$ | d. $\overleftrightarrow{PR}$ intersecting $\overleftrightarrow{PQ}$ at Point P |
| e. $\overrightarrow{MJ}$ intersecting $\overrightarrow{JT}$ at Point J | f. $\overline{HR}$ |

Use the line below to answer questions 4 and 5.


4. How many line segments are there? Write the names of any line segments you see.

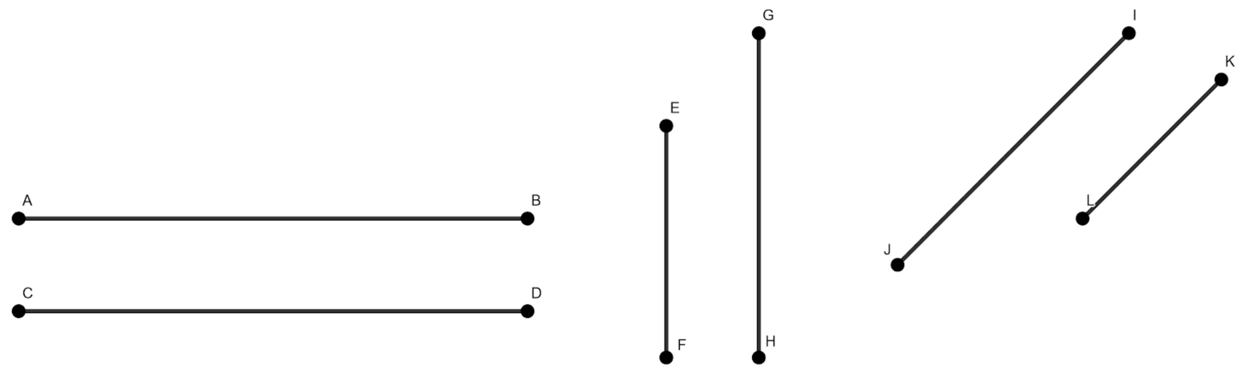
5. How many rays are there? Write the names of any rays you see.

# Parallel and Perpendicular Lines

Look around and chances are you will see parallel and intersecting lines.

**Parallel lines** are lines that lie on the same flat surface and do not intersect.

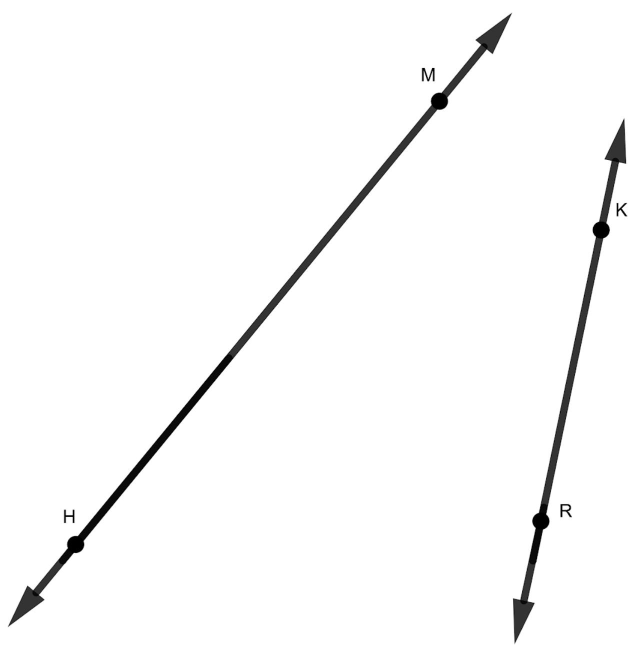
Below are some examples of lines that are parallel.




$\overline{AB}$ is parallel to $\overline{CD}$ which can be written as $\overline{AB} \parallel \overline{CD}$.

$\overline{EF}$ is parallel to $\overline{GH}$ which can be written as $\overline{EF} \parallel \overline{GH}$.

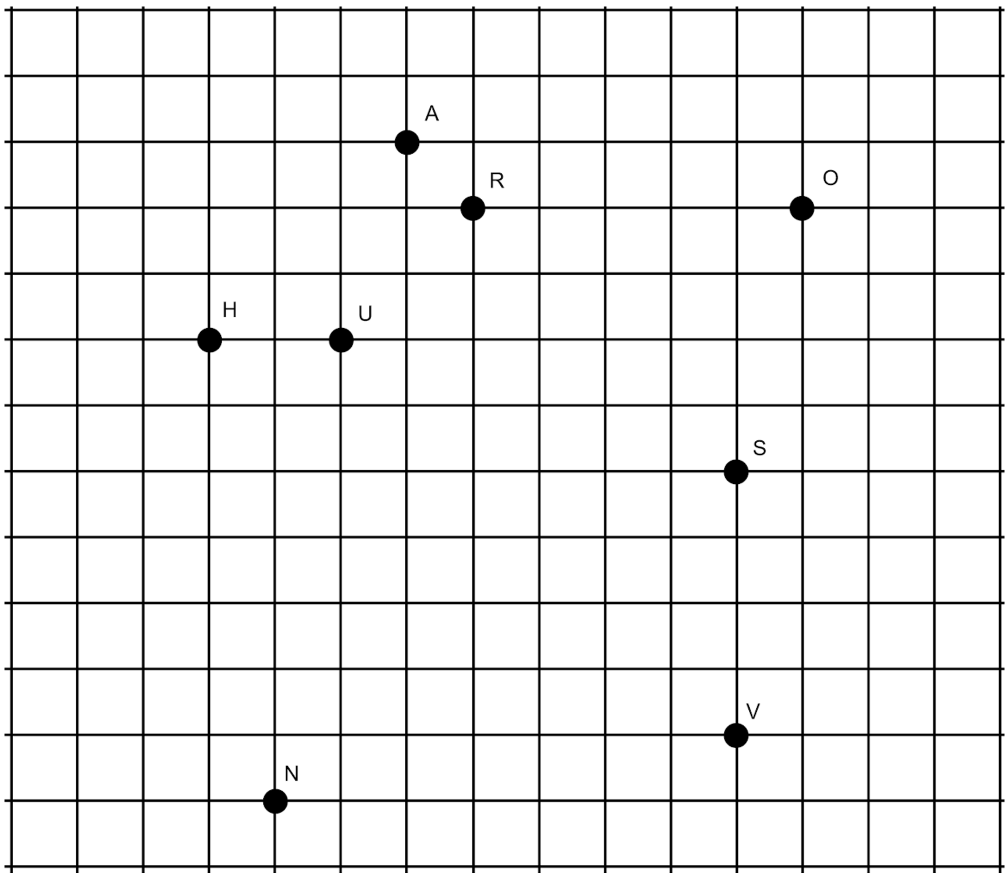
$\overline{JI}$ is parallel to $\overline{LK}$ which can be written as $\overline{JI} \parallel \overline{LK}$.



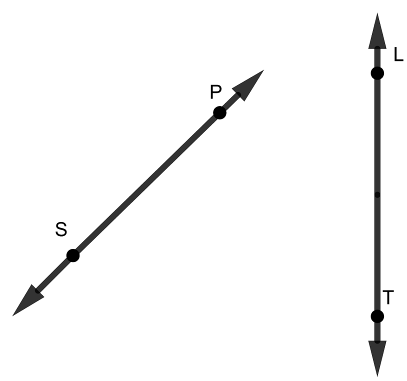

One important thing to remember is that just because drawings of lines do not intersect, it does not necessarily mean they are parallel.

For example, $\overleftrightarrow{HM}$ and $\overleftrightarrow{RK}$ are considered intersecting lines. If you extend the lines, they will eventually intersect at a single point.

6. Use the points below to draw pairs of parallel line segments. How many parallel lines can you find?




7. Which of the following statements is true about $\overleftrightarrow{SP}$ and $\overleftrightarrow{LT}$?



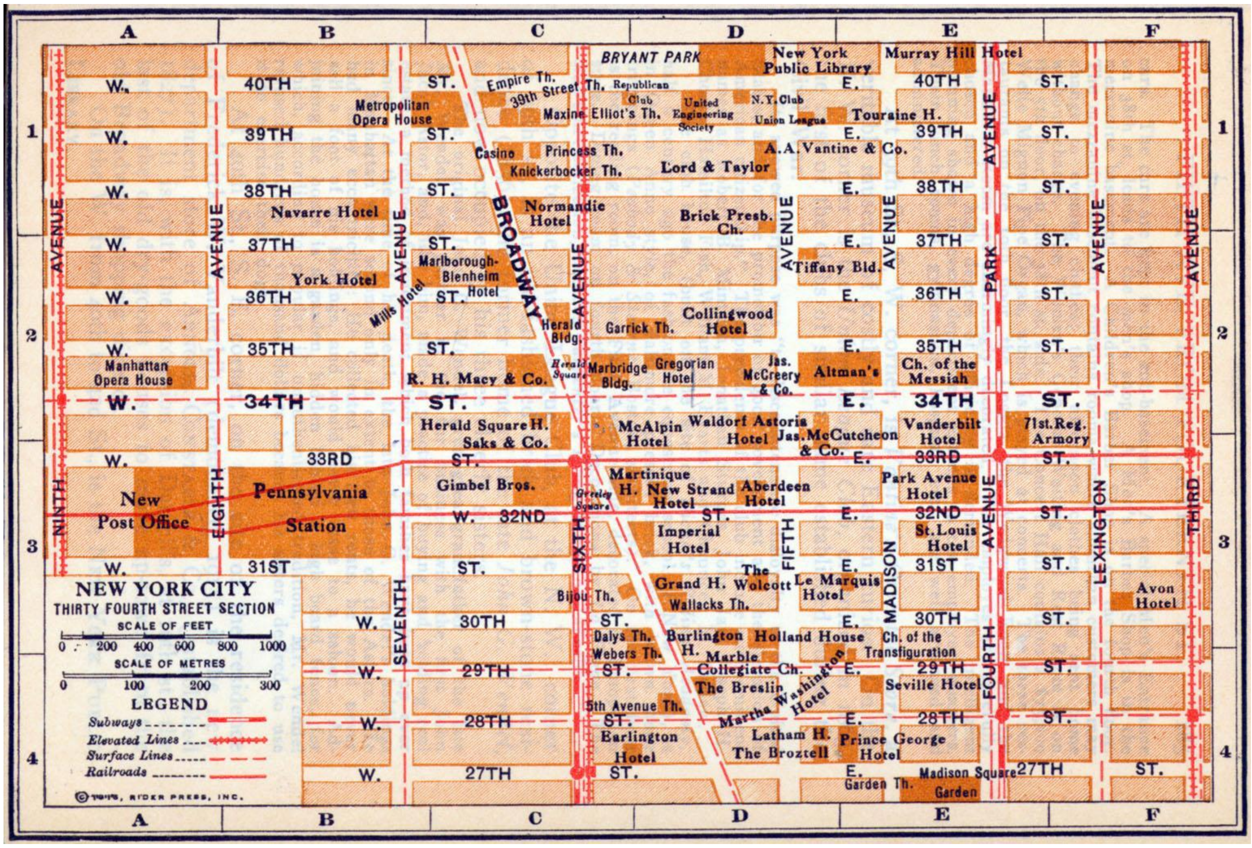

a. $\overleftrightarrow{SP}$ and $\overleftrightarrow{LT}$ are parallel because they do not intersect.

b. $\overleftrightarrow{SP}$ and $\overleftrightarrow{LT}$ are parallel because if you continue each line, they will intersect.

c. $\overleftrightarrow{SP}$ and $\overleftrightarrow{LT}$ are not parallel because if you continue each line, they will intersect.

d. $\overleftrightarrow{SP}$ and $\overleftrightarrow{LT}$ are not parallel because they do not intersect.

One place where we see parallel and intersecting lines are the streets and roads we use everyday. This map shows a section of Manhattan in New York City.




8. Use the names of the streets on the map to complete the following sentences.

   a. ______________________ is parallel to ______________________.

   b. ______________________ is parallel to ______________________.

   c. ______________ is parallel to ______________ and ______________.

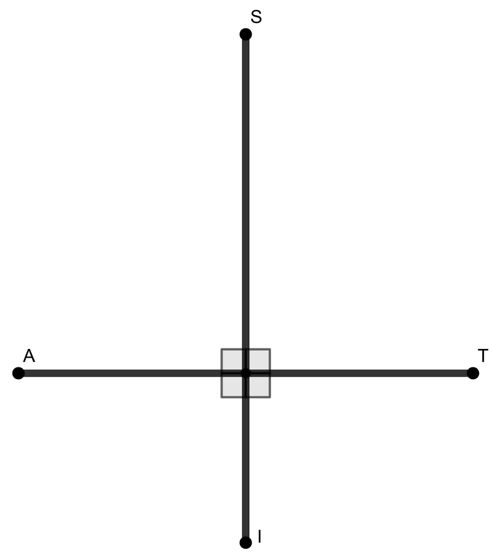
   d. __________________________ is not parallel to any other street on this map.

You may have noticed that many of the streets and avenues intersect in such a way that they form an L or a T.

When two lines intersect to form an L or a T, we call them **perpendicular lines.**

Perpendicular lines are lines that intersect to form right angles.

For example, $\overline{AT}$ and $\overline{SI}$ are perpendicular lines.




The mathematical symbol for perpendicular is $\perp$. Notation for showing that two lines are perpendicular can look like this:

$$\overline{AT} \perp \overline{SI}.$$

This notation can be read as "*Line AT is perpendicular to Line SI*".

9. Use the names of the streets on the map of Manhattan to complete the following sentences.

   a. ______________________ is perpendicular to ________________________.

   b. ______________________ is perpendicular to ________________________.

   c. ______________________ is perpendicular to ________________________.

There are examples of parallel lines and perpendicular lines all around us.

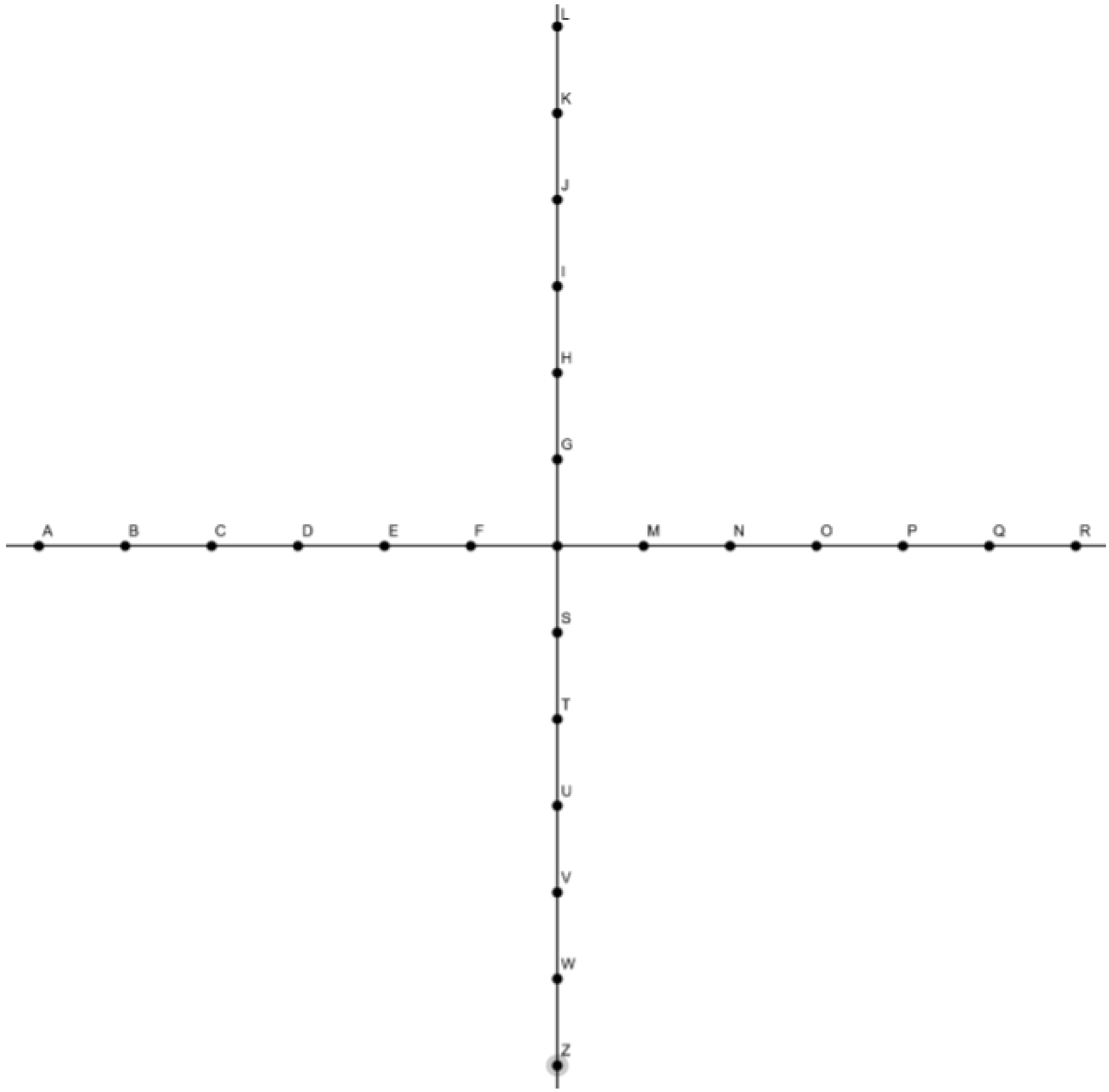
Tiles, brick walls, sports fields, windows, floors, trees, guitar strings, railroad tracks, telephone wires, tables—these are just a few examples.

10. Look around the room and find 5 examples of parallel lines and 5 examples of perpendicular lines.

| **Examples of Parallel Lines** | **Examples of Perpendicular Lines** |
|---|---|
| | |
| | |
| | |
| | |
| | |

11. Using a ruler or other straight edge (the side of a book will work), connect the dots to create the following line segments:

    a. $\overline{AG}$, $\overline{BH}$, $\overline{CI}$, $\overline{DJ}$, $\overline{EK}$, $\overline{FL}$
    b. $\overline{GR}$, $\overline{HQ}$, $\overline{IP}$, $\overline{JO}$, $\overline{KN}$, $\overline{LM}$
    c. $\overline{FZ}$, $\overline{EW}$, $\overline{DV}$, $\overline{CU}$, $\overline{BT}$, $\overline{AS}$
    d. $\overline{SR}$, $\overline{TQ}$, $\overline{UP}$, $\overline{VO}$, $\overline{WN}$, $\overline{ZM}$


L
K
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M
N
O
P
Q
R
S
T
U
V
W
Z

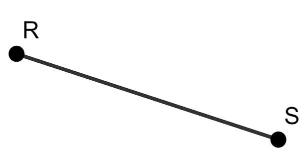
## *Lines, Rays and Segments - Answer Key*

1. Some things you might have noticed:
    a. Lines have arrows at both ends. Rays have an arrow at one end and a point at one end. Segments have a point at both ends.
    b. There are two ways to use notation for lines and segments, but only one way to use it for a ray.
    c. All three figures have two points.
    d. Lines keep going in both directions and rays keep going in only one direction.

2. Possible notations for each drawing:
    a. $\overline{MT}$ or $\overline{TM}$
    b. $\overrightarrow{EF}$ or $\overrightarrow{FE}$
    c. $\overleftrightarrow{DG}$ or $\overleftrightarrow{GD}$
    d. $\overrightarrow{AC}$
    e. $\overrightarrow{AN}$ intersecting with $\overleftrightarrow{KL}$ (or $\overleftrightarrow{LK}$) at Point A.
    f. $\overline{AC}$ intersecting with $\overrightarrow{AJ}$ at Point A.

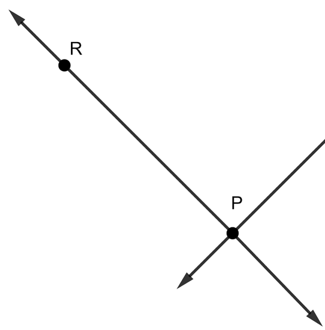
3. There are multiple possible ways to draw the given figures. Here are some examples:


a.

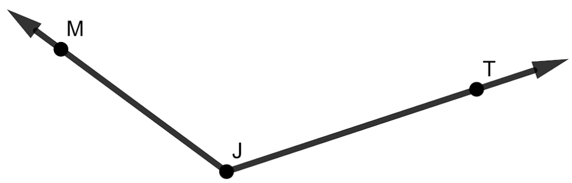



b.

c.




d.



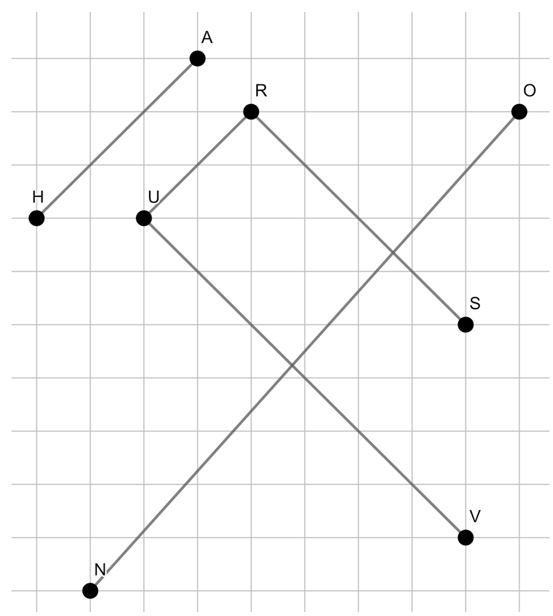

e.


f.

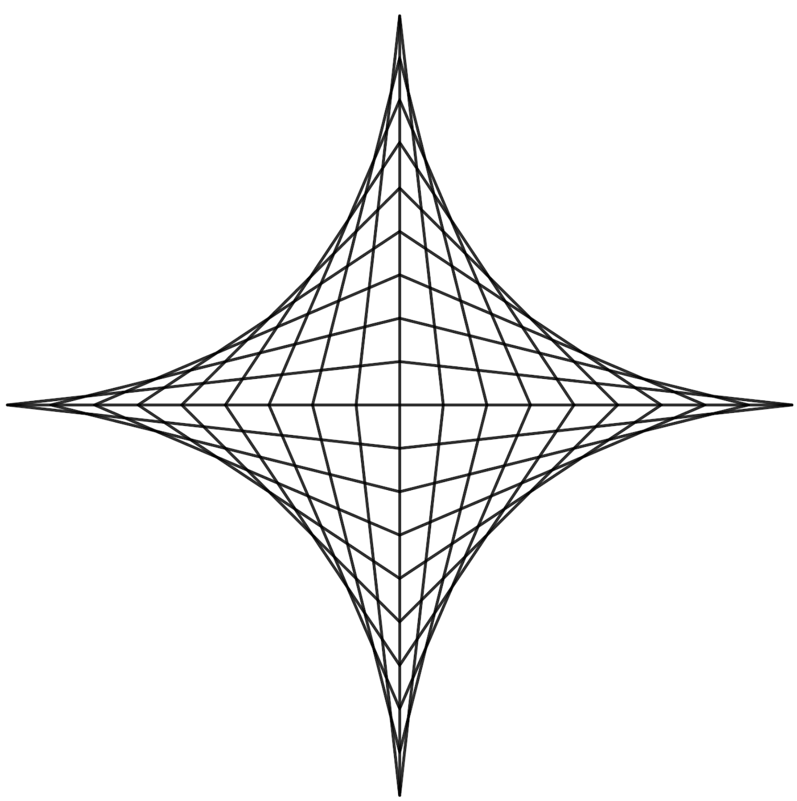
4. There are three segments: $\overline{HE}$, $\overline{EK}$, and $\overline{HK}$. ($\overline{EH}$, $\overline{KE}$, and $\overline{KH}$are also possible names for each of the three segments.)

5. There are two rays: $\overrightarrow{EH}$ and $\overrightarrow{HK}$. What is important is that the endpoints (point E and point H) are the first letter in each name.

6. There are several examples of parallel line segments. You can see them in the diagram on the right. $\overline{AH}$,$\overline{UR}$ *and* $\overline{NO}$ are all parallel to each other. $\overline{UV}$is parallel to $\overline{RS}$.



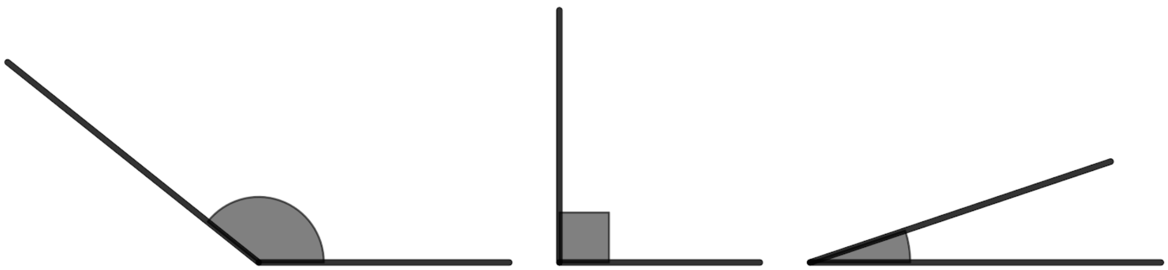
7. Choice C. $\overleftrightarrow{SP}$ and $\overleftrightarrow{LT}$ are not parallel because if you continue each line, they will intersect.
8. There are many possible correct answers. In general, Manhattan streets are parallel to other streets and avenues are parallel to other avenues. Broadway cuts across Manhattan, intersecting many streets.
9. There are many possible correct answers. In general on this map avenues are perpendicular to streets. So, for example, Sixth Avenue is perpendicular to W. 33rd Street.
10. There are countless examples of parallel and perpendicular lines around us. Be creative!
11. You should get a figure that looks something like the figure below. Notice that even though you drew straight lines, the intersections make them appear to curve.



# Angles

Along with points, lines, segments, and rays, another figure in geometry is the **angle**. Angles also help us describe the shapes in the world.

An angle is an "opening" formed when two segments, lines, or rays intersect.

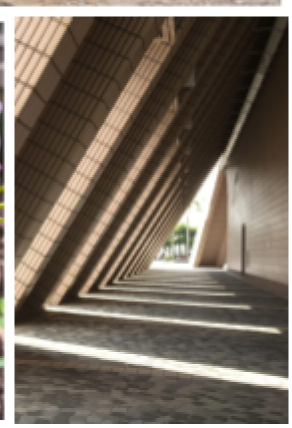


Here are just a few examples of angles in the world.

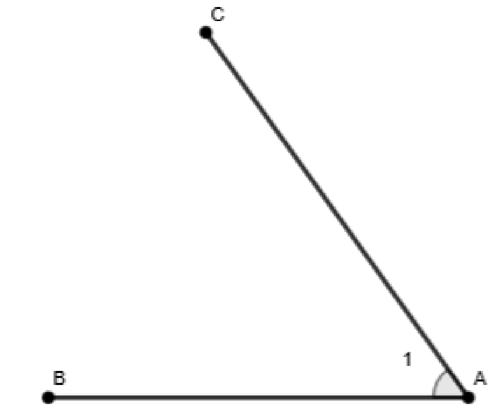
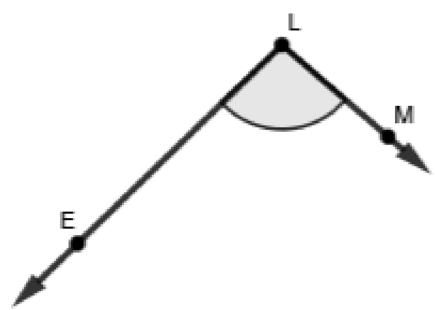
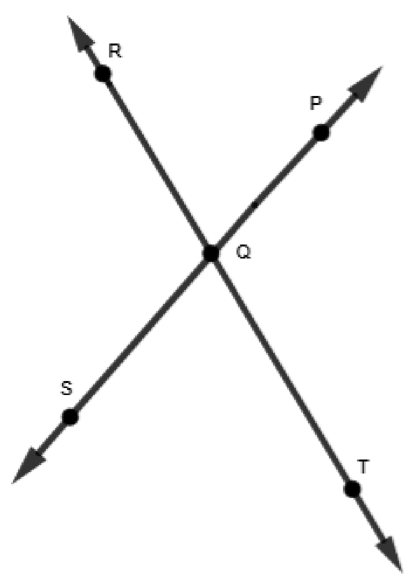








# Naming Angles

| | |
|---|---|
| In this diagram, we see an angle formed by the intersection of two line segment.<br><br>$\overline{AB}$ and $\overline{CA}$ intersect at point A. |  |
| In this diagram, we see an angle formed by the intersection of two rays.<br><br>$\overrightarrow{LE}$ and $\overrightarrow{LM}$ intersect at point L |  |
| In this diagram, we see several angles formed by the intersection of two lines.<br><br>$\overleftrightarrow{SP}$ and $\overleftrightarrow{RT}$ intersect at Point Q. |  |

There is a special name for the point where the segments, rays, or lines intersect to form an angle. We call this point the **vertex** of the angle. The vertex is important because when we want to name any angle, we need to identify the vertex first.

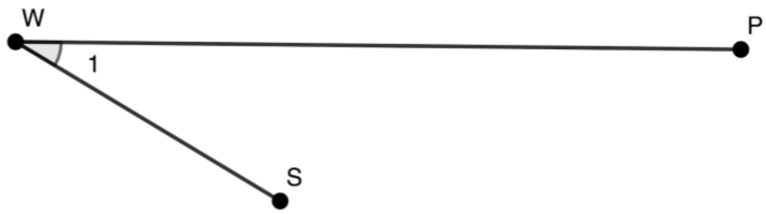
The angle to the right is formed by the intersection of two rays at point A. Point A is the vertex.

We call this angle $\angle$CAT or $\angle$TAC. Notice that the vertex, point A, is in the middle of the notation. You can start with either of the outside endpoints, but the vertex must always appear in the middle when we name an angle.


We can also refer to this angle as $\angle$A.

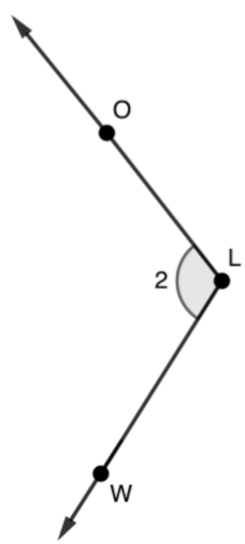
Name the numbered angles using letters.

1.



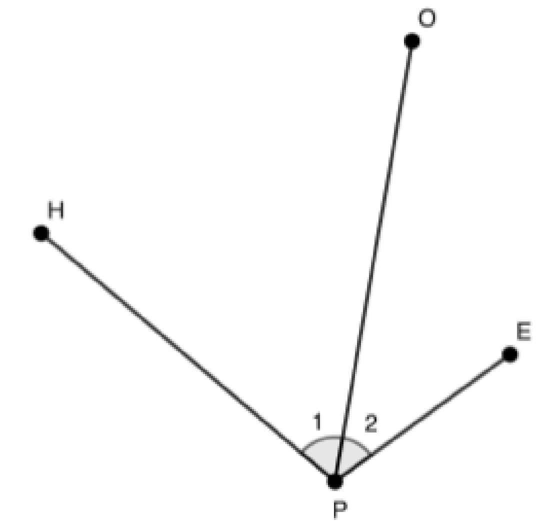

$\angle 1 =$

2.




$\angle 2 =$

3.



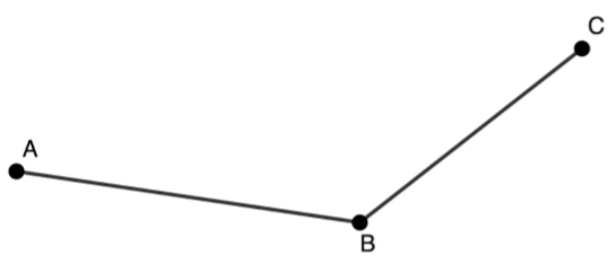

$\angle 1 =$

$\angle 2 =$

In the example above, $\angle 1$ is formed by line segments HP and PO. $\angle 2$ is formed by line segments OP and PE. But there is a third angle in the diagram. Can you see it?

It is the angle formed by line segments HP and PE, or $\angle$HPE.

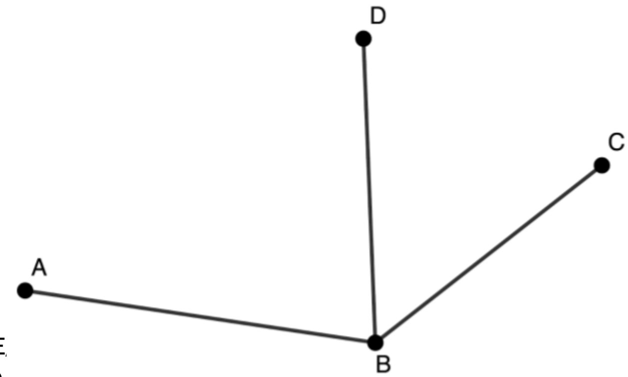
4. Here is $\angle$ABC.




If we add line segment DB, it forms three angles. Try to name them all.

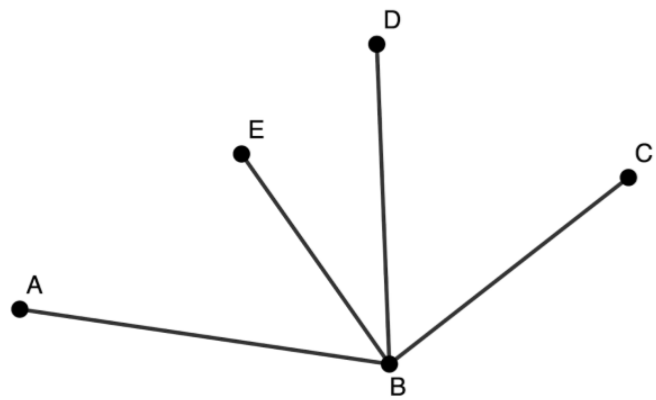
a. $\angle$

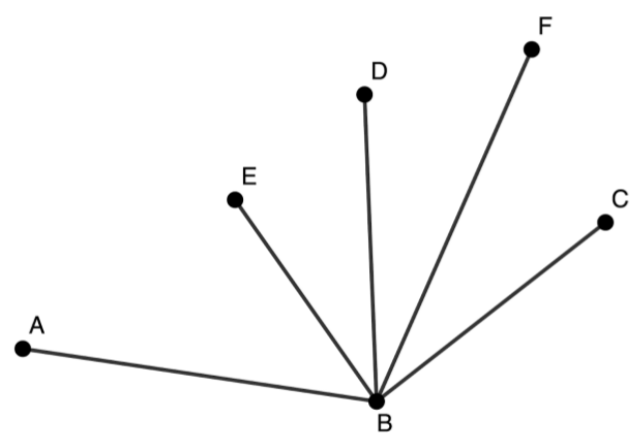
b. $\angle$



c. ∠

5. If we add another line segment, it forms six angles. Can you name all six?
   a. ∠
   b. ∠
   c. ∠
   d. ∠
   e. ∠
   f. ∠




6. If we add another line segment, how many angles are formed? Write the names of any angles you see.

7. The chart below shows the numbers of angles formed by different numbers of line segments. What patterns do you notice?

   Can you predict the number of angles made by adding additional line segments?

| Number of Line Segments | Number of Angles |
|---|---|
| 2 | 1 |
| 3 | 3 |
| 4 | 6 |
| 5 | 10 |
| 6 | |
| 7 | |
| 8 | |

   You may want to look back at the intersecting line segments on the previous pages.

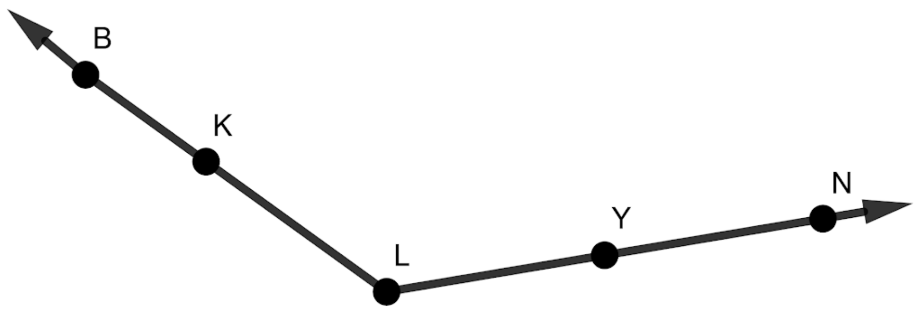
8. Which of the following notations for the angle to the right is <u>not</u> correct?

   a. ∠BLY

   b. ∠YLK

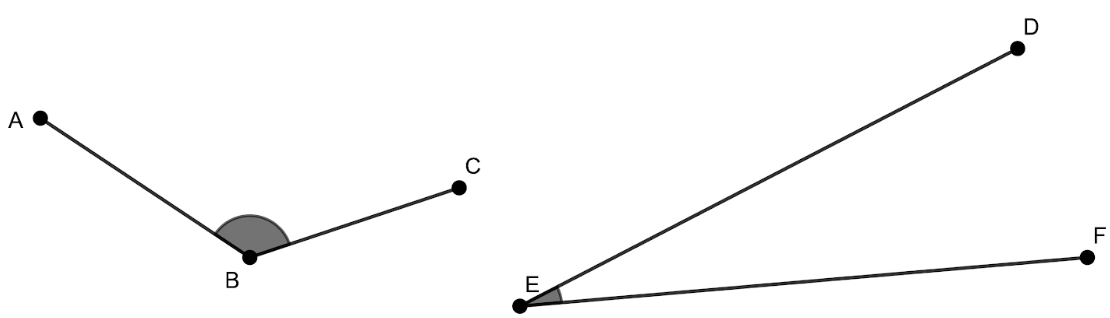
   c. ∠BLN

   d. ∠BKN




Explain why the notation is incorrect.

9. Compare ∠ABC and ∠DEF.



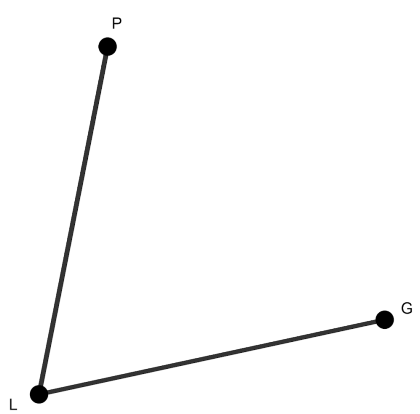

Which is the larger angle? Explain how you know.

# A Brief History of Angles and Degrees

Most people agree that the Babylonians were the first people to measure angles using **degrees**. Babylon was a key city in ancient Mesopotamia. The city was built between the Tigris and Euphrates rivers, just south of Baghdad in what is modern-day Iraq. The Babylonian system for measuring angles is one of the oldest forms of measurement still in use today. Using Fahrenheit to measure temperature is about 300 years old. Measuring temperature with Celsius is about 275 years old. The metric system (millimeters, centimeters, meters, etc) is about 225 years old. The Babylonian system of measuring angles was developed more than 3,500 years ago!

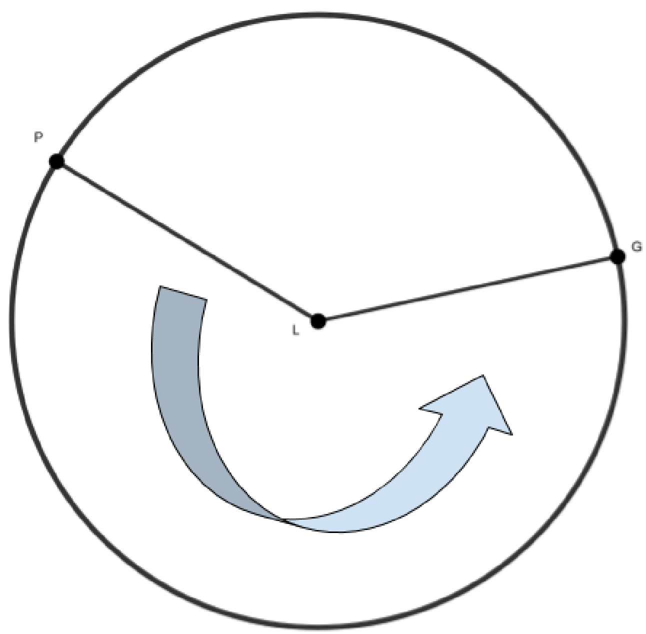

The Babylonians used a circle to describe the entire range of possible angles.

Here is ∠PLG.

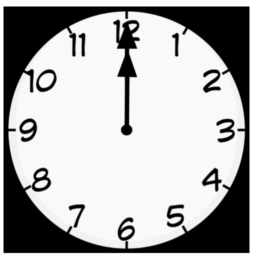
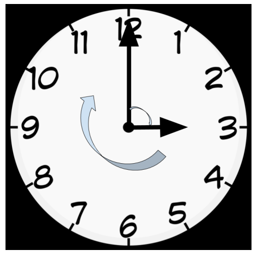
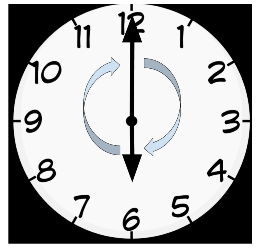
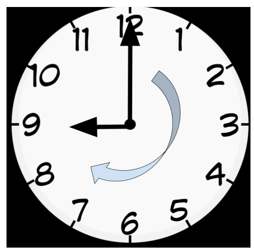
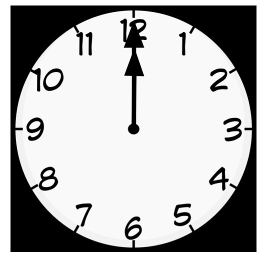



Eventually, $\overline{PL}$ would come all the way around to meet $\overline{LG}$.

Imagine if we made the opening bigger by rotating $\overline{PL}$ counterclockwise.



Let's think about the hands of a clock.

| | | |
|---|---|---|
| The clock below reads twelve o'clock. Let's imagine it is noon. We can think of the center of the clock as a vertex of the angle formed by the two arms of the clock.  | As the hour passes, the opening between the two arms gets larger.    | Once the hour hand returns to the 12 at midnight, it has moved all the way around the vertex.  |

The ancient Babylonians needed to decide what number of degrees they would use to measure the whole circle. They decided to use 360 degrees (also written as 360°) as the measure of an angle that goes all the way around a vertex.

You might be asking yourself, “Why 360? What is so special about that number?”

Well, for starters, it is very close to the number of days in a year. One year is defined as the amount of time it takes the Earth to complete one full rotation around the sun. It actually takes the Earth 365 days and 6 hours to complete its orbit around the sun, but we usually think of a year as 365 days.

*Babylonian scholars of 4000 years ago knew that there were 365¼* days in a year*. So why didn’t they decide that there should be 365 ¼ degrees in a circle?*

The answer to this question is very simple and very human: Who wants to do mathematics with the number *365* ¼? It’s an awkward number! The natural thing to do is to round it to a friendlier value.

*If we round the number 365 ¼ to the nearest five or the nearest ten we get 365 and 370, not the number 360. Why did we humans decide to round all the way* down *to 360?*

The answer here is also very simple and very human. Thousands of years ago there were no calculators and all arithmetic had to be done by hand or in one’s head. It is natural to want to work with numbers that are easier to calculate in your head. They also preferred working with numbers that could be divided with no remainders.

In life and in mathematics we often want to divide numbers by two and choosing 365 as the count of degrees in a circle is unfriendly. 365 divided by 2 is 182.5—no thanks!

370 and 360 are even at least.

We also often want to divide measurements by three as well. In that case, 360 is much better than 370. In fact, 360 is a much friendlier number for arithmetic over 370: you can divide it evenly into two equal pieces, three equal pieces, four, five, six, eight, nine, ten, twelve, fifteen, and many more. In fact there are 24 different ways to divide 360 into equal pieces.

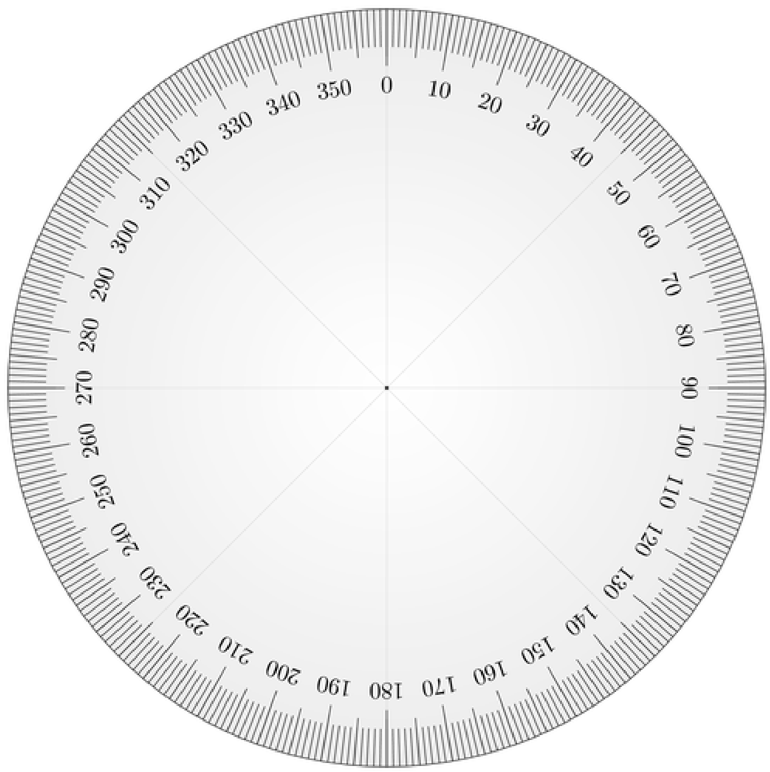
So, it was for two very human reasons—what we experience on this planet and our desire to avoid awkward work—that the Babylonians settled on the number 360 for the count of degrees in a circle.[2]

[2] Adapted from “Two Key - but ignored - Steps to Solving Any Math Problem” by James Tanton

# Dividing 360°

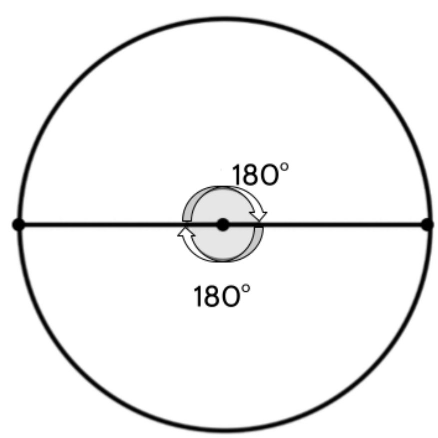
In the last section, you read that one of the reasons the Babylonians choose 360 as the number of degrees in a circle was because there are so many ways to divide 360 into equal groups. For this next activity, you'll identify some of those equal groups.

Remember a complete circle is 360°.

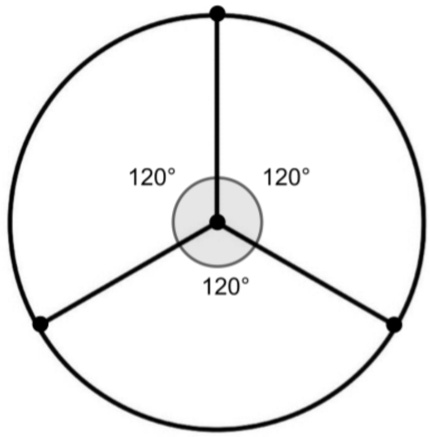



On the next few pages, there are circles. Each circle is divided into a different number of equal angles.

For example, this circle has two equal angles. If we divide 360° into two equal angles, each angle is 180°. Another way to think about this is that 180° halfway if you travel around the circle..

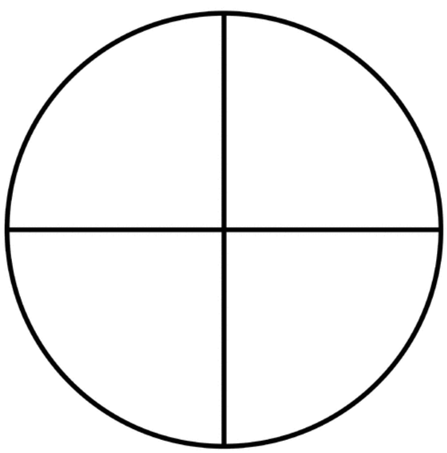


What if we divide the circle into three equal angles?

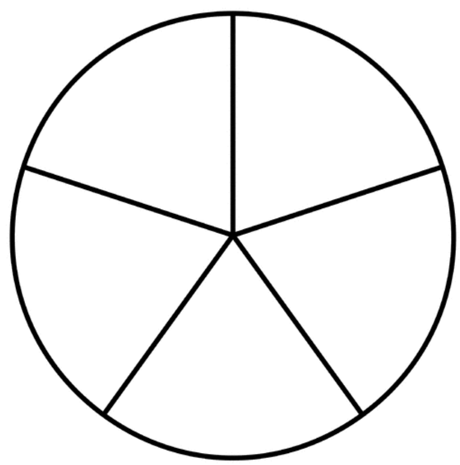



120 + 120 + 120 = 360

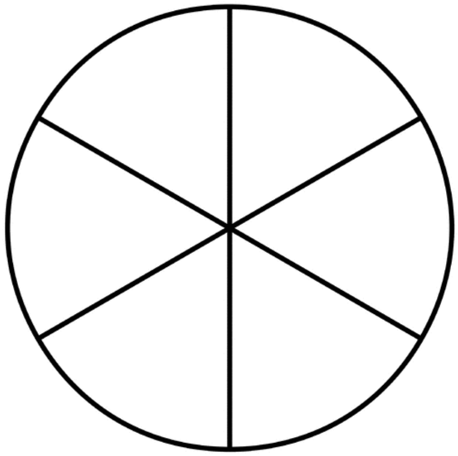
10. Find the value of and label each angle. Remember that each circle has to add up to a total of 360°.



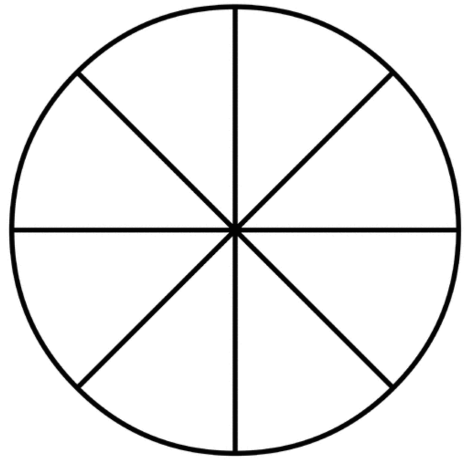
a.



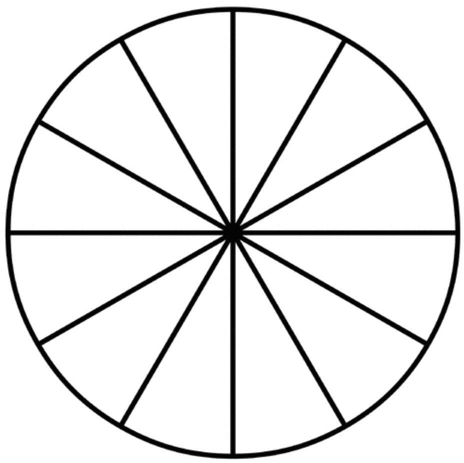
b.



c.



d.



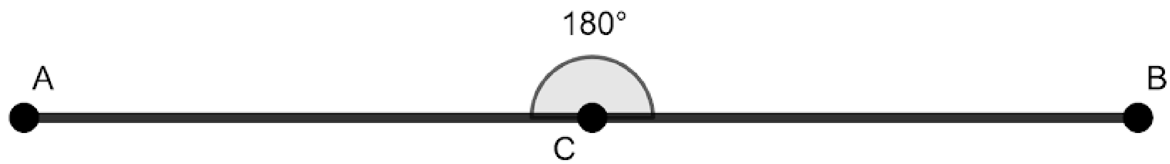
e.

Note: These aren't all the ways to divide up 360° into equal groups, but these are some common angles you might see.

# Measuring Angles

There are tools that we can use to measure angles, but there is a lot we can tell about them just by looking.

One angle that is easy to see is 180°.  As you saw in the Dividing 360° activity, a 180° angle is an opening that is half of a full circle, which is 360°. We can also see that an angle that is 180° is a straight line. For this reason,a 180° angle is often referred to as a **straight angle.**

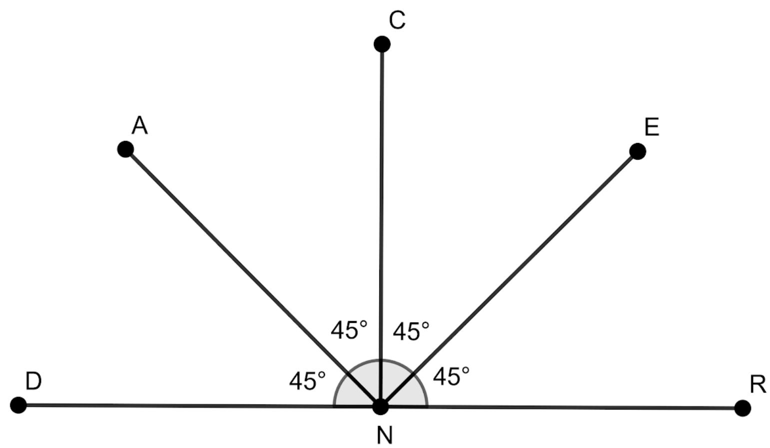



Look at this window. Do you see a straight angle? Do you see any other angles?

The bottom of the window is a straight line, so we can say it is a straight angle, and measures 180°. You might also notice that the window is divided up into 4 equal angles. If the whole opening is 180°, how much do you think each of those 4 angles would measure?

Take a minute to think about it before turning the page.

Even without any numbers being given, we can figure out all of the angle measurements. We know that the base of the window is a straight angle measuring 180°. If we divide 180° into 4 equal angles, we end up with four angles that measure 45° each.

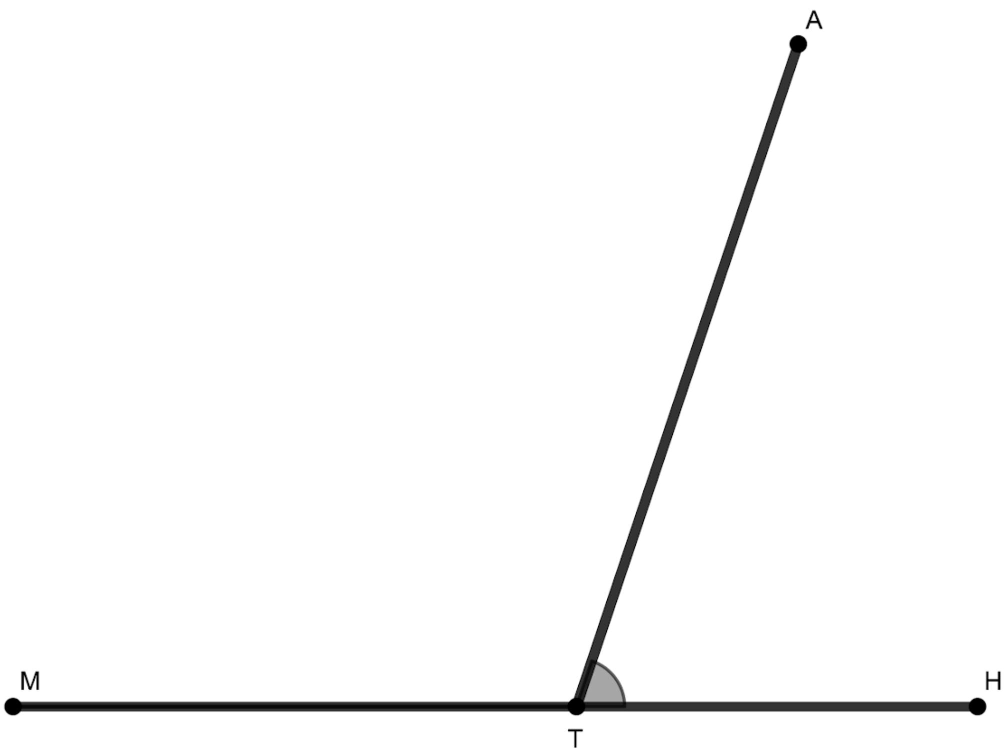



Straight Angle DNR can be divided into four 45° angles. 45° + 45° + 45° + 45° = 180°

11. There are three 90° angles in the diagram above. Try to name as many of them as you can below.

12. There are two 135° angles in the diagram as well. Name them.

In this diagram, straight angle MTH is divided into two angles by $\overline{AT}$.




The straight angle is divided into $\angle$MTA and $\angle$ATH.

Even without numbers, there is a relationship between those two angles. Whenever a straight angle is divided into smaller angles, we know all of those angles need to add up to 180°.

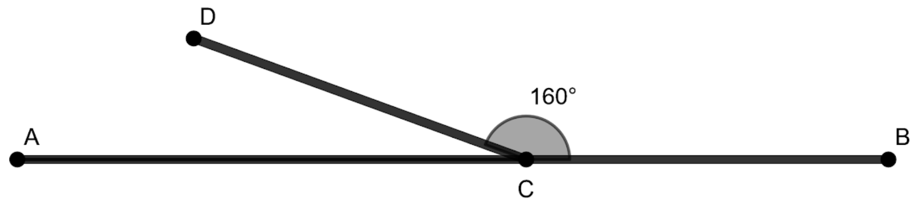
When a straight angle is divided into two angles, there is a special name for those two angles. We say they are supplementary angles.

**Supplementary angles** are any pair of angles that add up to 180°.

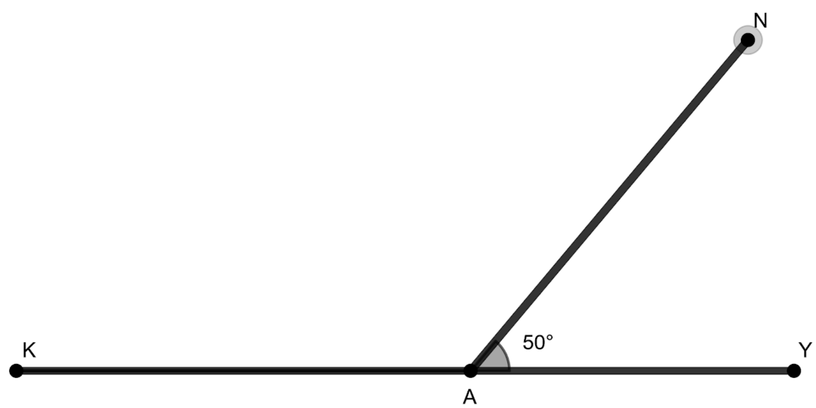
In the diagram above $\angle$MTA and $\angle$ATH are supplementary angles.

13. Use what you know about supplementary angles to find the measure of the missing angles in each of the examples below:

a.




b.




14. Which of the following pairs of numbers could **not** represent supplementary angles?

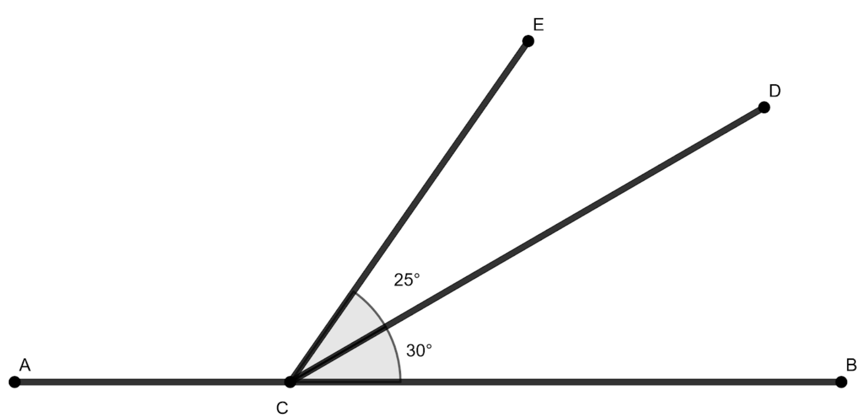
a. 155° and 25°

b. 90° and 90°

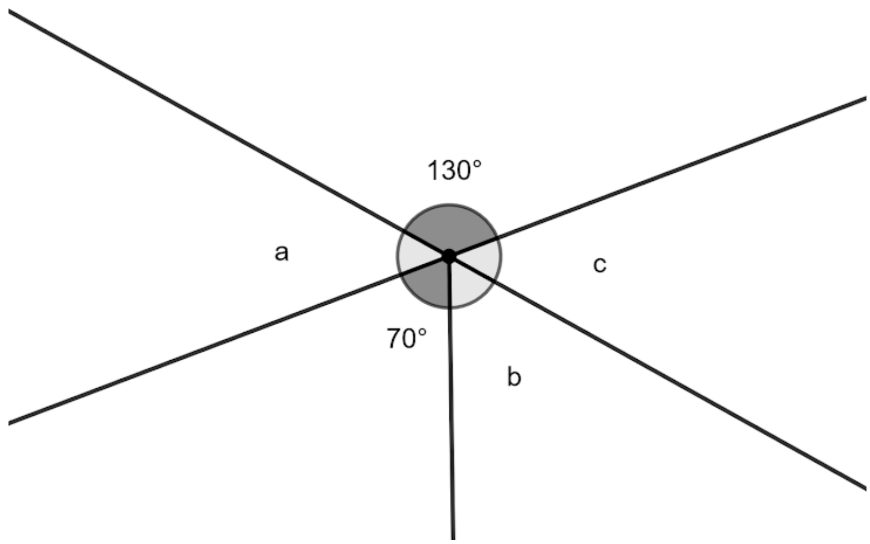
c. 178° and 12°

d. 70° and 110°

15. ∠ACF measures ___

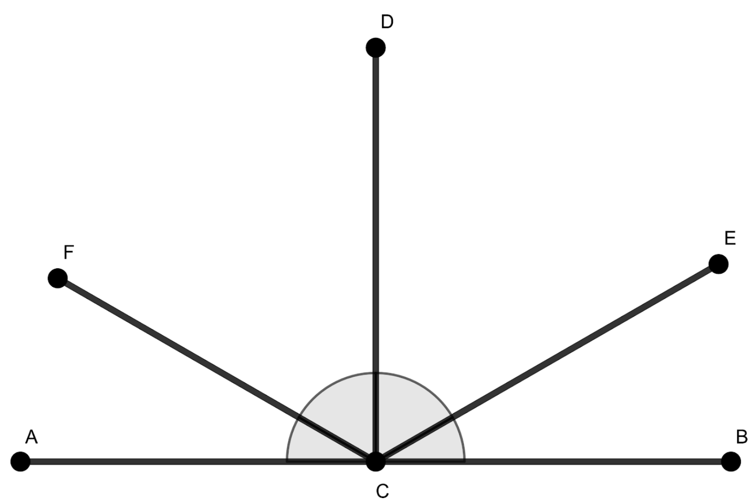


16. Find the measures of ∠a, ∠b, and ∠c.




17. ∠DCF is twice the size of ∠FAC.
∠DCE is twice the size of ∠ECB.
∠DCA is equal to ∠DCB.

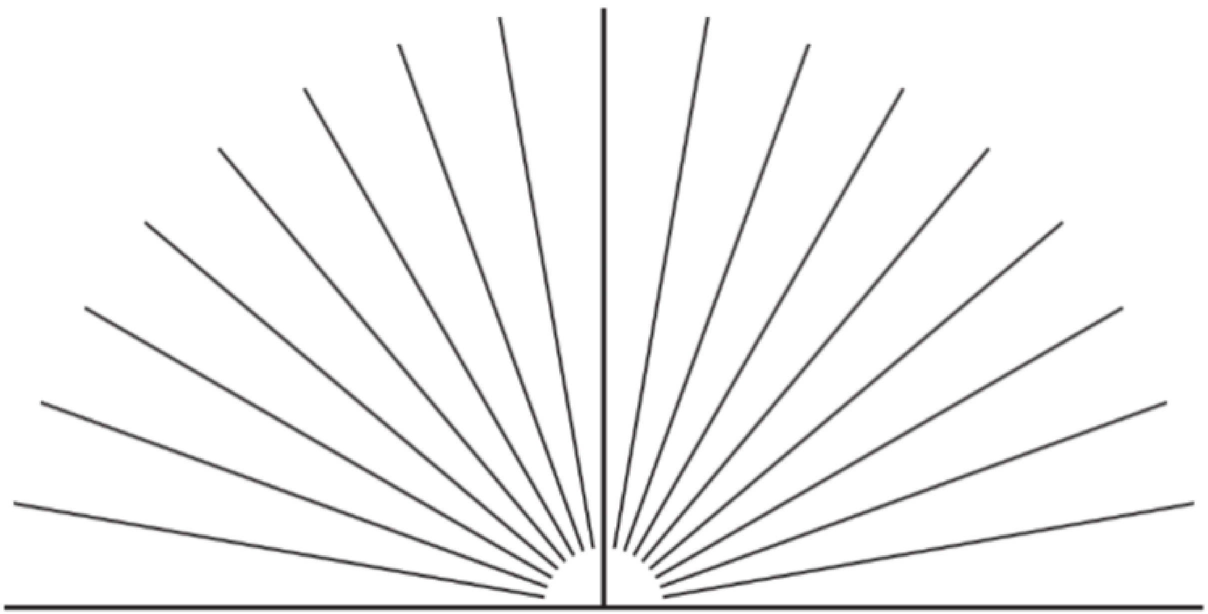
What is the measure of ∠FCE?



**Numberless Protractor**

There are different tools to measure angles depending on how precise or exact you need your measurements to be. One commonly used tool for measuring angles is called a protractor.

Many protractors have numbers, but this is a numberless protractor.



Write three things you notice about this numberless protractor.

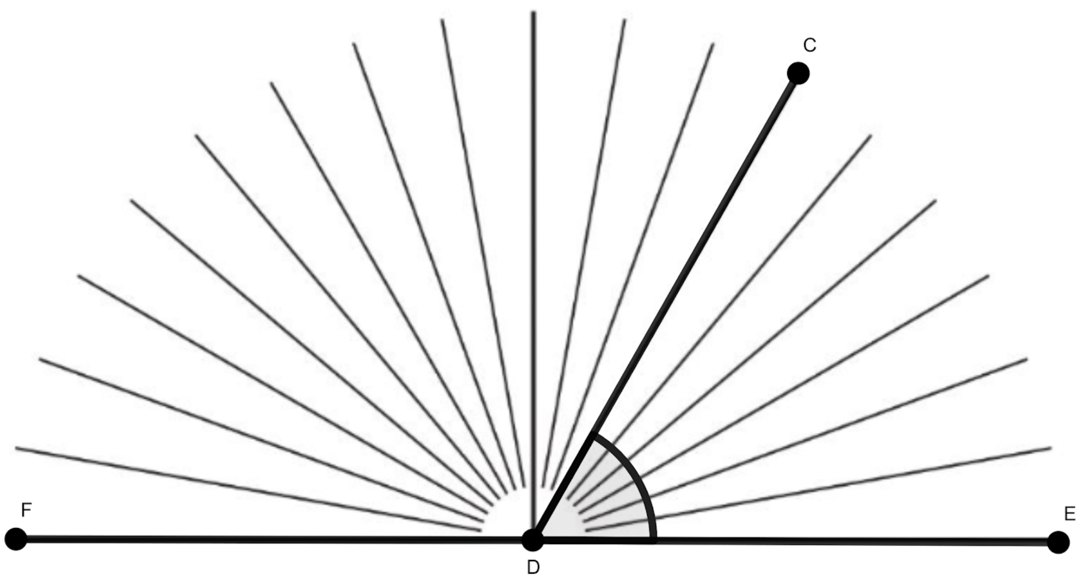
One thing you may have noticed is that there is a straight angle formed across the bottom of the numberless protractor. You may also have noticed that the straight angle is divided up into 18 equal angles.

18. How much does each of those smaller angles measure? How do you know?

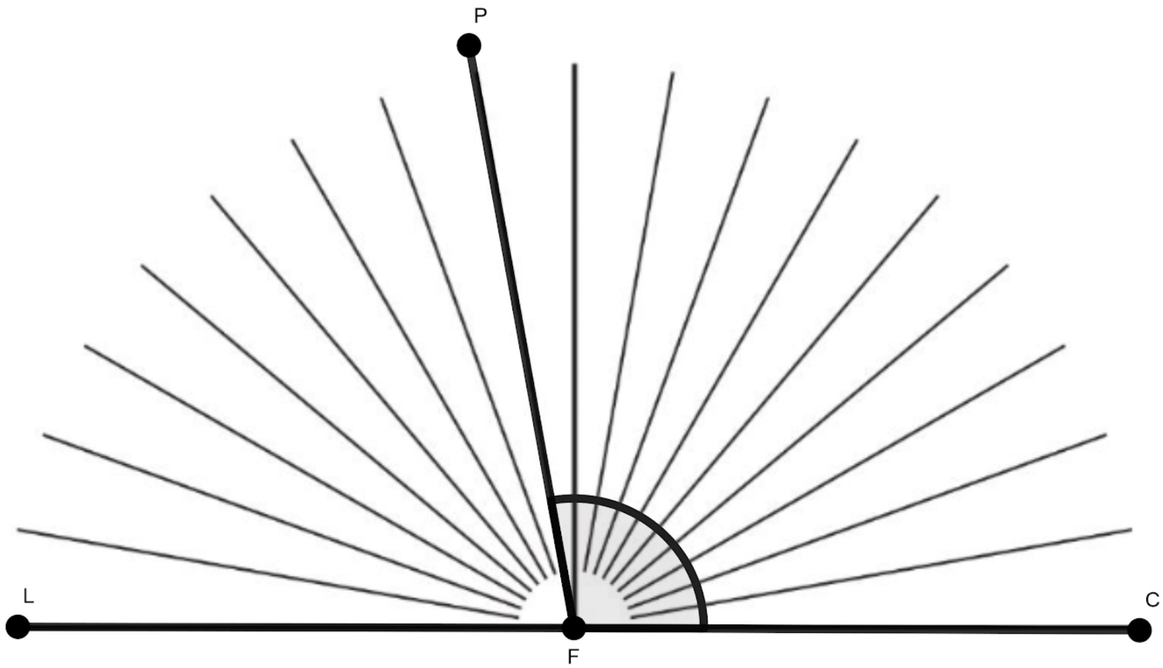
We can use a numberless protractor the same way we were able to look at the window and figure out the measurements of the smaller angles.

19. Use what you know about the numberless protractor to determine the measurements of ∠CDE and ∠FDC.

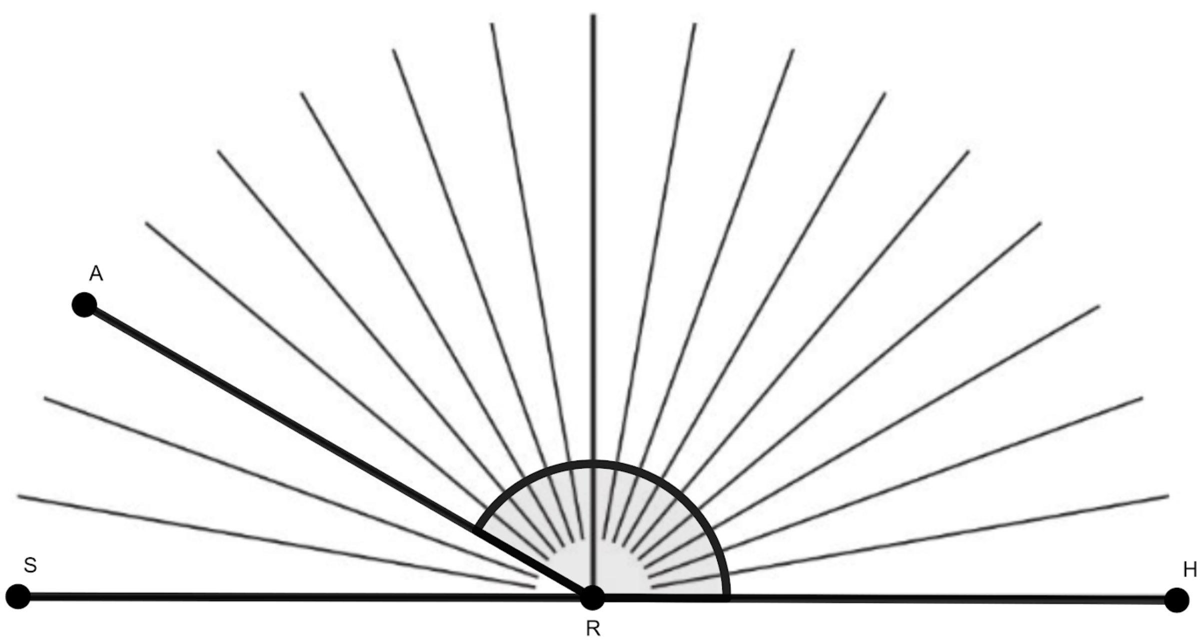
∠CDE is
∠FDC is



20. What is the measure of ∠LFP?
What is the measure of ∠PFC?

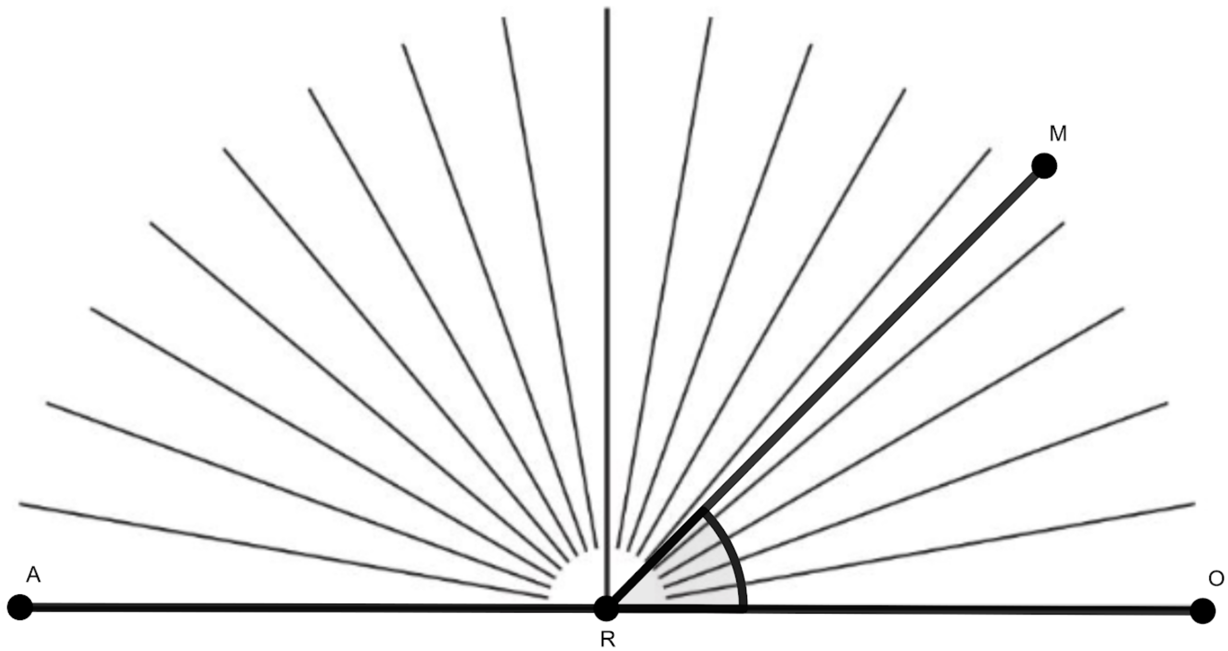



21. What is the measure of ∠SRA?
What is the measure of ∠HRA?

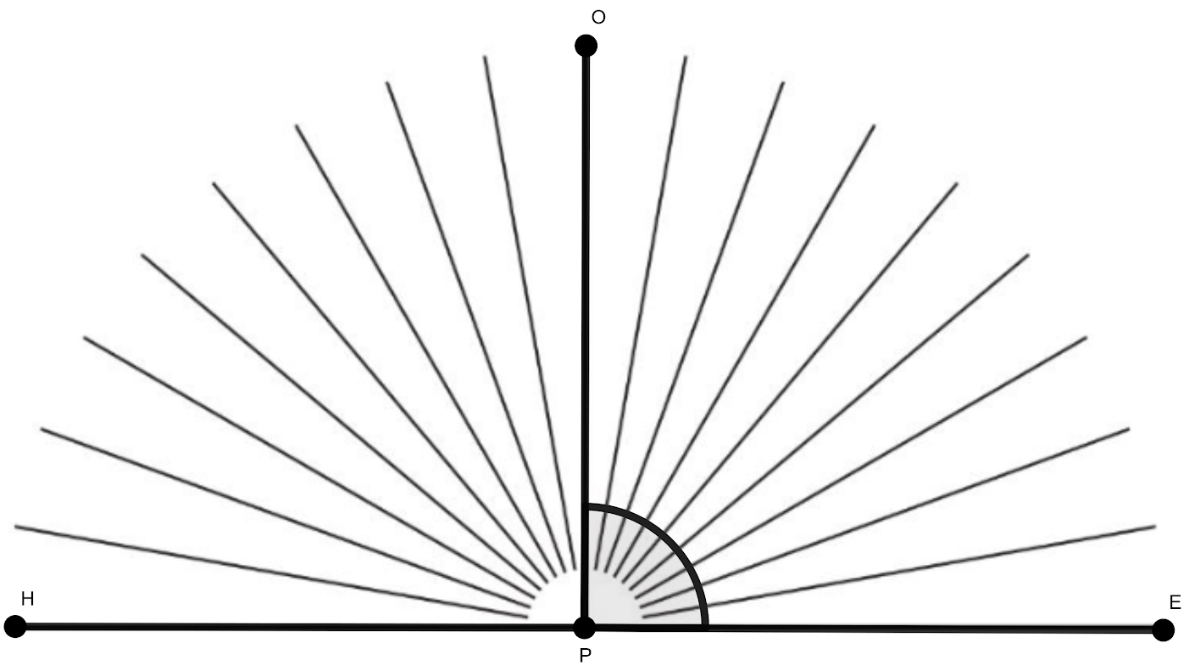


Here's one that is a little different from the others we've seen so far.

22. What is the measure of ∠MRO?
    What is the measure of ∠ARM?



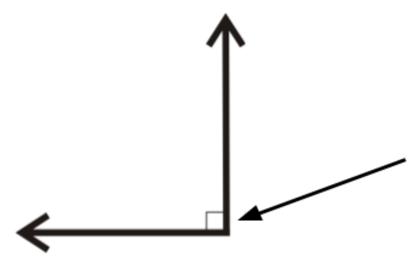

23. What is the measure of ∠HPO and ∠OPE?




In the last question you saw straight angle HPE divided in half by $\overline{OP}$, forming two 90° angles.

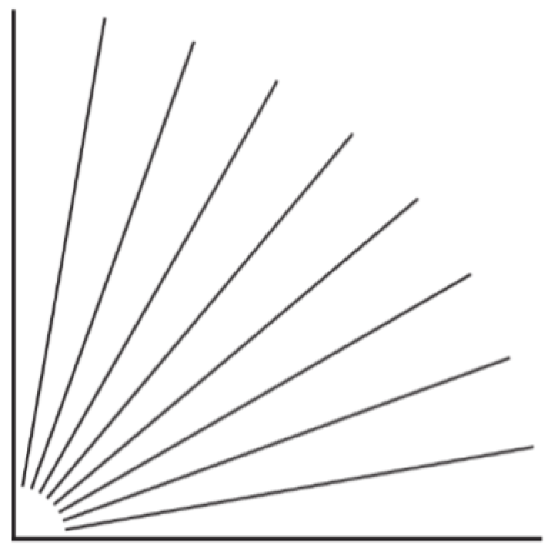
Up until now, our focus has been on 180° angles. We will now spend some time learning about 90° angles.

A 90° angle is called a **right angle**. Right angles are important, especially in construction. Right angles allow us to build perfect squares. You might see an angle with a small square in the angle. That means it is at a right angle and measures 90°.



Remember, any lines that intersect to form right angles are called perpendicular lines.

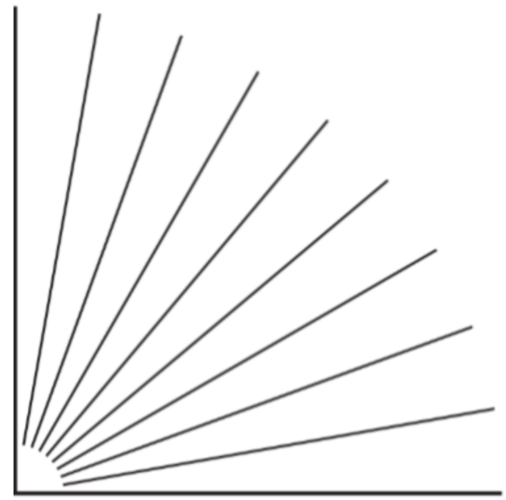
Here is a numberless protractor showing a right angle.



24. If you divide a right angle into <u>three</u> equal angles, what is the measure of each angle?

25. If you divide a right angle into <u>two</u> equal angles, what is the measure of each angle?

26. Use the numberless protractor on the right to draw a 60° angle.



27. What is the measure of the angle remaining after you drew your 60° angle?

When a right angle is divided into two angles, there is a relationship between those two angles. The two angles together need to add up to 180°. When a right angle is divided into two angles, there is a special name for those two angles. We say they are complementary angles.

**Complementary angles** are any pair of angles that add up to 90°.

The two angles you created above are complementary angles.

28. Which of the following pairs of numbers are complementary angles?

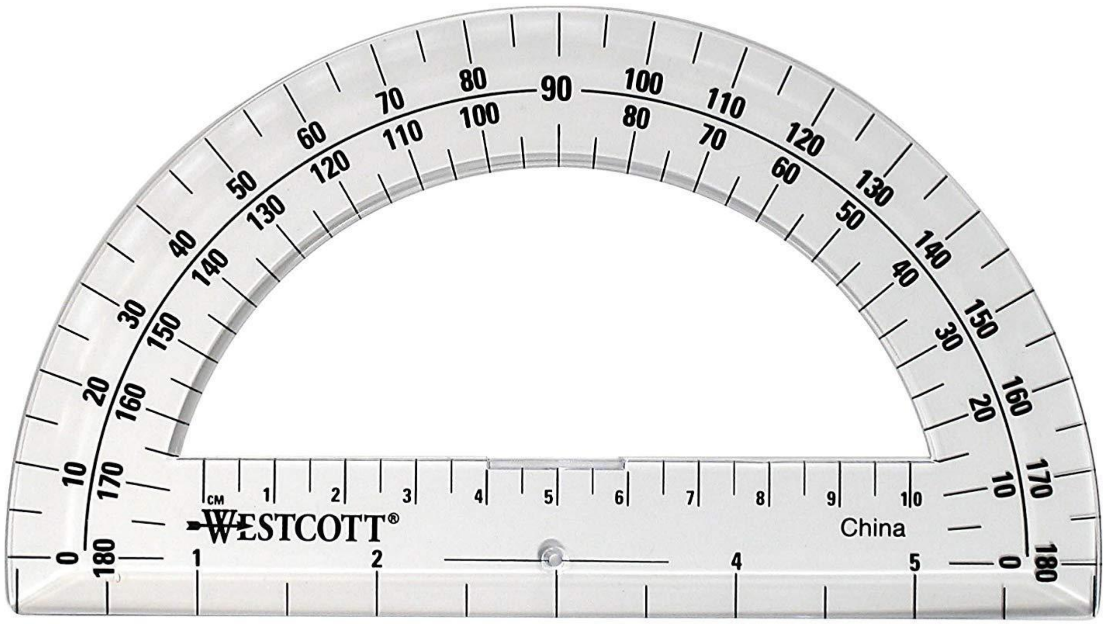
    a. 70° and 30°
    b. 65° and 25°
    c. 120° and 60°
    d. 110° and 70°

29. Which of the following pairs of numbers could **not** represent complementary angles?

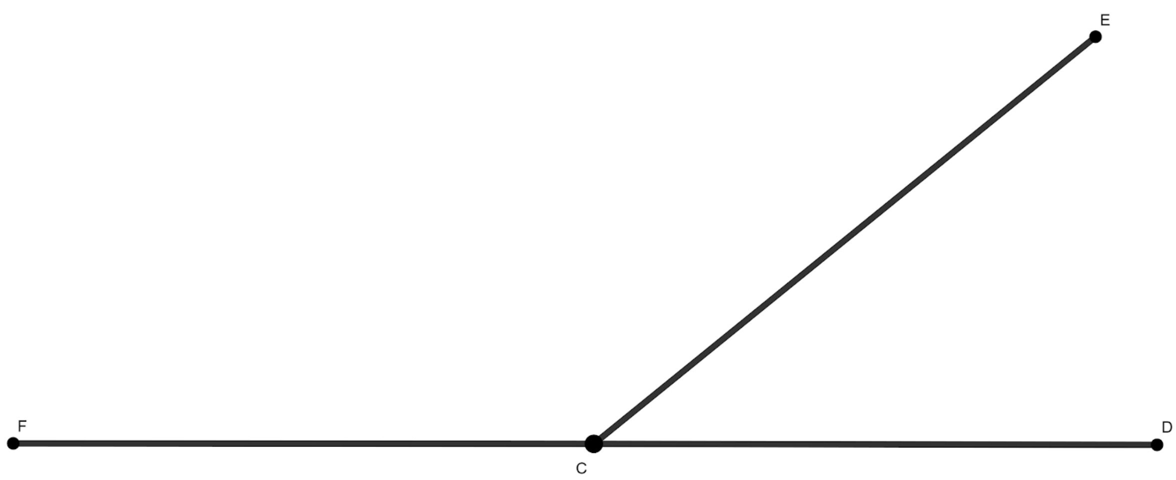
    a. 1° and 89°
    b. 14° and 76°
    c. 28° and 62°
    d. 90° and 1°

**Standard Protractor**

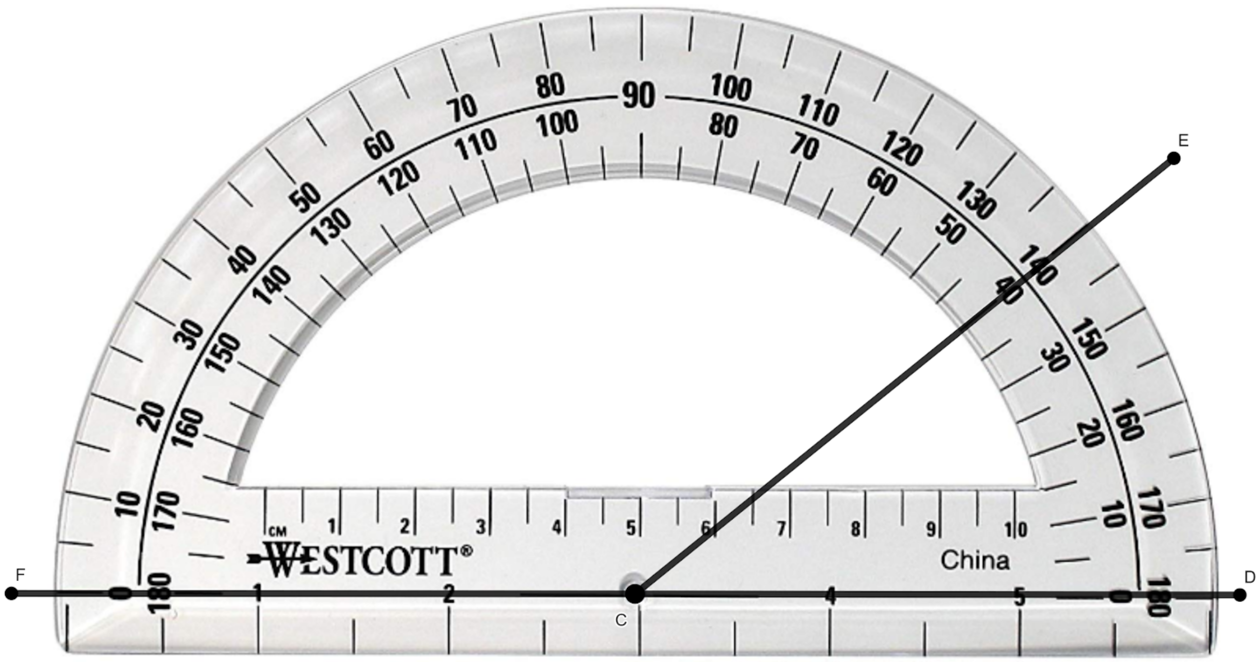
Here is a protractor with numbers.




Here are two supplementary angles.



Here are the angles and the protractor together.



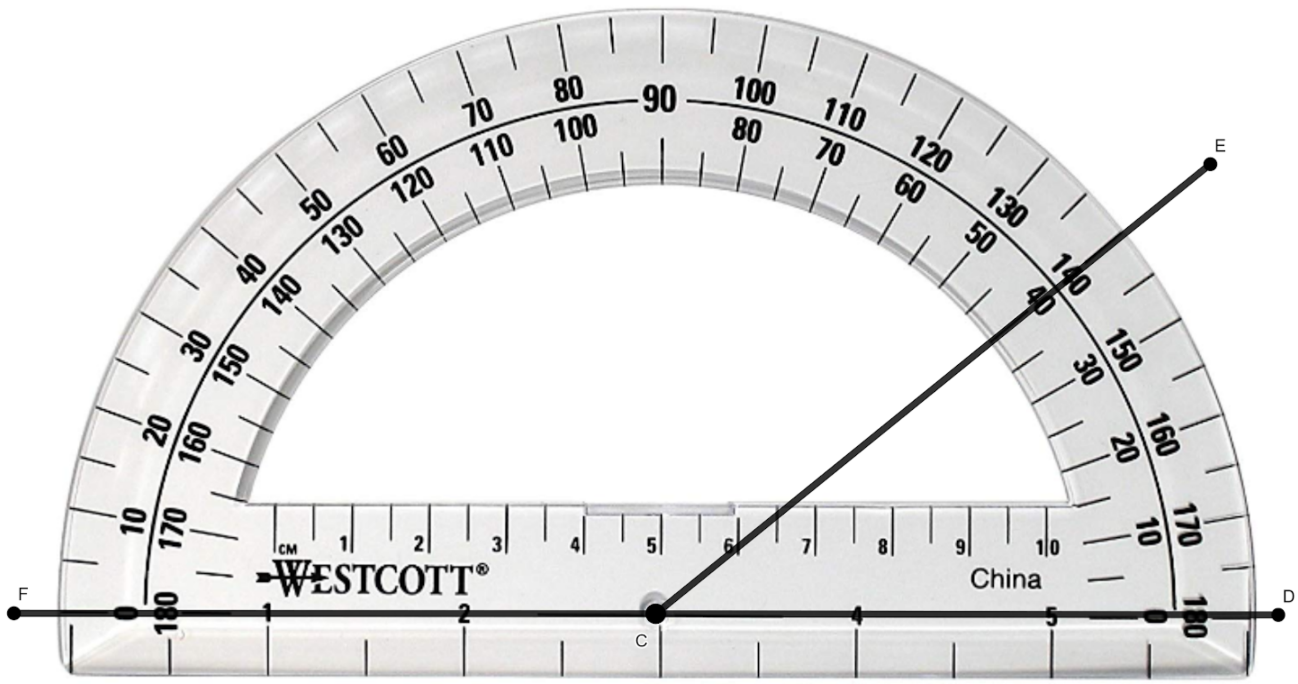

Write 4 things you notice.

| |
|---|
| |
| |
| |
| |

We can use protractors to measure angles the same way we use a ruler to measure length.

If you were using a tape measure to figure out how tall you are, you would not start measuring at your knees. You would line up the tape measure with the bottom of your foot and measure all the way up to the top of your head.

When we are measuring angles with protractors, we need to line them up properly. We do this by placing the small dot at the bottom of the protractor over the vertex of the angle.

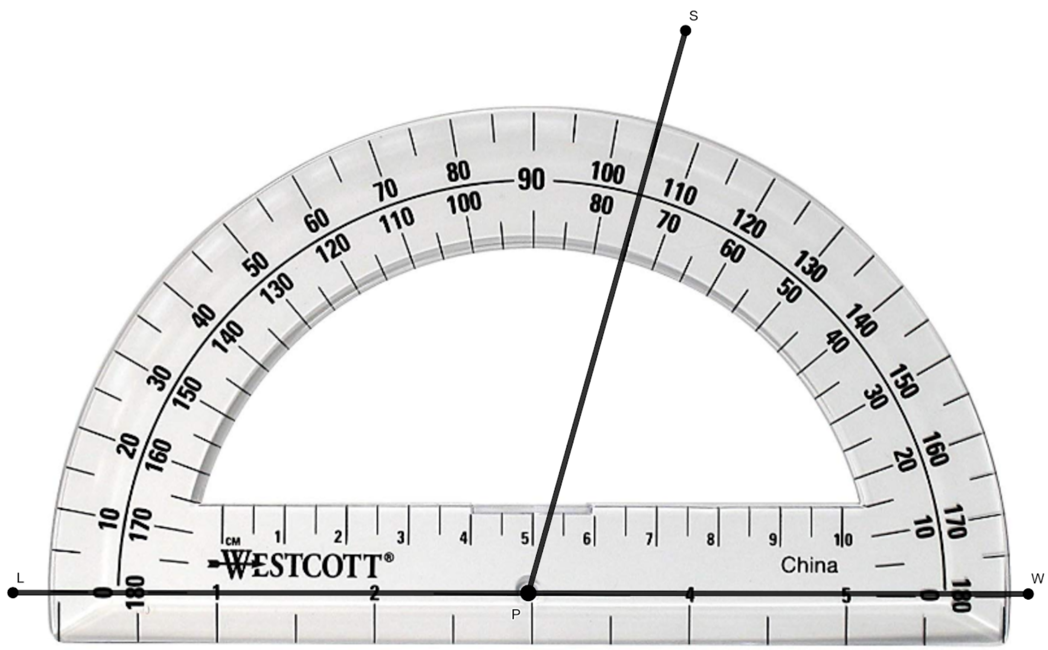



You may have noticed that there are pairs of numbers all the way around the protractor. You may have also noticed that each pair adds up to 180°. In the angle above, $\overline{CE}$ lines up with the measure for 140° and 40°. We can read this as the smaller angle (∠ECD) as 40° and the larger angle (∠FCE) as 140°.

For problems 27-30, use the protractor provided to find the measure of the angles.

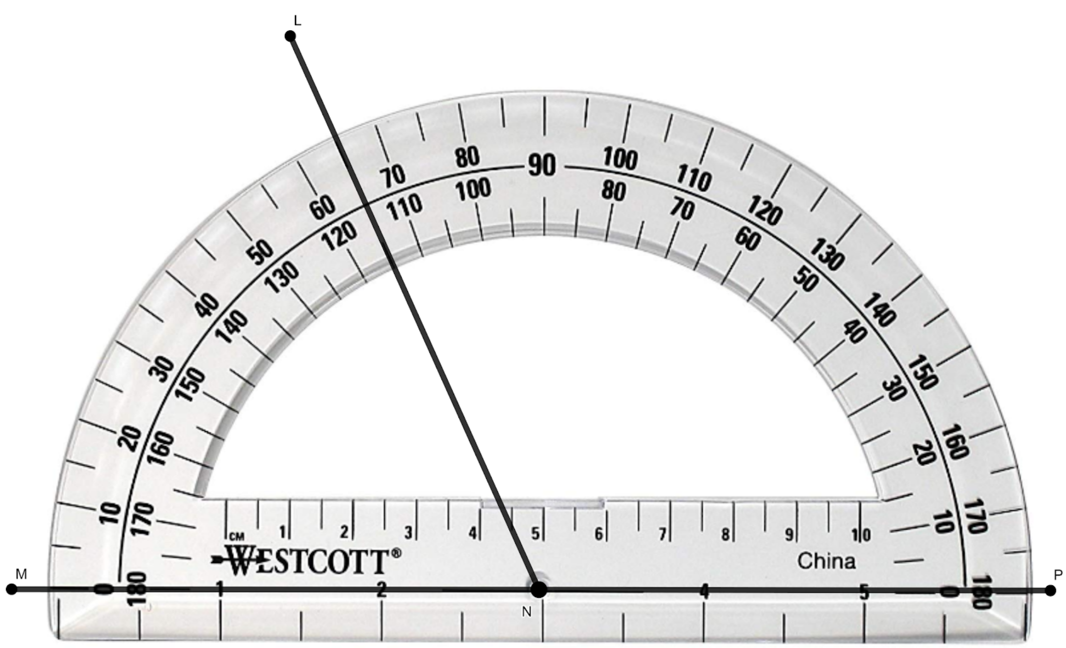
30. ∠SPW measures ___

∠LPS measures ___

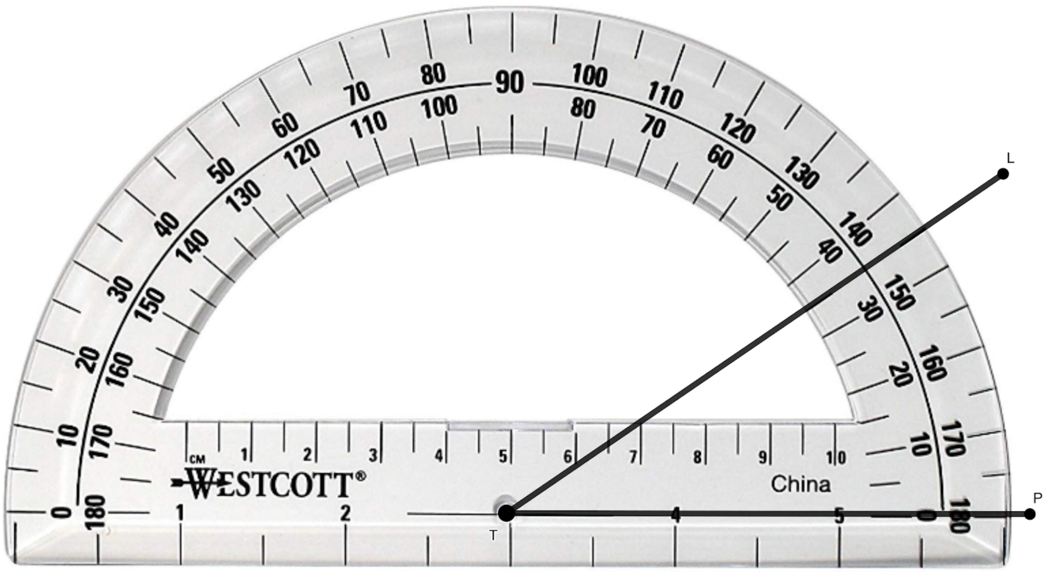



31. ∠MNL measures ___

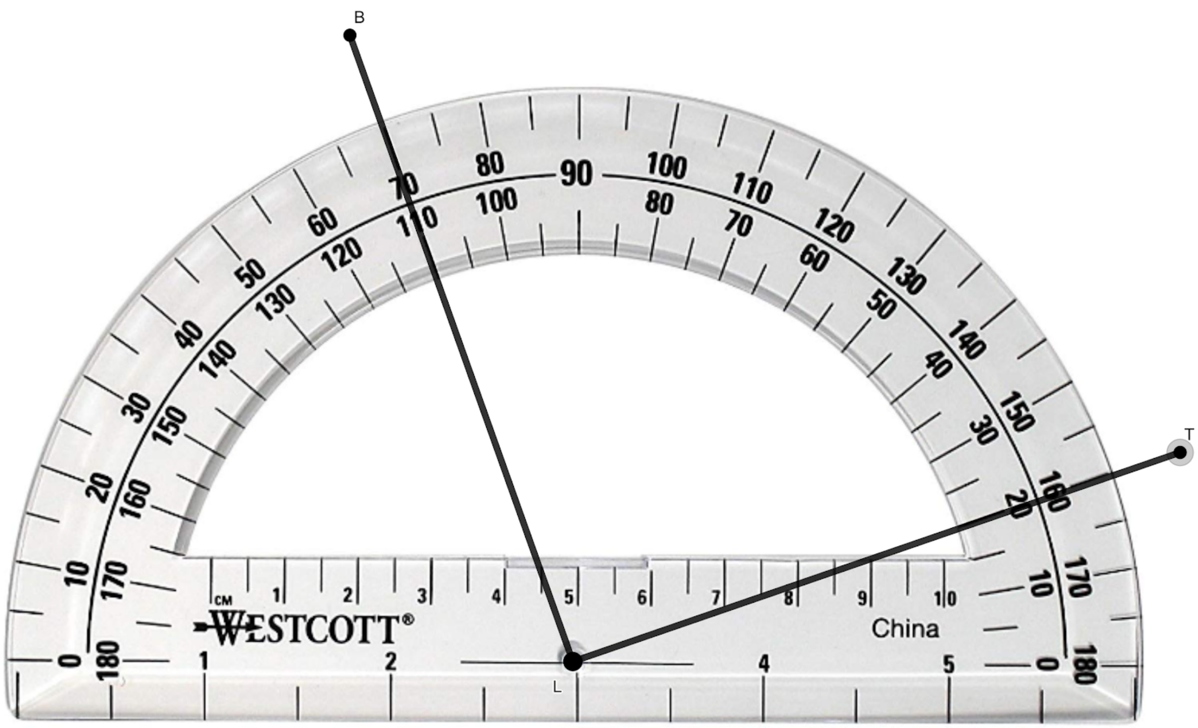
∠LNP measures ___



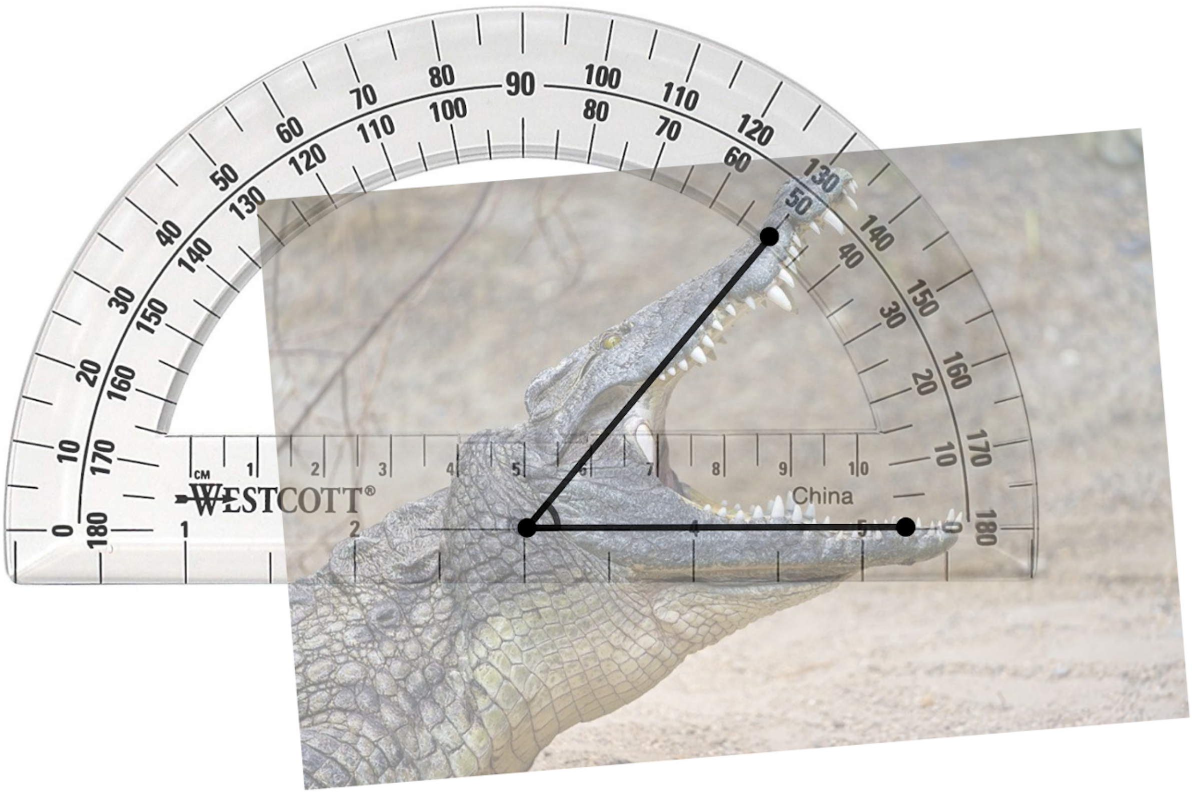
32. ∠LTP measures ___




33. ∠BLT measures ___

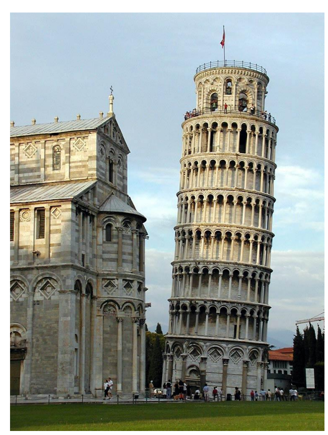


34. What is the angle of this alligator's open jaw?

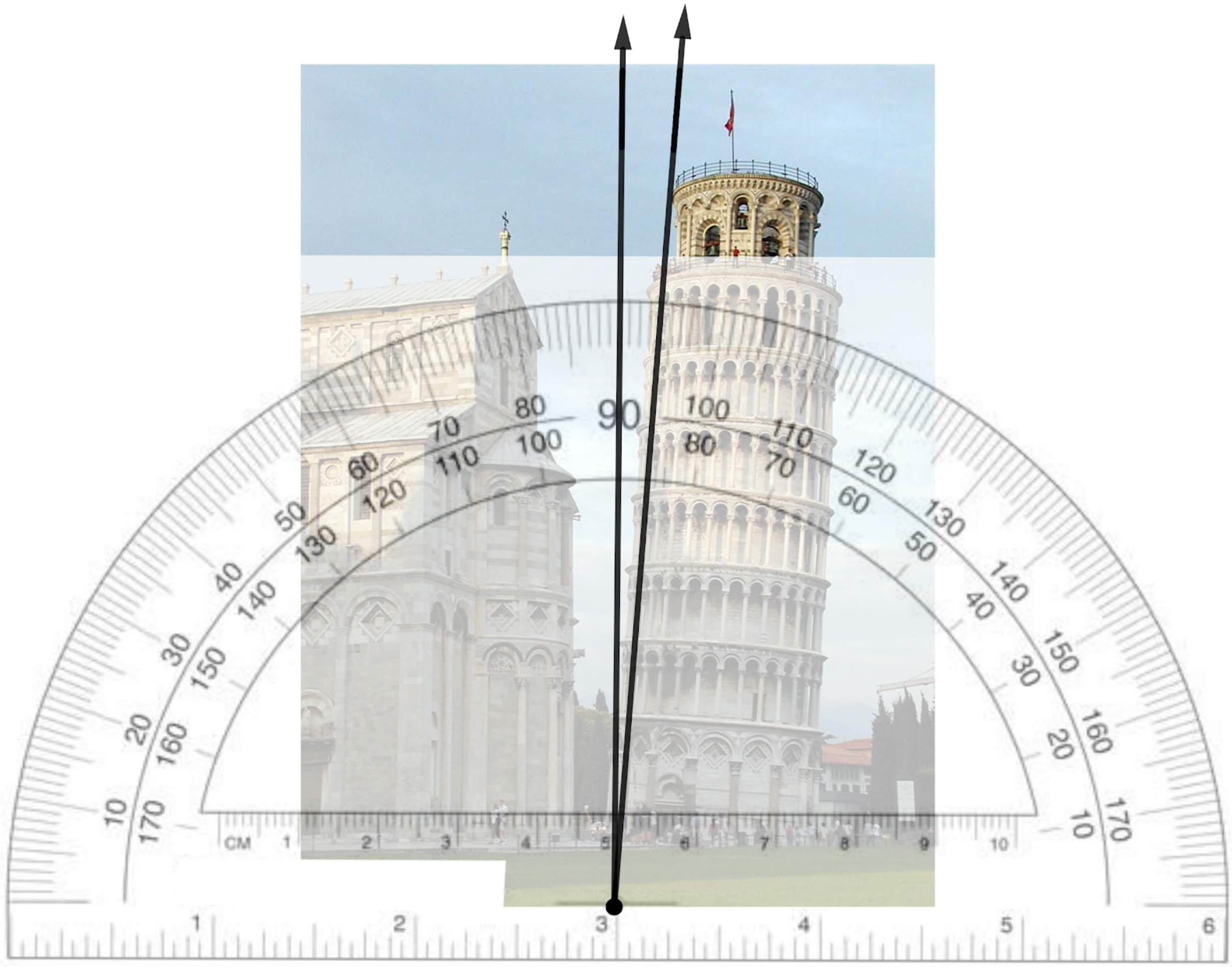


35. The Tower of Pisa is a bell tower located outside the cathedral of the Italian city of Pisa. It is famous because the building seems to lean due to an unstable foundation.

About how many degrees off-center is the Tower of Pisa?



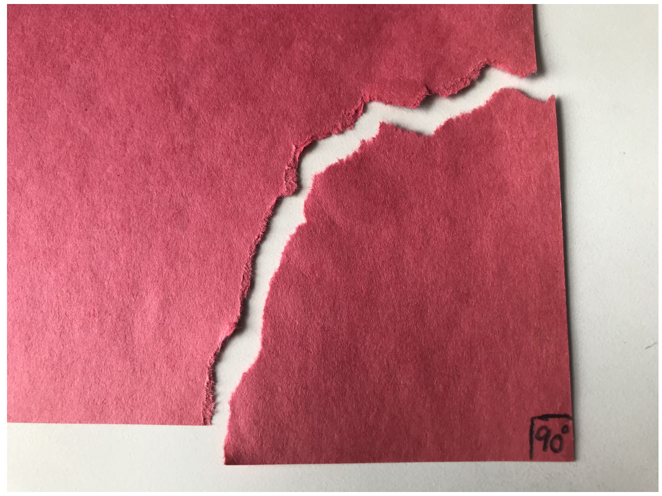
We can use an image of the tower together with a protractor to get a pretty good idea of how many degrees the tower is off-center.



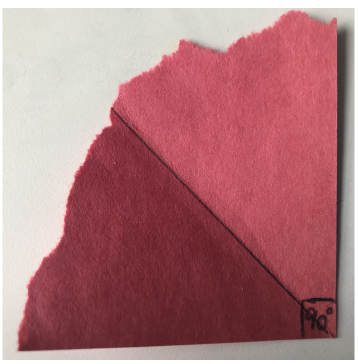
**Making Your Own Tool to Measure Angles**[3]

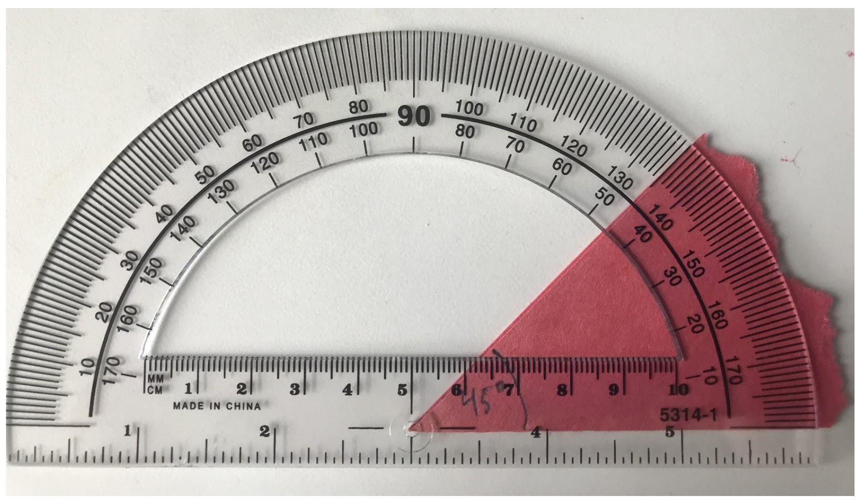
If you are like me, you might not carry a protractor with you at all times. So what can you do if you need to measure an angle?

Let's start with an angle that is easy to find. The corner of every piece of paper forms a right angle. Find a piece of paper and tear off a corner.



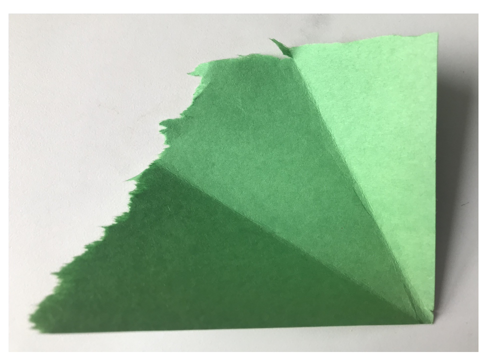

What happens if we fold that 90° angle in half?



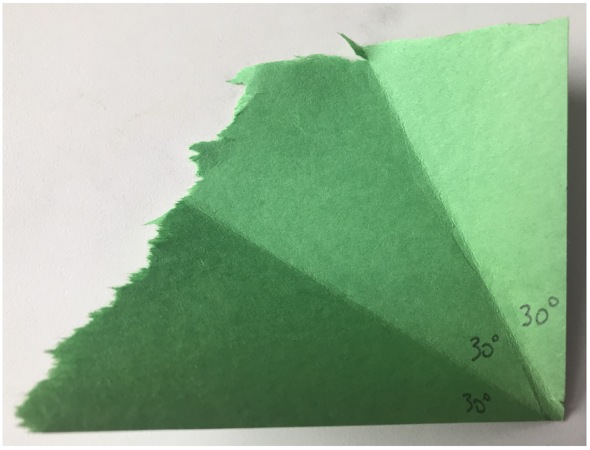

Now, tear off another corner and fold it into three equal angles.

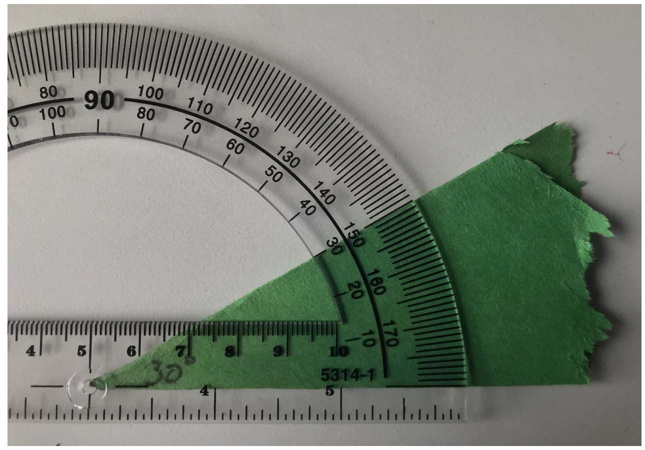
What is the measure of each of those angles?



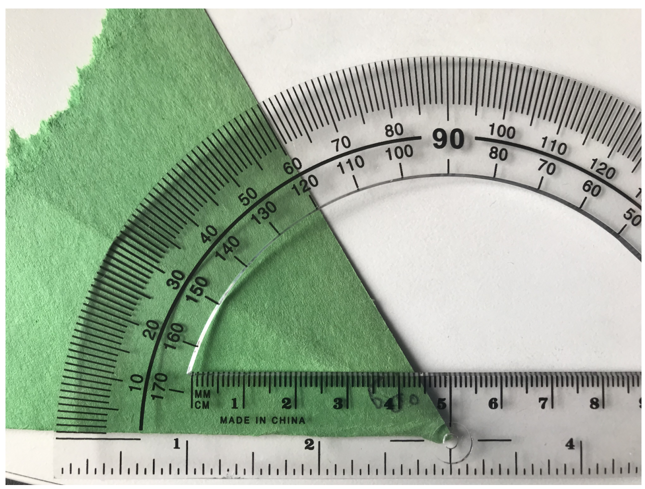
[3] This activity was inspired by the work of Vi Hart, viewable on YouTube.

If you divide 90° into three equal angles, each angle will equal 30°.



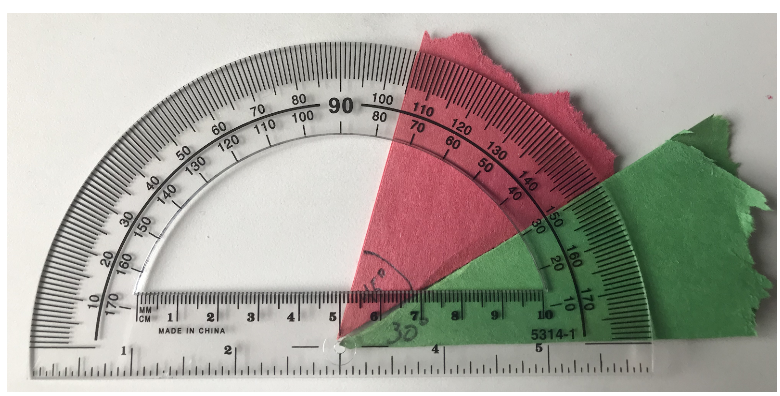

If you fold over one of those three sections, you can make a 60° angle.




With just two pieces of paper, we can now make angles that measure 90°, 45°, 30°, and 60°.

And if we combine those angles, we can make even more. Here's a combination of 45° and a 30° to make a 75° angle.

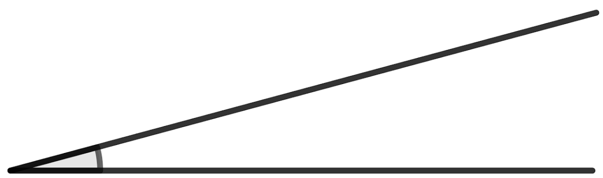
What other angles can we make using combinations of 90°, 45°, 30°, and 60°?



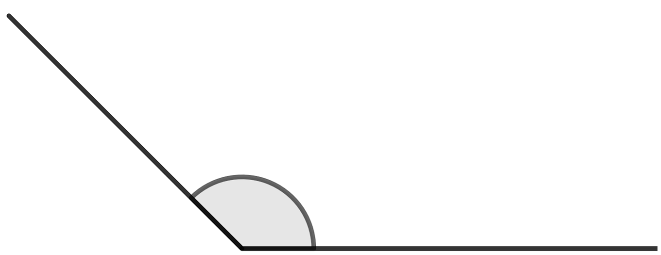
Use your paper tools and what you know about angles draw a line from each angle to their measurement in degrees.

36.

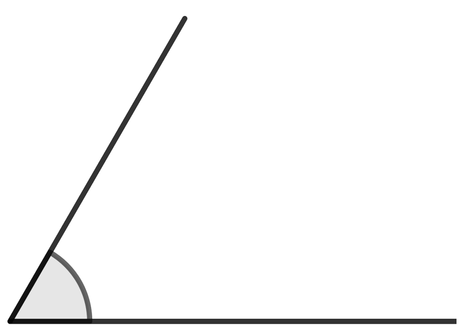
37.



38.



39.



60°

135°

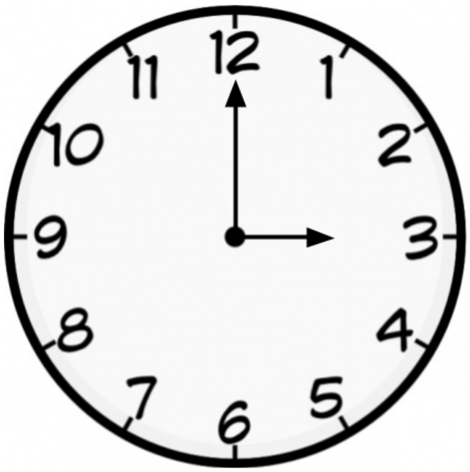
105°

15°

**Clock Angles**

Find the measure of the angle formed by the hands of each clock.

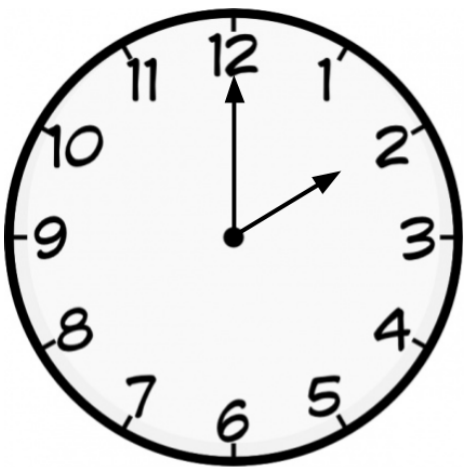
40.




When it is 3 o'clock, the hands of a clock form an angle that is _____ degrees.

Explain how you got your answer:

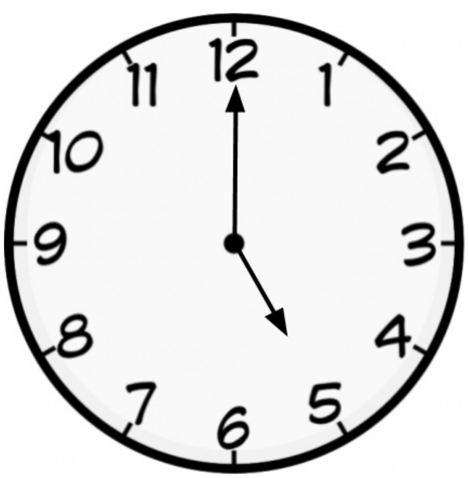
41.




When it is 2 o'clock, the hands of a clock form an angle that is _____ degrees.

Explain how you got your answer:

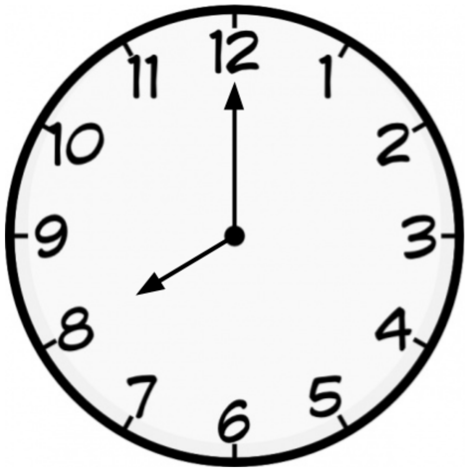
42.




When it is 5 o'clock, the hands of a clock form an angle that is _____ degrees.

Explain how you got your answer:

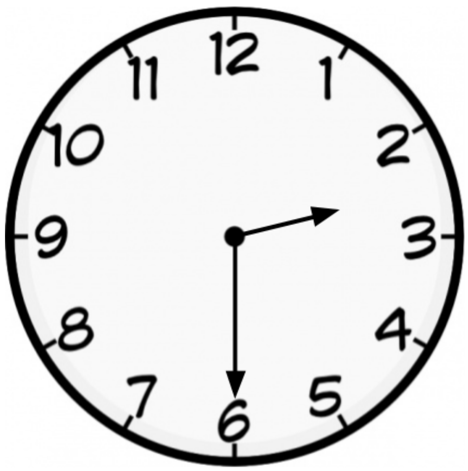
43.




When it is 8 o'clock, the hands of a clock form an angle that is _____ degrees.

Explain how you got your answer:

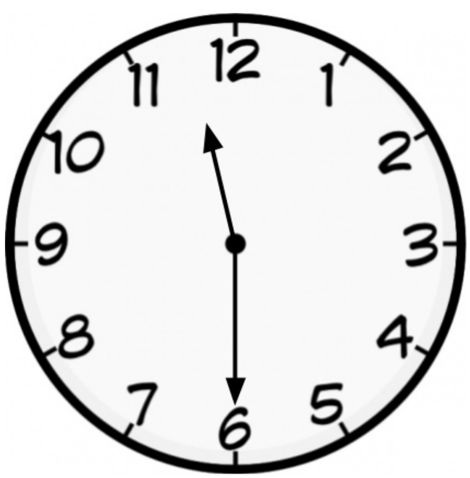
44.




When it is 2:30, the hands of a clock form an angle that is _____ degrees.

Explain how you got your answer:

45.



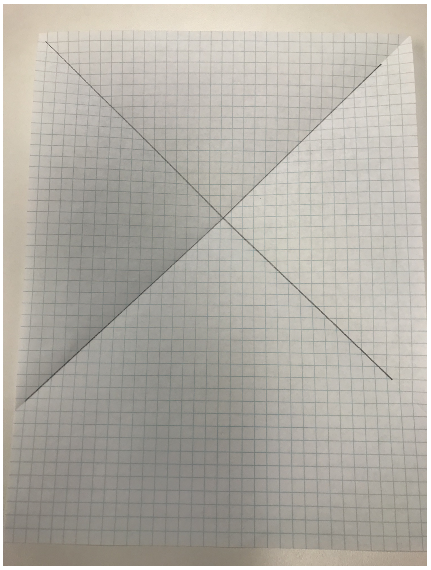

When it is 11:30, the hands of a clock form an angle that is _____ degrees.

Explain how you got your answer:

## Special Angle Relationships[4]

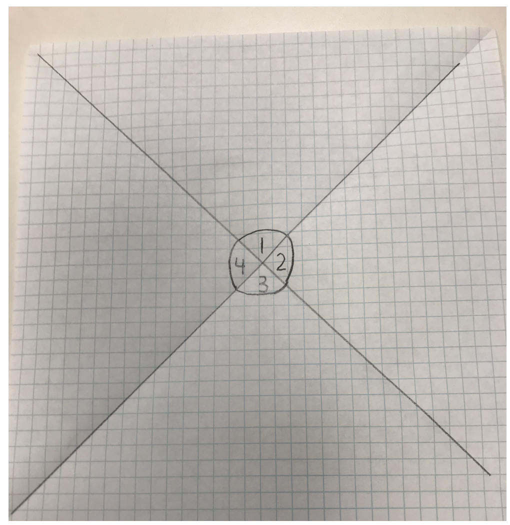
In the last section we looked at how to measure angles. In this section, we are going to discover some special relationships between angles.

For the activities in this section you will need some blank paper and a pair of scissors.



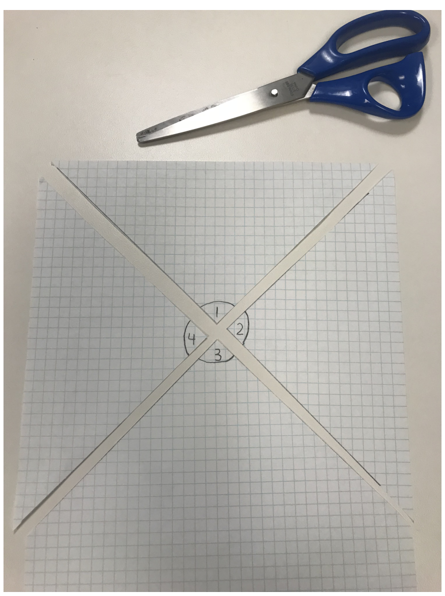
**Step One:**

Take one sheet of blank paper and fold it twice so that the folds create an "X". It will form 4 angles. It does not matter if the angles look exactly the same as the one in the example to the left.




**Step Two:**

Number the angles 1, 2, 3, and 4. Start with the angle at the top and number them clockwise.




**Step Three:**

Cut out the four angles. You might want to trace the folds with a pencil to help you cut in a straight line.

**Step Four:** Take a few minutes and move the angles around. What relationships do you notice?

---

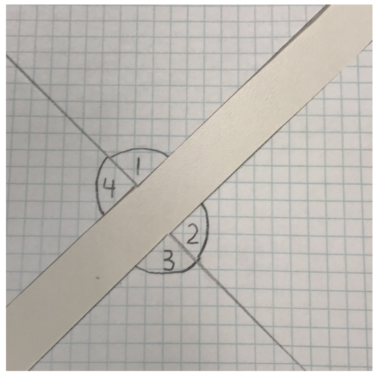
[4] This activity was inspired by the work of Kyle Pearce.

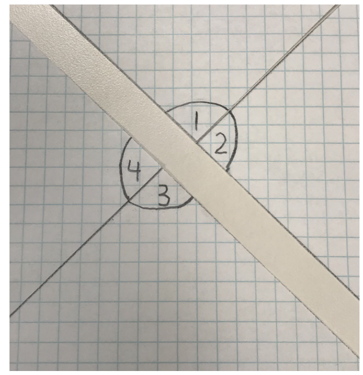
Some relationships you may have noticed:

- When you put some of the angles next to each other, they form straight angles of 180°.

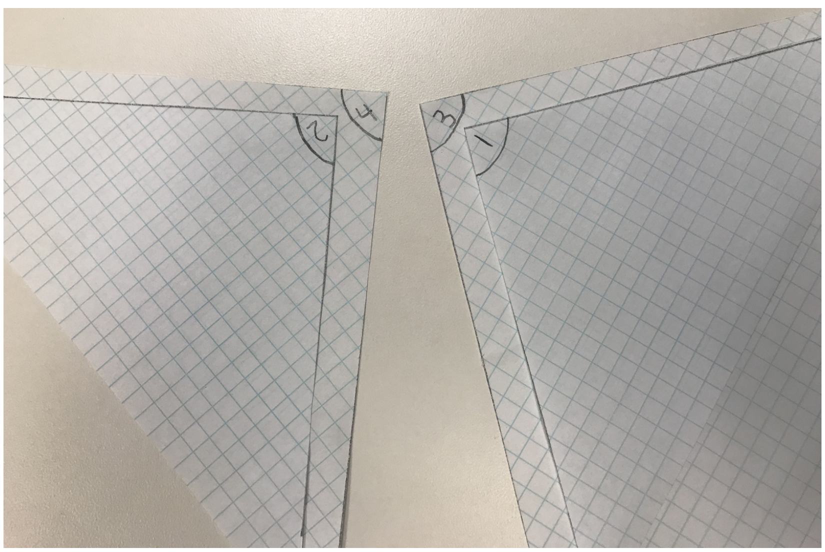
  In the example below, the following pairs of angles are supplementary:

  - ∠1 and ∠4
  - ∠2 and ∠3
  - ∠1 and ∠2
  - ∠3 and ∠4



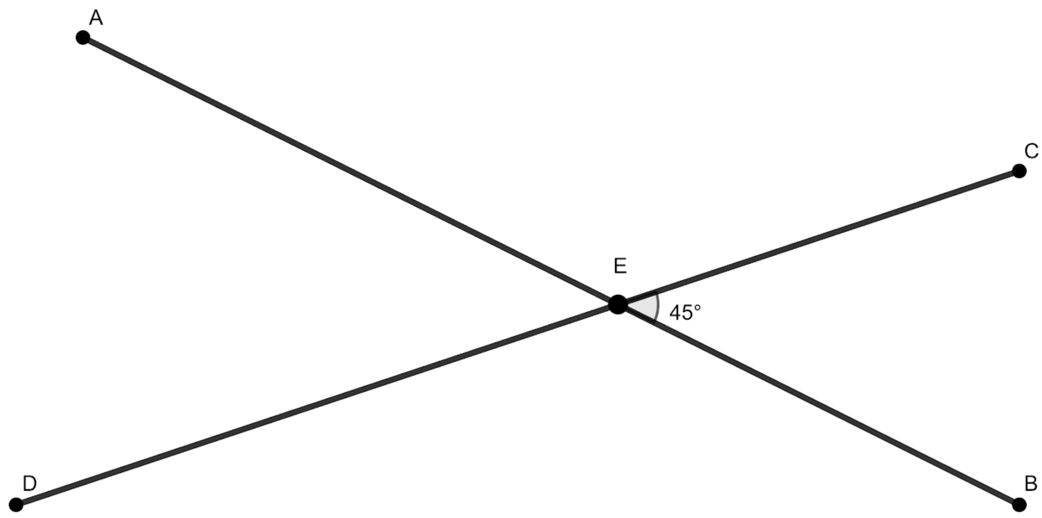




- The angles opposite each other are the same. You can take ∠2 and it fits exactly over ∠4. And you can take ∠1 and exactly cover ∠3.



When two lines intersect to form four angles, the opposite angles are equal.

46.

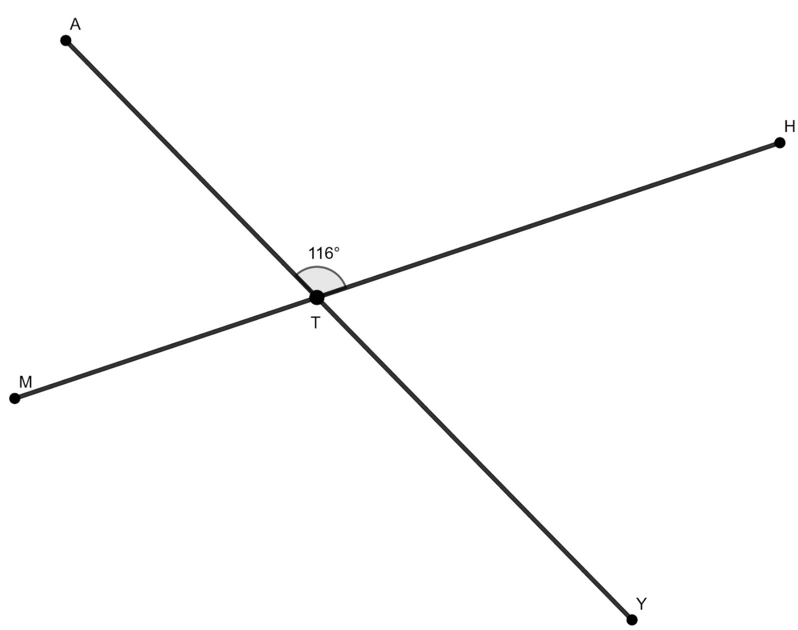



∠ CEA measures ______

∠ AED measures ______

∠ DEB measures ______

47.

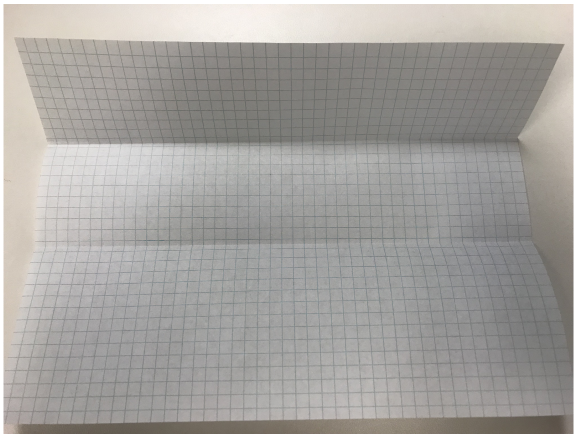



∠ MTA measures __

∠ HTY measures __

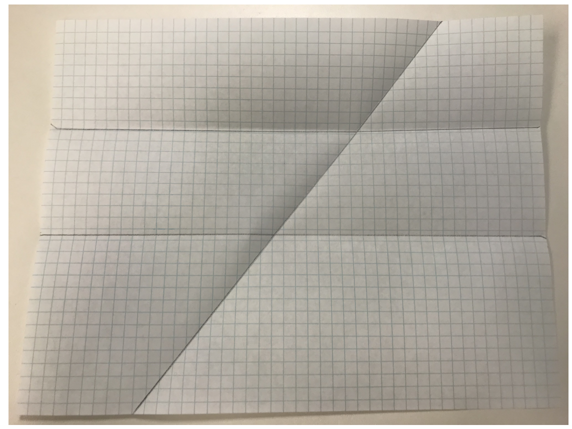
∠ MTY measures __

**Angles and Parallel Lines**



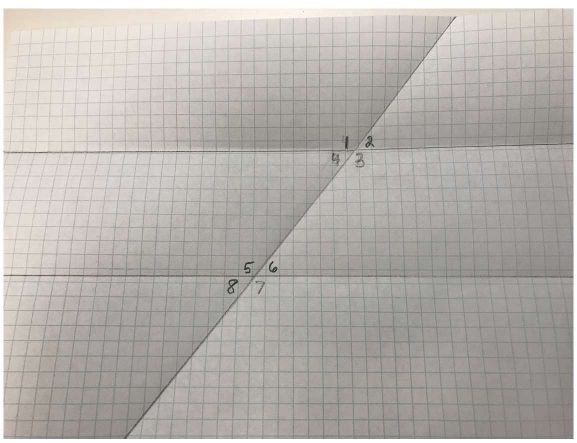
**Step One:**

Take another blank sheet of paper. Fold it twice to create two parallel lines with the folds.



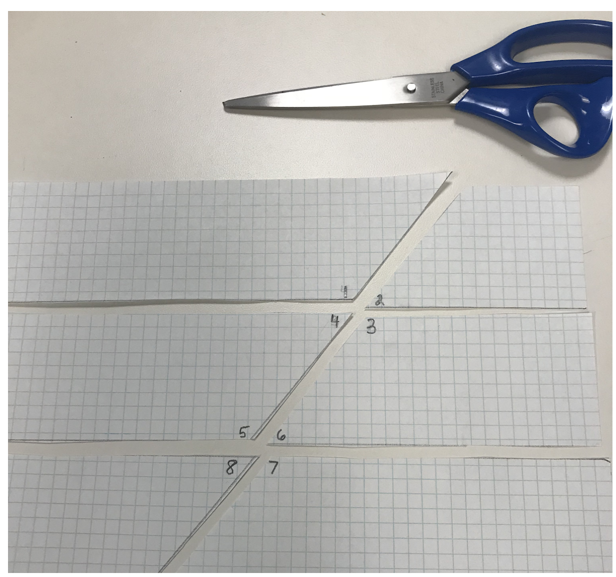
**Step Two:**

Fold the paper again so that the fold cuts through both of the parallel lines.




**Step Three:**

Number the angles in order, going clockwise.




**Step Four:**

Cut out the angles. You might also want to trace the folds with a pencil to help you cut straight.

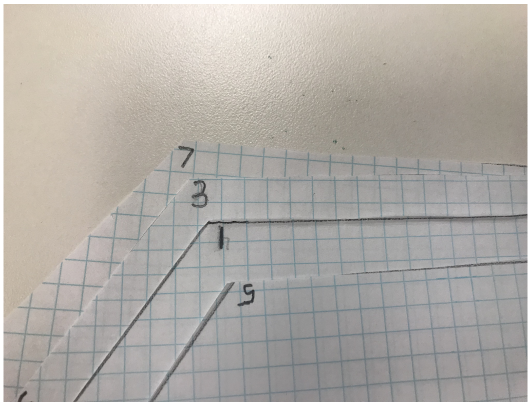
**Step Five:**

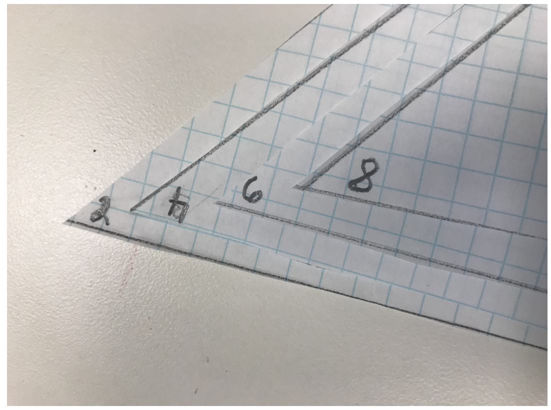
Take a few minutes and move the angles around. What relationships do you notice?

When you have two parallel lines that are both intersected by a straight line, it will make two groups of opposite angles.

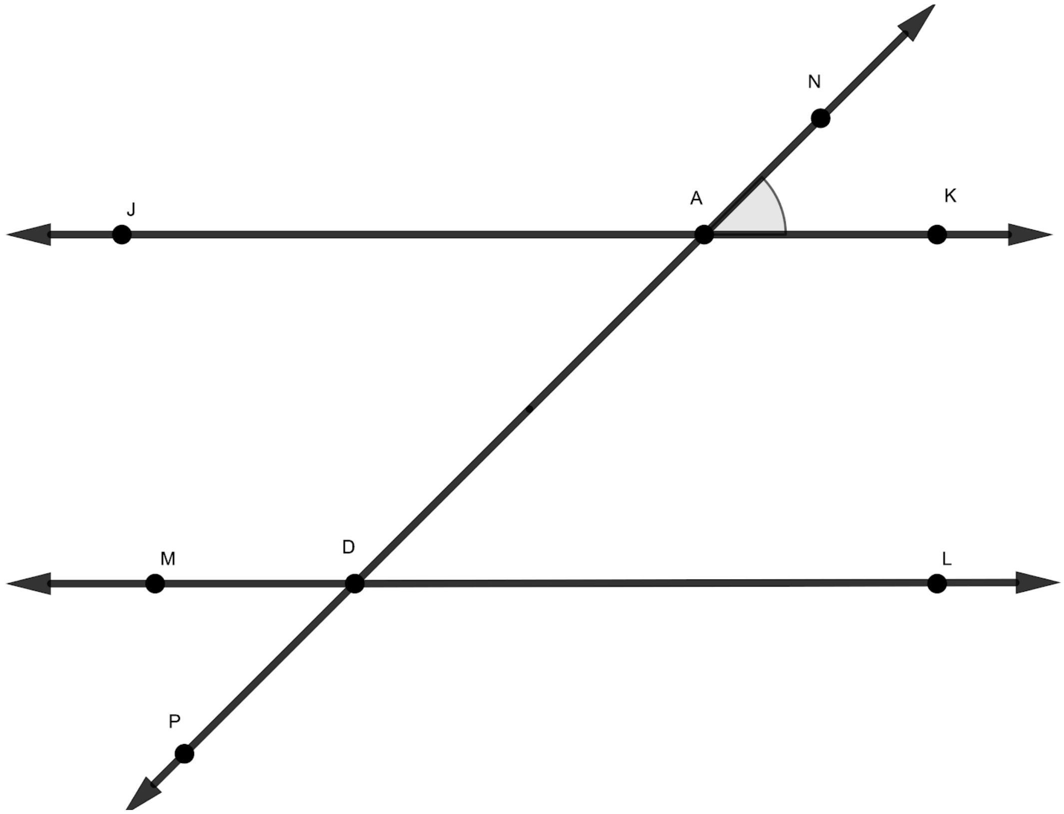
In the example above:

∠1, ∠3, ∠5, and ∠7 are all the same. ∠2, ∠4, ∠6, and ∠8 are all the same.



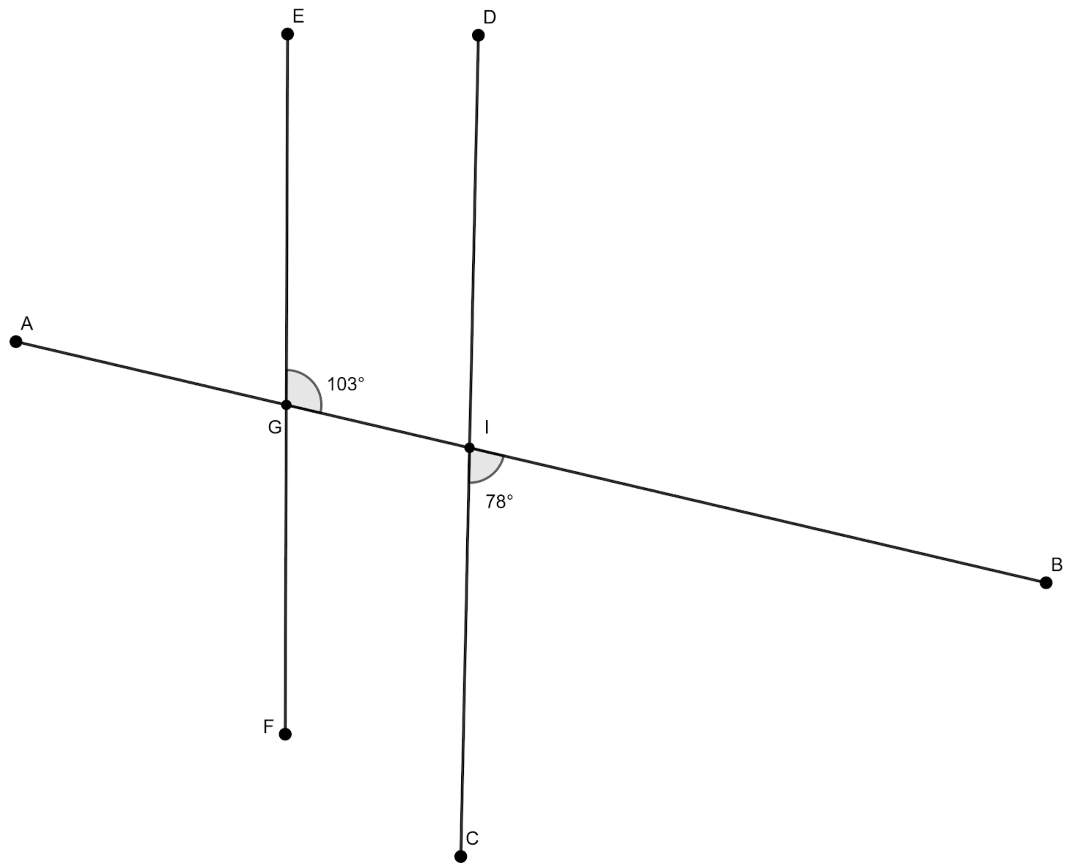




48. $\overleftrightarrow{JK}$ is parallel to $\overleftrightarrow{ML}$. If ∠NAK measures 52°, fill in the missing angles in the diagram.

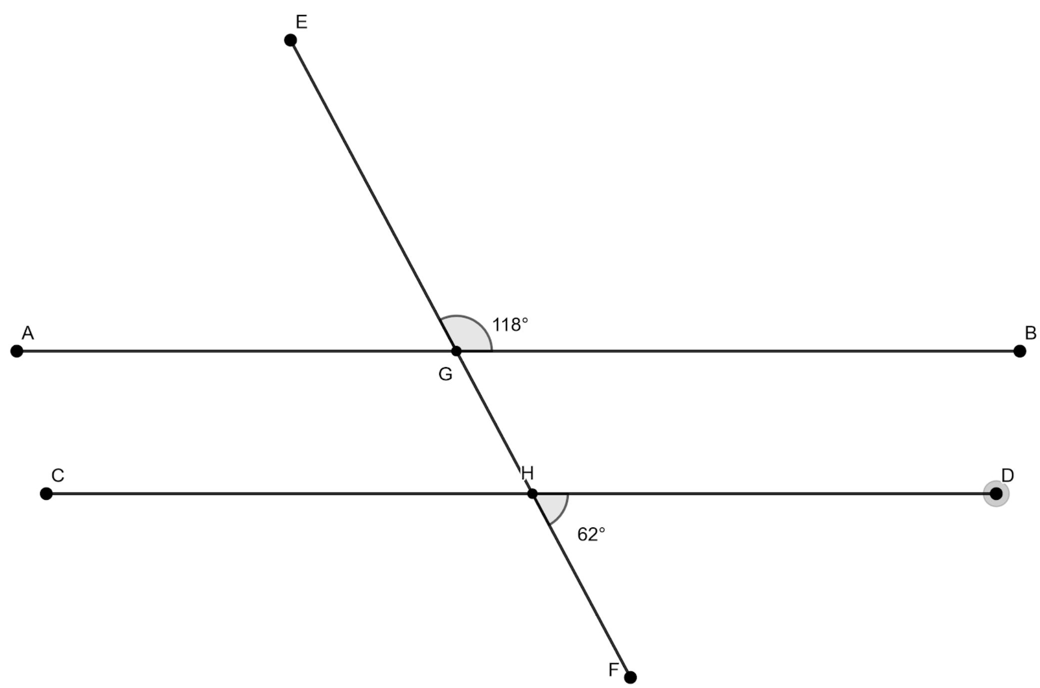

N
J
A
K
M
D
L
P

49. Which of these pairs of lines are parallel?

a.




b.



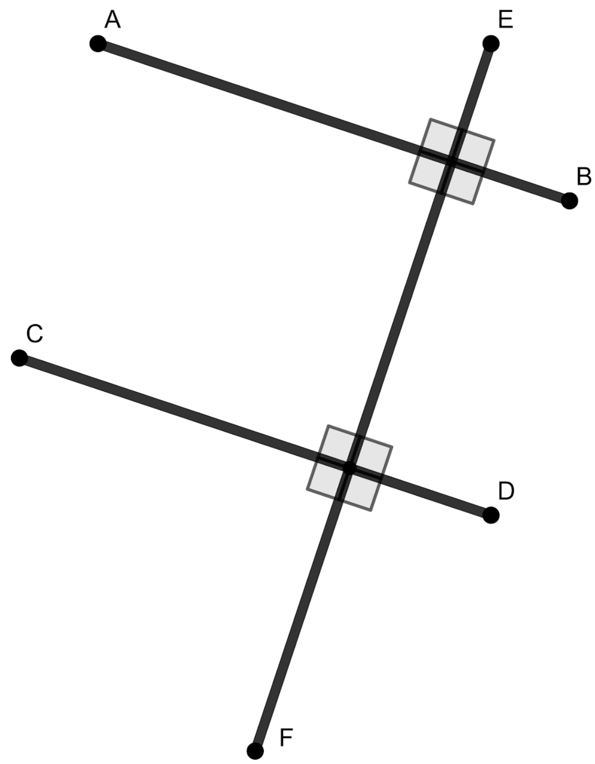
50. Which two statements are true?

a. $\overline{CD}$ is perpendicular to $\overline{AB}$

b. $\overline{FE}$ is perpendicular to $\overline{CD}$

c. $\overline{CD}$ is parallel to $\overline{FE}$

d. $\overline{AB}$ is parallel to $\overline{CD}$



# *Angles - Answer Key*

1. ∠SWP, ∠PWS, or ∠W
2. ∠OLW, ∠WLO, or ∠L
3. ∠1 is ∠HPO or ∠OPH. ∠2 is ∠OPE or ∠EPO. Note that because both ∠1 and ∠2 share P as a vertex, ∠P ***is not*** an acceptable name for either angle.
4. The three angles formed are ∠ABD (or ∠DBA) and ∠DBC (or ∠CBD) and ∠ABC (or ∠CBA).
5. ∠ABE, ∠ABD, ∠ABC, ∠EBD, ∠EBC, ∠DBC
6. There are ten angles formed when 5 line segments intersect at the same vertex. ∠ABE, ∠ABD, ∠ABF, ∠ABC, ∠EBD, ∠EBF, ∠EBC, ∠DBF, ∠DBC, ∠FBC
7.

| Number of Line Segments | Number of Angles | There are different patterns you might notice. Here's one pattern you might have seen:<br><br>With each new line segment, the number of angles formed increases.<br><br>It increases by 2, then 3, then 4, then 5, then 6, then 7... |
|---|---|---|
| 2 | 1 (+2) | |
| 3 | 3 (+3) | |
| 4 | 6 (+4) | |
| 5 | 10 (+5) | |
| 6 | 15 (+6) | |
| 7 | 21 (+7) | |
| 8 | 28 (+8) | |

8. Choice D is the incorrect name. It does not have the vertex in the middle of the name.
9. Angle ABC is the larger angle. Even though the line segments that form angle DEF are longer, angles are the openings between
10.
    a. There are four 90° angles in 360°.
    b. There are five 72° angles in 360°.
    c. There are six 60° angles in 360°.
    d. There are eight 45° angles in 360°.
    e. There are twelve 30° angles in 360°.

11. ∠DNC, ∠CNR, and ∠ANE are all 90° angles. They each contain two 45° angles.

12. ∠DNE, and ∠ANR are both 135° angles. They each contain three 45° angles.

13.
    a. We know straight angle ACB is 180°. If ∠BCD is 160° than we know ∠DCA measures 20° because together they need to add up to 180°.

    b. Similarly, if ∠KAY measures 180° and ∠NAY measures 50°, than we know ∠KAN measures 130°.

14. Choice C. 178° and 12° adds up to 190°. All of the other answer choices add up to 180°.

15. ∠ACF measures 125°.

16. ∠a measures 50° because it is supplemental to the 130° angle.
    ∠b measures 60°, because the 50° angle and the 70° angle form a straight angle with ∠b. 50° + 70° is 120°. Angle b has to be 60° because 120° + 60° is 180°.
    ∠c measures 50° because it is supplemental to the 130° angle.

17. There are many different ways to approach this problem. If ∠DCA is equal to ∠DCB than each of those angles has to measure 90° . If ∠DCA is 90° , we can ask ourselves, what number can we double and add to itself to get 90°. If you try different combinations, eventually you will find that 60° is twice as big as 30° and 30°+60° equals 90° . If ∠FCD is 60°, then ∠DCE is 60°, than ∠FCE is 120°.

18. Each of the smaller angles is a 10° angle. If you divide 180 into 18 equal sections, each section is 10. Another way to think about it is 10°+10°+10°+10°+10°+10°+10°+10°+10°+10°+10°+10°+10°+10°+10°+10°+10°+10°=180°

19. ∠CDE measures 60°. ∠FDC measures 120°.

20. ∠PFC measures 100°.∠LFP measures 80°.

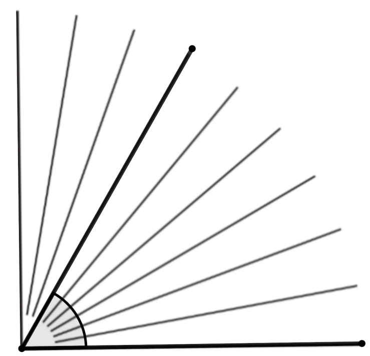
21. ∠SRA measures 30°.∠HRA measures 150°.

22. ∠MRO measures 45°. Line segment RM divides the 10° in half. So the opening is 10°+10°+10°+10°+5°. ∠ARM measures 135°.

23. ∠HPO and ∠OPE each measure 90°.

24. 30°

25. 45°

26. 

27. 30°

28. Choice B. 65° and 25° add up to 90°. Choices C and D are supplementary angles.

29. Choice D.

30. ∠SPW measures 75° and ∠LPS measures 105°.

31. ∠MNL measures 65° and ∠LNP measures 155°.

32. ∠LTP measures 35°.

33. ∠BLT measures 90°.

34. The alligator's jaw is open at a 50° angle.

35. The Leaning Tower of Pisa leans at about a 4° angle.

36. 105°. This angle is just a little bit wider than a right angle. It can be made by combining a 45° and a 60° angle.

37. 15°. This is the smallest angle. It is half of a 30° angle.

38. 135°. This is the largest angle on the page. It is a combination of a 90° and 45°.

39. 60°.

40. 90°. When the hands are at 12 and 3, they form a right angle.

41. 60°. One way you might answer this one is to use the measurement in the last question. If 3 o'clock is 90°, you can think about dividing that 90° into three equal angles. So at 1 o'clock, the hands would form an opening that was 30° and at 2 o'clock, the opening would be open 30° more. 30° + 30° is 60°.

42. 150°. There are a few ways you can answer this one. If you think of each hour as 30°, to form the opening for 5 o'clock you would need five 30° angles, which adds up to 150°. Another way is to start with what we know about 3 o'clock. If the angle of the hands at 3 o'clock is 90°, than opening it up two more hours would be another 60°. 90° + 60° is 150°.

43. 120°. This one might feel a little different because the angle is facing the other direction from the previous examples. One way to look at it is to say that when the hands are at 9 o'clock, they form a 90° angle. If you open the angle to 8 o'clock, that would add another 30°. 90° + 30° is 120°.

44. 105°. If the hands of the clock were at 3 and 6, they would form a 90° angle. Here the hour hand is halfway between the 2 and the 3, so the angle is open more than 90°. Since the time reads 2:30, we know the hour hand must be exactly halfway between the 2 and 3. Since each hour forms a 30° angle, half if that would form a 15° angle. 90° + 15° is 105°.

45. 165°. There are several different ways to answer this one. If the hands of the clock were at 12 and 6, they would form a 180° straight angle. Since the clock reads 11:30, we know the hour hand is halfway between the 11 and the 12. That means the opening of the angle is 15° less than 180°. 180° - 15° is 165°.

46. $\angle$CEA measures 135°. $\angle$AED measures 45°. $\angle$DEB measures 135°.

47. $\angle$MTA measures 64°. $\angle$HTY measures 64°. $\angle$MTY measures 116°.

48. $\angle$ADL, $\angle$MDP, $\angle$JAD all measure 52°. $\angle$JAN, $\angle$KAD, $\angle$MDA, $\angle$PDL all measure 128°.

49. Choice B is the only answer with parallel lines. If you fill in all the missing angles, choice B is the only one that has two groups of opposite angles.

50. Choices B and D are both true statements.