Two-Dimensional Geometry

Fast Track GRASP Math Packet

Part 1

Version 1.0

Released 6/21/2023

This Fast Track GRASP Math Packet was made possible through support from the New York State Education Department, Office of Adult Career and Continuing Education Services.

http://www.collectedny.org/ftgmp

Table of Contents

Overview

Prerequisites	There are no prerequisites to *Two-Dimensional Geometry, Part 1.* As long as you are able to read this packet independently, you don't have to study any other math packets first. Students should complete *Two-Dimensional Geometry, Part 1* before working on *Two-Dimensional Geometry, Part 2.*

In this packet, you will explore concepts in geometry and measurement. Geometry is the part of math that involves lines, shapes and space.

In Part 1, you will study the following topics:

- Two-dimensional shapes
- Measures and units of length (perimeter and circumference)
- Measures and units of area
- Geometric formulas
- The Pythagorean Theorem
- Scale factors

In Part 2, you will build on what you learned in Part 1, and study the following topics:

- Review of area
- Density
- Rates
- Population density (math in a social studies context)

In addition to the learning the topics above, you will find the following materials to help you:

- High School Equivalency Test Practice Questions. You will practice all the concepts you've learned from this packet to work on these questions. The answer key for this section explains the correct answers, and also some of the wrong answers.

- A graphic organizer to study vocabulary is included, along with a vocabulary activity to review concepts. A glossary with important terms from this packet is also included for your study.

- Concept Circles can help you make connections between the concepts you have learned and help you remember those connections.

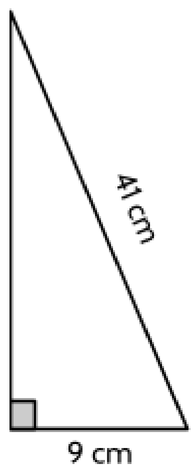
Assessment Questions

Calculator allowed

The following questions will help to see if this packet is right for you. Do your best to answer each question. If you can't answer, don't worry—this packet will help you answer questions like these and more. When you are finished with the questions, read our recommendations.

Question 1

What is the perimeter of this triangle?



A. 40 cm

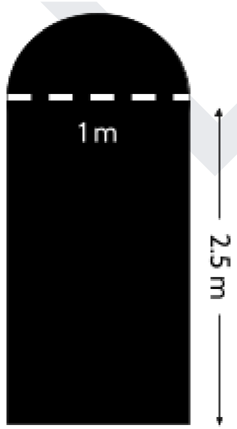
B. 42 cm

C. 50 cm

D. 90 cm

Question 2

Loretta is putting weather stripping around her door frame and needs to figure out the perimeter. The diagram below represents the dimensions of the doorway.



What is the approximate perimeter of Loretta's doorway?

A. 4.07 m

B. 7.57 m

C. 9.14 m

D. 10.14 m

Question 3

The length of one side of a rectangle is 22 cm and its perimeter is 72 cm. What is the area of the rectangle?

A. 308 sq. cm.

B. 528 sq. cm.

C. 616 sq. cm.

D. 1584 sq. cm.

Question 4

There are about 20 million people in New York State, with a total land area of about 47,000 square miles.

What is the population density of New York State?

A. 0.0004 people per sq. mile

B. 426 people per sq. mile

C. 2,350 people per sq. mile

D. 940,000 people per sq. mile

Answer Key

Question 1. Choice D. *90 cm*

Question 2. Choice B. *7.57 m*

Question 3. Choice A. *308 sq. cm.*

Question 4. Choice B. *426 people per sq. mile*

Recommendations

Consider the following when making a decision about working through this packet:

- Student has some difficulty with Question 1, 2, or 3: The student may choose to work through *Two-Dimensional Geometry, Part 1.*
- Student has some difficulty with Question 4: If a student comfortably answers Questions 1, 2 & 3, but has some difficulty with Question 4, the student may feel confident enough to skip *Two-Dimensional Geometry, Part 1* and go directly to *Two-Dimensional Geometry, Part 2.)*.
- Student comfortably answers all four questions: The student may choose to work on a different packet. However, it is recommended that students complete the Test Practice Questions in *Two-Dimensional Geometry, Part 1.* and *Two-Dimensional Geometry, Part 2*, for test questions that require students to interpret a variety of data representations before they take the HSE exam.

This assessment asks students to demonstrate understanding of:
Question 1 *(from Two-Dimensional Geometry, Part 1):* Pythagorean Theorem and perimeter of triangles (GED Algebraic Problem Solving Assessment Targets Content Indicators: Q.4.c and Q.4.e).

Question 2 *(from Two-Dimensional Geometry, Part 1):* Finding perimeter of composite shapes, including a semicircle. (GED Algebraic Problem Solving Assessment Targets Content Indicators: Q.4.b, Q.4.c and Q.4.d).

Question 3 *(from Two-Dimensional Geometry, Part 1):* Perimeter and area of rectangles. (GED Algebraic Problem Solving Assessment Targets Content Indicators: Q.4.a)

Question 4 *(from Two-Dimensional Geometry, Part 2):* Calculating population density (GED Algebraic Problem Solving Assessment Targets Content Indicators: Q.3.a)

Welcome!

Congratulations on deciding to continue your learning! We are happy to share this study packet on geometry, focusing on two-dimensional shapes and the measurement of area. We hope that these materials are helpful in your efforts to earn your high school equivalency diploma. This group of math study packets will cover mathematics topics that we see on high school equivalency exams. If you study these topics carefully, while also practicing other math skills, you will increase your chances of passing the exam.

Please take your time as you go through the packet. You will find plenty of practice here, but it's useful to make extra notes for yourself to help you remember. You will probably want to have a separate notebook where you can recopy problems, write questions and include information that you want to remember. Writing is thinking and will help you learn.

After each section, you will find an answer key. Try to answer all the questions and then look at the answer key. It's not cheating to look at the answer key, but do your best on your own first. If you find that you got the right answer, congratulations! If you didn't, it's okay. This is how we learn. Look back and try to understand the reason for the answer. Please read the answer key even if you feel confident. We added some extra explanations and examples that may be helpful. If you see a word that you don't understand, try looking at the *Vocabulary Review* at the end of the packet. There are some strategies for learning new words described in the next few pages.

We hope you share what you learn with your friends and family. If you find something interesting here, tell someone about it! If you find a section challenging, look for support. If you are in a class, talk to your teacher and your classmates. If you are studying on your own, talk to people you know or try searching for a phrase online. Your local library should have information about adult education classes or other support. You can also find classes listed here: **http://www.acces.nysed.gov/hse/hse-prep-programs-maps**

You are doing a wonderful thing by investing in your own education right now. You have our utmost respect for continuing to learn as an adult.

Please feel free to contact us with questions or suggestions.

Best of luck!

Mark Trushkowsky (mark.trushkowsky@cuny.edu) & Eric Appleton (eric.appleton@cuny.edu)
CUNY Adult Literacy and High School Equivalency Program

Vocabulary

It is important to understand mathematical words when you are learning new topics. The following vocabulary will be used a lot in this study packet: **area**, **square units**, **dimension**, **measure**, **perimeter**, **circumference**, **polygon**, and **scale**.

When we learn new vocabulary, it is good to think about your experience with the word. Asking questions like, "Have I heard this word before?", "When have I heard this word?", "What do I think this word means?" can help you build on what you already know.

Here's how it works. On the next page, you'll find a chart with each of the vocabulary words above. For each word, ask yourself how familiar you are with the word. For example - the word "*area.*" Which of these statements is true for you and your experience with the word "*area*"?

- I know the word "*area*" and use it in conversation or writing.
- I know the word "*area*," but I don't use it.
- I have heard the word "*area*" but I'm not sure what it means.
- I have never heard the word "*area*" at all.

In the chart on the next page, read each word and then choose one of the four categories and mark your answer with a ✔ (checkmark). Then write your best guess at the meaning of the word in the right column. If it's easier, you can also just use the word in a sentence.

Here's an example of how the row for "area" might look when you're done:

Word	I know the word and use the word	I know the word but don't use it	I have heard the word, but I'm not sure what it means	I have never heard the word	My best guess at the meaning of the word (or use the word in a sentence)
area	✓				A place or location, like a neighborhood or town

This activity is designed to help you start thinking about some of the important words you will find in this packet. As you go through the activities in this packet, you will learn more about these words, what they mean, and how to use them. You will learn more precise definitions that may come up during your high school equivalency exam.

There is a glossary with the definitions of useful vocabulary at the end of the packet.

Word	I know this word and use the word	I know this word but don't use it	I have heard this word, but I'm not sure what it means	I have never heard this word	My best guess at what this word means...
area					
square unit					
dimension					
measure					
perimeter					
circumference					
polygon					
scale					

Introduction to Geometry

Geometry is something humans have been learning and doing for thousands of years.

But geometry is not important because it is old. Geometry is old because it is important. It is old because it was a fundamental way for humans to understand their environment and to build a world around them.

Many of the words we use today in geometry and in measurement come from the Ancient Greeks. The word *geometry* comes from two Greek words, "*ge*" which means "land" or "earth" and "*metria*," which means "a measuring of." So the word *geometry* means "a measuring of the land" or "the measuring of the Earth."

When we talk about the Ancient Greeks, we are talking about people who lived from about 3,000 years ago until about 2,100 years ago. Even though many of the words we use today in geometry come from the Ancient Greek language, the study of geometry began thousands of years before the Greeks.

This clay tablet is over 4,600 years old. It comes from a city in Sumer. Sumer was a region in ancient Mesopotamia in what is today south-central Iraq.

<u>The tablet has a table with three columns.</u>

- The first two columns contain measurements of distances.
- The third column contains the area of rectangles with the dimensions from the first two columns.

The Ancient Egyptians used their knowledge of geometry to build the great pyramids. And about 4,500 years later, there are 138 pyramids still standing today. The oldest and tallest is called the Great Pyramid of Giza. It is about 200 feet taller than the Statue of Liberty and it would cover around 100 U.S. football fields! It is the only one of the Seven Wonders of the Ancient World still in existence today.

The Ancient Egyptians were practical in their use of geometry. Their study of geometry allowed them to collect taxes, divide land, make square corners, develop systems of measurement, and use blueprints. It allowed them to build wonders and organize their civilization.

Humans are still learning new ways to use geometry today. Geometry is at the heart of construction, architecture, navigation, global positioning systems (GPS), computer animation, and even how our governments are organized.

Geometry is also something that can happen in our own backyard. Liv's family wanted to convert a section of their yard into a vegetable garden. As we explore the geometry concepts in this packet, we will check back in on their progress and see how they put the geometry we are learning to good use.

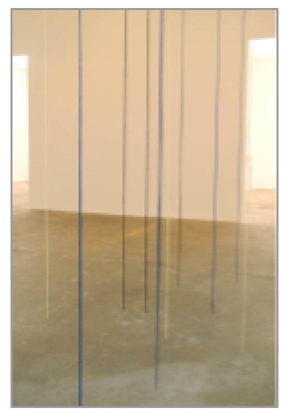
Points and Lines

Points and **lines** are two of the building blocks of geometry. They help us describe the things in the world that we want to measure.

Points are the most basic elements in geometry. Even though points do not exist in the natural world, we can use our experience of the natural world to understand them. Imagine using a pencil to draw the tiniest dot you can. Or a single tiny seed. Both of those examples can give us an idea of what we mean when we use the word *point* in geometry.

Like points, the **lines** of geometry do not exist in the natural world. We cannot see them or touch them, but we can use our experience in the world to understand them.

The horizon gives us an idea about what we mean when we use the world "*line*" in geometry. Train tracks are also useful physical models of lines.

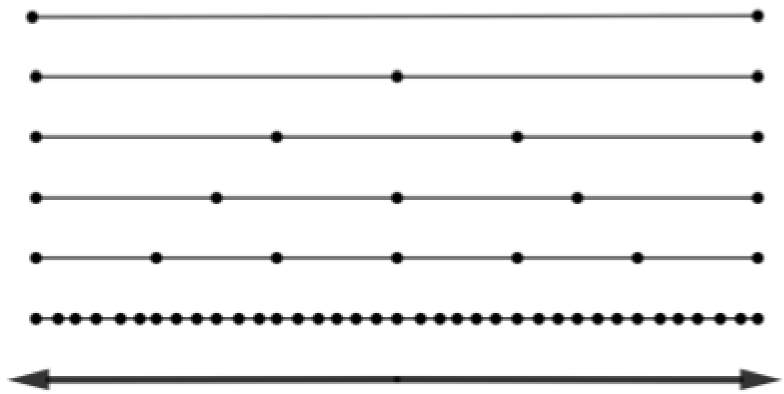


The artist Fred Sandback uses pieces of yarn to create the illusion of lines in physical spaces.

Lines can be named by using the letters of any two points on the line. For example, the name of the line below is $\overleftrightarrow{SP}$ **or** $\overleftrightarrow{PS}$.

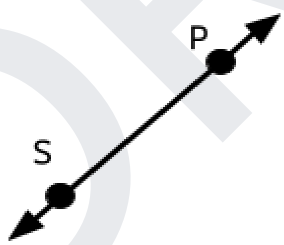
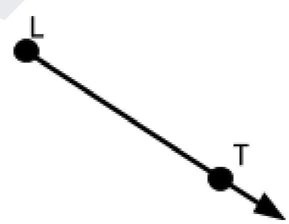
When we draw lines, we use arrows pointing in opposite directions to show that the line continues on forever in both directions. One thing to remember is that even though we only need two points to name a line, a line is made of an infinite series of points, one right after the other.

This drawing can help you visualize how lines are made up of many points. Start with the two points across the top. Visualize adding more and more points between them until you can't imagine any space between them. That is a line.



In this packet, we are going to be working with three figures that each look like lines.

1. Complete this table.

Name	Line	Ray	Line Segment
Figure			
Characteristics *(Briefly describe each figure. What makes it different from the other two?)*			

The **line** is the most basic of these figures which you already read about. In the table above, *Line SP* has arrows pointing in opposite directions. Lines in geometry are drawn this way to show that they continue in both directions forever.

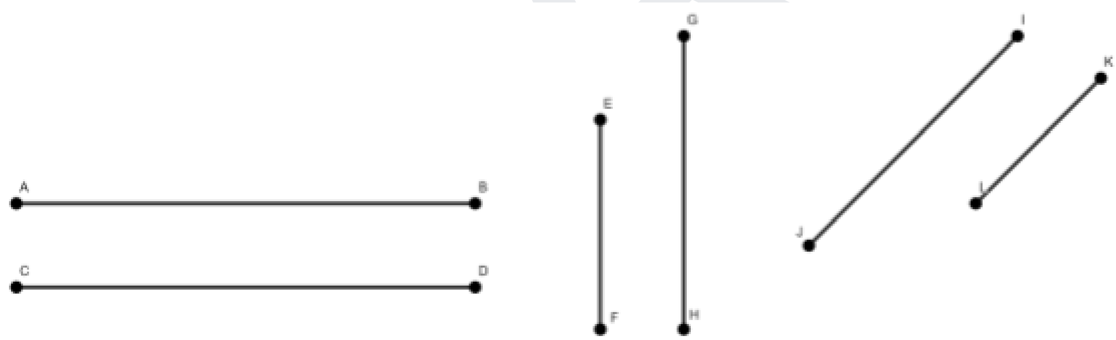
Most geometric figures, like shapes and angles, are made up of *parts* of a line.

A **segment** (or line segment) is part of a line, defined by two endpoints and all the points between them. The rungs of a ladder are physical examples of line segments. The sides of the shapes you will study in this packet are mostly line segments.

A **ray** is a part of a line, starting with one endpoint and made up of all the points on one side of that endpoint. A beam of light from a flashlight is a physical example of a ray.

It is useful in geometry to look for relationships between lines. One important way that two or more lines can be related is to be parallel.

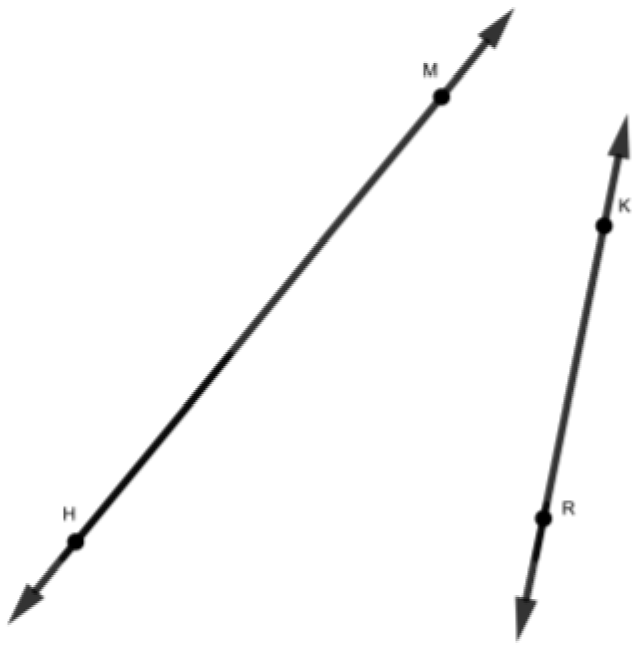
Parallel lines are lines that lie on the same flat surface and do not intersect.
Below are some examples of lines that are parallel.



The strings of a guitar are physical models of parallel lines.

One important thing to remember is that just because drawings of lines do not intersect, it does not necessarily mean they are parallel.

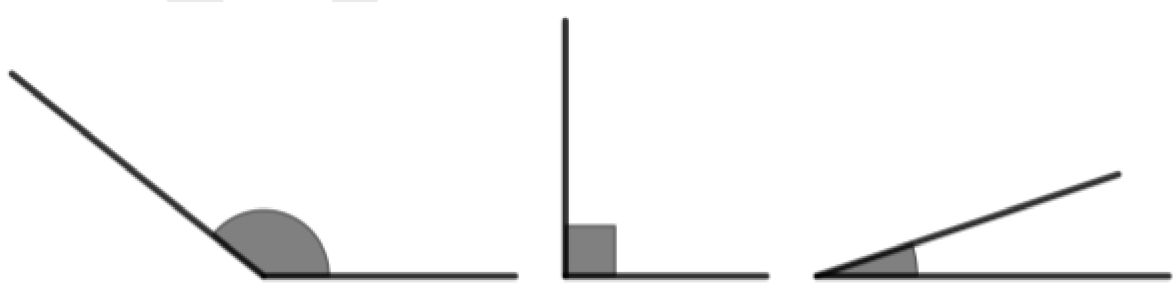
The two lines in the diagram below are not parallel lines. If you extend them, they will eventually intersect at a single point.



Angles

When lines, line segments, or rays intersect, they create another important figure in geometry called the angle.

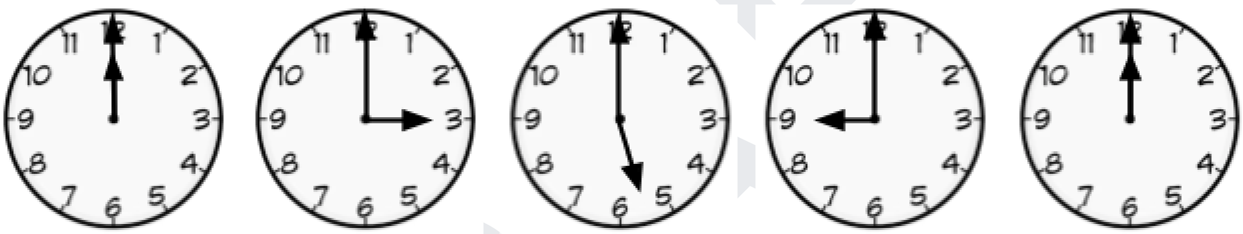
An **angle** is an “opening” formed when two segments, lines, or rays intersect.



How do we measure angles? This was an important question in the history of geometry. How could we measure the opening between two lines?

The system we use to measure angles today was developed more than 3,500 years ago by the Babylonians. Babylon was a city in ancient Mesopotamia. The city was built between the Tigris and Euphrates rivers, just south of Baghdad in what is modern-day Iraq. The Babylonian system for measuring angles is one of the oldest forms of measurement still in use today.

The Babylonians used a circle to describe the entire range of possible angles. A clock can help us visualize their thinking.



Imagine a clock that reads 12 pm

If you move the hour hand clockwise, the opening between the minute hand and the hour hand gets larger.

Eventually the hour hand will get back to where it started.

The ancient Babylonians needed to decide what number of degrees they would use to measure the whole circle. They decided to use 360 degrees (also written as 360°) as the measure of an angle that goes all the way around.

This is the degree symbol: °

Degrees are the unit we use to measure angles. (We also use the symbol ° for degrees of temperature).

You might be asking yourself, "Why 360? What is so special about that number?"

First of all, 360 is very close to the number of days in a year. One year is defined as the amount of time it takes the Earth to complete one full rotation around the sun. We usually think of a year taking 365 days. It actually takes the Earth 365 days *and 6 hours* to go around the sun.

Even though it was 4,000 years ago the Babylonians knew that there were 365¼ days in a year. Why didn't they decide that there should be 365 ¼ degrees in a circle?

The answer to this question is very simple and human: Who wants to do math with the number *365 ¼*? It's an awkward number! The natural thing to do is to round it to a friendlier value.

If we round the number 365 ¼ to the nearest five or the nearest ten we get 365 and 370, not the number 360. Why did we humans decide to round all the way down *to 360?*

The answer here is also very simple and human. Thousands of years ago there were no calculators. All arithmetic had to be done by hand or in one's head. It is natural to want to work with numbers that are easier to calculate in your head. They also preferred working with numbers that could be divided with no remainders.

In life (and in math) humans like to divide numbers by 2. Choosing 365 is unfriendly. 365 divided by 2 is 182.5—no thanks!

370 and 360 are both even. Why did they choose 360 and not 370?

We like to divide things by 2. We also like to divide things by 3. If you divide 370 by 3, you get 123 ⅓ . If you divide 360 by 3, you get 120.

In fact, 360 is a much friendlier number than 370: you can divide it evenly into 2 equal pieces, 3 equal pieces, 4, 5, 6, 8, 9, 10, 12, 15, and many more. In fact there are 24 different ways to divide 360 into equal pieces!

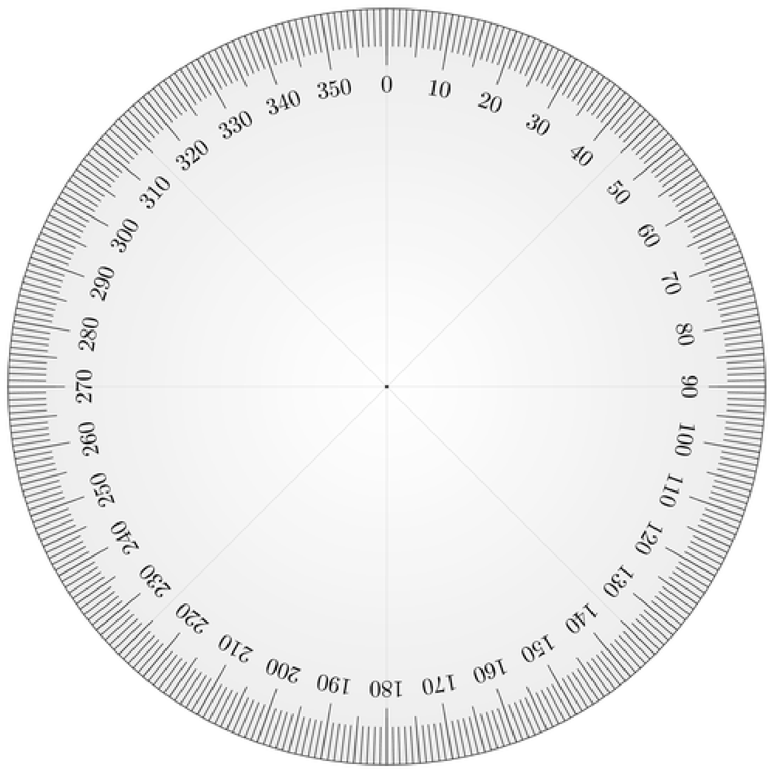
So, it was for two very human reasons—what we experience on this planet and our desire to avoid awkward work—that the Babylonians settled on the number 360 for the count of degrees in a circle.[1]

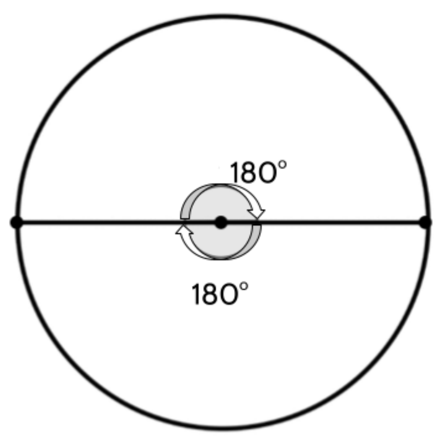
[1] Adapted from "[Two Key - but ignored - Steps to Solving Any Math Problem]" by James Tanton

Dividing 360°

In the last section, you read that one of the reasons the Babylonians chose 360 as the number of degrees in a circle was because there are so many ways to divide 360 into equal groups. For this next activity, you'll identify some of those equal groups.

Remember a complete circle is 360°.

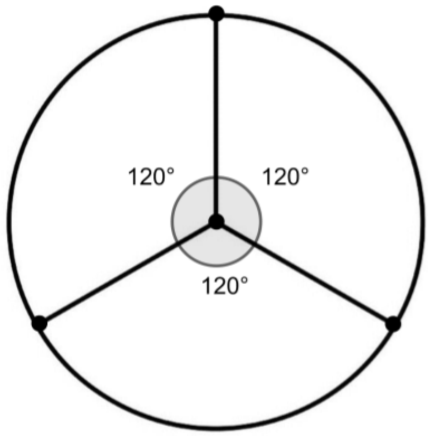




On the next few pages, there are circles. Each circle is divided into a different number of equal angles.

For example, this circle has two equal angles. If we divide 360° into two equal angles, each angle is 180°. Another way to think about this is that 180° is halfway if you travel around the circle.

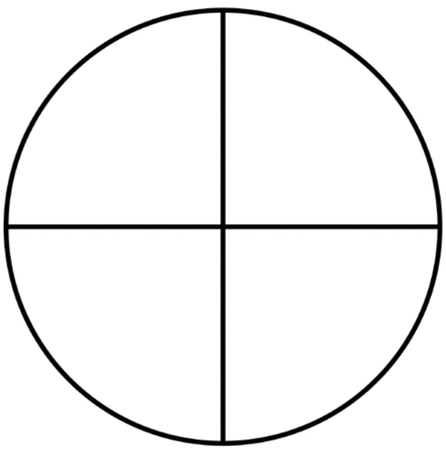
What if we divide the circle into three equal angles?



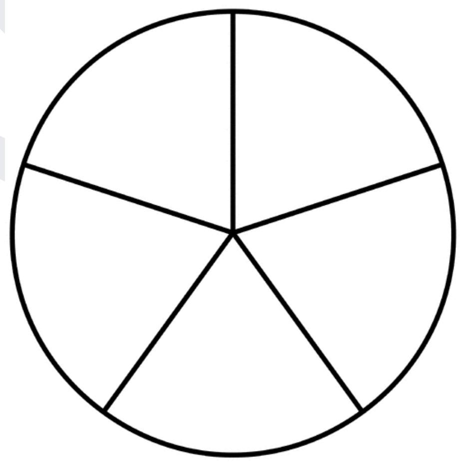
120 + 120 + 120 = 360

2. Find the value of and label each angle. Remember that each circle has to add up to a total of 360°.

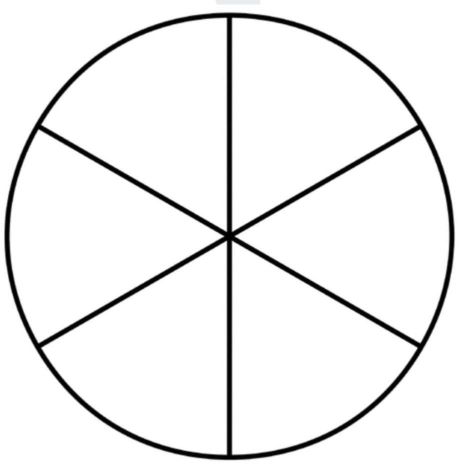
a.



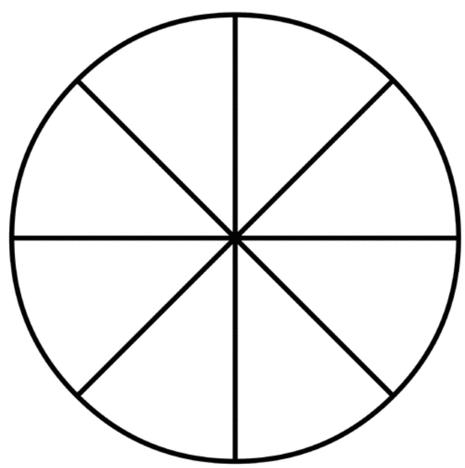
b.

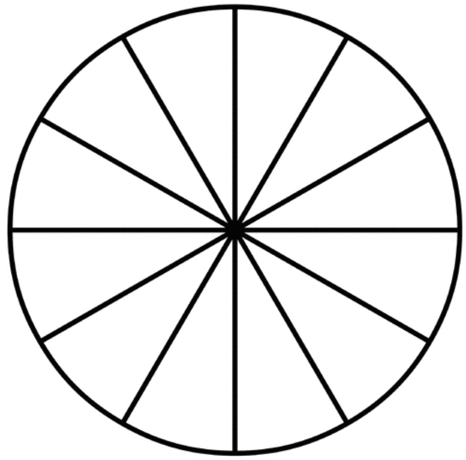


c.



d.





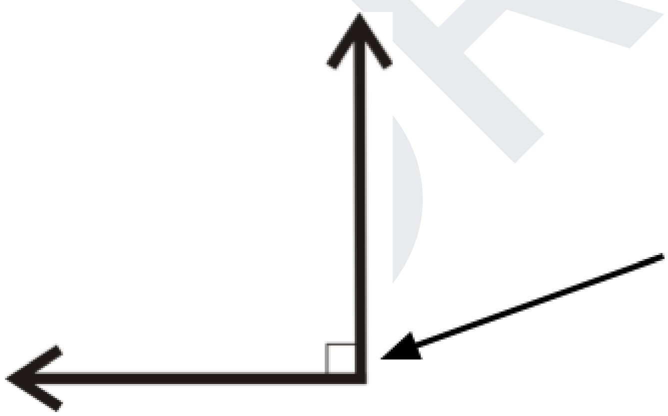
e.

Note: These are not all the ways to divide up 360° into equal groups, but these are some common angles you might see.

When you divide 360 into 4 equal sections, each section is 90. That particular angle is very important, especially in construction. That angle is called a **right angle.**

A ***right angle*** is an angle with an opening of 90°.

Right angles allow us to build perfect squares.



You will sometimes see a small square in the angle. That means the two lines are at a right angle and the angle measures 90°.

Lines that intersect to form right angles are called ***perpendicular lines.***

Introduction to Geometry - Answer Key

1. There are several things you may have noticed. Lines have arrows that point in both directions. Rays have arrows that point in one direction. Line segments don't have arrows.

2. Dividing 360 into equal sections.

    a. There are four 90° angles in 360°.

    b. There are five 72° angles in 360°.

    c. There are six 60° angles in 360°.

    d. There are eight 45° angles in 360°.

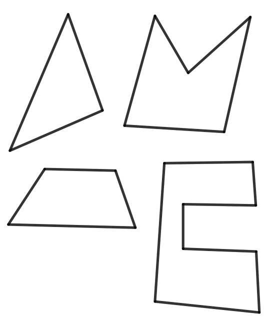
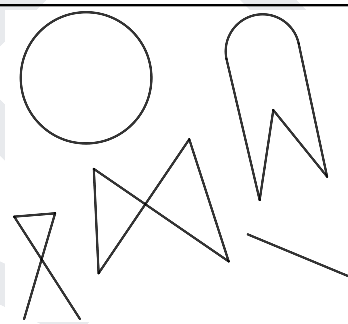
    e. There are twelve 30° angles in 360°.

Polygons

Introduction to Polygons

The world is full of shapes. One group of important shapes are called polygons. You may or may not know the word "polygon" but you definitely know many polygons already.

Polygons are two-dimensional shapes that have straight sides. Circles or anything with a curve are not considered polygons. Each side in a polygon intersects with exactly two other sides. If any sides intersect with more than two sides, the shape is not a polygon.

These are a few examples of polygons	**These shapes are *not* polygons**
	

Humans have studied polygons for a long time. Even children enjoy looking at and categorizing them. In order to study polygons, we split them into categories based on the number of sides they have.

Number of Sides	English Name
3	**Tri**angle
4	**Quad**rilateral
5	**Pent**agon
6	**Hex**agon
7	**Hept**agon
8	**Oct**agon
9	**Non**agon
10	**Dec**agon

Do you know the names of any of these polygons in other languages?

1) You can find examples of polygons all around you. The photos below show some examples. Under each photo, write the name of any polygons you see.

soccer ball / fútbol

a.

farmland

b.

pedestrian walking sign

c.

U.S. government building in Washington, DC

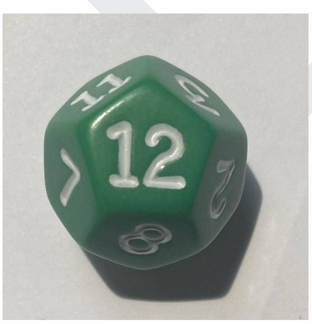
d.

2 Piso coin from the Philippines

e.

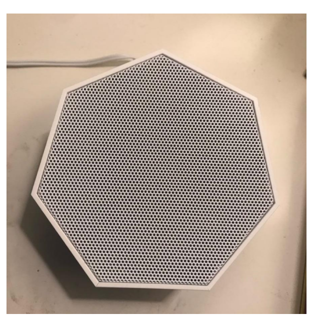
window, Yuyuan gardens in Shanghai, China

f.



one side of a 12-sided die

g.

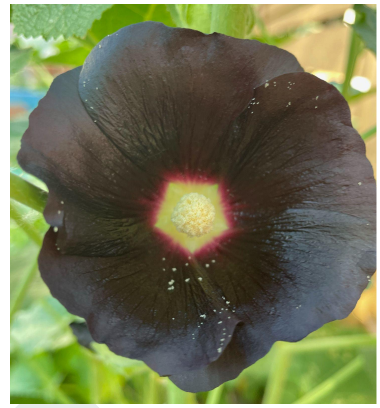


a white noise machine

h.

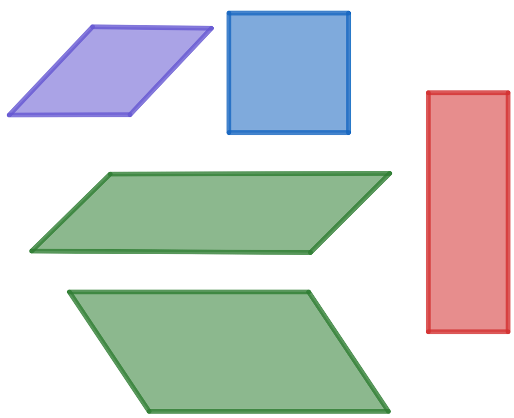
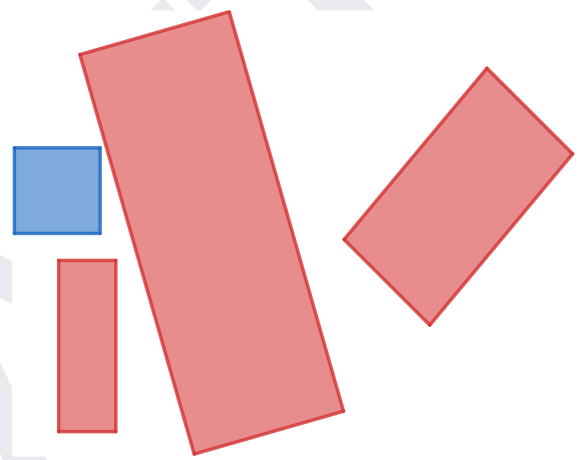
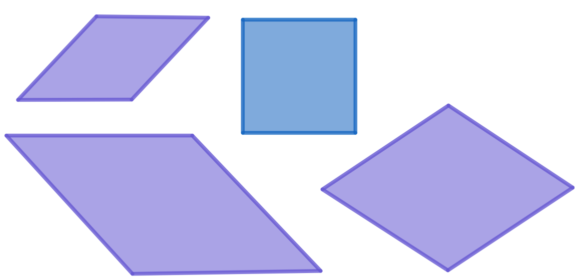
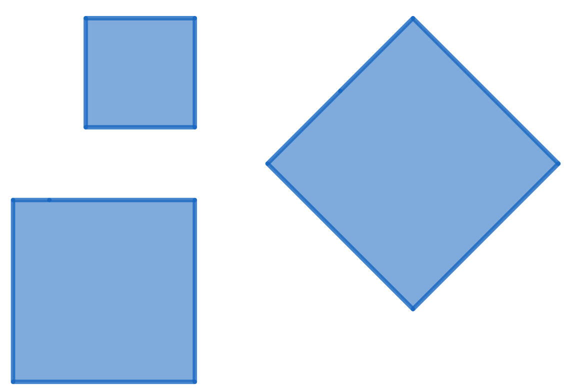
ice cream cup

i.

 *bottom of a drinking glass* j.	 *bottom of a glass jar* k.	 *a hollyhock blossom* l.
 *a morning glory blossom* m.	 *floor tiles* n.	

Quadrilaterals

One of the most common polygons is the quadrilateral. A **Quadrilateral** is any polygon with 4 sides and 4 angles. Within the quadrilateral family, there are smaller families. The chart below shows four common types of quadrilaterals - the parallelogram, the rhombus, the rectangle, and the square.

Some Examples of Common Quadrilaterals	
Parallelogram	Rectangle
	
Rhombus	Square
	

When we look at these figures, we might ask: What relationships are there between these 4 quadrilaterals? What do they all have in common? How are they different?

To answer these questions, let's gather some data. Below you will see a chart for studying quadrilaterals. The first column has a list of defining characteristics. If that characteristic is true for any of the 4 types of quadrilaterals, write an "X." For example, for all parallelograms, opposite sides are always parallel, so we wrote an "X" under "Parallelogram."

2) Use the pictures on the previous page to complete the rest of the chart.

Defining Characteristics	Parallelogram	Rhombus	Rectangle	Square
Number of sides	4	4	4	4
Number of angles	4	4	4	4
Opposite sides are parallel	X			
Opposite sides are the same length				
4 right angles				
All sides are the same length				

Once you have completed your chart, look down each column, and review the characteristics that define each quadrilateral. If a figure meets all of the characteristics in any column, it is part of that quadrilateral family.

3) Mark each of the following statements TRUE or FALSE.

a) All squares are rectangles. ________________

b) All rectangles are squares. ________________

c) All rectangles are parallelograms. ________________

d) Some rectangles are squares. ________________

Explain your answers. ____

____

____

Between these four quadrilaterals, the parallelogram is the easiest family to be a part of. Any shape that has 4 sides, 4 angles, and opposite sides that are parallel and equal in length is a parallelogram. That means all squares and rectangles are also parallelograms.

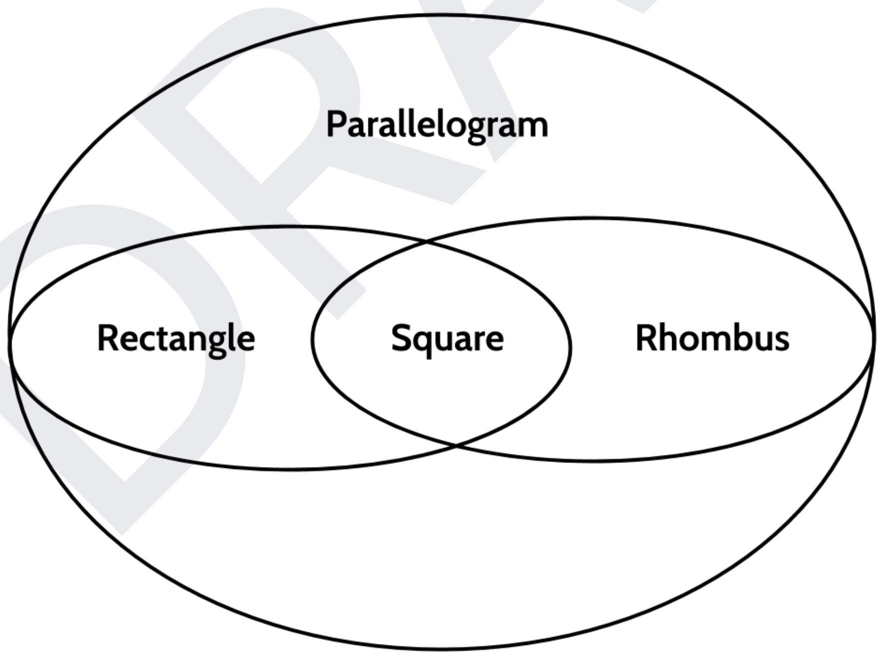
If a figure meets all of the characteristics of a parallelogram, *and* has 4 right angles, it is also considered a rectangle. This means that a figure can be a parallelogram and a rectangle. A rectangle is a special kind of parallelogram.

If a figure meets all of the characteristics of a parallelogram *and* has 4 sides that are equal in length, it is also considered a rhombus. A figure can be a parallelogram and a rhombus at the same time. A rhombus is another special kind of parallelogram.

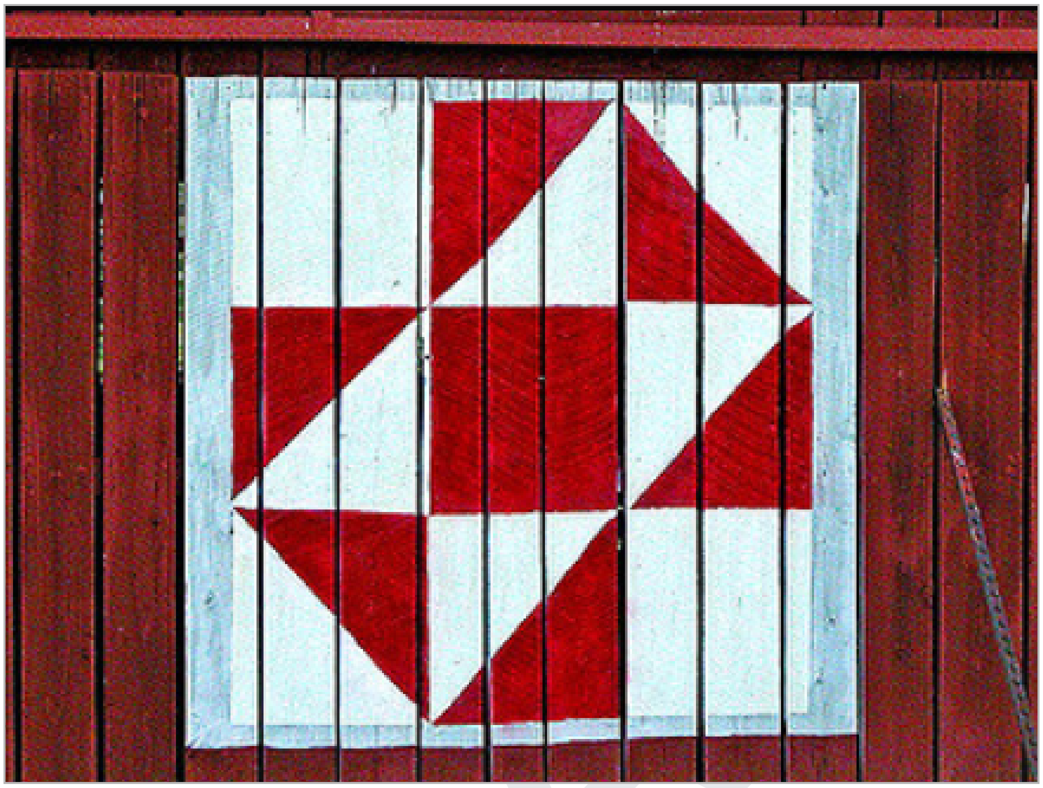
Of these four quadrilaterals, the square has the most defining characteristics.

- A square is a special rhombus with right angles.
- A square is a special rectangle with *4* equal sides.
- A square is a special parallelogram with 4 right angles and 4 equal sides.

So, a square is also considered a parallelogram, a rhombus, and a rectangle.



4) How many different polygons can you find in the image below?



In the space below, redraw each polygon you find and label it with its name.

Polygons - Answer Key

1) Identifying polygons in photos.

   a) The surface of a soccer ball is made of pentagons and hexagons.

   b) The farmland is divided up into quadrilaterals.

   c) The pedestrian walking sign is a pentagon.

   d) The Pentagon is the headquarters of the US Department of Defense—it is named for the shape of the building. Within the pentagon, you might also see quadrilaterals and triangles.

   e) The coin from the Philippines is in the shape of a decagon.

   f) This window from the Yuyuan gardens in Shanghai, China is a heptagon.

   g) The face of a 12-sided die is a pentagon.

   h) The white noise machine is a heptagon.

   i) The bottom of the ice cream cup is an octagon.

   j) The bottom of the drinking glass is a nonagon.

   k) The bottom of the glass jar is a decagon.

   l) There is a pentagon in the center of the hollyhock blossom.

   m) The petals of the morning glory blossom form a pentagon.

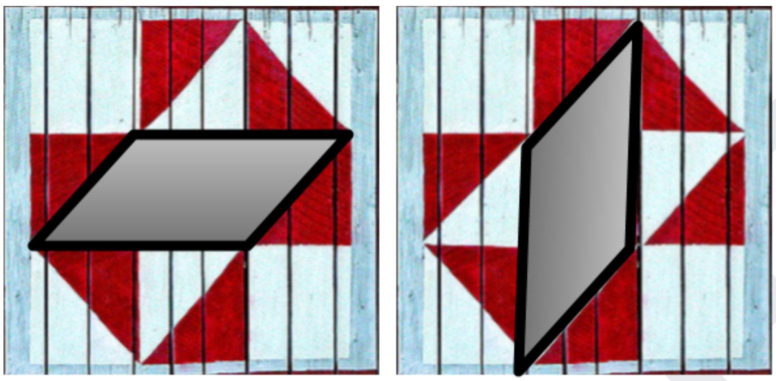
   n) The floor tiles are made up of hexagons.

2) The characteristics of quadrilaterals

Defining Characteristics	Parallelogram	Rhombus	Rectangle	Square
Number of sides	4	4	4	4
Number of angles	4	4	4	4
Opposite sides are parallel	X	X	X	X
Opposite sides are the same length	X	X	X	X
4 right angles			X	X
All sides are the same length		X		X

3)
   a) True
   b) False
   c) True
   d) True

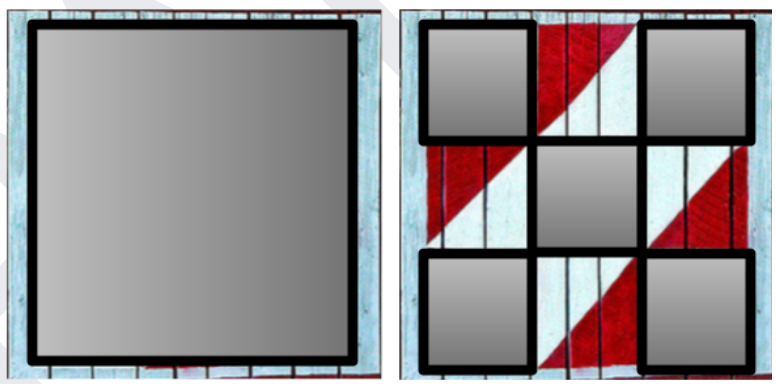
4) There are many, many polygons hidden in the photo of a barn door. Here are a few:



Parallelograms

Triangles

Rectangles



Squares

Length and Area

Attributes of Polygons

Now that you've learned the names of some polygons, we will look at some attributes of polygons. An **attribute** is an aspect of an object that can be measured. If someone asked you to measure this pizza box, you would need to know what attribute of the box they wanted you to measure. Here are some possible attributes of the pizza box:

ATTRIBUTE	What are we measuring?
Weight	How much does the box weigh?
Volume	How much can the box hold inside?
Length	What is the distance from one point on the box to another?
Area	How much surface is there on the box?

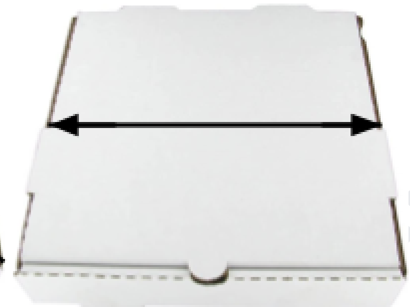
This packet focuses on the attributes of **length** and **area** for polygons and circles.

Length

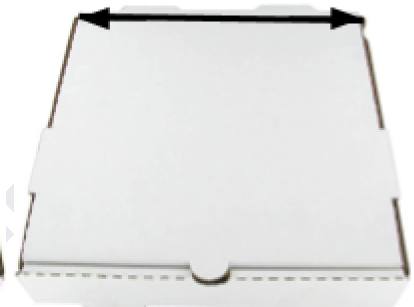
Length is the measure of the distance between two points.

These are some different ways to measure the length of the pizza boxes.

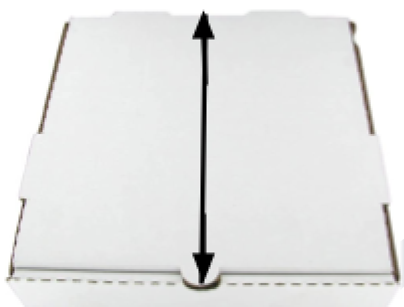
Diagonal of the lid



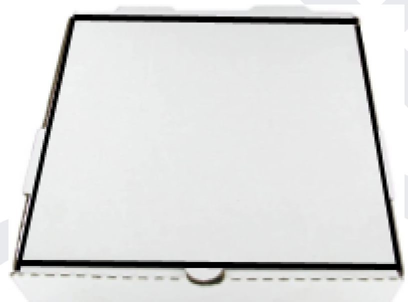
Width of the lid



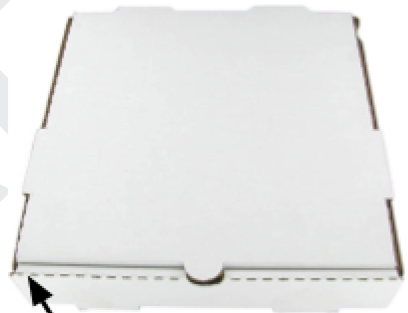
Edge or *side* of the lid



Length of the lid



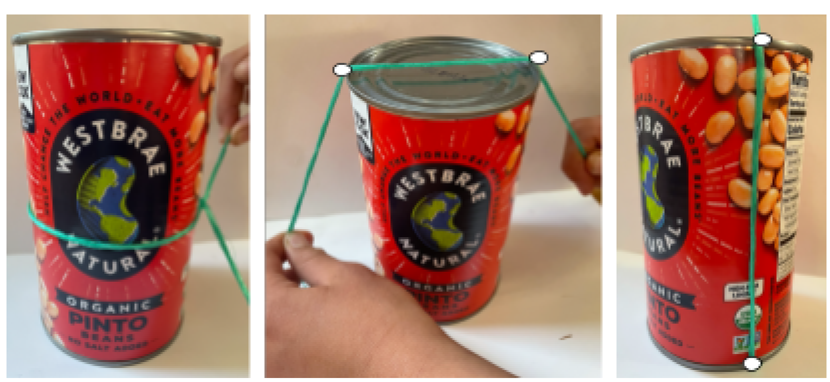
Distance around the box.
We call this the perimeter.



Height of the box

We use many words to talk about length. Distance, height, width, edge, diagonal, side, and perimeter are all measures of length. If we get confused, it can be helpful to think about a piece of string. <u>If you can imagine measuring something with string, it is a measure of length.</u>

This piece of string demonstrates different length measurements on a can of beans. The string helps us visualize that we are looking at the distance between two points.



Even if you have to close your eyes to imagine a really long piece of string, We can't really hold a string between Earth and the moon, but we can imagine it, because that distance is a measure of length.

We use **units** of measurement to help us describe the attributes of things. The word *unit* comes from the word for one. It means one of something that is being measured. For example, some familiar units we use to describe time are seconds, minutes, hours, days, weeks, months, and years.

COMMON UNITS OF LENGTH

There are many units that we use to describe length. These are some of the most common.

UNIT	One of these units is about the length of...	RELATIONSHIP BETWEEN UNITS
Inches (in.)	• The distance across the knuckle of your thumb • The distance across a quarter • A standard paperclip	1 in is about 2 ½ cm 12 inches = 1 foot
Feet (ft.)	• A 2 liter soda bottle • 2 US dollar bills	1 foot = 12 inches 1 foot = about 30 ½ cm 3 feet = 1 yard
Yard (yd.)	• The width of a doorway • A 3 year old child	1 yard = 3 feet
Mile (mi.)	• 20 NYC blocks • The length of the Brooklyn Bridge	1 mile = 5,280 feet 1 mile = 1.6 km
Meter (m.)	• A guitar • A baseball bat • The height of a doorknob	1 meter = 100 cm 1000 meters = 1 km 1 m is about 3.28 ft
Centimeter (cm.)	• 2 pencil erasers • 1 green pea • The distance across your little finger.	2 ½ cm = about 1 in 100 cm = 1 m
Kilometer (km)	• A little more than half a mile	1 km = 1000 m 1 km is about 0.62 miles

Which would you use?

1. Use the table above to determine which unit would make the most sense to use if you were measuring the following:

    a. The distance from New York City to Buffalo, NY

    b. Your height

    c. The height of a tall building

    d. The depth of a pool

    e. The depth of the ocean

    f. The distance from Puerto Rico (in the Caribbean) to Poland (in Europe)

    g. The width of a pizza

    h. The distance across a dime (10 cent coin)

    i. The height of a picture frame

    j. The thickness of a cell phone

DID YOU KNOW?

An inch worm gets its name because the way it moves looks like it is measuring length.

Area

Another attribute of the pizza box we can measure is its area.

We use **area** to measure the size of a surface.

When it comes to area and measuring the size of a surface, it can be helpful to think of paint.

<u>If you can imagine painting something with a paint brush, it is a measure of the size of that surface.</u>

We can imagine painting the entire surface of a wall, a door, a floor, a window, a tv screen. What we are imagining painting is the area.

We can even imagine painting the surface of the lid of a pizza box.

But how do we measure the size of a surface? What exactly are we counting?

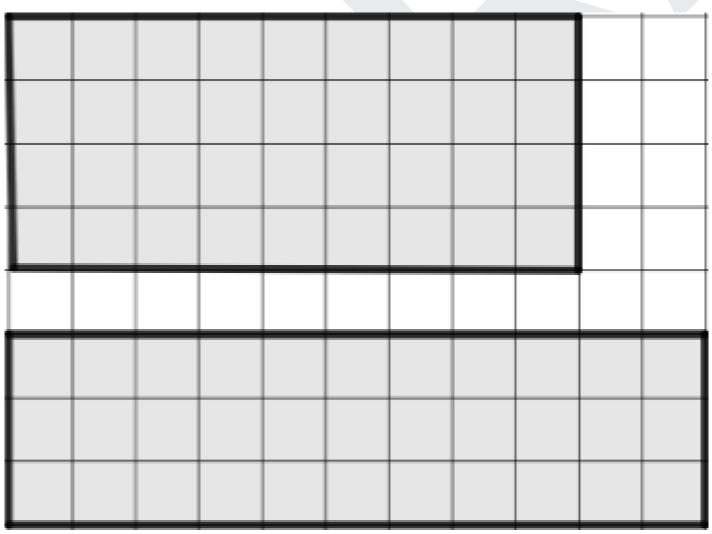
The ancient Egyptians came up with an answer to that question, and we still benefit from their creativity today.

Priests served a very important role in Ancient Egyptian civilization. One of their jobs was to collect taxes from the farmers at harvest time. The amount of tax depended on the size of the farm. The bigger the farm, the more taxes they had to pay.

But what do you do if you had two farms shaped like these two?

The top one looks wider, but the bottom one looks longer. Which one is bigger? Which farm should pay more in taxes? And how much taxes should they each pay?

To answer these questions, the Egyptians came up with an idea. Some people say they were inspired by the square tiles that covered the floors of their temples. They imagined covering the farms with squares and counting the squares.

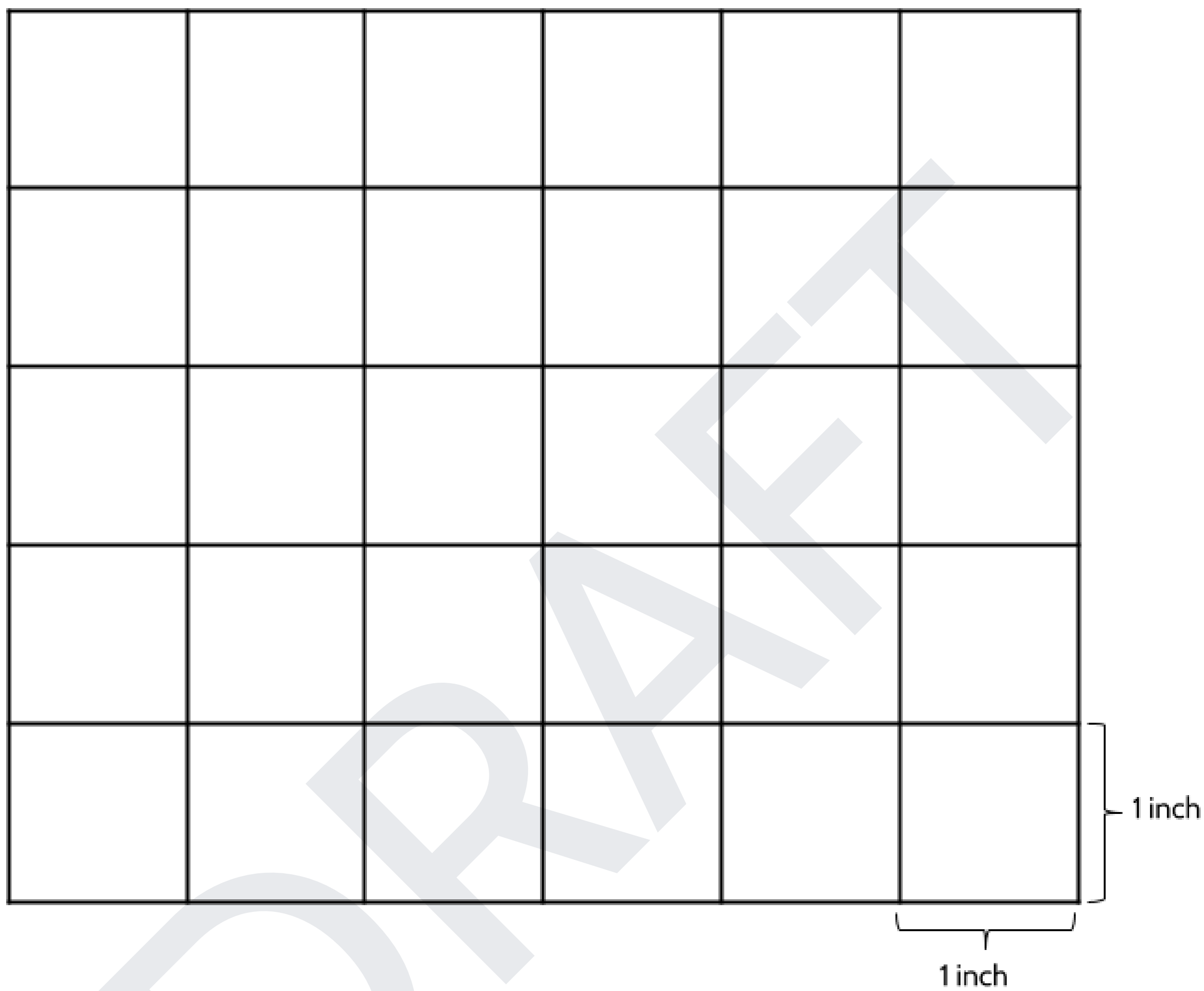


It takes 36 squares to cover the surface of the farm on the top. It takes 33 squares to cover the surface of the farm on the bottom.

So the farm on the top has a larger area.

You read earlier that **area** is the size of a surface. The Ancient Egyptians gave us a way to measure surfaces by asking how many squares it takes to cover that entire surface. <u>That is the key question of area.</u>

We measure area based on how many <u>squares</u> it takes to completely cover a surface.



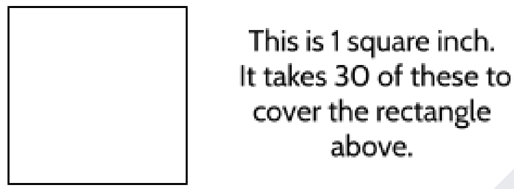
The area of the rectangle above is <u>30 square inches</u>.

- What do you think the 30 stands for?
- Why do you think it says “square inches”?

The 30 stands for the number of squares it takes to cover the entire rectangle.

The word “square” is used because we are covering the entire rectangle with squares.

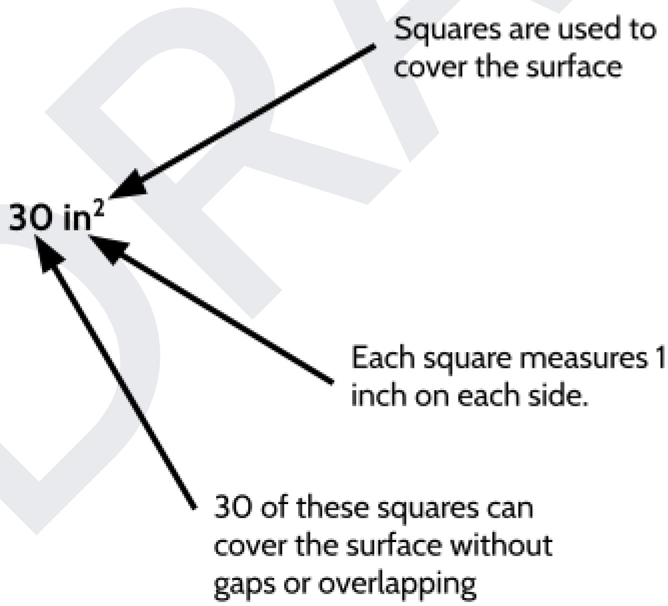
We use the word square inches, because each square measures 1 inch on each side.



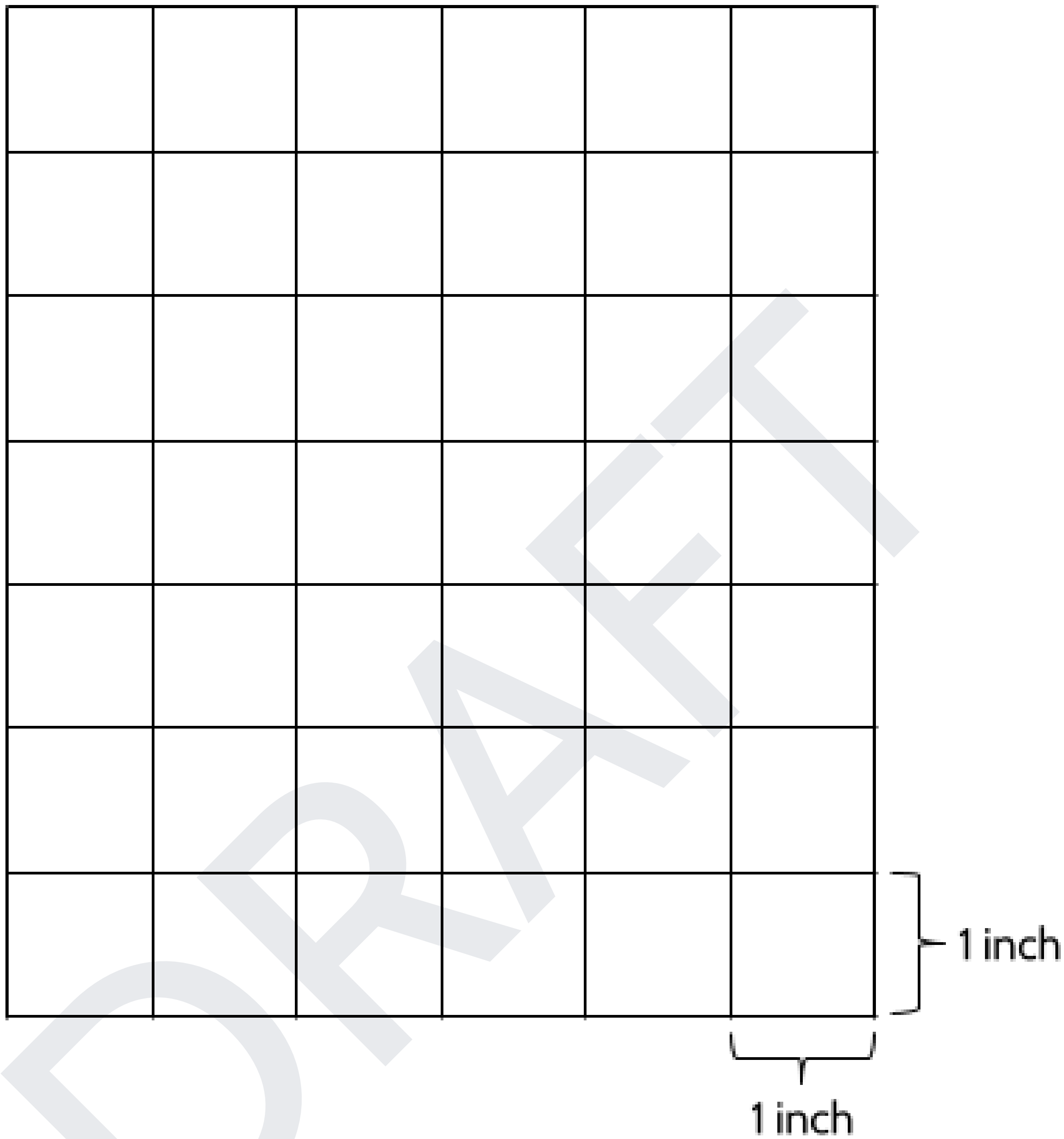
There are a few ways we can write 30 square inches:

30 square inches   30 sq. in.   30 in^2

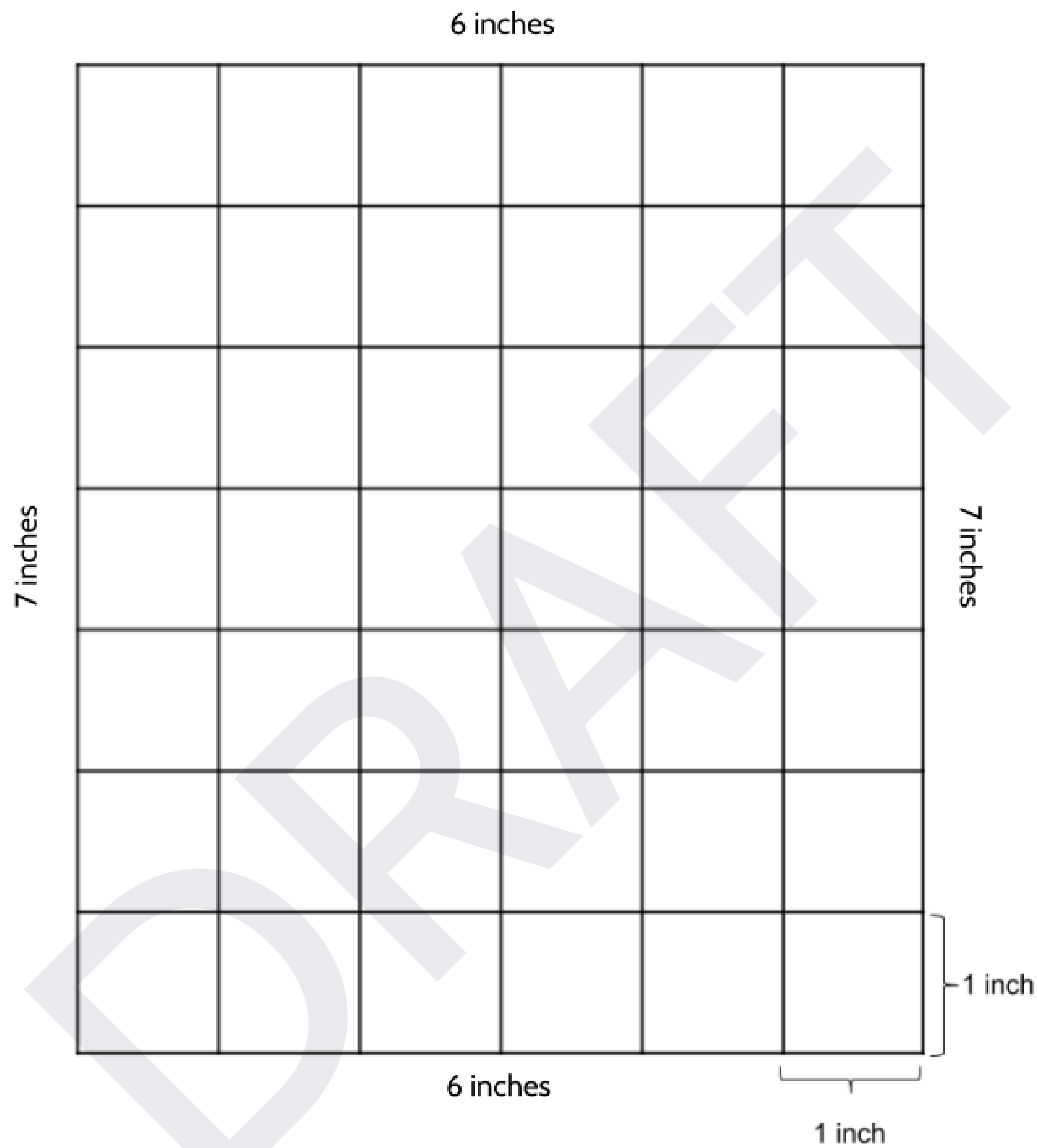
Any measurement in area will have a number, a unit (in this example the unit is “inches”) and the word “square”. Each part of that measurement means something.



2. What is the area of the rectangle on the next page? Write the area in all three ways.


1 inch
1 inch

In addition to the area, we can also measure the length of each side of the rectangle. Since we know that the side of each square is 1 inch, we can measure the length all the way around the rectangle by counting the inches.



If we add up the length of each side of this rectangle, we get 26 inches.

7 inches + 6 inches + 7 inches + 6 inches = 26 inches

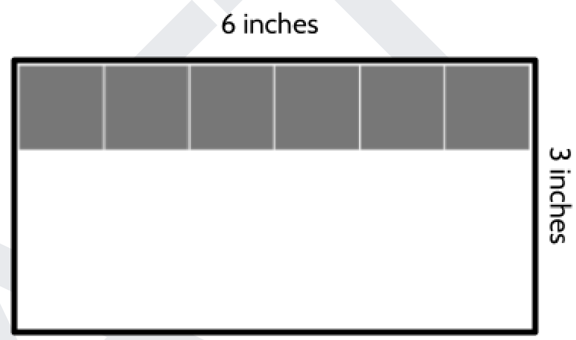
In geometry we have a special word for this measurement. *Perimeter* is the whole length of the border around an area or shape. It comes from the Greek word *peri* meaning "around" and *metron* meaning "measure."

In all the examples we have looked at so far, you have had squares drawn in to cover the surface of each rectangle. But what happens when there are no squares drawn in?

How could we figure out how many square inches it would take to cover the surface of this rectangle?

Here is one method:

We know the rectangle measures 6 *inches* across. We can imagine a row of 6 *square inches* along the top.



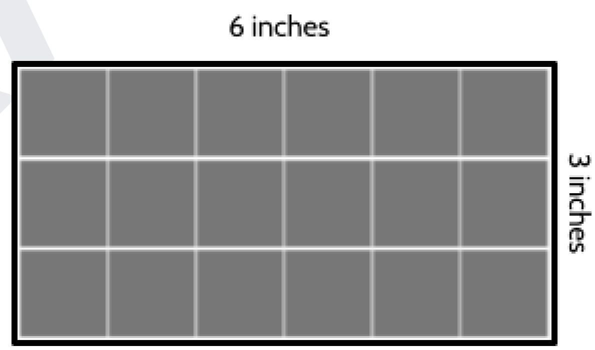
Since the rectangle measures 3 inches down, we can fit a total of three rows of square inches.

➔ 6 square inches

➔ 6 square inches

➔ 6 square inches

For a total of 18 square inches!



One thing you might notice about 6 and 3 is that when you multiply them you get 18. Multiplying 6 ✕ 3 means adding 3 groups of 6. When we are working with the area of rectangles, that means 3 rows of 6 square inches for a total of 18 square inches.

On the language of measurement: We can use the word "*by*" to describe the side lengths of rectangles. This rectangle measures 6 inches on the longer side and 3 inches on the shorter side, so we can describe this rectangle as measuring "6 by 3" or "6 inches by 3 inches." You may also see the side lengths described using a multiplication symbol. For example, the side lengths of this rectangle are 6x3, which we read as "6 by 3"

COMMON UNITS OF AREA

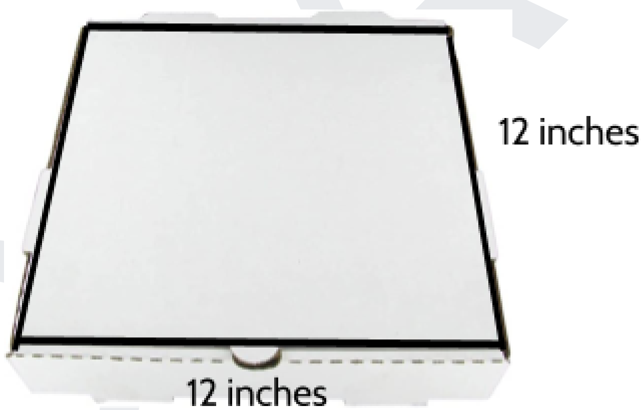
When we measure area, our units must have area too. We can't use the units of length you read about (inches, feet, meters, etc) to measure area. Remember, finding the area of a surface is asking how many squares it would take to cover a surface. To measure area, we use square inches, square feet, square meters, etc. In the table below you will find some common units of area and a few facts about each to help you think about how big they are.

A square inch is the area equal to a square that measures 1 inch on each side.
● A US quarter has an area just a little smaller than a square inch. ● A dollar bill has an area of about 16 square inches.
A square foot is the area equal to a square that measures 1 foot on each side.
● An average parking space has an area of about 160 square feet. ● A 12 inch pizza box has an area of 1 square foot.
A square yard is the area equal to a square that measures 1 yard on each side.
● The average parking space has an area of about 18 square yards. ● It would take about 97 dollar bills to cover a square meter.
A square mile is the area equal to a square that measures 1 mile on each side.
● The state of New York has an area of about 54,554 square miles. ● The United States has an area of almost 4 million square miles. ● The island of Puerto Rico has an area of about 3,515 square miles.
A square meter is the area equal to a square that measures 1 meter on each side.
● An average parking space has an area of about 15 square meters. ● It would take about 200 Post-it Notes to cover a square meter.
A square centimeter is the area equal to a square that measures 1 centimeter on each side.
● A US postage stamp has an area of about 4 square centimeters. ● A US quarter has an area of about 4 ½ square centimeters.
A square kilometer is the area equal to a square that measures 1 kilometer on each side.
● Central Park is almost 3½ square kilometers. ● The state of New York has an area of about 141,300 square kilometers.

Write at least 3 things you noticed as you read about the common units of area.

-
-
-

This pizza box is for a 12 inch pizza which means it has a length of 12 inches on each side.



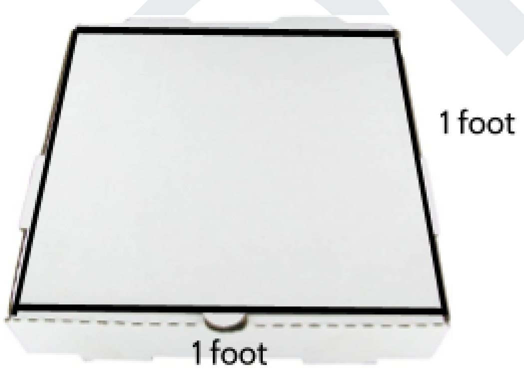
What is the area of the top of the pizza box?

You might have figured out that it would take 144 square inches to cover the top of this box.

That's 12 rows with 12 square inches in each row. The area of the top of the pizza box is 144 square inches.

Another way you might have answered is that it would take 1 square foot to cover the top of the pizza box.

Each side of the pizza box has a length of 12 inches. 12 inches is equal to 1 foot. So we can also describe the length of each side of the pizza box as 1 foot. The definition of a square foot is "a square that measures 1 foot on each side."



A 12 inch pizza box is a good visual model of one square foot. When we say something has an area of 16 square feet, it means we can cover its surface with 16 12-inch pizza boxes.

As a review, imagine the following situations and decide whether each is a measure of length or area.

3. Which would you use to measure the following: length or area?

    a. The distance from the floor to the ceiling

    b. Your height

    c. The distance between your car and the car in front of you on the highway

    d. The number of tiles needed to cover a bathroom floor

    e. The wrapping paper necessary to wrap a gift

    f. The distance between The Empire State Building and Niagara Falls

    g. The amount of skin on your body

    h. The total land in New York State

    i. The size of a room

Length and Area - Answer Key

1. Which would you use? *(Keep in mind that there are multiple reasonable answers for most of these)*
    a. miles or kilometers
    b. feet, inches, meters
    c. feet, meters
    d. feet, meters
    e. For the deep part of the ocean: miles, kilometers. For shallow parts of the ocean: feet, meters, and even inches.
    f. miles, kilometers
    g. inches, centimeters
    h. centimeters
    i. inches, centimeters, feet (for a large picture frame)
    j. centimeters, inches

2. 42 square inches. 42 sq. in. 42 in^2

3. Is it a measure of length or area?
    a. length
    b. length
    c. length
    d. area
    e. area
    f. length
    g. area
    h. area
    i. The size of a room can actually be a measure of both. If you are talking about the length of a room, you could be talking about the distance from one wall to the other or the height of the room. If you were talking about the area, you might be talking about how much floor space there is in the room.

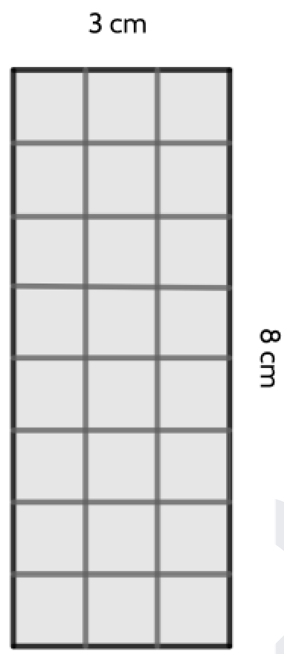
Area and Perimeter of Rectangles

Practice with Rectangles

In this section, you will apply what you have learned about area and perimeter to rectangles.

1. Find the area and perimeter of each rectangle.

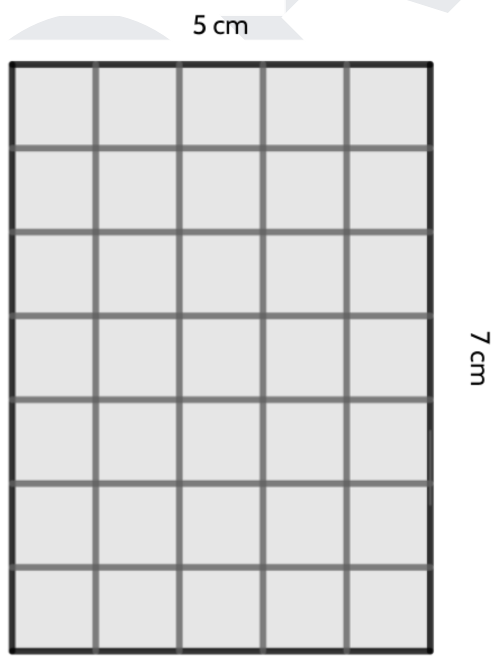
a.



Area = ______ square centimeters

Perimeter = ______ centimeters

b.



Area = ______ square centimeters

Perimeter = ______ centimeters

2. On the next page, there is a grid made up of square centimeters. Each square measures 1 centimeter on each side.

   Please draw the following figures on that grid and label each rectangle:

   a. Create a rectangle with an area of 28 square centimeters.

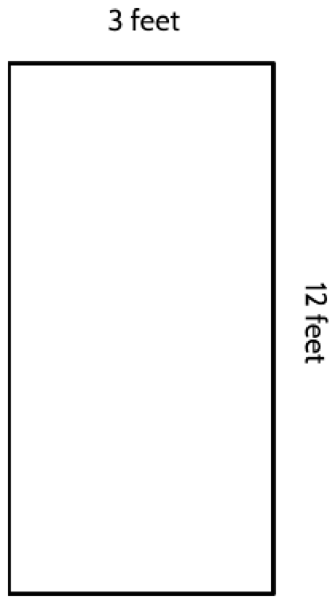
   b. Create two rectangles with different perimeters that both have an area of 24 sq. cm.

   c. Create a figure that is <u>not a rectangle</u> that has an area of 10 cm^2.

   d. Create a rectangle that has an area of 36 square centimeters and a perimeter of 26 centimeters.

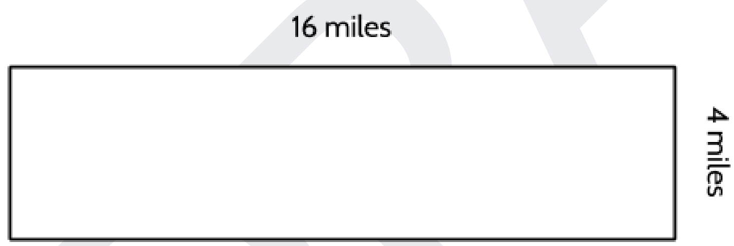
   e. Create a rectangle with two sides that each measure 2 centimeters and whose area is exactly 13 cm^2.

3. What is the area and perimeter of this rectangle?



A. Area = 36 feet, Perimeter = 30 feet
B. Area = 30 square feet, Perimeter = 36 feet
C. Area = 36 square feet, Perimeter = 30 feet
D. Area = 36 square feet, Perimeter = 15 feet

4. What is the area of this rectangle?
   What is the perimeter of this rectangle?



5. What is the greatest perimeter you can make with a rectangle that has an area of 24 square feet? (Use whole numbers only for the lengths of the sides).

AREA	SHORTER SIDE	LONGER SIDE	PERIMETER
24 square feet			
24 square feet			
24 square feet			
24 square feet			

6. Fill in this chart with the missing measurements.

Rectangle	Longer Side	Shorter Side	Area	Perimeter
1	10 cm	6 cm		
2	12 cm	5 cm		
3	4 in		12 in^2	
4	6 ft			20 ft
5			16 sq in	20 in
6		2 cm	30 sq in	
7		20 cm		100 cm
8	8 ft		56 ft^2	
9			24 sq in	22 in
10			100 sq feet	58 ft
11			144 sq cm	48 cm
12			180 mi^2	98 mi
13	4.5 cm	4 cm		
14		2 in		21 in

Liv's family wanted to grow vegetables. The rectangular space they wanted to dedicate to growing food is outlined in white in the picture to the right.

The long side measures 24 feet.

The short side measures 12 feet.

7. The perimeter of this rectangular space is ____________________ feet.

8. The area of this rectangular space is ____________________ square feet.

The family built four areas for planting, called raised beds.

- The largest bed was 12 ft by 4 ft.
- The two smaller beds were 4 ft by 4 ft.
- The middle sized bed was 10 ft by 4 ft.

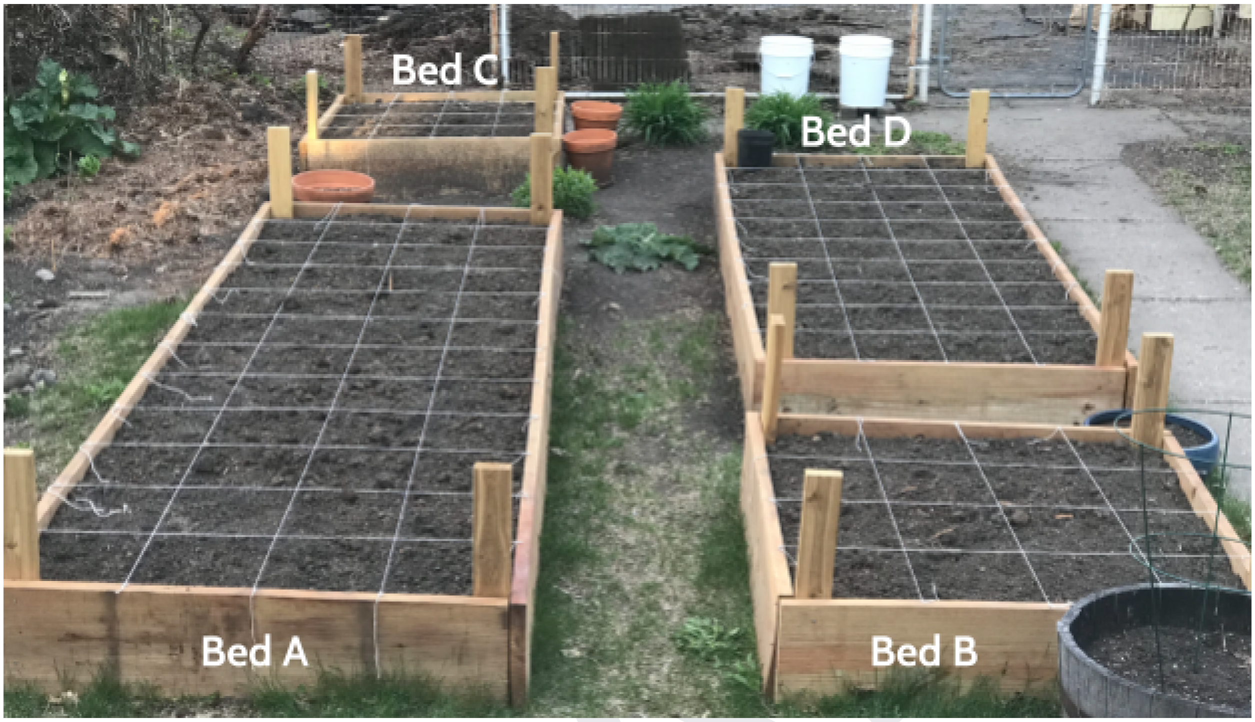
9. What was the perimeter of each bed, in feet?

   Perimeter of largest bed = ____ ft.

   Perimeter of smaller bed = ____ ft.

   Perimeter of middle sized bed = ____ ft.

Once they filled the beds with soil, they divided each bed into square feet areas.



10. What is the area of each bed?

Bed A: ____________________ square feet

Bed B: ____________________ square feet

Bed C: ____________________ square feet

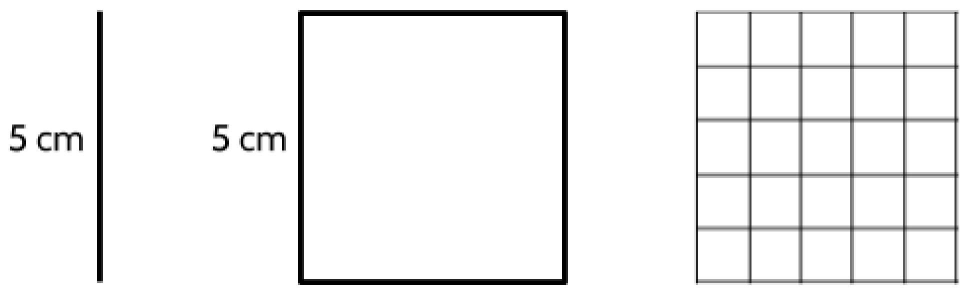
Bed D: ____________________ square feet

Building Squares & Finding Square Roots

You may have heard of numbers being "squared". For example: Five "squared" equals 25.

The idea of numbers being squared comes from area.

"Five squared" means building a square from a length of 5 units and finding the area of that square. Since all the sides of the square equal 5, then 5 squared equals 25.



5 cm is the length of the segment

To "square" this segment, it means we use the segment as one side and draw a full square.

The area of the square is 25 square cm.

Another way to write **five squared equals 25** is using an exponent. An exponent tells us how many times to multiply a number by itself. Exponents are written next to the number, but raised.

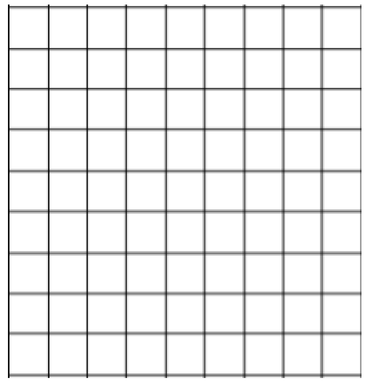
For example, 5^2 means 5×5.

2 is the exponent that tells us to multiply 5 by itself twice.

$$5^2 = 25$$

$$5 \times 5 = 25$$

11. Find the area of a square whose sides each measure 9 cm.



We can also go the other way. Starting with a square, we can figure out the length of one side of that square. The length of that side is called the *square root*.

The area of this square is 49 square cm.

What is the length of one side of this square?

We can write this as $7^2 = 49$, which can be read as "7 squared equals 49,"

OR

We can write $\sqrt{49} = 7$ which is read as "The square root of 49 is 7."

Sometimes we need to figure out the square root of a number that is too big to draw. There are a few ways we can do that.

Let's say we want to find the square root of 441. Another way to say this is: If we have a square with an area of 441 square centimeters, what is the length of one side of the square? To find the square root of a number we can ask, "What number times itself would give me the original number?"

Especially if you have a calculator, this strategy of guess-and-check can be quick.

Try a few different guesses to see how many tries it takes you to find what number times itself is 441. Record each attempt in the space below. After each attempt, ask yourself, "Should my next guess be smaller or larger than the number I just tried?"

Guess (Square Root)	Number Squared
12	144

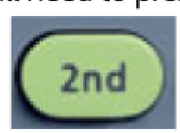
Guess (Square Root)	Number Squared

Another strategy is to use the square root function on a calculator. Depending on the calculator, you will use one of the following buttons:

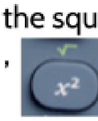
On most calculators you will see a button that looks something like this. Enter the number that represents the area of the square and then press this button. The calculator should display the square root.

If you have a smartphone, you also have a square root button, but it looks a little different. Go into the calculator mode. Then turn your phone on its side and you will see more options. One of them will look like the symbol above. Enter the number first and then press the button.

The TI-30XS is the calculator you are allowed to use for the HSE exam. For the TI-30XS, you will need to take an extra step. Notice that the square root symbol is printed above the x^2 button. You will need to press the **2nd** button…



… and then x^2 to tell the calculator to use the function written above the button.

Another difference is you need to enter the square root symbol first. So to find the square root of 144, press  ,  , **144**, then **enter**.

12. Use a calculator to find the square root of the following numbers.

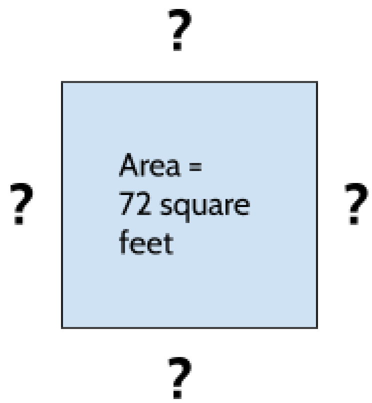
	Number	Square Root
	36	
	625	
	1225	
	0.25	
	250,000	
Try your own:		

13. Complete this table.

Length of one side	Length of one side squared	Area of the square
3	3^2	9
4	4^2	
5	5^2	
6	6^2	
7	7^2	
8	8^2	
9	9^2	
10	10^2	
11	11^2	
12	12^2	
13	13^2	
14	14^2	
15	15^2	
16	16^2	
17	17^2	
18	18^2	
19	19^2	
20	20^2	

The numbers in this table are called perfect squares. Perfect squares are made when we square whole numbers. For example, 441 is a perfect square because 21 × 21 is 441.

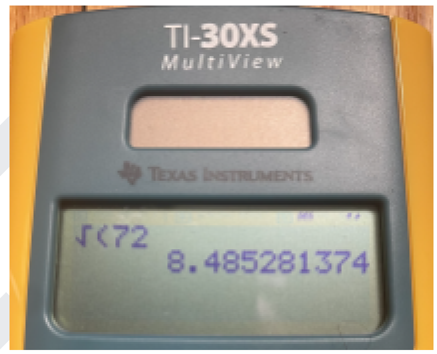
We can also find the square root of numbers that are *not* perfect squares.



Say we have a <u>square</u> that has an area of 72 square feet and you want to find the length of its sides.

There is no whole number that you can multiply by itself to get 72. But there *is* a square root of 72.

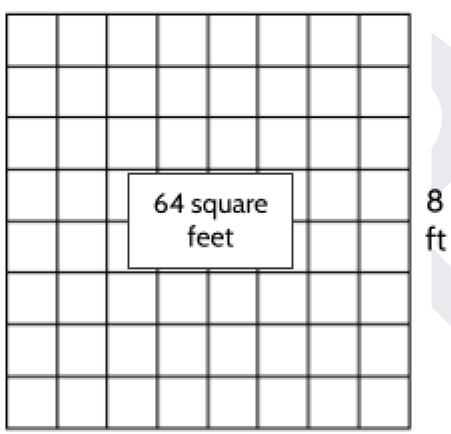
You could use your calculator to find a precise answer.

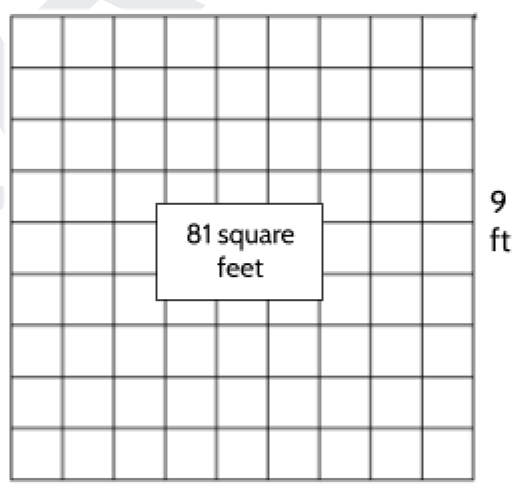


A square root of 8.485281374 feet means that each side of this square has a side length of about 8.5 feet.

We can use the perfect squares to *estimate* non-perfect squares.

The square root of 81 is 9 and the square root of 64 is 8.





If we are looking for a square with an area of 72 square ft, a side of 8 feet is too small and a side of 9 feet is too big. So we can estimate that the square root of 72 is between 8 and 9.

Practice with perfect squares and non-perfect squares.

14. Find the square root of each of the following numbers:

    a. The square root of 576 is _______ .

    b. If the area of a square is 625, the length of one side of the square is _______ .

    c. What number times itself is 676? _______

    d. _______ × _______ = 900.

    e. The square root of 1024 is _______ .

15. The square root of 85 is between:

    A. 6 and 7

    B. 7 and 8

    C. 8 and 9

    D. 9 and 10

16. A square has an area of 355 square inches. What is the approximate length of each of its sides?

    A. between 16 inches and 17 inches

    B. between 17 inches and 18 inches

    C. between 18 inches and 19 inches

    D. between 19 inches and 20 inches

17. Which of these is a non-perfect square number with square root between 5 and 6?

    A. 22

    B. 23

    C. 28

    D. 38

Estimation and Precision

As you work on the activities in this packet, you will make observations and learn how to make precise calculations and measurements. You will also practice answering the kinds of geometry questions that you will see on the high school equivalency exam.

Sometimes in life, precise measurements are required. For example, you can't build a strong foundation of a building without being able to make a proper square. But in our everyday lives, we often estimate. Estimating means finding a number that is not exact, but close enough to the right answer. When we estimate, we are not trying to get the exact right answer. When we estimate, we are looking for an answer that is good enough.

There is a symbol we use in math when we estimate	We use this symbol to represent things that are **equal**	We use this symbol to say things are **approximately equal**
	=	**≈**
Examples	$\sqrt{81} = 9$ $\sqrt{100} = 10$	$\sqrt{90} \approx 9.5$ $\sqrt{72} \approx 8.5$

Estimation is something we get better at the more we do it. Look for opportunities to estimate in your life. As you are shopping, estimate how much the total cost will be. Estimate how long it will take you to get somewhere. Estimate how many people are in the same place as you. Estimate the temperature before looking to see what it is. Compare your estimate to the actual result. When we compare the result with our estimate, it teaches us to make an even better estimate next time.

You can estimate as you walk. An average adult has a stride length of *about* 2 to 2½ feet. A stride is the length of two natural steps. You can think of a stride as each time your left foot touches the ground. Now you can estimate a distance and test your estimations by counting how many strides it takes you to walk that distance. With a little more work, you can measure your *own* personal stride length and use that to measure your world.

You may find yourself estimating without numbers at all. When you are cooking, do you use measuring cups and spoons or do you just estimate? After dinner, when you look through your tupperware to choose the right size container for your leftovers, how do you know which one to choose?

Area and Perimeter of Rectangles - Answer Key

1.
    a. Area = 24 square centimeters. Perimeter = 22 centimeters.
    b. Area = 35 square centimeters. Perimeter = 24 centimeters.

2.
    a. There are several different rectangles you might draw that could have an area of 28 sq cm. Some possible answers include: a rectangle that is 7 cm by 4 cm, a rectangle that is 14 cm by 2 cm.
    b. A rectangle that is 2 cm by 12 cm has a perimeter of 28 cm. A rectangle that is 3 cm by 8 cm has a perimeter of 22 cm. A rectangle that is 4 cm by 6 cm has a perimeter of 20 centimeters. A rectangle that is 1 cm by 24 cm has a perimeter of 50 cm. All have an area of 24 square cm.
    c. There are many correct answers. The important thing is that whatever shape you have, it takes 10 square centimeters to cover that shape.
    d. A rectangle that is 9 cm by 4 cm has a perimeter of 26 centimeters and an area of 36 sq cm.
    e. This one is tricky. For a rectangle to have two sides that measure 2 cm and an area of 13 cm^2 you need the other sides to measure 6 ½ centimeters.

3. Choice C is the correct answer. Area = 36 square feet. Perimeter = 30 feet. Choice A has the correct numbers, but the area is given as 36 feet, which is a measure of length and not of area. Choice B has the measurement for area and perimeter reversed. Choice D has the correct area, but only half of the perimeter. Remember when calculating the perimeter, you need to add the length of every side, not just the sides that are given.

4. Area = 64 square miles. Perimeter = 40 miles.

5. There are 4 possible rectangles with an area of 24 sq ft. 50 ft is the largest possible perimeter.

Area (sq ft)	Shorter Side	Longer Side	Perimeter (ft)
24 sq ft	1	24	**50 ft**
24 sq ft	2	12	28 ft
24 sq ft	3	8	22 ft
24 sq ft	4	6	20 ft

6.

Rectangle	**Longer Side**	**Shorter Side**	**Area**	**Perimeter**
1	**10 cm**	**6 cm**	60 sq cm	32 cm
2	**12 cm**	**5 cm**	60 sq cm	34 cm
3	**4 in**	3 in	**12 in^2**	14 in
4	**6 ft**	4 ft	24 ft	**20 ft**
5	8 in	2 in	**16 sq in**	**20 in**
6	15 in	**2 in**	**30 sq in**	34 in
7	30 cm	**20 cm**	600 cm	**100 cm**
8	**8 ft**	7 ft	**56 ft^2**	30 ft
9	8 in	3 in	**24 sq in**	**22 in**
10	25 ft	4 ft	**100 sq feet**	**58 ft**
11	12 cm	12 cm	**144 sq cm**	**48 cm**
12	45 miles	4 miles	**180 mi^2**	**98 miles**
13	**4.5 cm**	**4 cm**	18 sq cm	17 cm
14	8.5 in	**2 in**	17 in^2	**21 in**

7. The perimeter is 72 feet.

8. The area is 288 square feet.

9. Perimeter of largest bed = 32 ft.

   Perimeter of the two smaller beds = 16 ft.

   Perimeter of middle sized bed = 28 ft.

10. Bed A: 48 square feet

    Bed B: 16 square feet

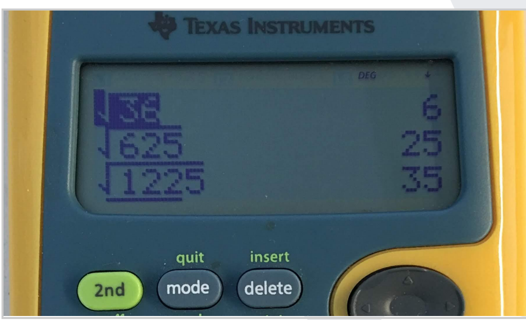
    Bed C: 16 square feet

    Bed D: 40 square feet

    The area of the 12x4 bed is 48 sq ft. The 10x4 bed is 40 sq ft. The 4x4 beds are 16 sq ft.

11. A square with a side length of 9 cm has an area of 81 square cm.

12. Square roots with a calculator



13. Completed chart

Length of one side of the square	The side squared	Area of the square
2	2^2	4
3	3^2	9
4	4^2	16
5	5^2	25
6	6^2	36
7	7^2	49

8	8^2	64
9	9^2	81
10	10^2	100
11	11^2	121
12	12^2	144
13	13^2	169
14	14^2	196
15	15^2	225
16	16^2	256
17	17^2	289
18	18^2	324
19	19^2	361
20	20^2	400

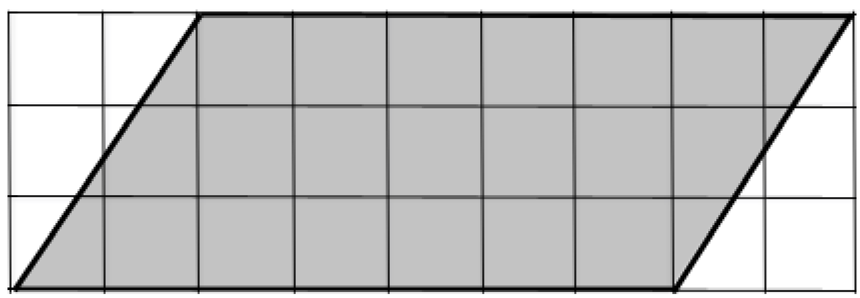
14. Finding square roots
    a) The square root of 576 is **24.**
    b) If the area of a square is 625, the length of one side of the square is **25**.
    c) What number times itself is 729? **27**
    d) **30** × **30** = 900.
    e) The square root of 1024 is **32.**
15. Choice D. You could estimate that the square root of 85 is between 9 and 10. One way to estimate this answer is to see that 85 is between the perfect squares 81 and 100. 9 is the square root of 81 and 10 is the square root of 100, so the square root of 85 will be between 9 and 10.
16. Choice C. The square root of 18 is 324 and the square root of 19 is 361, so the square root of 335 will be between 18 and 19.
17. Choice C. The square of 5 is 25 and the square of 6 is 36, so we are looking for a number between 25 and 36.

Area of Other Polygons

Square units fit nicely when we use them to cover the surface of a rectangle or a square. But what about the other polygons?

In this section, we will use what we know about finding the area of rectangles to learn how to find the areas of other shapes.

Parallelograms

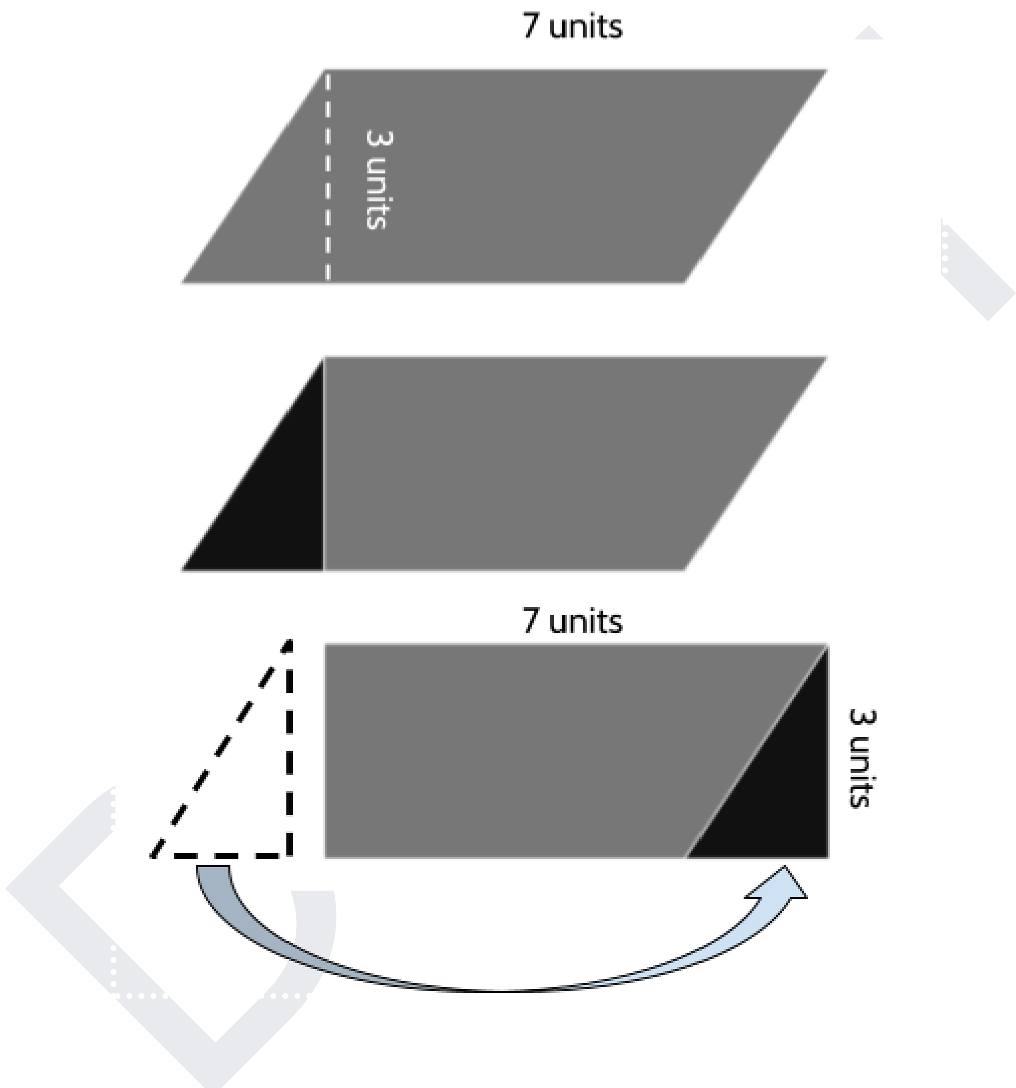


1. What is the approximate area of this parallelogram? (*How many square units do we need to cover its surface?*)

   Explain how you know.

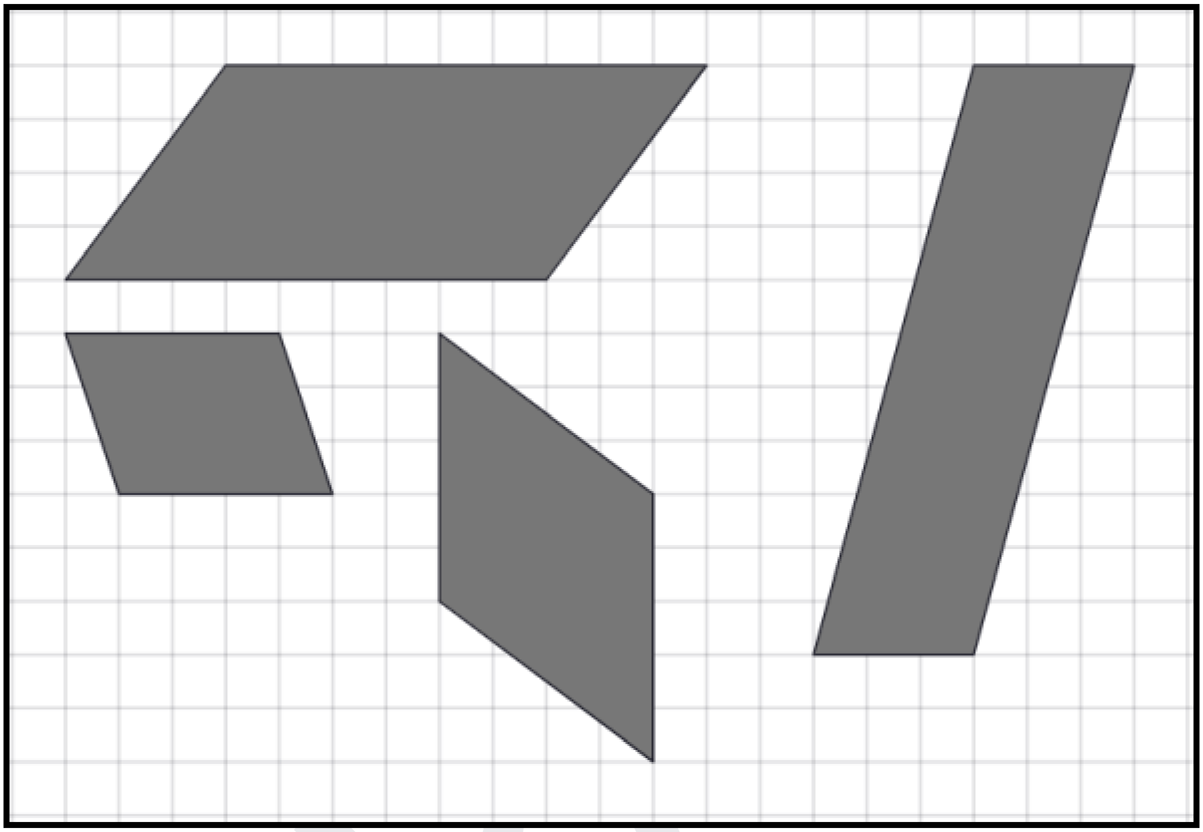
We can use what we learned about rectangles to find the area of this parallelogram.

Imagine cutting a triangle off of the parallelogram. We can move that triangle to the other side of the parallelogram to make a rectangle with the same area as the parallelogram. The new rectangle is 7 units by 3 units which makes an area of 21 square units. We didn't change the area of the parallelogram. We just moved it a piece around to make a rectangle.



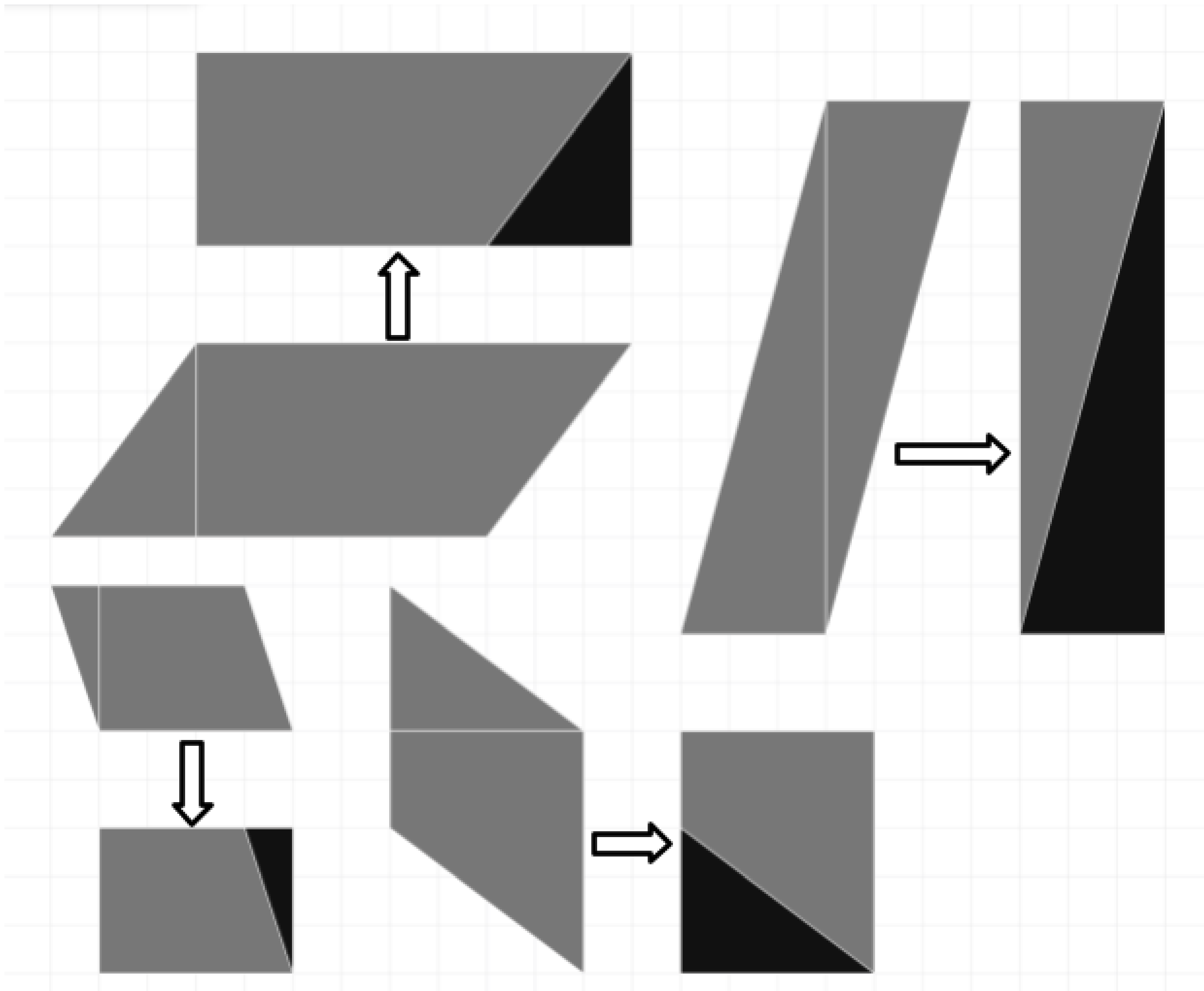
You may be wondering if this method will work for all parallelograms. Try it!

Turn each of the parallelograms below into a rectangle.



Does each rectangle you created have the same area as the parallelogram?

Here are some possible ways to cut a triangle section from each parallelogram and rearrange them into rectangles:



As you can see in these examples, each parallelogram can always be rearranged to form a rectangle that has the same height. Because of this, the formula for finding the area of a parallelogram is exactly the same as it is for rectangles. We multiply the base times the height.

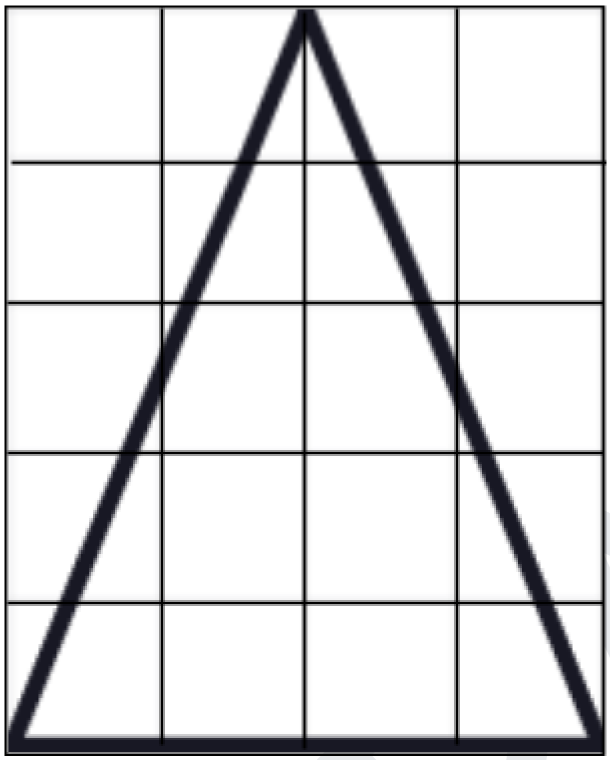
We can write a formula to help find the area of a parallelogram:

$$A = bh$$

where A represents the area, b represents the length of the base and h represents the length of the height.

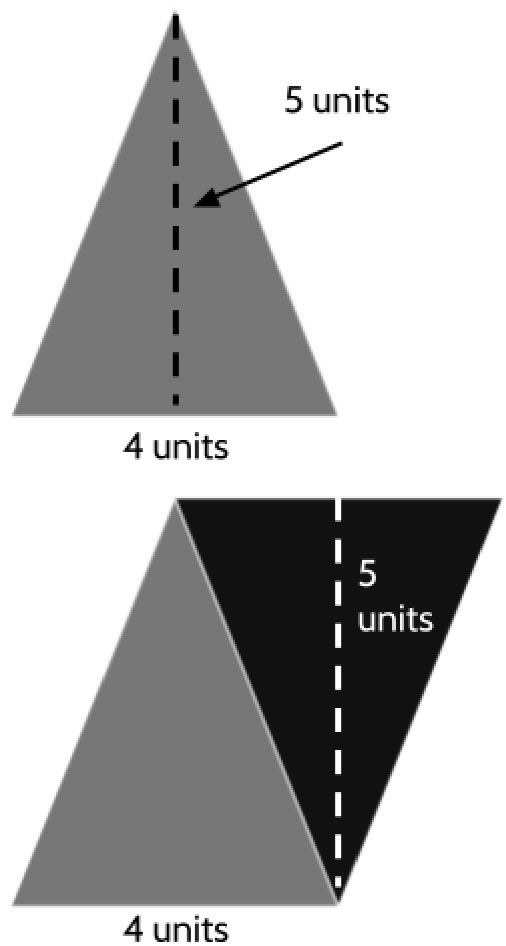
Triangles

2. What is the area of this triangle? *(How many square units do we need to cover its surface?)*



Explain how you came up with your estimate.

One way to connect what we know about finding the area of rectangles and parallelograms to triangles is to imagine **doubling** the triangle.

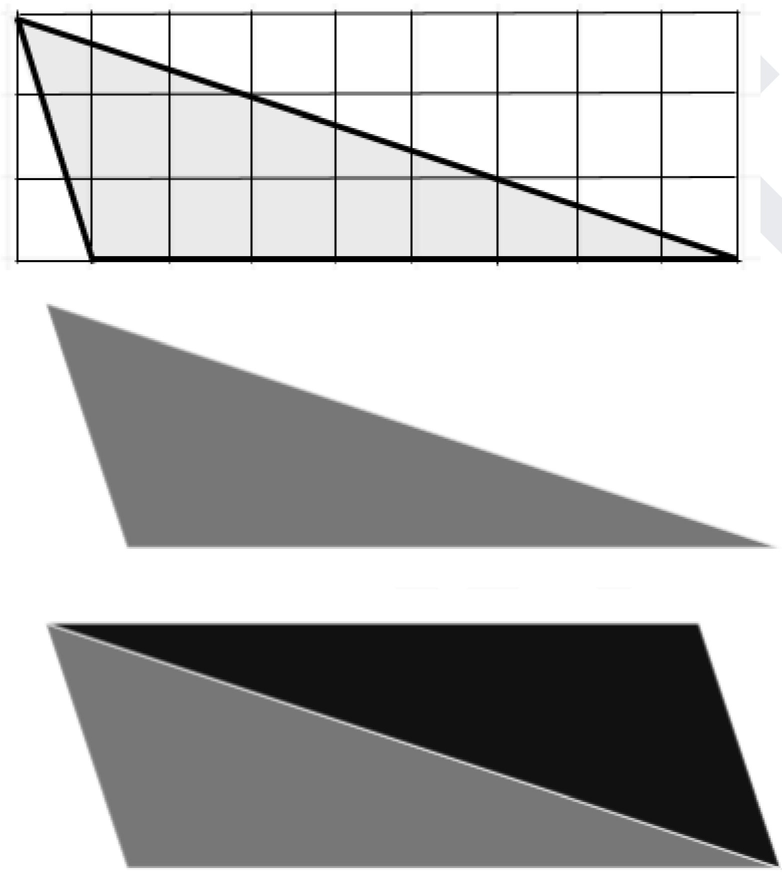


Two copies of a triangle can always be rearranged to form a parallelogram. We can find the area of the parallelogram by multiplying the base times the height. 5 × 4 is 20, so the area of the parallelogram is 20 square units. But that is not the area of the triangle. Can you figure out why?

This parallelogram does not have the same area as the triangle we started with. Remember, we doubled the triangle to form the parallelogram. The area of the parallelogram is double the area of the triangle. If the area of the parallelogram is 20 square units, the area of the triangle is 10 square units.

Let's see what happens with a different triangle. This triangle has a base of 8 units and a height of 3 units. If we double the triangle, we can make a parallelogram that has two times the area of the triangle. In this case, the area of the parallelogram is 21 square units.

If we divide 21 square units by 2, we get 10.5 units. The area of the triangle is 10.5 square units.



We have already learned we can use the formula ***A = bh*** to find the area of a parallelogram. To find the area of a triangle, we can use that same formula and divide by 2 (because we doubled the triangle earlier).

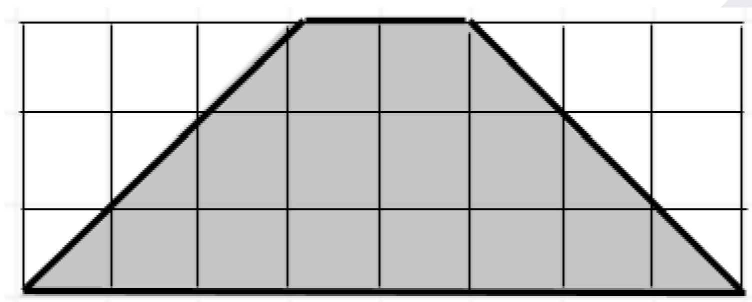
We can also use a formula for finding the area of any triangle. We can write it as:

$$A = \frac{bh}{2} \quad \text{or} \quad A = \tfrac{1}{2} bh$$

Trapezoid

We are now going to find the area of a shape called a *trapezoid*.

A trapezoid is a flat shape with four straight sides that has *at least* one pair of opposite parallel sides.

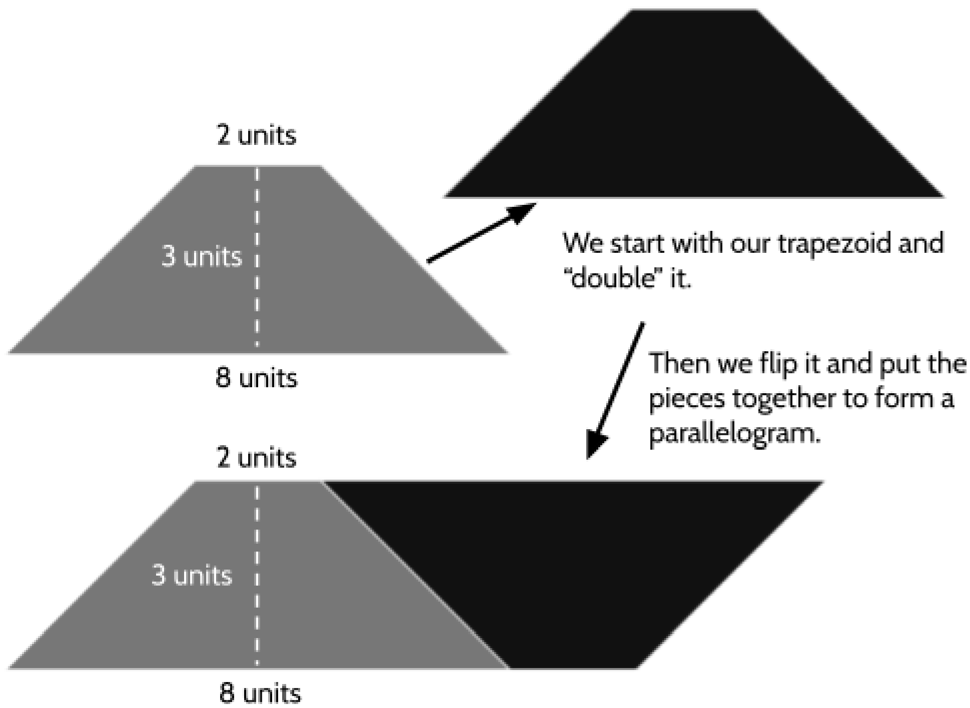


3. Use what you know about area to find the area of this trapezoid.

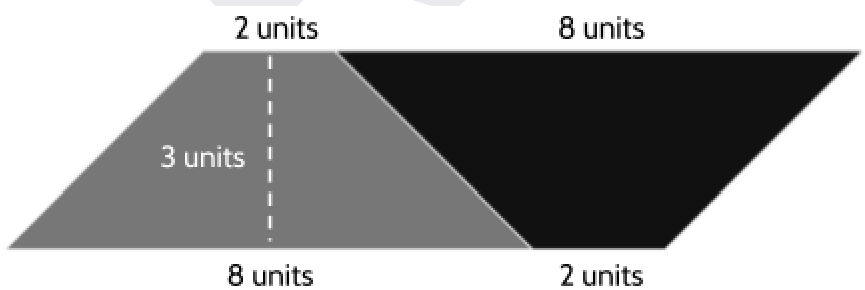
   (How many square units do we need to cover its surface?)

There are many ways to find the area of the trapezoid.

One way is similar to what we did with triangles.



We know the height of the trapezoid is 3, but what is the base? Remember that we doubled the trapezoid, *and then flipped it*, to make the parallelogram. That means the base is the length of the shorter base *plus* the length of the longer base.



The area of this parallelogram: 10 units × 3 units = 30 square units.

Since we doubled the trapezoid, we need to divide that area by 2 to find the area of the trapezoid. 30 ÷ 2 is 15 square units.

To summarize, this is one way to find the area of a trapezoid:

- Double the trapezoid and rearrange it so you have a parallelogram.
- The base of the parallelogram is the length of the parallel sides of the trapezoid.
- Find the area of the parallelogram.
- Remember that you doubled the area of the trapezoid, so you need to divide the area of the parallelogram by 2.

We can represent this method with a formula.

$$A = \frac{h\,(base\ 1 + base\ 2)}{2} \quad \text{or} \quad A = \tfrac{1}{2}\, h(base_1 + base_2)$$

A WORD ON FORMULAS

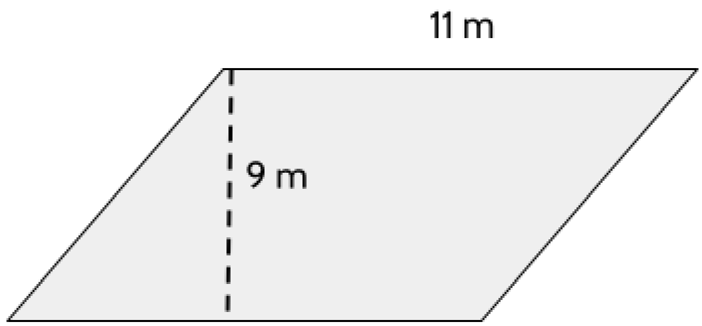
When you take the HSE exam, you will be given a math formula sheet that has various formulas. These are the area formulas for polygons that you will be given if you take the HSE exam. Each of these formulas represents one way to find the area of these polygons. You don't have to remember the formulas for the test, but it is helpful to understand how they are all connected. It is more important that you understand the formulas than it is to memorize them.

Area of a:

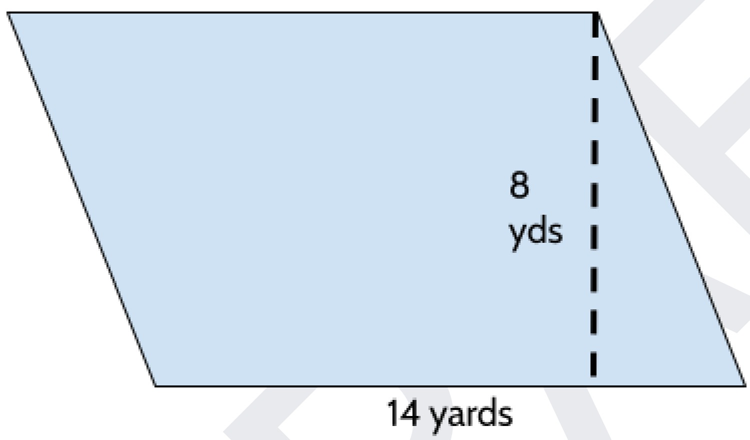
square	$A = s^2$
rectangle	$A = lw$
parallelogram	$A = bh$
triangle	$A = \frac{1}{2}bh$
trapezoid	$A = \frac{1}{2}h(b_1 + b_2)$

It is a strange that the formula they give for finding the area of a rectangle is *A = lw* when you can use the same formula when finding the area of a parallelogram. Remember that *s (side), l (length), w (width), b (base)*, *and h (height)* are different words for things that are similar.

4. What is the area of this parallelogram?

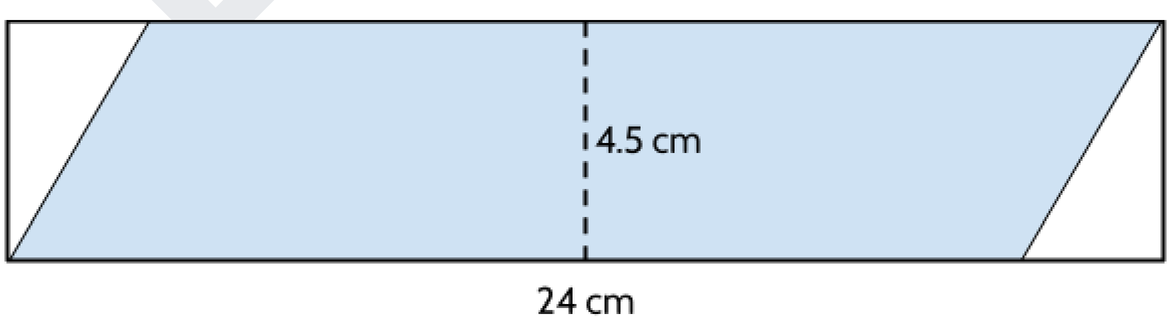


5. What is the area of the parallelogram below?

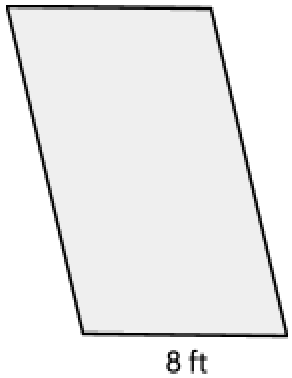


A. 44 square yards
B. 56 square yards
C. 112 square yards
D. 224 square yards

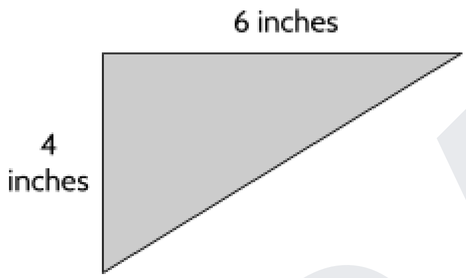
6. What is the area of the shaded section of the rectangle below?



7. The area of this parallelogram is 96 square feet. If its base is 8 feet, what is the height of this parallelogram?



8. Find the area of the triangle below.



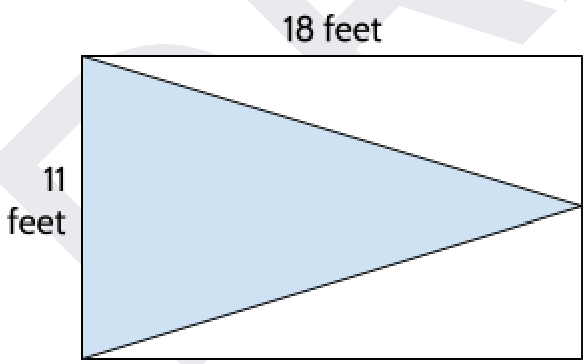
A. 12 inches

B. 12 square inches

C. 24 inches

D. 24 square inches

9. What is the area of the shaded part of the rectangle below?



A. 58 sq. ft.

B. 99 sq. ft.

C. 198 sq. ft

D. 396 sq. ft.

10. The triangle below has an area of 276 square inches.

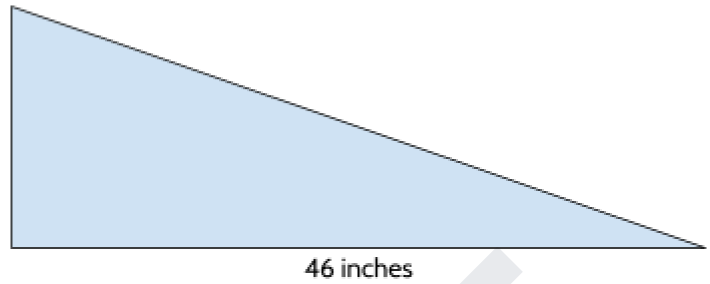
    What is the height of the triangle in inches?

    A. 3 inches

    B. 6 inches

    C. 12 inches

    D. 24 inches



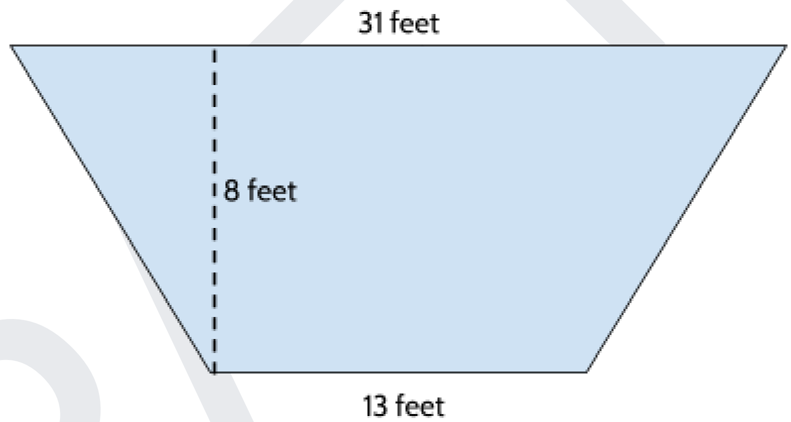
11. What is the area of the trapezoid in the diagram below?

    A. 124 sq. ft.

    B. 176 sq. ft.

    C. 248 sq. ft.

    D. 352 sq. ft.



12. The area of a trapezoid is 164 sq. ft. If the height is 8 ft and the length of the longer base is 24 feet, what is the length of the shorter base?

    A. 3.5 ft

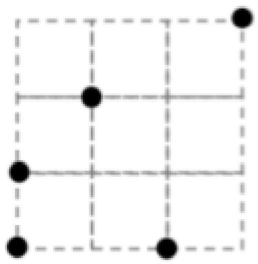
    B. 17 ft

    C. 44.5 ft

    D. 54.7 ft

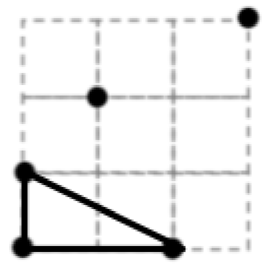
13. Here are some area puzzles. On each grid, connect three dots that form a triangle with the specified area.[2]

For example:



Area = 1

For this puzzle, we need to connect 3 dots to form a triangle with an area of 1 square unit.

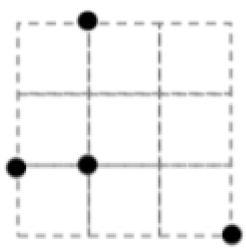


Area = 1

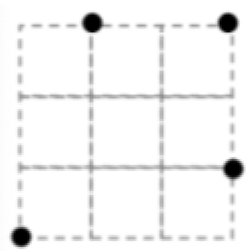
This triangle has a base of 2 and a height of 1.
2 x 1 = 2 and if we divide 2 by 2, we get 1.

A=½ bh
A = ½ (2 x 1)
A = ½ (2)
A = 1

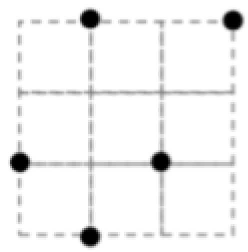
Have fun!



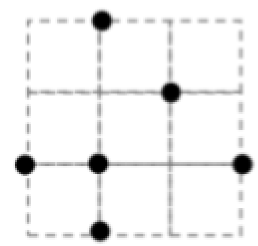
Area = 1



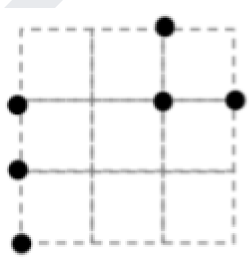
Area = 2



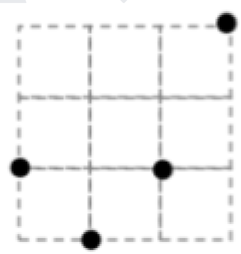
Area = 3



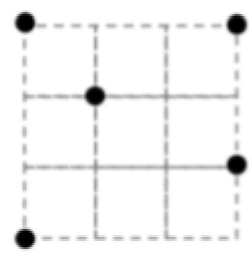
Area = 2



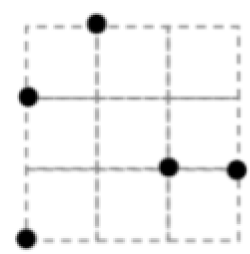
Area = 3



Area = 1



Area = 2

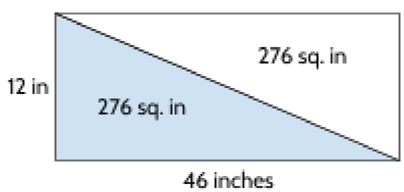


Area = 3

[2] These are called Sankaku Puzzles. They were created by Naoki Inaba. Learn more here: https://mathequalslove.net/sankaku-puzzles/

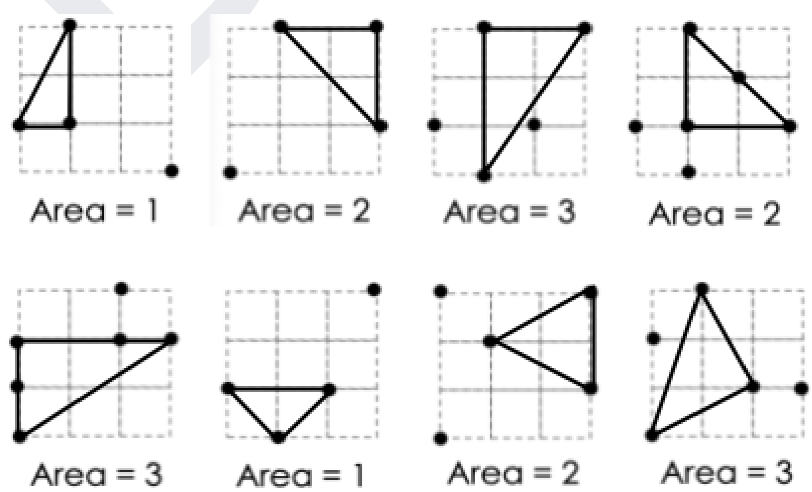
Area of Other Polygons - Answer Key

1. 21 square units
2. 10 square units
3. 15 square units
4. 99 square meters
5. Choice C. 112 square yards
6. 108 sq. cm.
7. 12 feet
8. Choice B. It would take 12 square inches to cover the surface of the triangle.
9. Choice B. The area of the rectangle is 198 sq. ft., so the area of the triangle is 99 sq. ft.
10. Choice C. This question is a little different. We are given the area and one side of the



triangle and asked to find the length of one of the other sides.It can help to remember where the formula for finding the area of triangles comes from.
If we double the 276 sq., we can imagine a parallelogram with an area of 552 sq. in. It can help to sketch the doubled triangle.

11. Choice B.
12. Choice A. The area of the trapezoid is 164 sq ft. If we double that in order to imagine a parallelogram, the area of the parallelogram would be 328 square ft. We can find the area of a parallelogram by multiplying the base by the height. We want to figure out the number we could multiply by 8 to get 328. That number is 41. The length of the longer side is 24, and 41 - 24 = 17.
13.

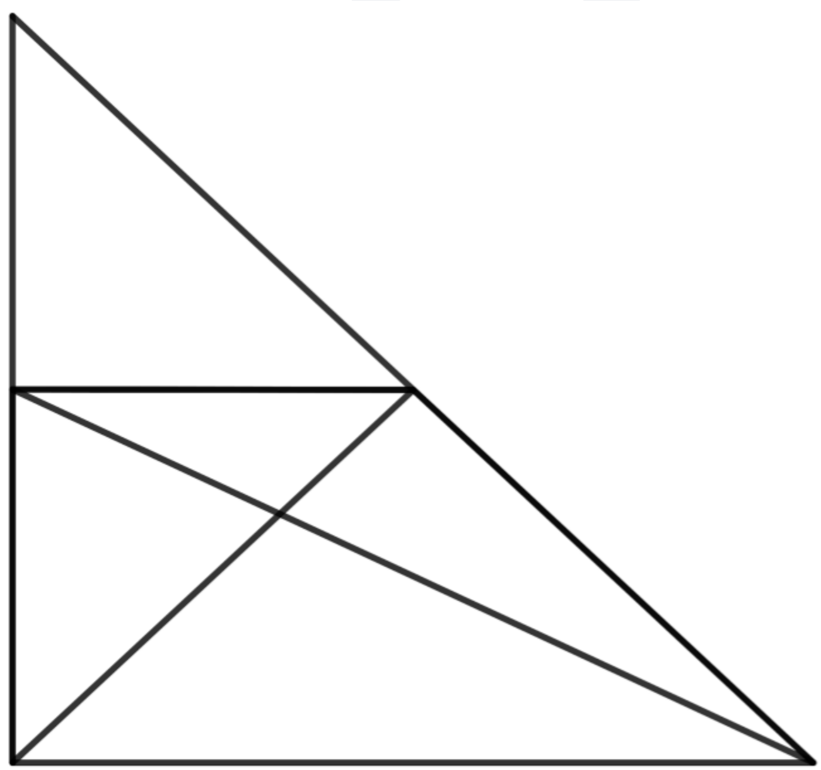


Triangles

Triangles are polygons with three sides that form three angles.

We often see triangles in structures like bridges and high-rise buildings. Triangles in construction provide strength and stability.

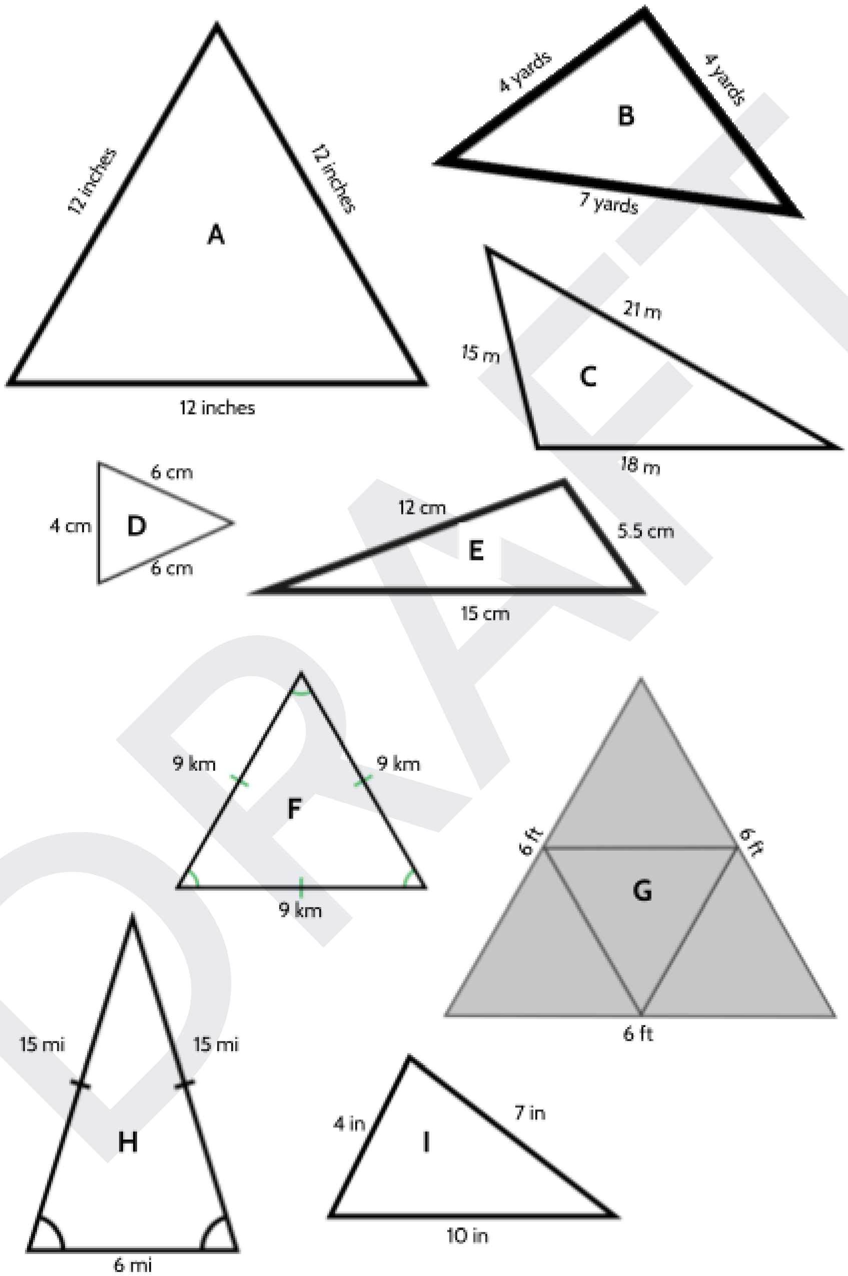
1) How many triangles can you find in this picture? (Hint: There are more than 5.)



Types of Triangles

There are different ways to categorize triangles. One way is by their sides.

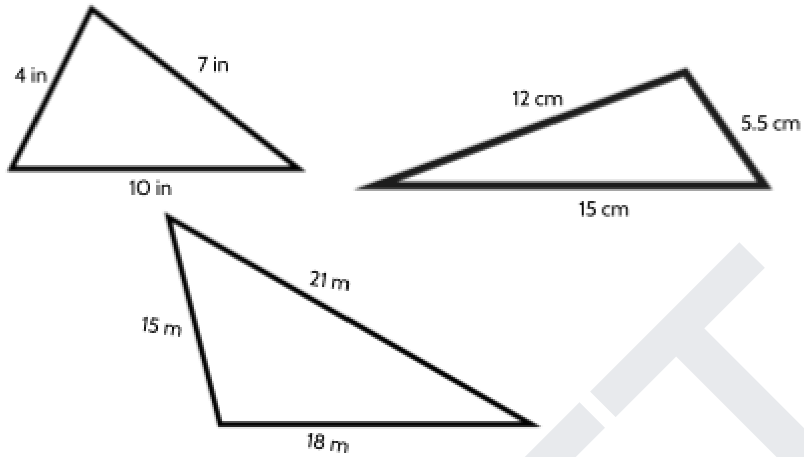
Examine the 9 triangles below. Which ones could be grouped together?



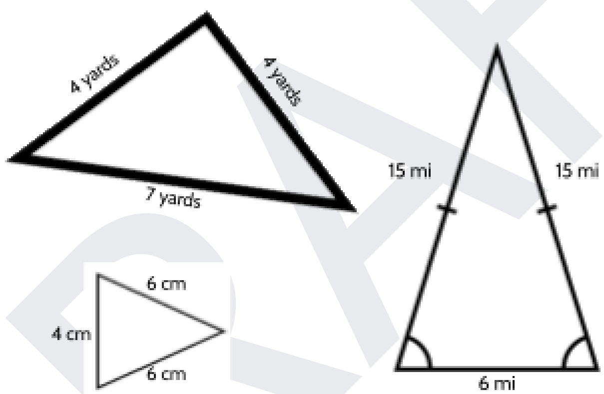
[None of these triangles are drawn to scale.]

There are multiple ways to classify triangles, depending on what we are interested in looking at. One way to categorize triangles is by the number of sides with the same length.

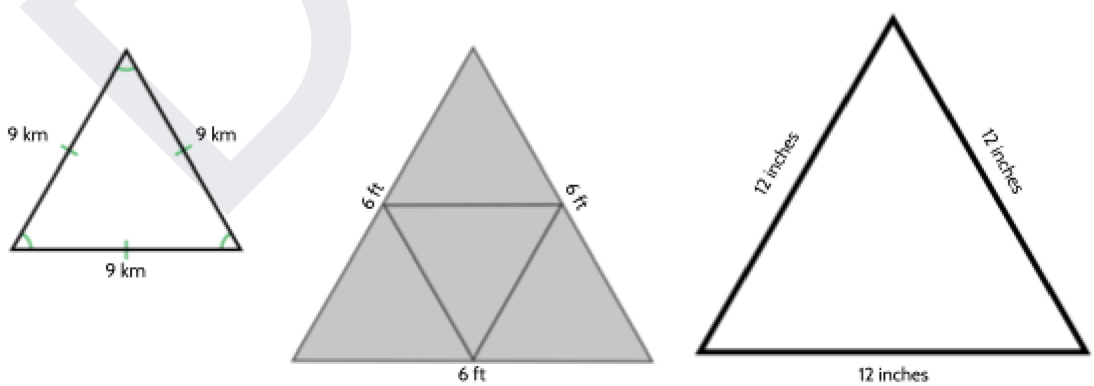
Scalene Triangles have no sides with the same length.



Isosceles Triangles have two sides with the same length.



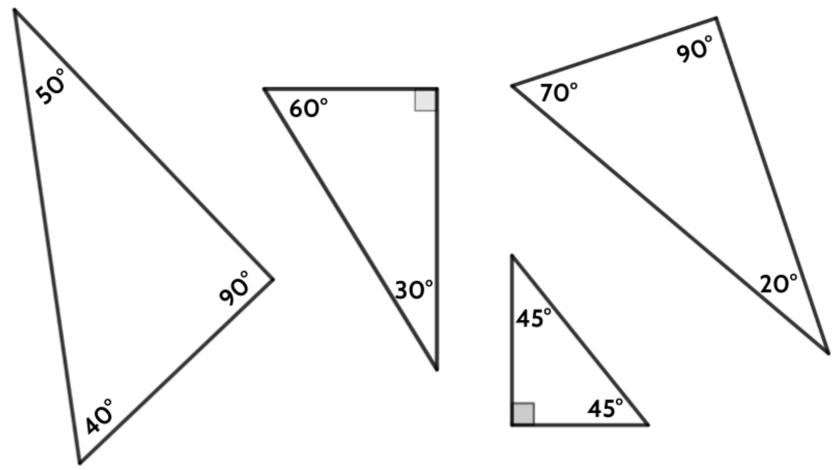
Equilateral Triangles have three sides with the same length.



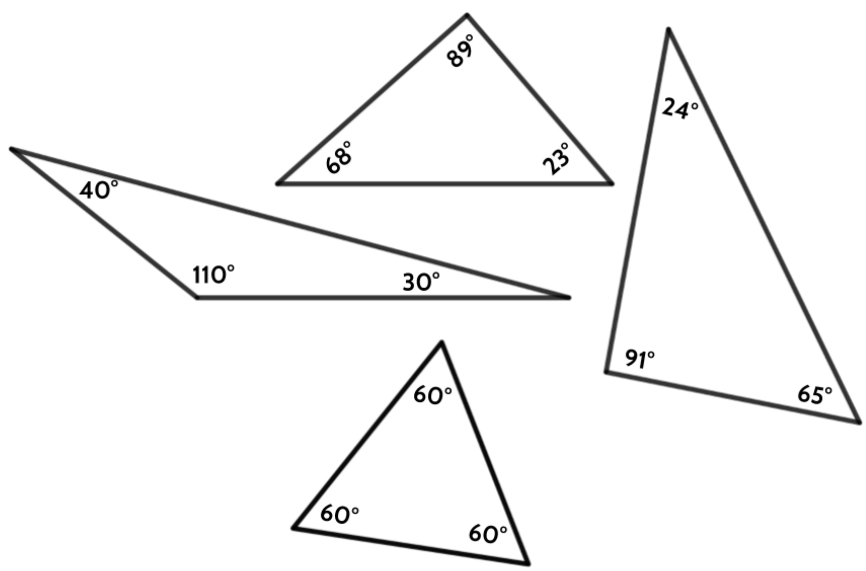
Right Triangles

Another way to classify triangles is by their angles. One common triangle classified in this way is called a right triangle. Use the examples below to complete a definition for a right triangle.

These are right triangles.



These are ***not*** right triangles.



2) I think a right triangle is a triangle that ____________________________________

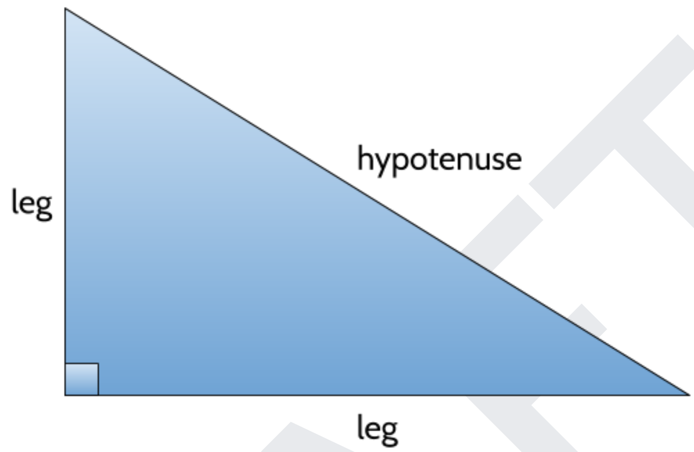
____

____.

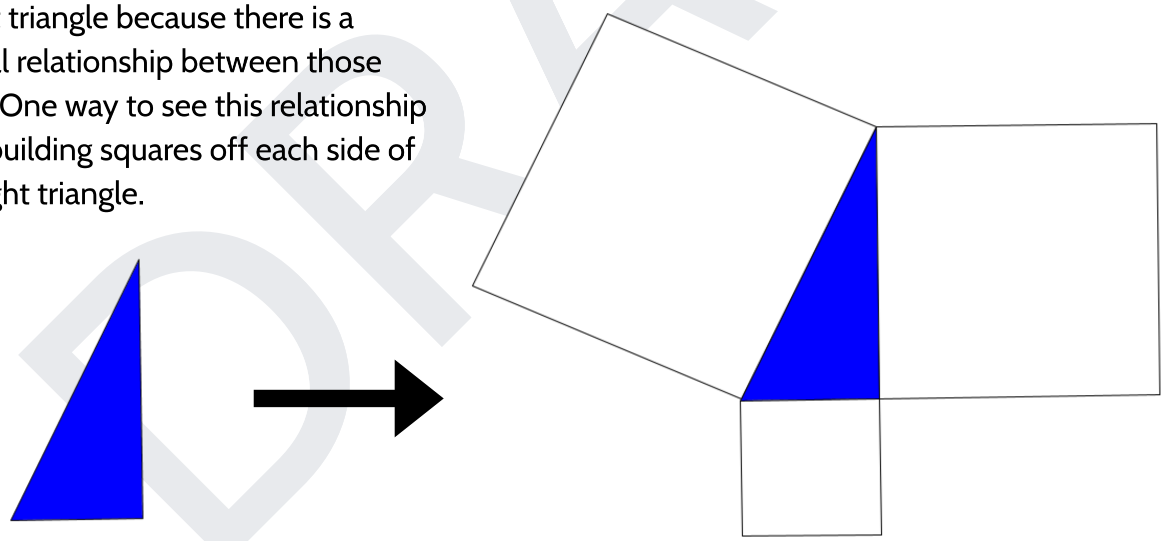
A right triangle is any triangle that has a 90 degree (90°) angle in it.

In a right triangle, we call the two sides that form the right angle, legs.

We call the side opposite the right angle, the hypotenuse. The hypotenuse is always the longest side of a right triangle.



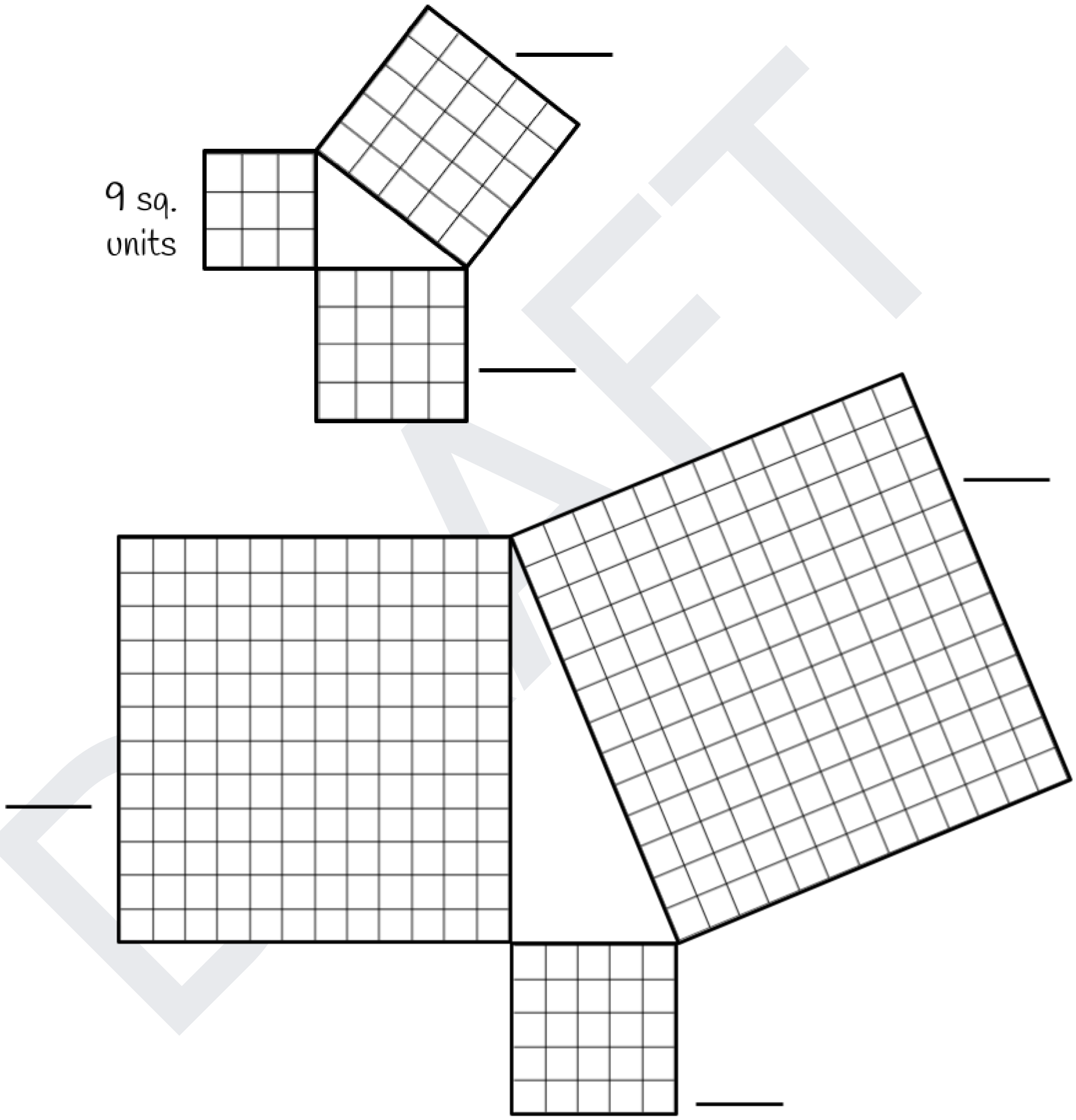
We have special words for the sides of a right triangle because there is a special relationship between those sides. One way to see this relationship is by building squares off each side of the right triangle.



The Pythagorean Theorem

Do you see the triangles inside the squares below? Both are right triangles. There is a square built off the sides of each right triangle.

3) Find the area of each square.



After you find the area of each square: What relationships do you see between the areas of the three squares?

One relationship you might have noticed is that if you put them together, the areas of the two smaller squares are equal to the area of the larger square.

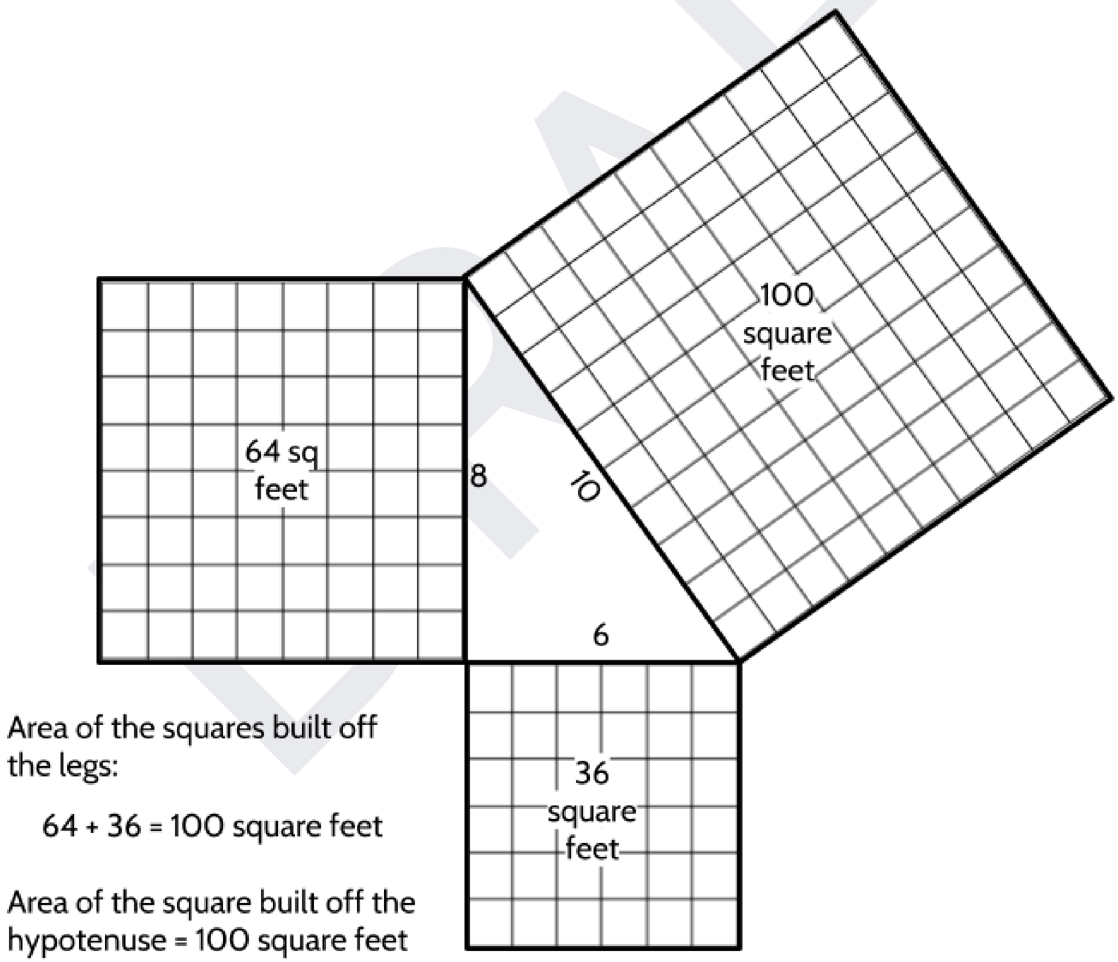
For example 9 + 16 = 25 and 25 + 144 = 169.

You may have also noticed that if you subtract the area of one of the smaller squares from the area of the largest square, you are left with the area of the other square.

For example, from the triangles on the previous page,

25 - 16 = 9   25 - 9 = 16   169 - 25 = 144   169 - 144 = 25

Let's see what happens when we build squares off the sides of a right triangle with legs of 6 feet and 8 feet and a hypotenuse of 10 feet.



We call this relationship the Pythagorean Theorem.

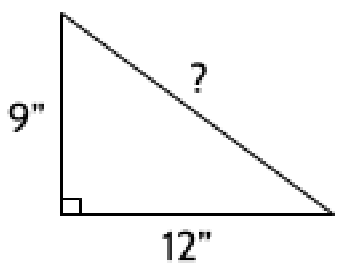
It is true for every right triangle and is incredibly helpful in problem-solving.

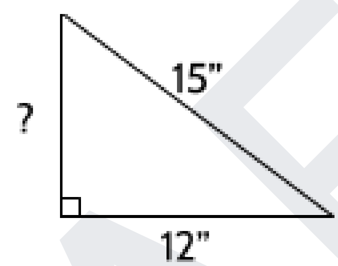
<u>The Pythagorean Theorem</u>: In a right triangle, the combined areas of the squares built off the legs are equal to the area of the square built off the hypotenuse.

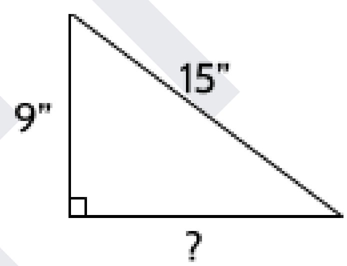
The two most common situations where we use the Pythagorean Theorem are:

- If we know the lengths of any two sides of a right triangle, we can use this relationship to find the missing length.

  In this way, the Pythagorean theorem can be used to determine distance.

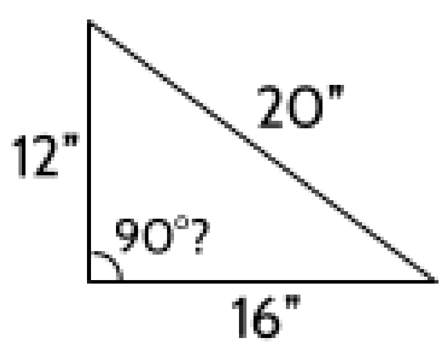






- We can also use this relationship to determine if a triangle is a right triangle.

  If we know the lengths of the three sides of a triangle, we can use the relationship to test whether the triangle has a 90 degree angle. If the combined areas of the squares built off the legs is equal to the area of the square built off the hypotenuse, then the triangle is a right triangle with a 90° angle.



In the next section, you will practice using the Pythagorean Theorem to solve both types of problems.

4) Larry and Joe Haun are brothers and professional contractors. They have a series of YouTube videos on how to build a house. In the photos below, they are using the Pythagorean Theorem to make sure the corners of a house's foundation are at right angles (90°).

The brothers use tape measures to create the legs of a right triangle.

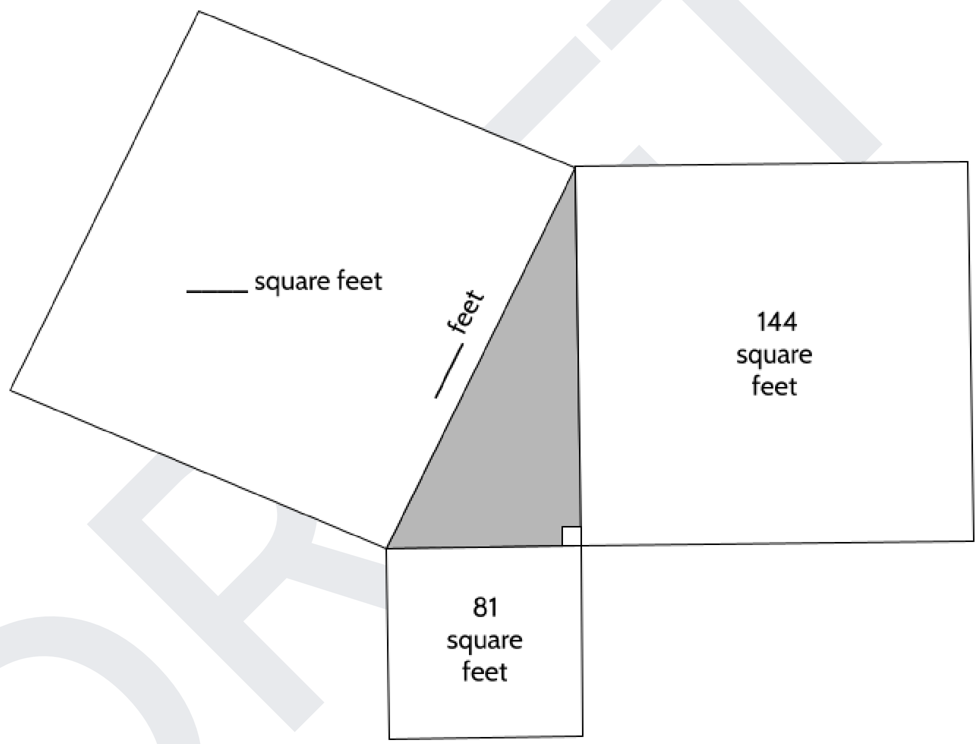
One of the brothers is measuring a length of 6 feet and the other brother is measuring a length of 8 feet.

Finally, they measure the hypotenuse. If they were successful in building a right angle in the foundation, what should the length of the hypotenuse be?

5) In the diagram below, the area of the smallest square is 81 square feet and the area of the middle-sized square is 144 square feet.

   a. What is the area of the largest square?

   b. What is the length of the hypotenuse?



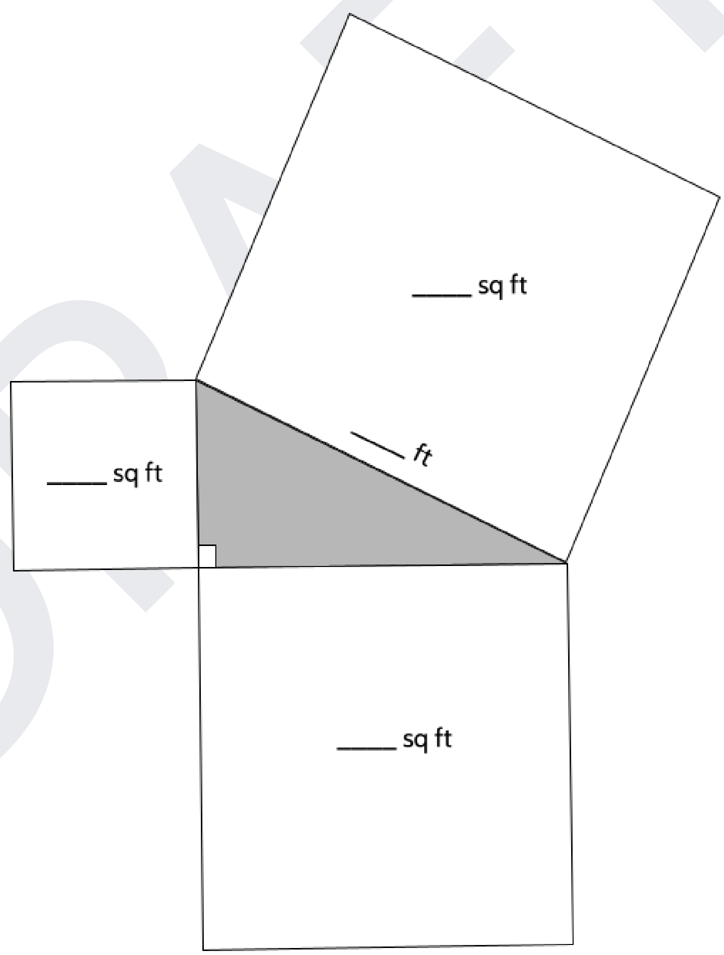
6) In the diagram, the length of the shorter leg is 12 feet and the length of the longer leg is 16 feet.

   a) What is the area of the smallest square?

   b) What is the area of the middle-sized square?

   c) What is the area of the largest square?

   d) What is the length of the hypotenuse?



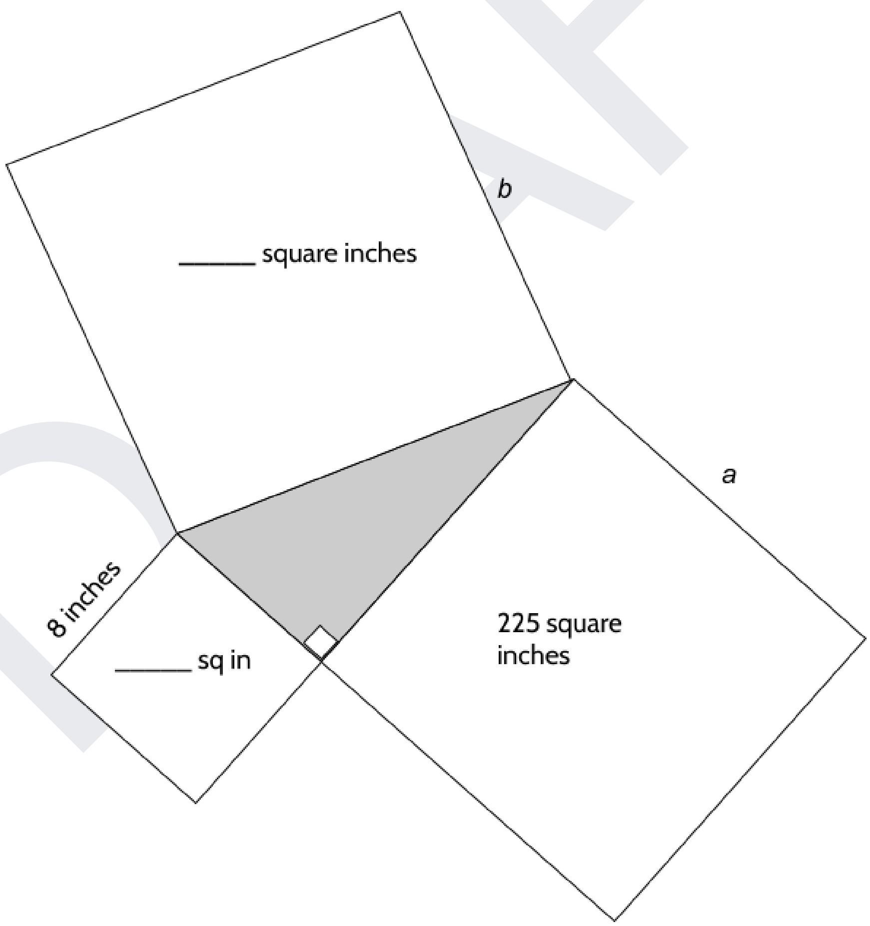
7) In the diagram, one side of the smallest square is 8 inches. The area of the middle-sized square is 225 square inches.

    a) Find the area of the smallest square.

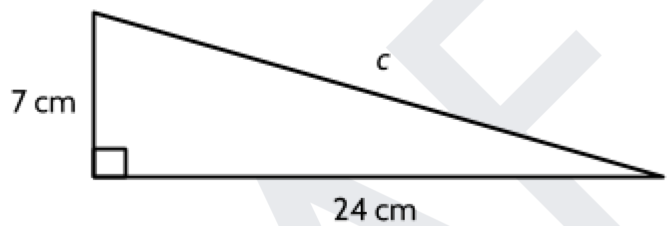
    b) Find length *a*.

    c) Find the area of the largest square.

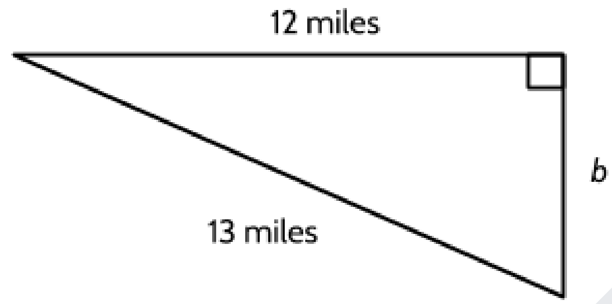
    d) Find length *b.*



8) Sketch squares off each side of the triangle below and use their areas to find length *c*.



9) Side *b* = ______ miles



10) Which of the following choices could not represent the side lengths of a right triangle?

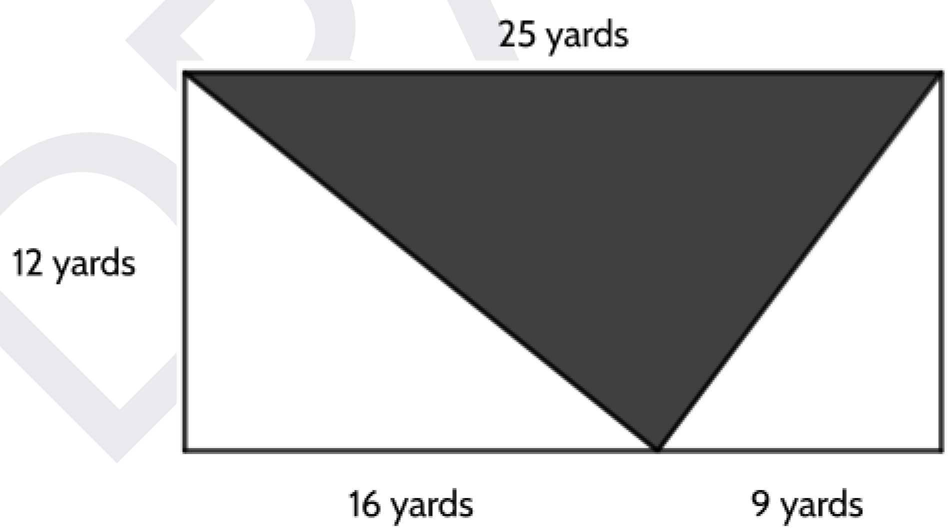
A. 6 feet, 9 feet, and 12 feet

B. 6 feet, 8 feet, and 10 feet

C. 8 feet, 15 feet, and 17 feet

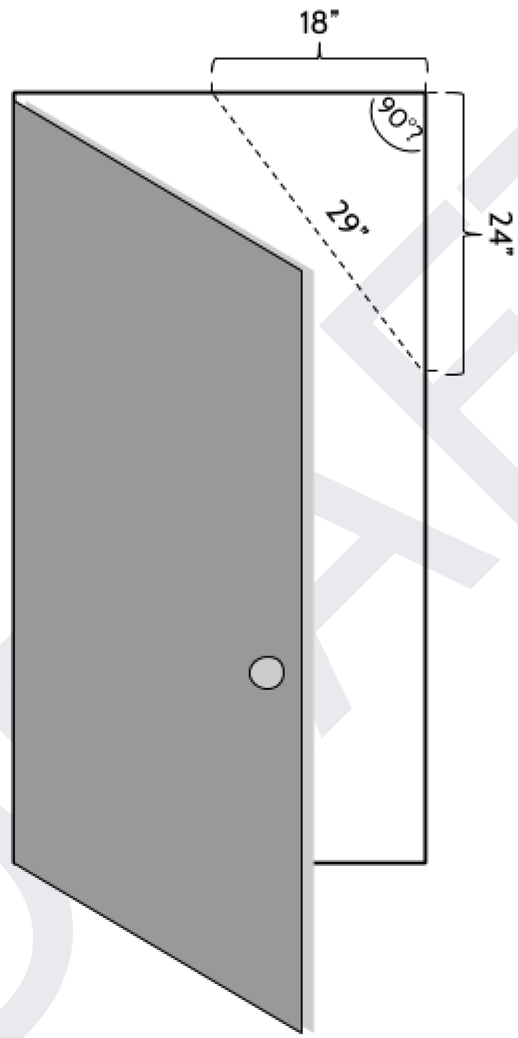
D. 18 feet, 24 feet, and 30 feet

11) This rectangle is 12 yards by 25 yards. It has been divided into triangles. Is the shaded triangle a right triangle? Explain your answer.



12) A carpenter is checking to see if a door frame has a 90° angle at the corner. This will show whether the frame is installed correctly.

    Is the corner 90 degrees? How do you know?



13) The ladder leaning against the building makes a right triangle. The base of the ladder is 5 feet away from the wall. It is 13 feet from the bottom of the ladder to where the ladder hits the top of the building. How tall is the building?

14) Liv's family built raised beds to grow vegetables. One of the beds (pictured below) was a square that measured 4 feet by 4 feet. They measured the diagonal to see if the sides formed right angles.

Which of the following approximate measurements would show that the sides form a 90 degree angle?

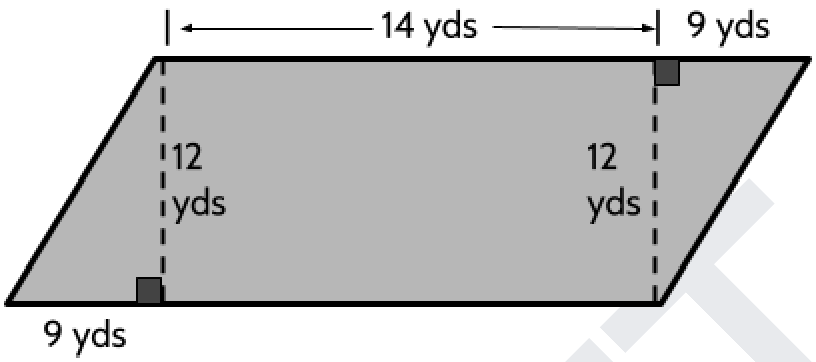
A. Between 3 and 4 feet

B. Between 4 and 5 feet

C. Between 5 and 6 feet

D. Between 6 and 7 feet

15) What is the perimeter of the parallelogram in the diagram below?

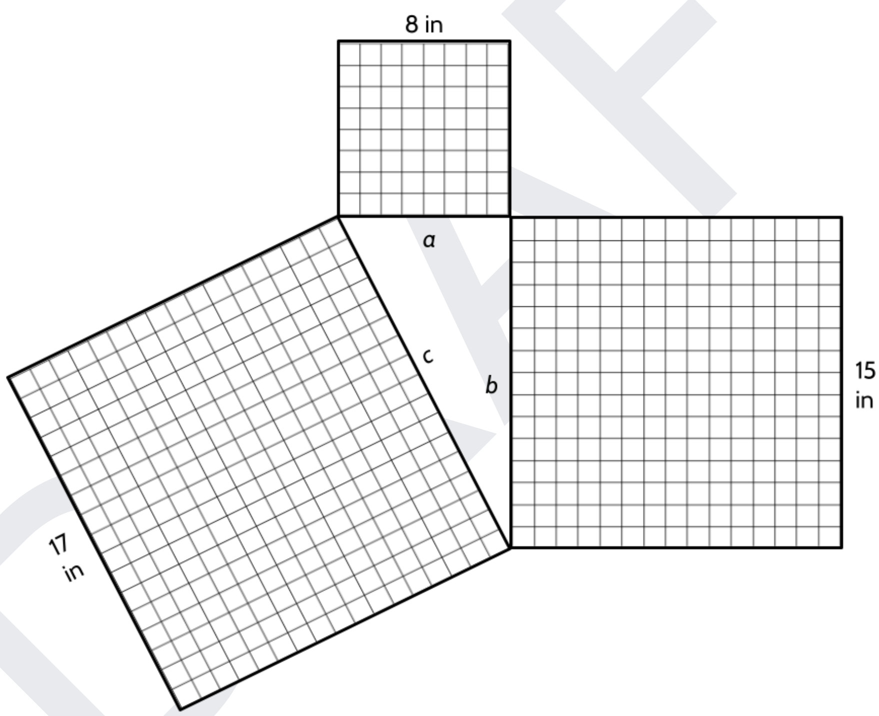


The Pythagorean Theorem states, in a right triangle, the combined areas of the squares built off the legs is equal to the area of the square built off the hypotenuse. You may have also heard the Pythagorean Theorem expressed as an equation:

$a^2 + b^2 = c^2$, *where a and b are the legs and c is the hypotenuse of a right triangle.*

The formula comes from the relationship you have been studying. The formula is a way of summarizing the relationship, which is good, as long as we understand where it comes from.

> The Pythagorean Theorem states, in a right triangle
>
> the combined areas of the squares built off the legs | is equal to | the area of the square built off the hypotenuse.
>
> $a^2 + b^2$ | $=$ | c^2



Area of smallest square		Area of middle-sized square		Area of largest square
64	+	225	=	289
8^2	+	15^2	=	17^2
Leg^2	+	Leg^2	=	Hypotenuse^2
a^2	+	b^2	=	c^2

The "Pythagorean" Relationship before Pythagoras

Humans have known about this relationship for thousands of years. The most common name for the relationship today is the Pythagorean Theorem, named for Pythagoras. Pythagoras was a Greek philosopher born about 2,600 years ago. Many people argue that Pythagoras was not the first person to discover the relationship. Many cultures, especially those in North Africa, the Middle East and Asia were using the relationship before that.

The ancient Egyptians used it in construction over 4,500 years ago. Without understanding it, they could not have built the pyramids. There are several ancient Babylonian stone tablets that are almost 4,000 years old that describe the same relationship. In China, it is called the GouGu Theorem, named for the *Gou* (the shorter leg of the right triangle) and the *Gu* (the longer leg of a right triangle). The Gougu Theorem was first written about 2,000 years ago.

This clay tablet was created around 3,700 years ago in Sumeria. Sumeria was located in southern Mesopotamia in what is today south-central Iraq.The rows and columns in this table are filled with what are called *Pythagorean triples*. Pythagorean triples are three numbers that represent the 3 sides of a right triangle. 3, 4, and 5 is a Pythagorean triple because $3^2 + 4^2 = 5^2$. The numbers 5, 12, and 13 are another example of a Pythagorean triple.

This tablet was created over 1,000 years before Pythagoras was born.

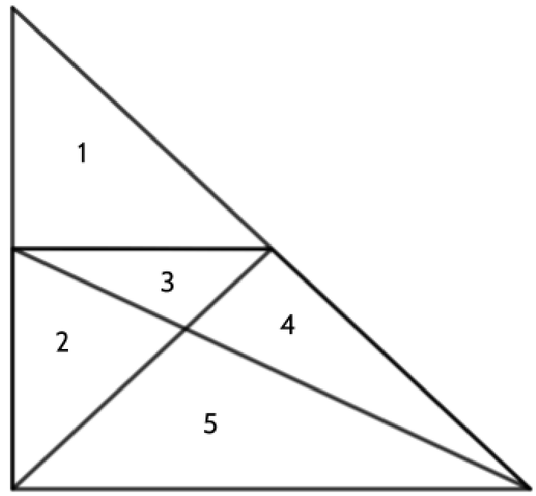
This tablet was created between 3,800 and 3,600 years ago in ancient Babylon, a city in ancient Mesopotamia.

The geometric diagram here shows a square with two diagonals.The side length of the square is given as 30. The numerals also show how to find the length of the diagonal, which is a little more than 42 units.

This tablet was most likely a “hand tablet,” used by a student, like yourself, for their rough work.

Triangles - Answer Key

1) There are 12 triangles in this picture.



- 1
- 2
- 3
- 4
- 5
- 2 & 5 together
- 2 & 3 together
- 4 & 5 together
- 3 & 4 together
- 1, 2, & 3 together
- 1, 3, & 4 together
- 1, 2, 3, 4, & 5 together

2) A right triangle is any triangle that has a 90 degree angle in it.

3) Areas: 9, 16, 25 and 25, 144, 169.
Did you notice that the area of two smaller squares adds up to the area of the large square? 9 + 16 = 25 and 25 + 144 = 169

4) The brothers made legs that were 6 feet and 8 feet in length. If we imagine a square built off those legs would have areas of 36 square feet and 64 square feet. That means the area of the square build off the hypotenuse would be 100 square feet (36 + 64 = 100). The side length of a square with an area of 100 square feet would be 10 feet. If their measure of the hypotenuse is 10 feet, then the corner of their foundation is a right angle. Using a right triangle with side measures of 6, 8, and 10 or of 3, 4, and 5 are commonly used in construction to make sure that corners are 90°.

5)

a) 144 square feet + 81 square feet = 225 square feet

b) $\sqrt{225}$ = 15 feet

6)
   a) 144 square feet
   b) 256 square feet
   c) 400 square feet
   d) 20 feet

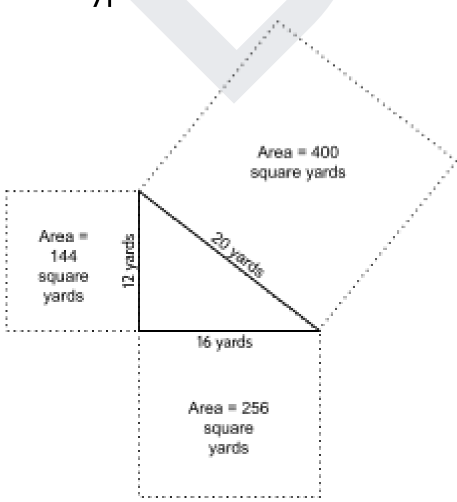
7)
   a) 64 square inches
   b) 15 inches
   c) 289 square inches
   d) 17 inches

8) Length *c* is 25 cm

9) Side *b* is 5 miles. Did you notice that this problem is different from the others you have done so far. For this problem, we are given the hypotenuse and one of the legs. If we build a square off the hypotenuse, its area would be 169 square miles. If we build a square off the given leg, the area is 144 square miles. We know the sum of the areas of the squares is equal to the area of the square built off the hypotenuse. We need to add 25 square miles to 144 square miles to match the area of the largest square. A square that is 25 square miles has a side that is 5 miles in length.

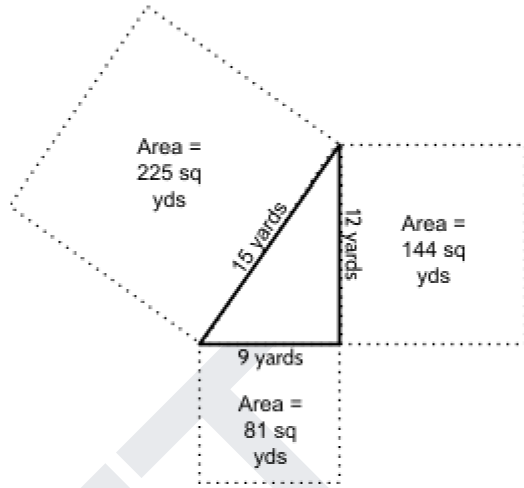
10) Choice A. We know the hypotenuse is the longest side of every right triangle. If we square 6 feet and 9 feet, we get 36 sq. ft and 81 sq. ft which combine to 117 square feet. Since the square of 12 feet would be 144 square feet, we know this combination of side lengths cannot be a right triangle.

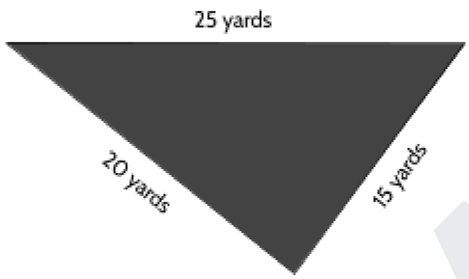
11) The shaded triangle <u>*is*</u> a right triangle. The legs of the shaded triangle are made up of the hypotenuses of the two unshaded triangles.



Starting with the triangle on the left: If we build a square off of the side that is 12 yards, we get a square with an area of 144 square yards. If we build a square off the side that is 16 yards, we get a square that has an area of 256 square yards. 144 square yards plus 256 square yards is 400 square yards. A square that has an area of 400 square yards will have sides that are 20 yards in length. So the hypotenuse of the unshaded triangle on the left is 20 yards.

Working with the unshaded triangle on the right: If we build a square off of the side that is 12 yards, we get a square with an area of 144 square yards. If we build a square off the side that is 9 yards, we get a square that has an area of 81 square yards. 144 square yards plus 81 square yards is 225 square yards. A square that has an area of 225 square yards will have sides that are 15 yards in length. So the hypotenuse of the unshaded triangle on the right is 15 yards.





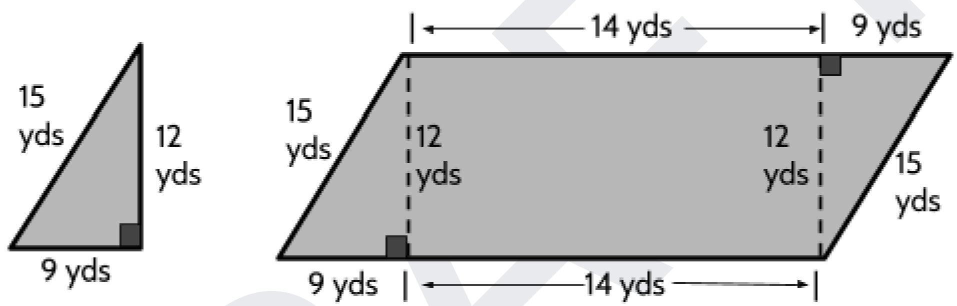
The legs of the shaded triangle are 20 yards and 15 yards. The square built off the 20 feet side has an area of 400 sq yards. The square built off the 15 feet side has an area of 225 sq yards. That means the area of the square built off of the hypotenuse would have to be 625 square yards (400 + 225). A square that has an area of 625 square yards will have sides that are 25 yards, which matches the length of the hypotenuse of the shaded triangle.

12) The corner is not 90°. We know this because $18^2 + 24^2 \neq 29^2$ or $324 + 576 \neq 841$. $18^2 + 24^2$ doesn't equal 29^2, but they do equal 30^2. The diagonal measurement would have to be 30 inches if the angle is 90°.

13) The ladder, building, and ground form a right triangle with sides of 5 ft, 12 ft, and 13 ft.

14) Choice C. $4^2 + 4^2 = 32$. $C^2 = 32$. We are looking for the square root of 32. You can use your calculator, but it is also between the perfect squares 25 and 36, which means it will be between 5 and 6 feet.

15) 76 yards. We can use the relationship between the sides of a right triangle to find the length of the diagonal sides. If we imagine squares built off each leg, we find that the area of the square built off the hypotenuse is 225 sq yds because $9^2 + 12^2 = 225$. The square root of 225 sq yds is 15 yds. Because this is a parallelogram, we know the opposite side is also 15 yds. To find the length of the longer side we need to combine what we know. Part of the length is 14 yards and the other part is 9 yards. Combined, we find the length of the longer side is 23 yards. 23 yds + 23 yds + 15 yds + 15 yds = 76 yds.



Circles

Introduction to Circles

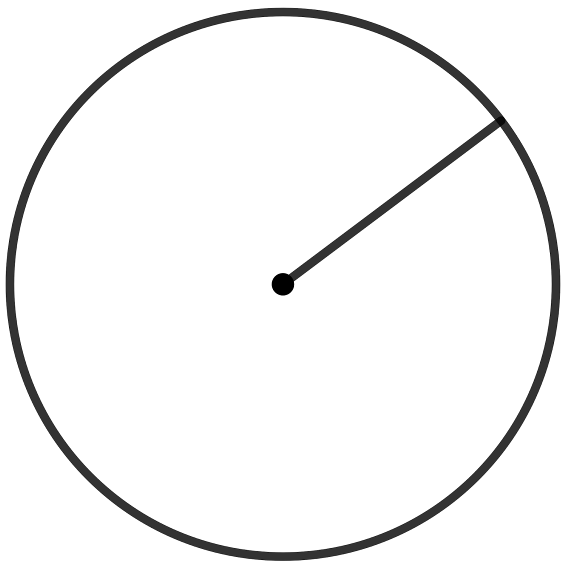
Humans have a long history with the circle. From the first time we looked up and saw the sun and moon in the sky, the circle has captivated our imaginations. We see hundreds of circles everyday, from the plates we eat from to the wheels and clocks that move around us. Spend a minute writing down all the circles you can think of in your everyday life.

So, what exactly is a circle?

In the space below, draw a perfect circle. Try at least four times. Which circle do you think is your best?

Chances are what you just drew is not a circle. It may be “circular,” meaning shaped *like* a circle, but it is really hard to draw a perfect circle by hand.

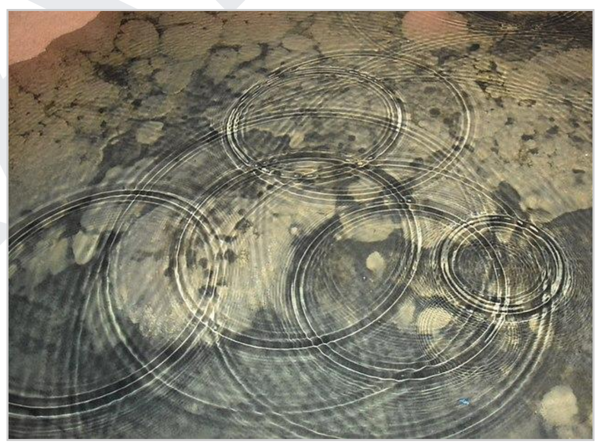
A *circle* is the set of all points at a given distance from a center point.



In a perfect circle, any line segment that goes from the center point to the edge of the circle will be the same length. Because it helps us define what a circle is, we give that line segment a special name.

The *radius* of a circle is the line segment from the center point to the edge of the circle. When we are talking about one line segment, we say “radius.” When we are talking about more than one, we say “radii.” Every circle has an infinite number of radii. And each radius inside a circle is equal in length.

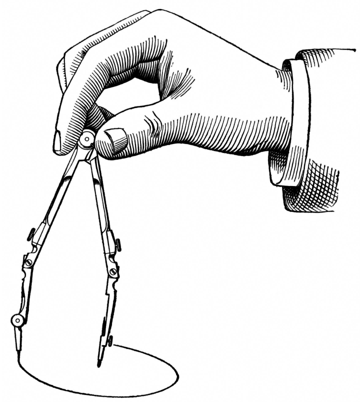
Imagine a drip of water creating a ripple in a lake. The place where the drop hits the water is the center of the circle. The ripples form perfect circles because they move away from that center at the same speed and travel the same distance.

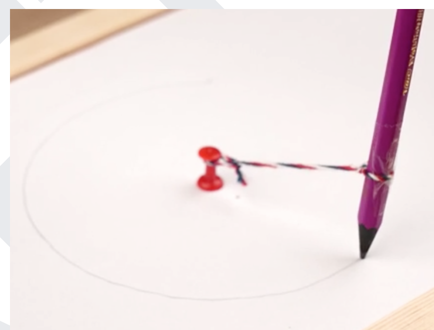


Observe the grass in the photo on the left. Its roots keep it in the same spot, creating a center point. When the wind blows, it moves the blades of grass and creates several circles around that center point. In this example, the length of each blade of grass would be the radius of the circle it creates in the sand.

Thousands of years ago, one of the tools humans used to draw circles was similar to the grass example. They would tie two posts to either end of a piece of rope. They would sink one post into the ground to serve as the center point. Then they would pull the rope tight and walk around the center point, dragging the other post in the sand. By this method, they could draw all the points at the distance they wanted around the center point.

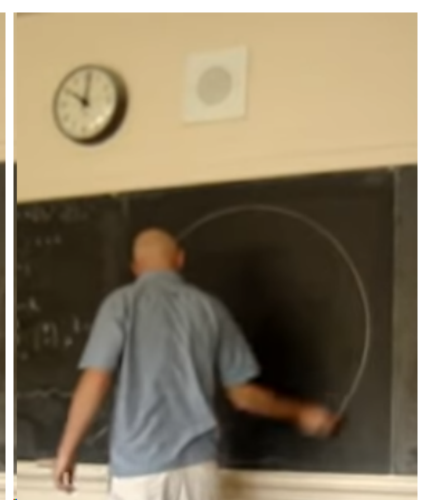
You may have seen this tool, called a *compass*, used to draw a perfect circle. It has a sharp end that serves as the center point of the circle. You adjust the pencil to be the distance you want from the center and turn the compass around the center. You can also use a string and pin.



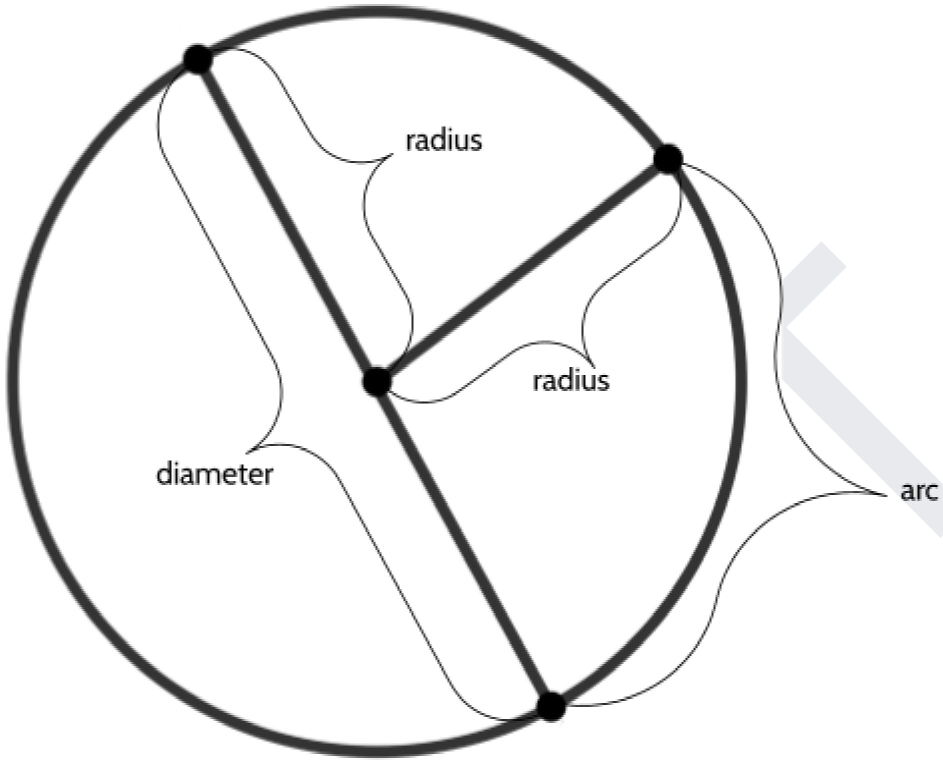


It is not impossible to draw a circle free hand.

Alexander Overwick, is a math teacher and a "world freehand circle drawing champion." One of Overwick's students posted a video of him on YouTube, showing his technique. The video has been viewed over 13 million times.

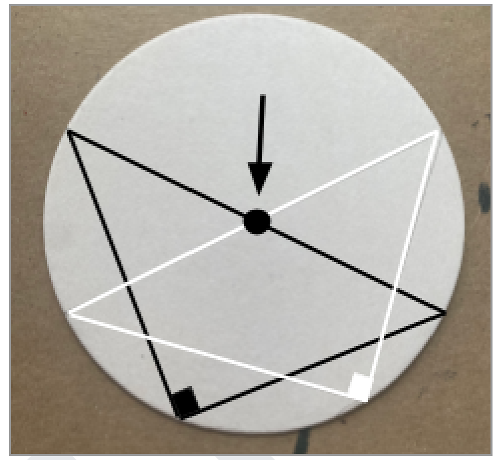


Parts of a Circle



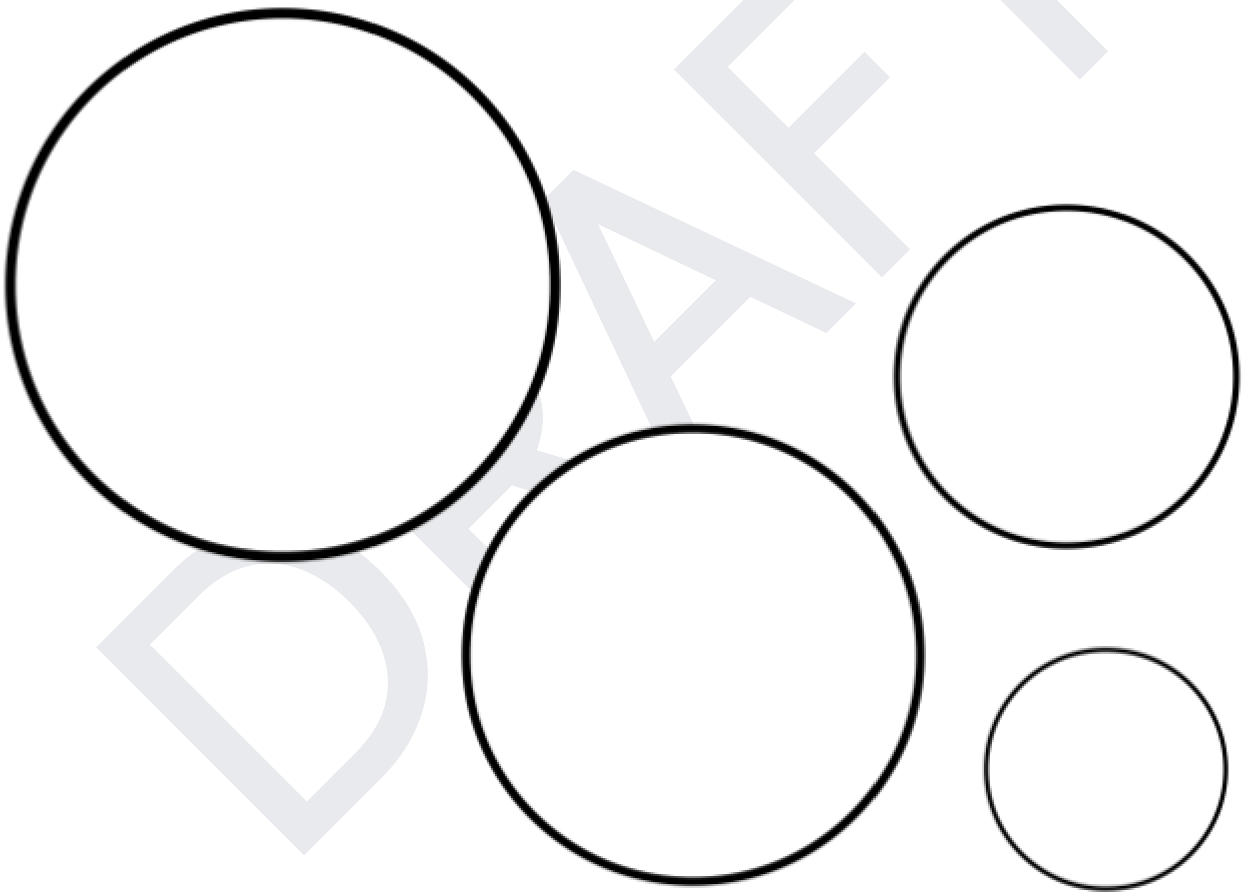
- The *radius* of a circle is the line segment from the center point to the edge of the circle.
- A *diameter* of a circle is a line segment that goes from one edge of the circle to another, passing through the center point. A diameter is equal to the lengths of two radii. Put another way, the diameter is twice as long as a radius. And a radius is half the length of a diameter.
- We have used the word *perimeter* to describe the whole length of a border around a shape. Circles have a special word to describe the distance around the circle. That word is *circumference*. We do not refer to the length around a circle as a perimeter.
- A *semicircle* is half of a circle. The length of a semicircle is half the length of the circumference.
- An *arc* is a section of the circumference.

Finding the Center of a Circle



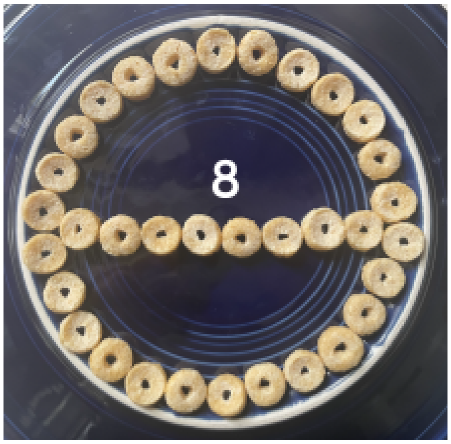
There are many methods that people use to find the center of a circle. One way is to use right triangles. Draw two right triangles with the right angles touching the circumference of the circle. The place where the two hypotenuses intersect is the center of the circle.

Try it for yourself with the circles below. You can use the corner of a book or box to make the right angle.

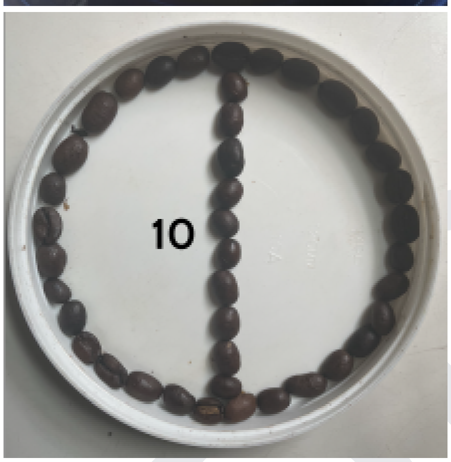


Circumference of a Circle

The circles below were made out of household materials (Cheerios, coffee beans, and frozen peas). They were arranged using the bottom of two circular plates and a circular lid. To help with your estimations, the diameter is written on each photo.



Without counting each one, estimate how many Cheerios you think there are in the circumference of this circle. (*The diameter is 8 Cheerios in length.*)



Without counting each one, estimate how many coffee beans you think there are in the circumference of this circle. (*The diameter is 10 coffee beans in length*.)

Without counting each one, estimate how many frozen peas you think there are in the circumference of this circle. (*The diameter is 12 frozen peas in length*.)

Once you write your estimate, count the objects in each circumference. Were you close? Are you surprised by any of the numbers?

Let’s look at the diameter and circumference of other circles from our everyday lives.

The chart below has measurements for several circular objects of all different sizes. (If you are able, measure circles around you and add your own measurements in the blank rows below.)

Circular Object	Length of the Diameter	Length of the Circumference[3]
A quarter	1 in	3 ¼ in
A can of tuna	8 cm	24.5 cm
A plate	10 in	31 in
A vinyl record (LP)	12 in	39 in
A CD	5 ¼ in	15 ¾ in
A wall clock	33 cm	101 cm
An orange slice	9 ½ cm	29 cm
A frisbee	10 ⅕ in	34 in
A hula hoop	35 in	112 in
A car tire	25 in	78 in
A circle of Cheerios	8 Cheerios	26 Cheerios
A circle of coffee beans	10 coffee beans	31 coffee beans
A circle of frozen peas	12 frozen peas	43 frozen peas

What do you notice? What patterns do you see?

What relationships do you see between the diameters and circumferences of these circles?

[3] All of these measurements are approximate. They were made by placing a string around the circumference. Then the length of string was pulled straight and measured with a ruler.

One thing you might have noticed is that the circumference of each object is about 3 times bigger than the diameter. If you didn't notice, see for yourself.

Circular Object	Length of the Diameter	The diameter multiplied by 3	Length of the Circumference
A quarter	1 in	3 in	3 ¼ in
A can of tuna	8 cm	24 cm	24.5 cm
A plate	10 in	30 in	31 in
A vinyl record (LP)	12 in	36 in	39 in

In every circle, the circumference is a little bit more than 3 times longer than the diameter. The relationship is true <u>of any circle,</u> <u>of any size,</u> <u>in the entire universe</u>.

Because this relationship is true for any circle, there is a special symbol we use to describe it. That symbol is the Greek letter, π, which is sometimes written as *pi* and is pronounced like "pie". π is the 16th letter in the Greek alphabet.

π is also the first letter in the Greek word for "perimeter" and the Greek word for "periphery" (meaning the outside edge of something).

You may have heard pi described as 3.14. Saying the circumference is "a little bit more than 3 times longer" than the diameter is a good way to estimate. Using 3.14 gives a more precise answer.

3.14 is still only an estimate of pi. The first 30 digits of pi are 3.14159265358979323846264338327... and it keeps going. Pi actually has an infinite number of digits. Using computers people have been able to calculate pi to TRILLIONS[4] of digits.

Most of the time, however, in our everyday lives, 3.14, or even 3, will be precise enough for our calculations.

It's important to remember that pi is not just a number. Pi is the relationship between the length of the diameter and the circumference in any circle.

This relationship can be described with the equation $C = \pi d$

[4] A trillion is the same as one million million and can be written as a 1 followed by 12 zeros (1,000,000,000,000).

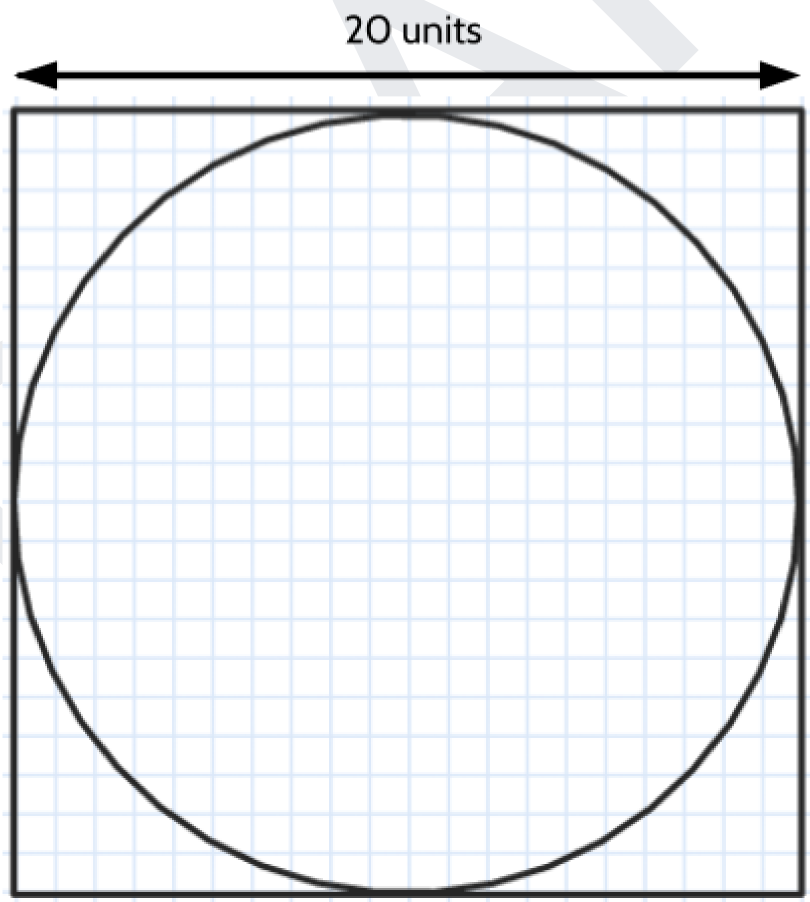
Area of a Circle

Area is the size of a surface. It is a measure of how many square units it would take to cover that surface. What do we do if the surface we want to measure is a circle?

This is a serving dish that has a diameter of 20 inches.

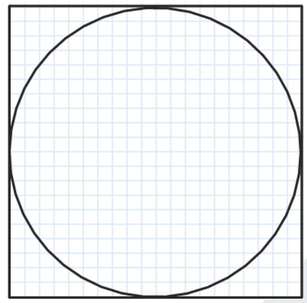
Below is a diagram of our serving dish.

1) Estimate the area of the circle. Approximately how many square units does it take to cover its surface?

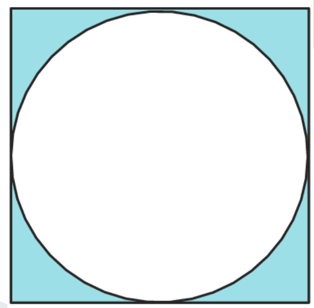


There are many ways to estimate the area of a circle.

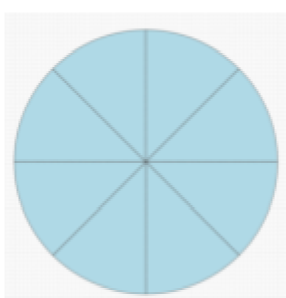
We can draw it on grid paper and estimate what to do with the incomplete squares.



We can draw a square around it with a side length equal to the diameter and estimate how much we need to take away.

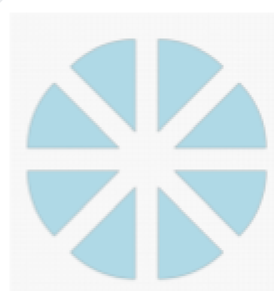


There is another way that humans have found to measure the area of a circle. This method builds on what we know about how to find the area of other shapes. This method can give us a very good *approximate* answer for the area of any circle.

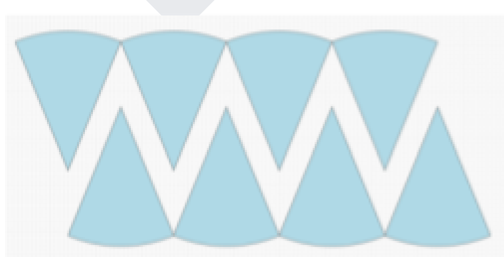


Start by dividing a circle up into equal pieces like a cake.

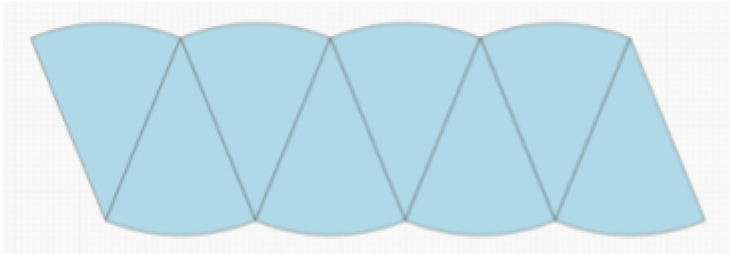
This circle has been divided into 8 equal sections.



Pull the pieces apart.



Point half of the pieces down and point the other half of the pieces up. Since we have 8 sections, we have 4 pointing up and 4 pointing down.

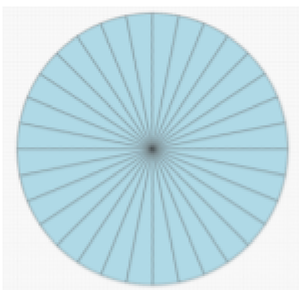


We end up with a shape that looks like this.

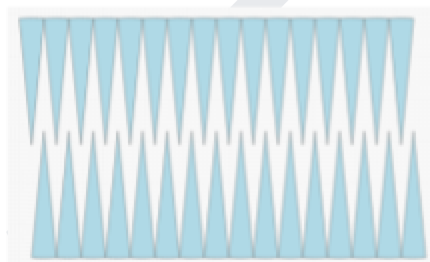
Because it is made from the same 8 sections we started with, the area of this shape is equal to the area of our circle.

If we look at this shape, we might say that it is kind of similar to a rectangle or a parallelogram, but it looks strange.

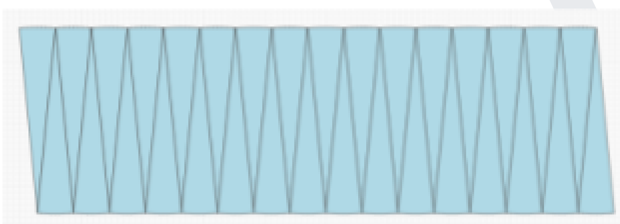
What happens if we divide our circle up into smaller sections?



This circle has been divided up into 32 equal sections.



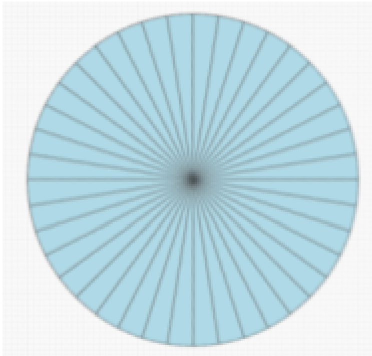
We arrange half of the sections to point down and half of the sections to point up.

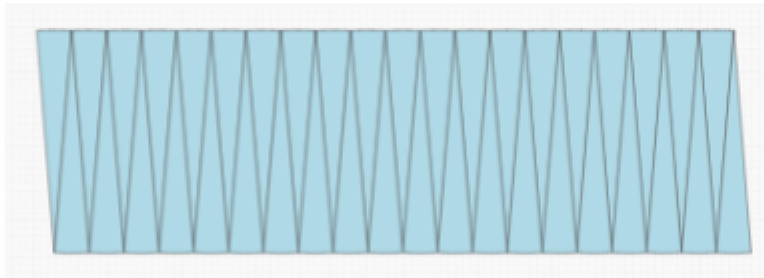


When we put the sections back together, the shape is starting to look more like a real parallelogram. And because it is made up of the same sections as the circle, it has the same area of the circle.

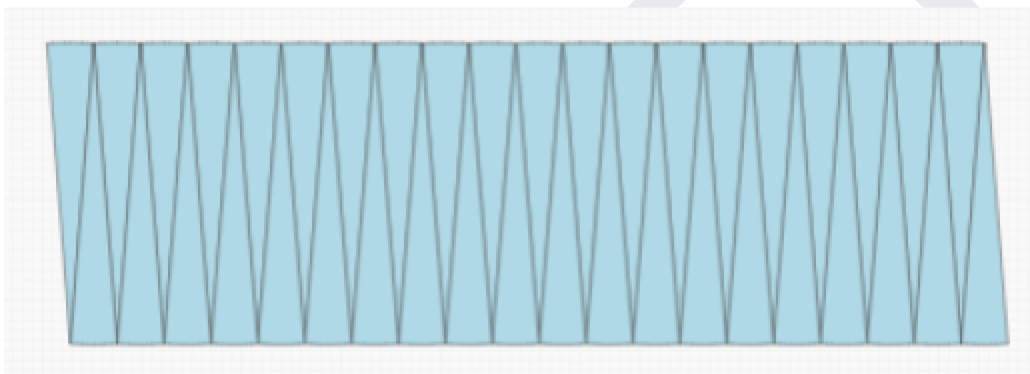
Here is one more circle. This one has been divided into 40 sections. Those 40 sections have been rearranged, half of them on top and half of them on the bottom. We won't continue this process, but we could. The more sections we divide the circle into, the more the rearranged sections will look like a parallelogram.[5]

[5] These images were created using this free, interactive, online demo - https://academo.org/demos/circle-area-rearrangement-method/

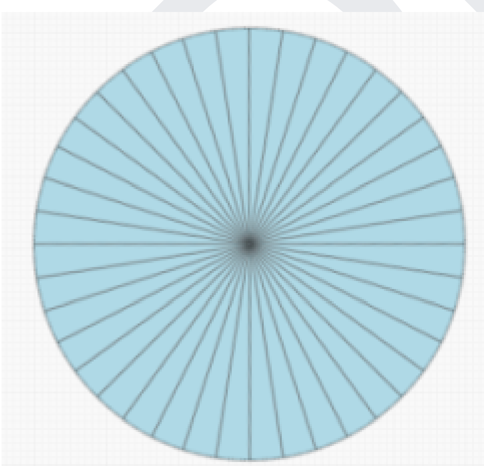


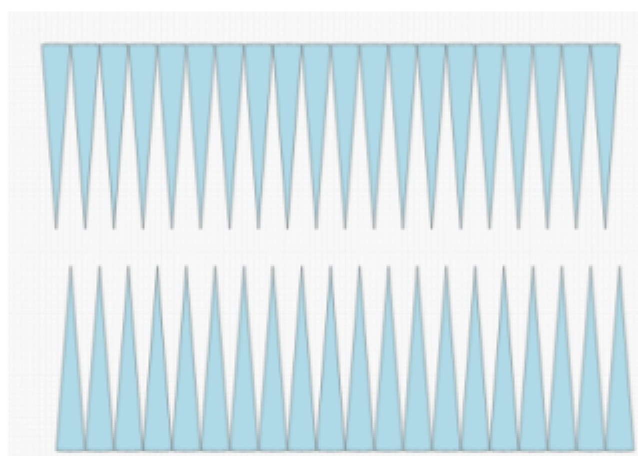


We explored how to find the area of parallelograms in an earlier section. Similar to a rectangle, one method for finding the area of a parallelogram is to multiply adjacent sides. But what are the lengths of the sides of this parallelogram?



What relationships do you see between the parts of the circle and the side lengths of the parallelogram?

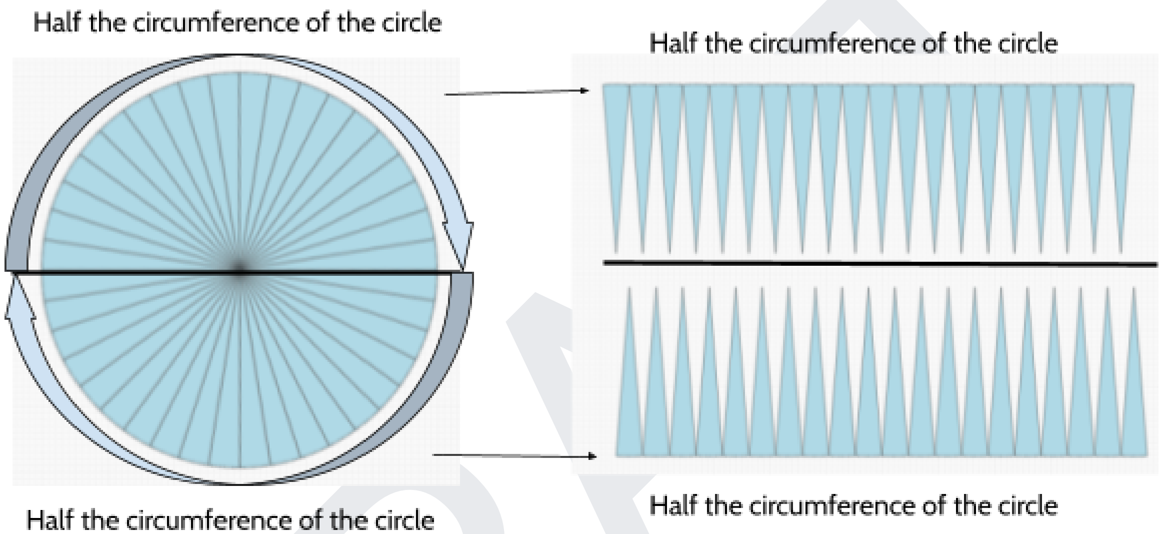




Let's look back at how we rearranged the sections.

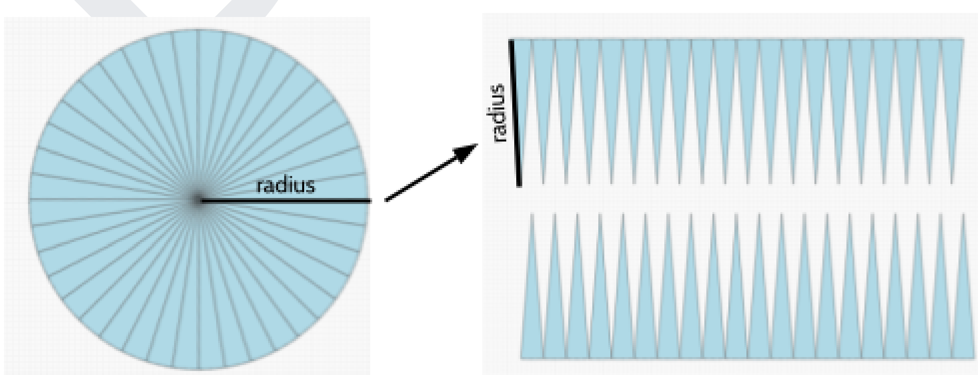
<u>The longer side.</u>
We divided the circle into sections and then arranged half of them to be pointing down and half of them to be pointing up.

Each section includes a piece of the circumference. And half of those pieces end up on the top of our parallelogram. That means that each of the long sides of the parallelogram is half the length of the circumference of the circle.



We learned the whole circumference of a circle is approximately equal to π times the diameter. We need to know *half* of that. We have a word for *half* of the diameter - the radius. So we can say that the longer side of this parallelogram is π times the radius or $\pi \times$ r or π r.

<u>The shorter side</u>
The circle is divided into sections and each of those divisions is a radius. Imagine our circle as a cake cut into 40 pieces. Each cut goes from the edge of the circle to the center.



The parallelogram that we created has the same area as the circle.

The lengths of the sides of the parallelogram are πr *and r.*

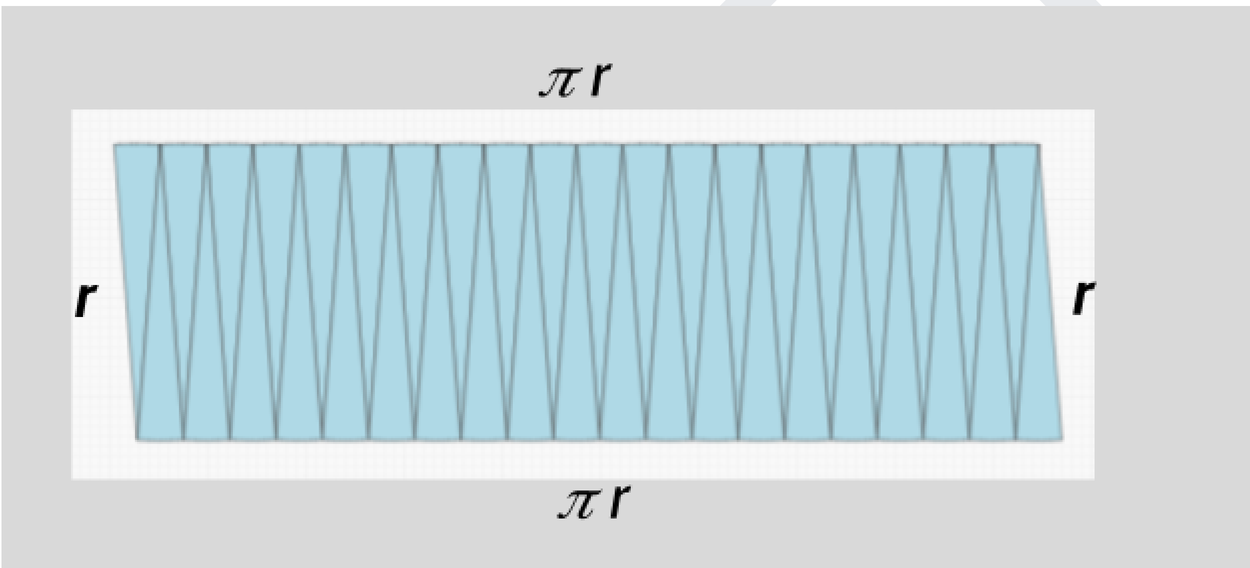
We can find the area of a parallelogram by multiplying the adjacent sides.

We can find the area of any circle by multiplying $\pi \times r \times r$.

We can represent this relationship using an equation.

$A = \pi \times r \times r$, *where A represents the area of the circle, and r represents the radius of the circle.*

The equation can also be written as $A = \pi r^2$



The serving plate we started with had a diameter of 20 inches and we wanted to find its area.

If the diameter is 20 inches, then the radius is 10 inches.

$$A = \pi r^2$$

$$A = \pi \times 10^2$$

$$A = \pi \times 100$$

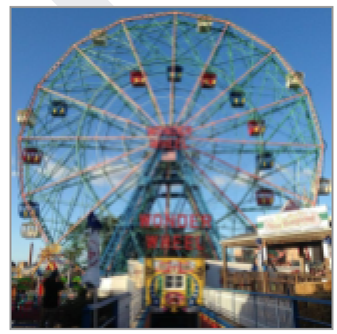
Using 3.14 for π, the area of the plate is approximately 314 square inches. How close was your estimate?

Circles in Our World

Practice using what you have learned about the area and circumference of circles.

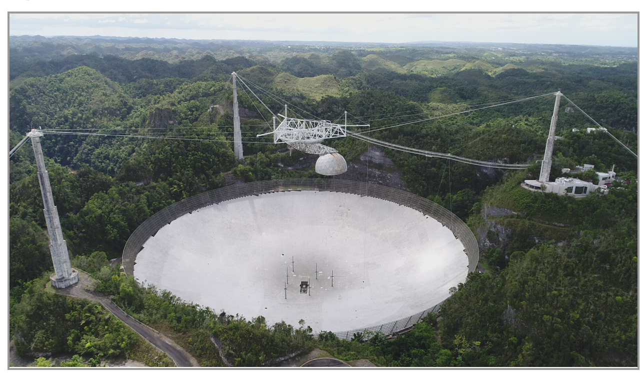
2) Mardgrina walked around a circular pond and estimated the distance to be around 558 feet. What is the approximate diameter across the pond?

3) The Wonder Wheel in Coney Island Brooklyn, pictured here, has a diameter of 41 meters. What is the approximate circumference of the Wonder Wheel?



4) For 53 years, the Arecibo radio telescope in Puerto Rico was the largest single-aperture telescope in the world. Among other uses, it was part of the Search for Extraterrestrial Intelligence (SETI) program. The telescope was damaged by Hurricane Maria in 2017 and decommissioned in 2020.

The main collecting dish had a diameter of approximately 305 meters. What is the area of the main collecting dish?



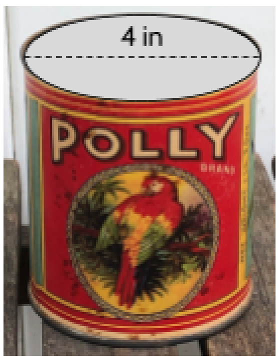
5) Find the difference in the areas of a square pizza with a side length of 16 inches and a circular pizza with a diameter of 16 inches?

6) Vinyl records come in 3 standard sizes for singles: 12 inches, 10 inches, and 7 inches. The size represents the diameter of the record. What is the area of each standard size vinyl record?

7) "Parade" is a sculpture by Mike Rathbun in the shape of a circle. It is on display at the Franconia Sculpture Park. If the sculpture is 40 feet tall, what is its approximate circumference?

8) What is the circumference of a circle with a radius of 1 yard?

9) The Polly company is designing a rectangular label for a can of peeled tomatoes. What is the circumference of the can?



10) Which has a greater area: two 12 inch pizzas or one 18 inch pizza?

Circles in Our World - Answer Key

1) There are many ways to estimate the area of the circle.

2) The circumference is about 3 times bigger than the diameter. We can divide 558 feet into 3 equal parts and get a diameter of about 186 feet as a rough estimate. If you divide 558 feet by 3.14, you get a more precise measurement of 177.7 feet for the diameter.

3) Using 3.14 gives an approximate circumference of 129.74 meters.

4) The area of the main collecting dish is approximately 957.7 square meters.

5) The area of the square pizza is 16x16, or 256 square inches. We can find the area of the circular pie by multiplying 3.14 by 8^2 or 3.14×64. The area of the circular pizza is approximately 200.94 square inches. The difference in area is 55.04 square inches.

6) A 12" record has an area of 113.04 square inches.
A 10" record has an area of 78.5 square inches.
A 7" record has an area of 38.47 square inches.

7) If you use 3, the circumference of the sculpture is about 120 feet. If you use 3.14, the sculpture is about 125.6 feet.

8) If the radius is 1 yard, we know the diameter is 2 yards. If you use 3, the circumference is 6 yards. If we use 3.14, the circumference is 6.28 yards.

9) If you use 3, the circumference is about 12 inches. If you use 3.14, the circumference is about 12.56 inches. The interesting thing is that the length of the longer side of the rectangular label will *also* be 12.56 inches. Can you figure out why?

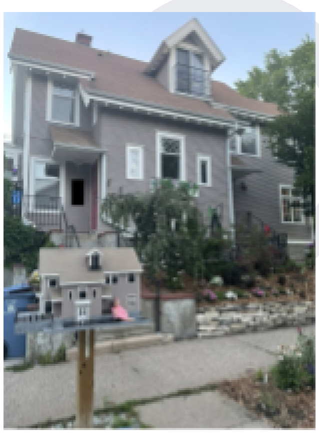
10) The 18 inch pizza has a larger area.
18 Inch Pizza: $9 \times 9 \times \pi$ = 254.34 square inches of pizza.
12 Inch Pizza: $6 \times 6 \times \pi$ = 113.04 square inches of pizza. Two of those make a total area of 226.08 square inches of pizza.

Scale Factors & Dilation

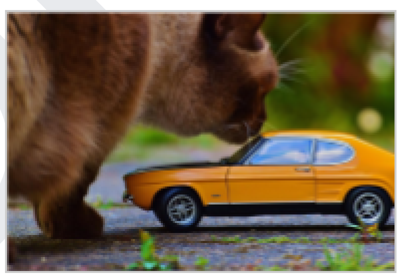
Making Things Bigger and Smaller

In the Queens Museum in New York City, there is an exhibit that has been on display since the 1964 World's Fair. It is called the Panorama of the City of New York. It is a scale model of the entire city where one inch in the model equals 100 feet in real life. For example, in real life, the Empire State Building is 1500 feet tall. The model of the Empire State Building in the Panorama is 15 inches tall.

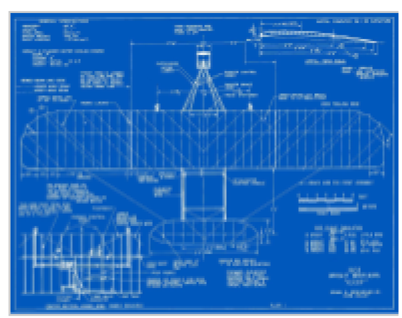
We use scale when we draw or create a model of an object that is larger or smaller than the actual object. Models, maps, and blueprints all use scale. Scale allows us to practice, plan, and study things.



The mailbox of this house is a *scale* model of the house.



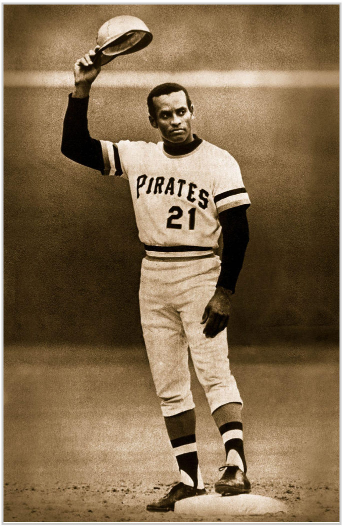
Either that is a giant cat or that is a scale model of a 1974 Ford Capri!



These blueprints were used by the Wright Brothers to build their Wright Flyer in 1903.

But what does it mean to make something *to scale*?

Let's say I want to enlarge this photograph of Pittsburgh Pirate legend Roberto Clemente, tipping his cap to the crowd after collecting his 3,000th hit in 1972.



Look at the enlargements on the next page. Which one do you think is the best enlargement?

Explain why you think so.

____

____

____

____

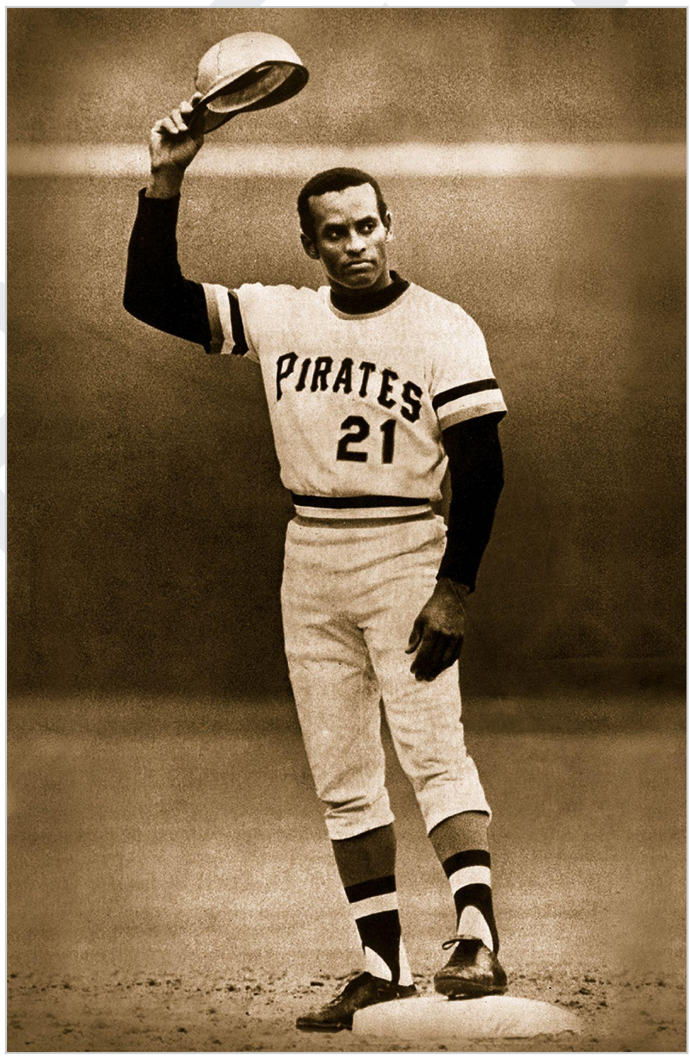
____

____

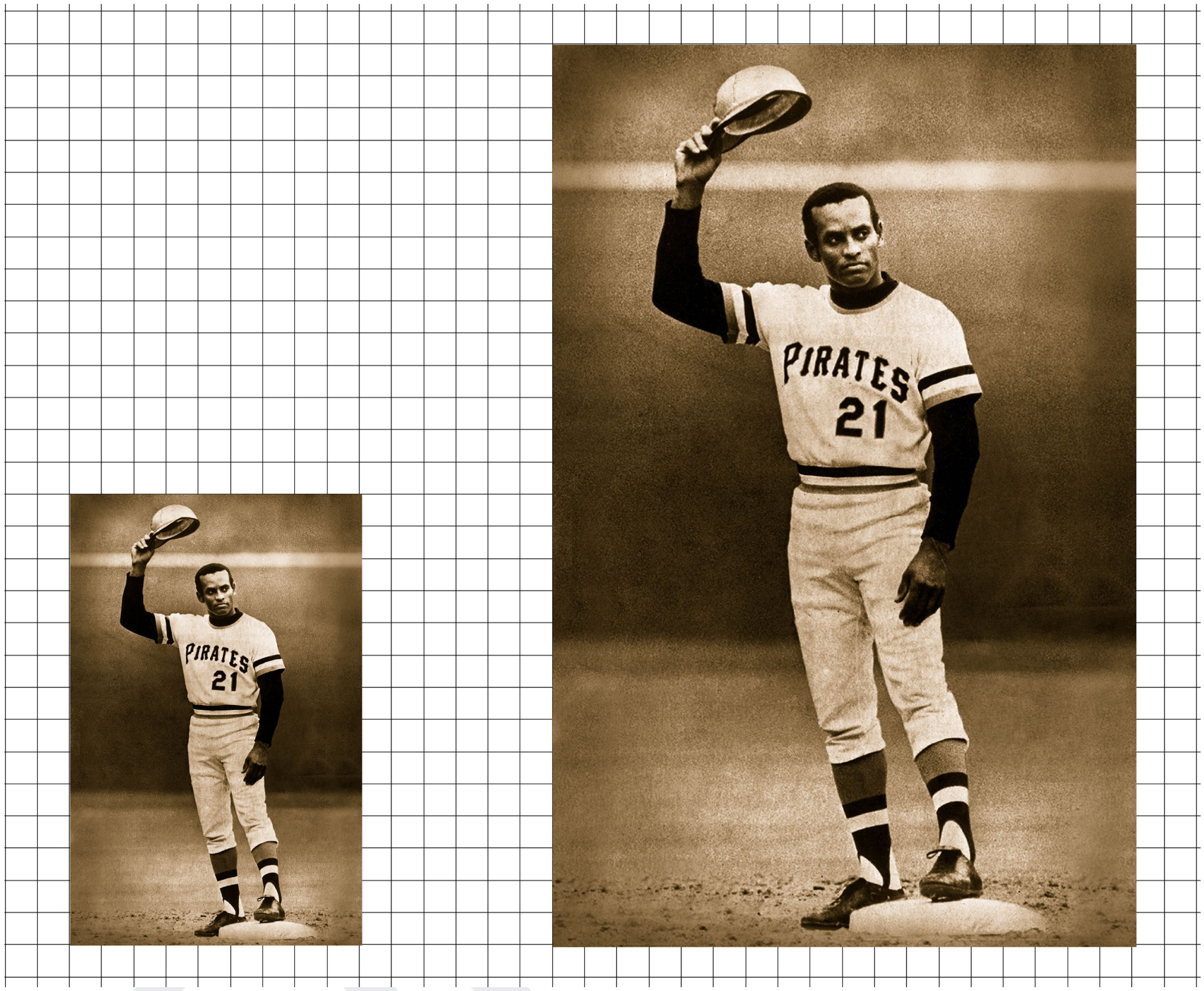
____

____

Which is the best enlargement?



When we enlarge or shrink photographs, we don't want them to look stretched or flattened. Below is a correct enlargement of the photograph of Roberto Clemente.



A grid has been added to help you compare the two images.

What do you notice about the length of the sides and the perimeter of these two images?

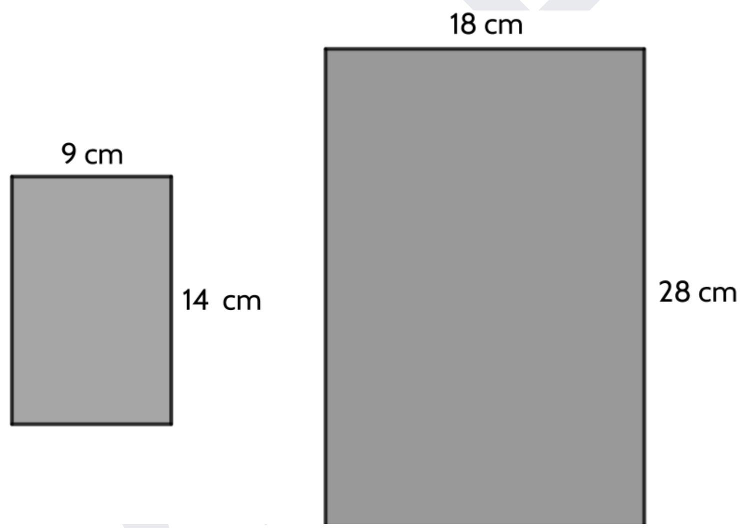
One thing you may have noticed is that the smaller photograph is 9 boxes wide and 14 boxes tall. The bigger photograph is 18 boxes wide and 28 boxes tall.

We can say that these images are *dilations* of each other. A dilation makes a figure larger or smaller without changing its shape. Dilations can be described by the term *scale factor*.

For example, the image of Roberto Clemente on the right is two times larger than the image on the left. We can say that we dilated the smaller image by a scale factor of 2. That means that it is two times larger.

We can also say that the smaller image is a dilation of the larger image by a scale factor of ½. That means it is half the size as the larger image.

If we look at the dilations as simple rectangles, we can say that the two rectangles are mathematically similar. **Similar** figures are the same shape, but they are different sizes. You can enlarge or shrink the figure so that it is large or small enough to cover the other figure.



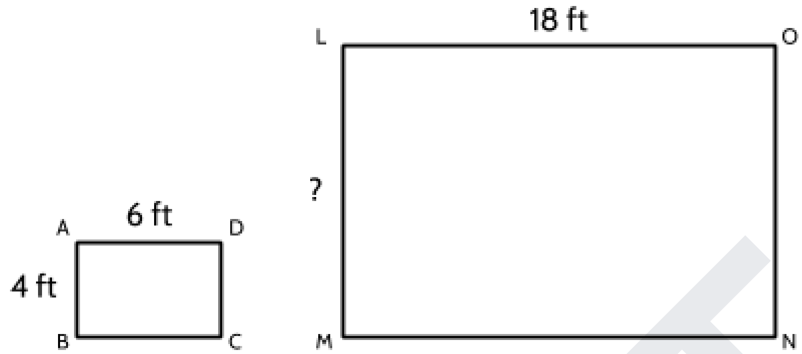
Notice how each length is related to the corresponding length in the dilated image.

To go from the rectangle on the left to the rectangle on the right, the scale factor is 2.

18 is two times bigger than 9.   28 is two times bigger than 14.

Each pair of corresponding sides has the same scale factor.

Rectangle ABCD and Rectangle LMNO are similar.



1) Which sides in Rectangle ABCD correspond to which sides in Rectangle LMNO?

   $\overline{AB}$ corresponds to $\overline{LO}$ How do you know?

   $\overline{BC}$ corresponds to $\overline{\quad}$

   $\overline{CD}$ corresponds to $\overline{\quad}$

   $\overline{AD}$ corresponds to $\overline{\quad}$

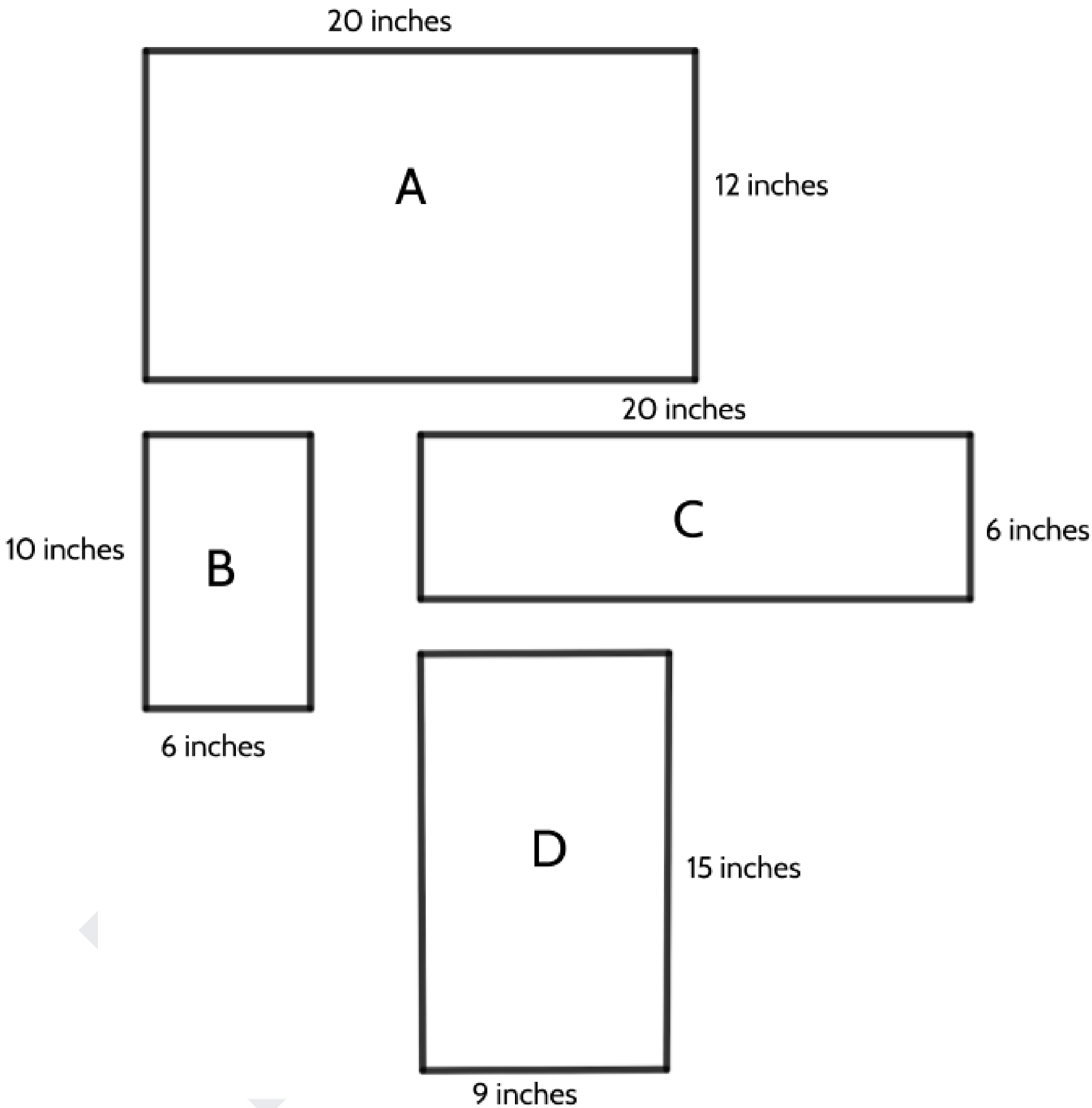
2) What is the scale factor to go from Rectangle ABCD to Rectangle LMNO? How do you know?

3) What is the length of $\overline{LM}$? How do you know?

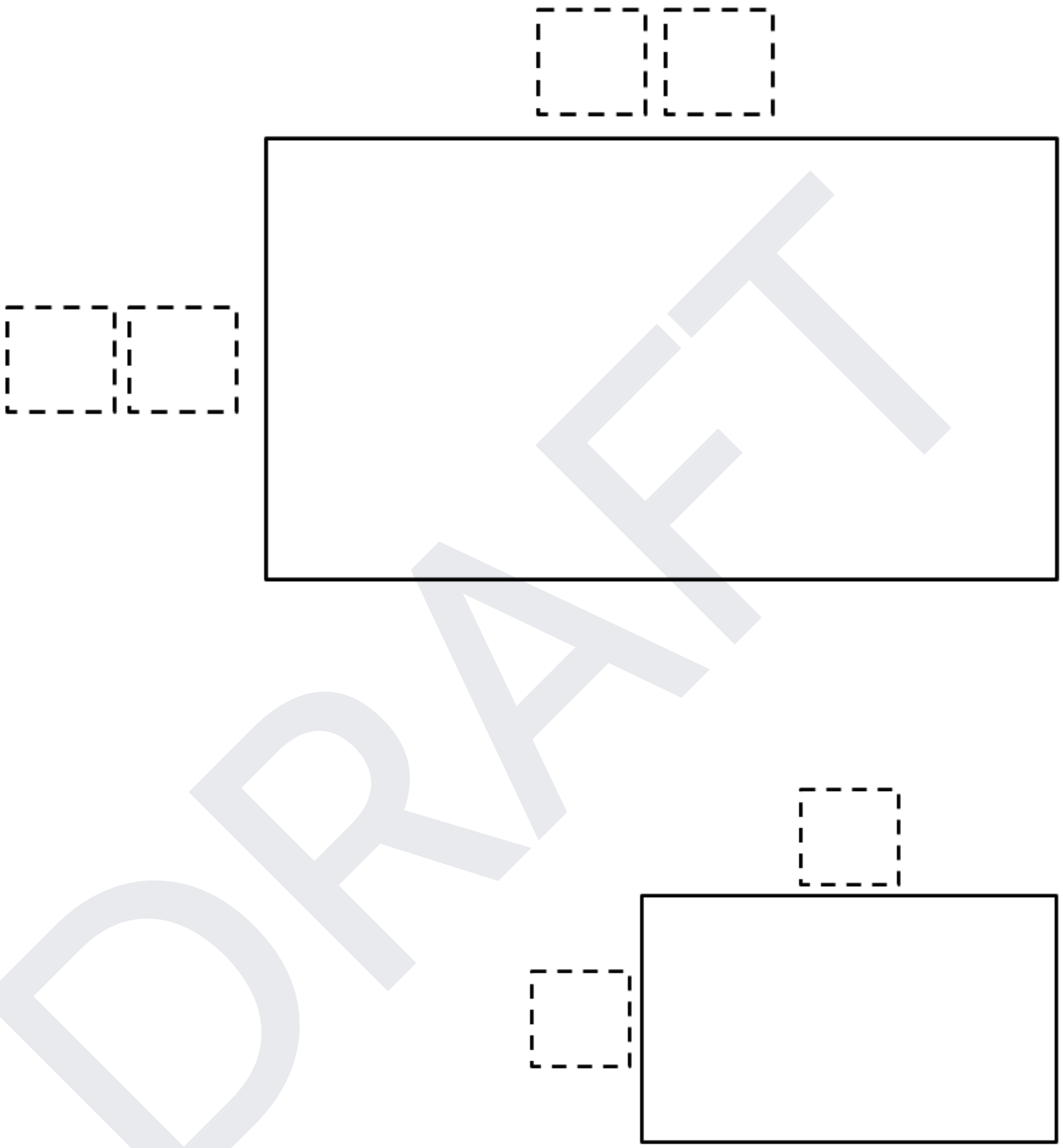
4) What is the perimeter of each rectangle? What is the area of each rectangle?

Rectangle ABCD	Rectangle LMNO
Perimeter = ____________________	Perimeter = ____________________
Area = ________________________	Area = ________________________

5) Which rectangles are similar?



6) Using the digits 0-9, at most one time each, fill in the boxes so that one rectangle is a scaled drawing of the other.[6]

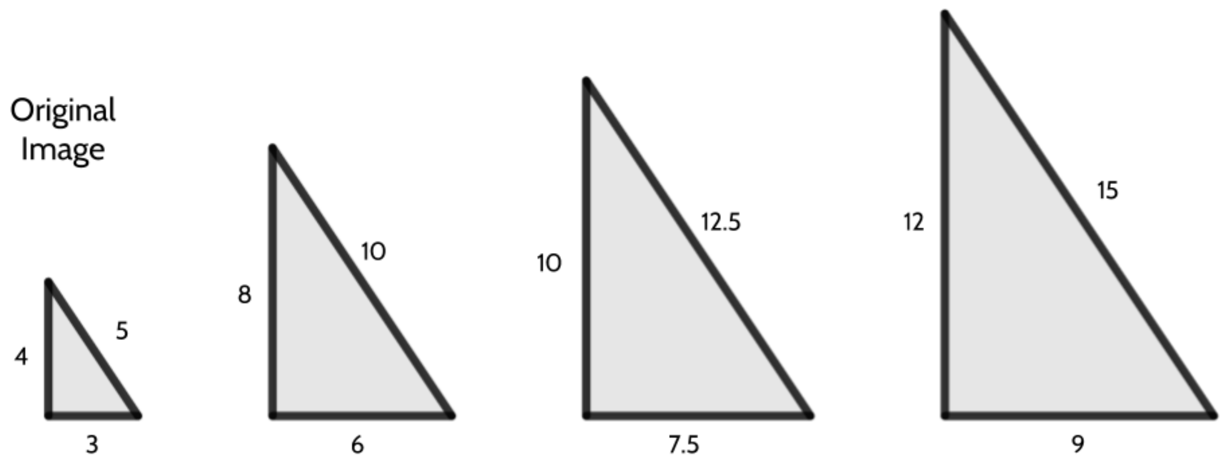


[6] Inspired by an activity from Open Middle - openmiddle.com

Similarity within Triangles

The same ideas of scale, dilation, and similarity apply to other shapes as well.

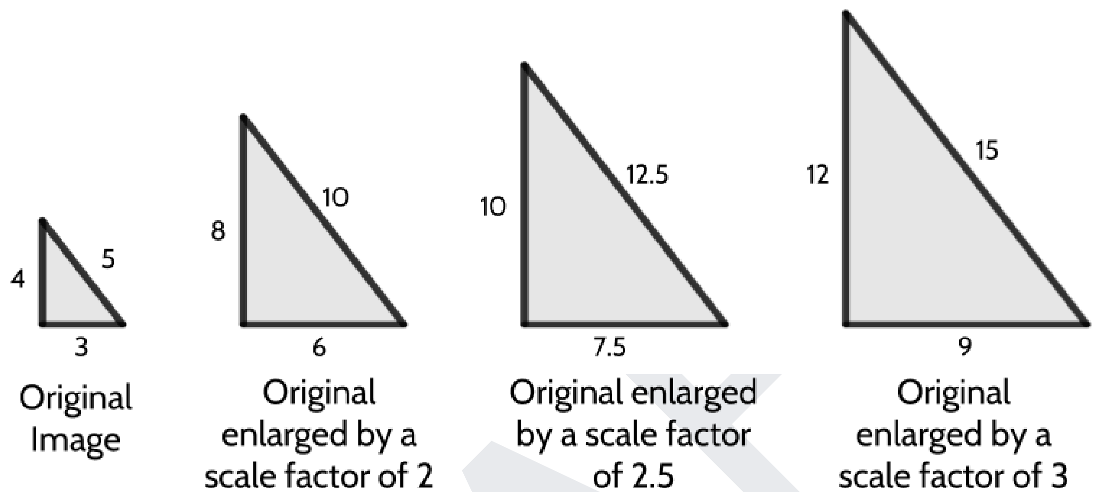
Here are four similar right triangles.



Write 5 things you notice about these shapes.

One thing you may have noticed is that these are all right triangles. The original image is a 3-4-5 right triangle. Since dilation only changes the size and not the shape of a figure, the enlargements are also all right triangles.

But don't take our word for it. Choose one of the enlarged triangles and use the Pythagorean Theorem to test whether it is still a right triangle.



The largest triangle in this diagram is a dilation of the original image by a scale factor of 3. To go from the original image to this triangle, we multiply the length of each side by 3.

3 * 3 = 9

4 * 3 = 12

5 * 3 = 15

What if we needed to reduce the 9-12-15 triangle? What scale factor could we use to reduce the largest triangle back to the size of the original image?

When two shapes are similar, if we divide the lengths of the larger shape by the scale factor, we get the lengths of the smaller shape.

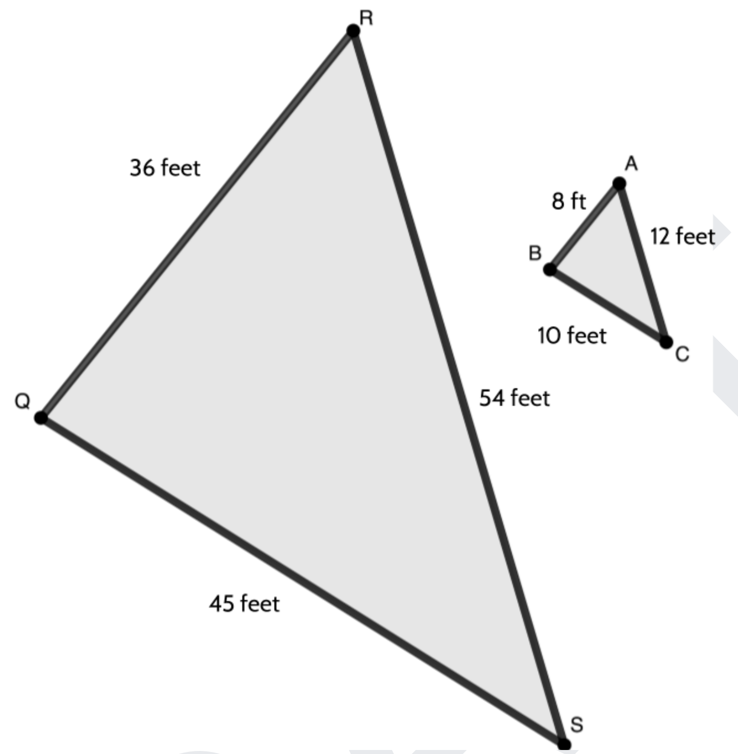
$9 \div 3 = 3$

$12 \div 3 = 4$

$15 \div 3 = 5$

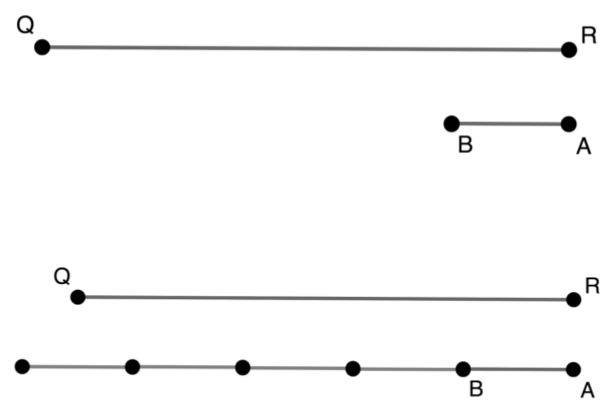
And if we need to figure out the scale factor?

In the diagram below △QRS and △ABC are similar. We have all of the corresponding lengths of each triangle. What is the scale factor?



One question that can help is, "How many of the smaller lengths fit in each corresponding larger length?"

For example, how many 8 foot sections are there in 36 feet? How many 10s are there in 45? How many 12s are there in 54?



We can use division to answer that question. If we divide 36 by 8, we can see how many 8's fit in 36. 36 feet ÷ 8 feet = 4.5

Since the triangles are similar, that same scale factor must describe the relationship between all the corresponding lengths.

12 feet × 4.5 = 54 feet

10 feet × 4.5 = 45 feet

7) Complete this chart of lengths from dilated right triangles.

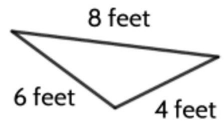
In the final row, choose your own dilation and scale factor (enlarge/reduce).

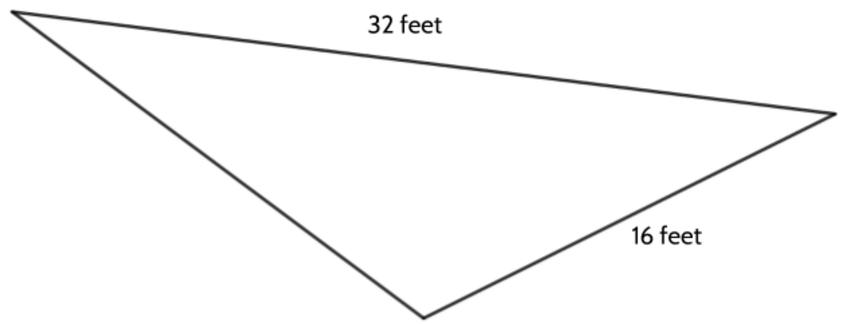
	Corresponding Sides		
Original	**3**	**4**	**5**
Enlarged by scale factor of 2	6	8	10
Enlarged by scale factor of 2.5	7.5	10	12.5
Enlarged by scale factor of 3	9	12	15
Enlarged by scale factor of 4			
Enlarged by scale factor of 5			
Enlarged by scale factor of _______	30	40	50
Enlarged by scale factor of _______	45	60	75
Reduced by a scale factor of ½			

Understanding dilation allows us to solve different kinds of problems. You solved two kinds in the chart above. On the high school equivalency test, you might be given the lengths of a shape and the scale factor and asked to figure out the lengths of the new figure. You might also be given the lengths of an original image and a dilated image and be asked to figure out the scale factor.

A third kind of dilation problem might ask you to figure out the scale factor between two similar shapes and then use it to figure out one of the lengths. Questions 8-10 fit into the third category.

8) The two triangles below are similar triangles.

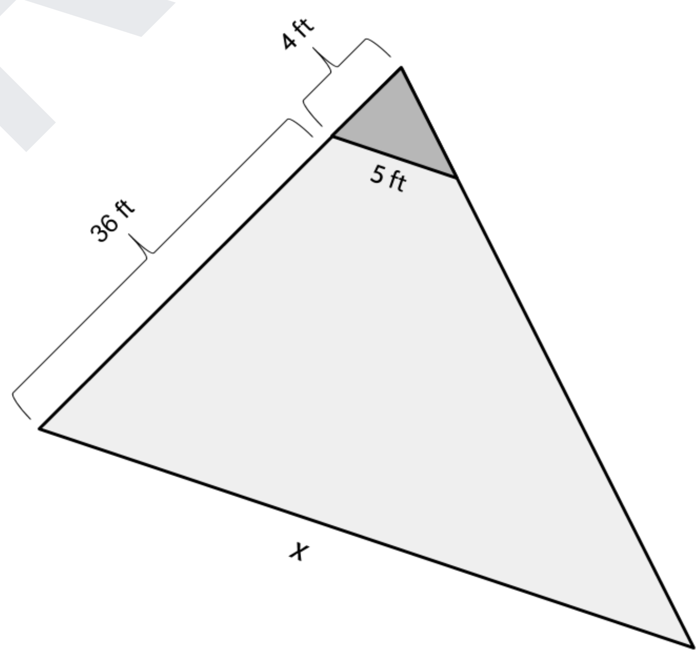




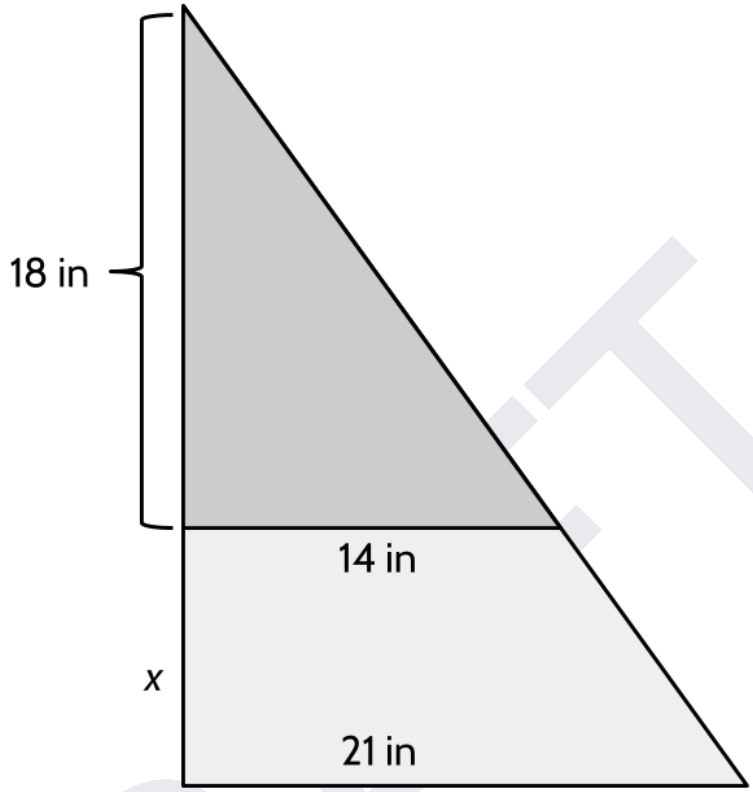
What is the perimeter of the larger triangle?

Sometimes similar triangles will overlap each other. The first question to ask yourself is how many triangles there are in the diagram. Do you see the two similar triangles in question 9?

9) What is the length of side *x*?



10) The two triangles below are similar. What is the length of *x*?



Scale Factors & Dilation - Answer Key

1) $\overline{AB}$ corresponds to $\overline{LM}$. $\overline{CD}$ corresponds to $\overline{NO}$. $\overline{AD}$ corresponds to $\overline{LN}$. $\overline{BC}$ corresponds to $\overline{MO}$.

2) Rectangle LMNO is enlarged from Rectangle ABCD by a scale factor of 3. Corresponding sides $\overline{AD}$ and $\overline{LO}$ are 6 and 18. 18 is 3 times larger than 6.

3) $\overline{LM}$ is 12 feet. The corresponding side to $\overline{LM}$ is $\overline{AB}$and the scale factor is 3. Since $\overline{AB}$ has a length of 4 feet, $\overline{LM}$will be 3 times 4, which is 12.

4) The perimeter of Rectangle ABCD is 20 feet. The area is 24 sq ft.
   The perimeter of Rectangle LMNO is 60 feet. The area is 216 sq ft.

5) Rectangles A and B are similar. They are rotated, but we can still see the relationship between the corresponding sides. The shorter sides in A and B are 12 and 6. The longer sides are 20 and 10. By looking at the relationship between the corresponding sides, we can see that Rectangle B is dilated by a scale factor of 2 to give us Rectangle A.

   Did you also notice that Rectangle D is also similar to Rectangles A and B? It is harder to see, but there are a few ways to think about it.

   Rectangle B is 6 inches and 10 inches. Rectangle A is 12 inches and 20 inches. For every 6 inches on the shorter side, the longer side needs to gain 10 inches.

   Rectangle D is 9 inches on its shorter side. That is 6 and then half of 6. Rectangle D is 15 inches on its longer side. That is 10 and then half of ten. So Rectangle D is 1 and a half times bigger than Rectangle B.

   Another way to think about it is to ask, “How many 6 are there in 9?” If you divide 9 by 6 you get 1.5. Then we can look at the other corresponding sides. If you divide 15 by 10 you also get 1.5. That means both sets of corresponding sides have been enlarged by the same scale factor.

6) There are many possible answers. Here are two of them:

   The smaller rectangle is 2 by 8. The larger rectangle is 14 by 56. The scale factor would be 7.

   The smaller rectangle could be 3 by 8. The larger rectangle could be 15 by 40. The scale factor would be 5.

7)

	Corresponding Sides		
Original	**3**	**4**	**5**
Enlarged by scale factor of 2	6	8	10
Enlarged by scale factor of 2.5	7.5	10	12.5
Enlarged by scale factor of 3	9	12	15
Enlarged by scale factor of 4	12	16	20
Enlarged by scale factor of 5	15	20	25
Enlarged by scale factor of 10	30	40	50
Enlarged by scale factor of 15	45	60	75
Reduced by a scale factor of ½	1.5	2	2.5

8) The perimeter of the larger triangle is 72 feet. Since we are told the triangles are similar, we can look at the corresponding sides to find the scale factor. The larger triangle is 4 times larger than the smaller triangle. We can use that information to see that the missing side of the larger rectangle is 24 feet. Its corresponding side in the smaller triangle is the side with a length of 6.

32 feet + 24 feet + 16 feet = 72 feet.

9) 50 feet.

There are two triangles here. The smaller triangle on top is similar to the entire triangle. The 36 ft and the 4 ft combine to make the left side of the triangle 40 feet. The corresponding length to that is the side that is 4 ft. From the corresponding lengths of 4 and 40, we know there is a scale factor of 10. Since 5 ft and *x* are corresponding sides and the scale factor is 10, *x* would be 50 feet because that is 10 times longer than 5 ft.

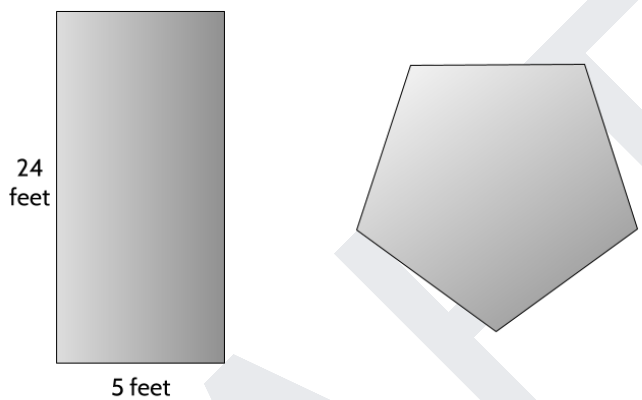
10) 9 inches.

The base of the triangles are corresponding sides. If we ask ourselves, “How many 14s fit in 21?” we take 21 divided by 14, which gives us a scale factor of 1.5. If the side of the smaller triangle measures 18 inches than its corresponding side would have to be 1.5 times bigger than that, which is 27. Since we already have 18 inches of the 27 inches, the missing length is 9 inches.

Test Practice Questions

Answer the following questions. You can check your answers in Test Practice Questions - Answer Key. These questions will review what you have learned in this packet.

1) Each side of the pentagon below is 13 feet long. How much longer is the perimeter of the pentagon than the perimeter of the rectangle?



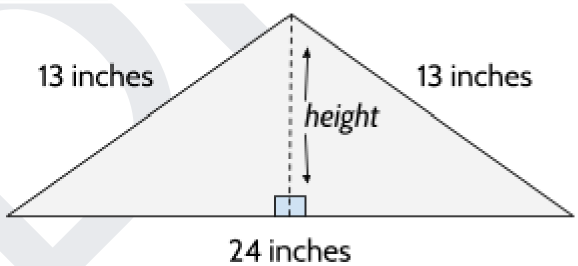
A. 7 feet

B. 11 feet

C. 16 feet

D. 36 feet

2) The dotted line below represents the height of the triangle. If the dotted line divides the base of the triangle in half, what is the height of the triangle?



[Not drawn to scale]

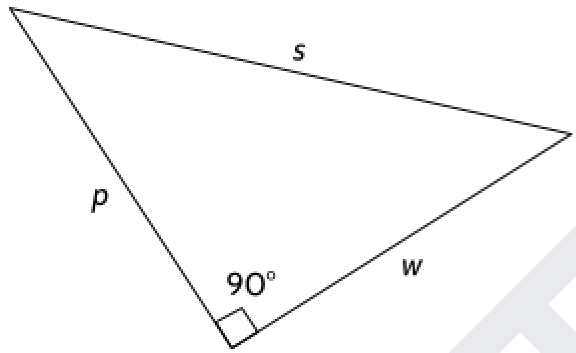
A. 5 inches

B. 12 inches

C. 20 inches

D. 50 inches

3) *s*, *w*, and *p* represent the sides of the right triangle below.



Which of the following statements is not true?

A. $p^2 + w^2 = s^2$

B. $s^2 - p^2 = w^2$

C. $s^2 + p^2 = w^2$

D. $s^2 - w^2 = p^2$

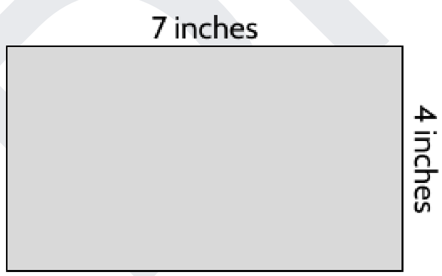
4)

Which of the rectangles below has the same area but a <u>different</u> perimeter from the rectangle above?

A.

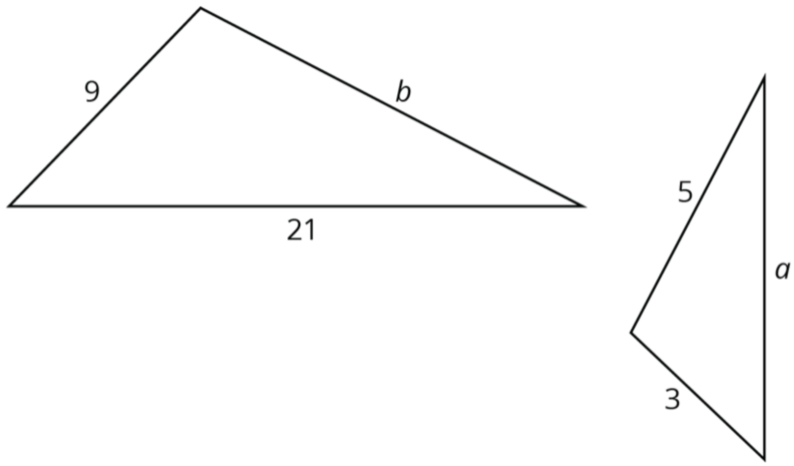
B.

C.



D.

5) These two triangles are similar. What is the perimeter of the larger triangle?



[Not drawn to scale]

A. 15 cm

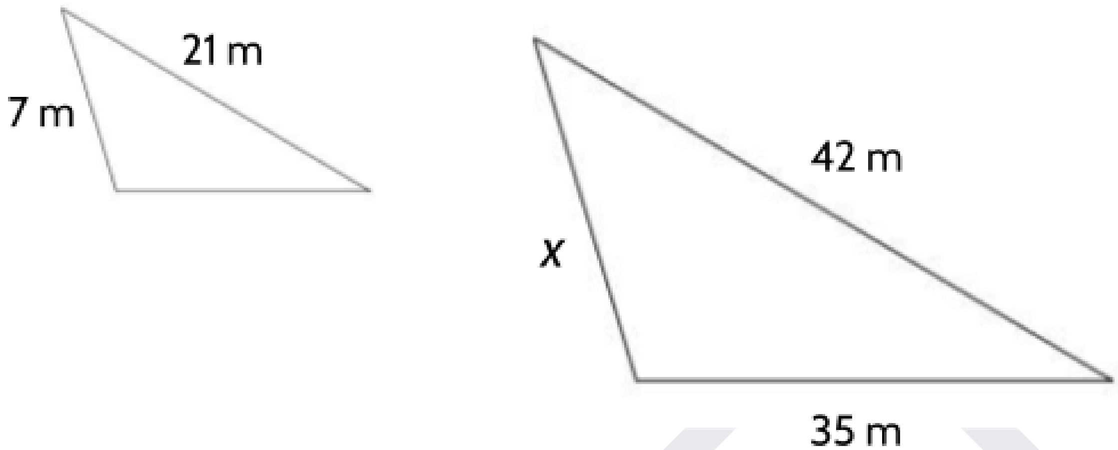
B. 30 cm

C. 45 cm

D. 60 cm

Explain why you chose your answer.

6) Below are two similar triangles.



[Not drawn to scale]

What is the length of side *x*?

A. 3.5 m

B. 7 m

C. 14 m

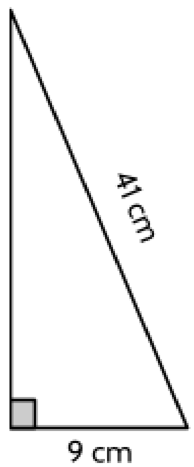
D. 28 m

Explain why you chose your answer.

7) Which of the following could represent the lengths of the sides of a right triangle?

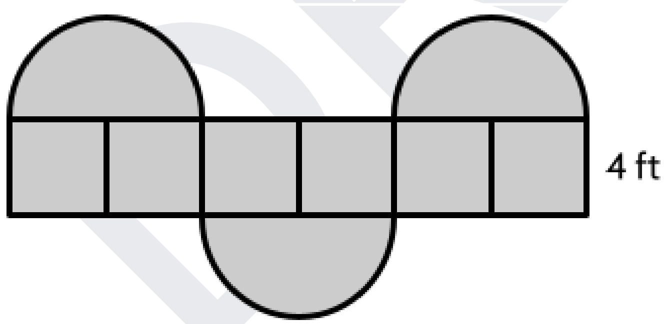
A. 9, 16, 25
B. 5, 12, 12
C. 15, 30, 45
D. 24, 32, 40

8) What is the perimeter of the triangle in the diagram below?



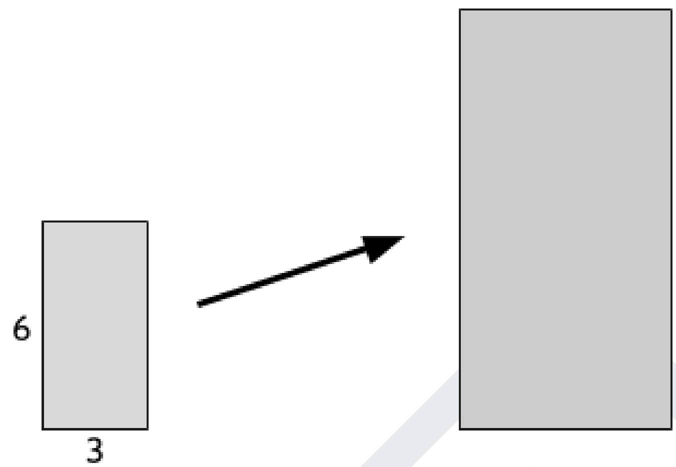
A. 40 cm
B. 42 cm
C. 50 cm
D. 90 cm

9) What is the approximate perimeter of this shape, composed of squares and semicircles?



A. 50.84 ft
B. 69.68 ft
C. 107.36 ft
D. 133.68 feet

10) The smaller rectangle below has been increased by a scale factor of 2 to create the larger rectangle.



Part One:

How does enlarging a rectangle by a scale factor of 2 affect its area?

A. The area doesn't change.

B. The area is twice as big in the larger rectangle.

C. The area is three times as big in the larger rectangle.

D. The area is four times as big in the larger rectangle.

Part Two:

How does enlarging a rectangle by a scale factor of 2 affect its perimeter?

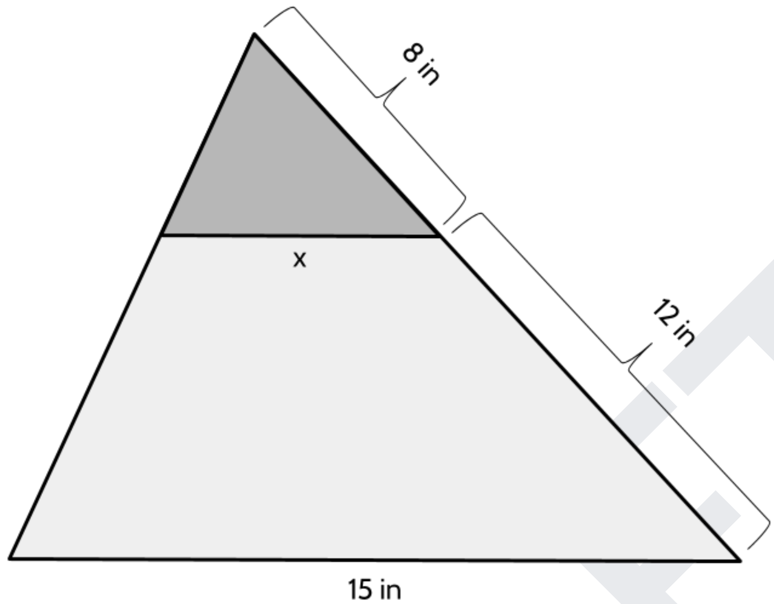
A. The perimeter doesn't change.

B. The perimeter is twice as long in the larger rectangle.

C. The perimeter is three times as long in the larger rectangle.

D. The perimeter is four times as long in the larger rectangle.

11) The small triangle has been dilated to create the large triangle.



[Not drawn to scale]

Part One:

What is the scale factor of the dilation?

A. 1.5

B. 1.6

C. 2.5

D. 11

Part Two:

What is the value of *x*?

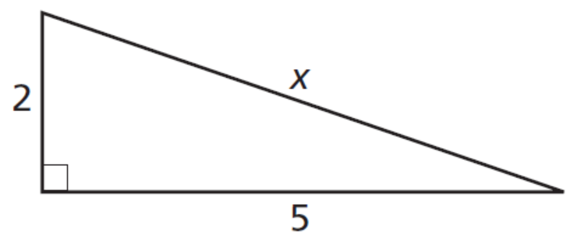
A. 6 in

B. 7 in

C. 10 in

D. 20 in

12) What is the length of side *x* in the triangle below?



[not drawn to scale]

A. 7

B. $\sqrt{7}$

C. 29

D. $\sqrt{29}$

13) The length of one side of a rectangle is 22 cm and its perimeter is 72 cm. What is the area of the rectangle?

A. 308 sq. cm

B. 528 sq. cm.

C. 616 sq. cm

D. 1584 sq. cm

14) Loretta is putting weather stripping around her door frame and needs to figure out the perimeter. The diagram below represents the dimensions of the doorway.

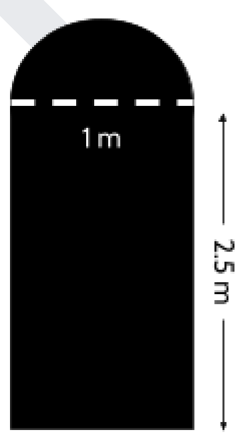
What is the approximate perimeter of Loretta's doorway?

A. 4.07 m

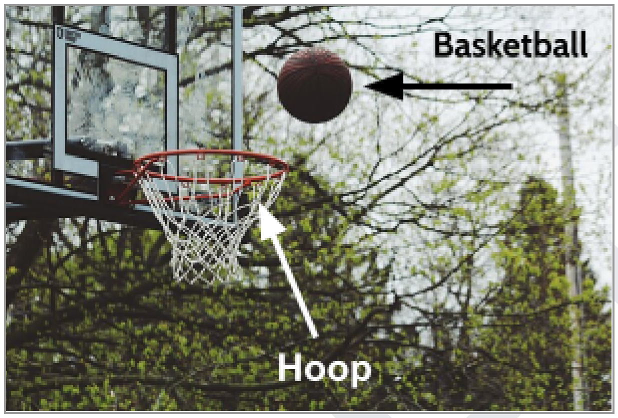
B. 7.57 m

C. 9.14 m

D. 10.14 m



15) According to rules of the National Basketball Association (NBA), a basketball hoop must have a circumference of about 56.52 inches. If an NBA basketball has an approximate diameter of 9.4 inches, what is the approximate difference between the diameter of a basketball hoop and the diameter of a basketball?



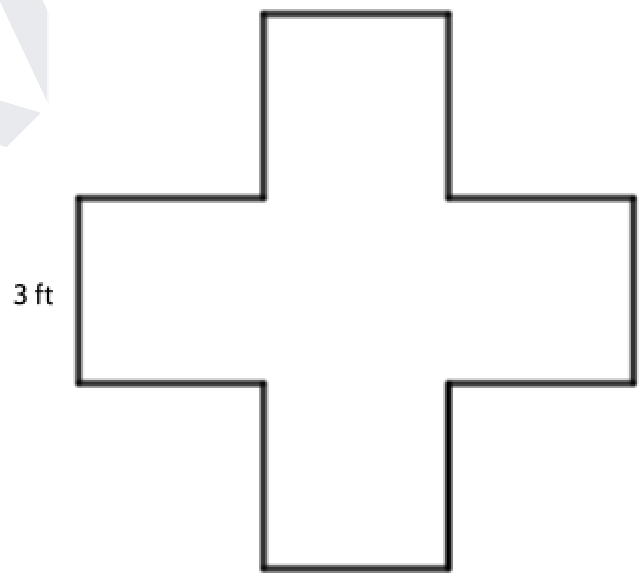
A. 8.6 inches

B. 18 inches

C. 29.5 inches

D. 47 inches

16) All segments in the diagram below measure 3 feet in length and all the angles are right angles.



What is the area of this figure?

A. 27 square feet

B. 36 square feet

C. 45 square feet

D. 56 square feet

17) Jesse drew a triangle with sides 10 cm, 26 cm, and 18 cm. Is Jesse's triangle a right triangle?

A. Yes, because 26 is bigger than 10 and 18.

B. Yes, because 10 + 18 is bigger than 26.

C. No, because 10 + 18 doesn't equal 26.

D. No, because 100 + 324 equals 424.

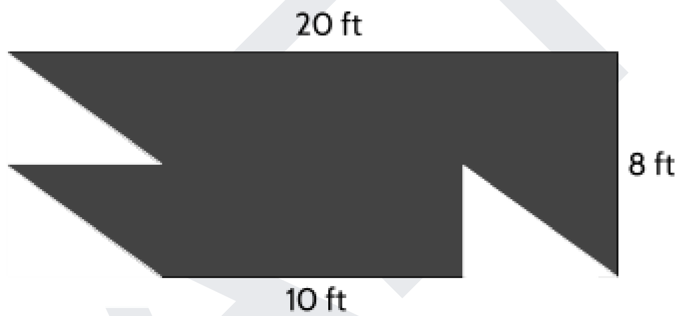
18) Isuri made a design by cutting three identical triangles out of a rectangle. What is the area of the piece they had left?

A. 30 sq ft

B. 80 sq ft

C. 130 sq ft

D. 160 sq ft



19) A circular shield has an area of 706.5 square inches. Using $\pi \cong 3.14$, what is its approximate diameter?

A. 15 inches

B. 30 inches

C. 94.2 inches

D. 225 inches

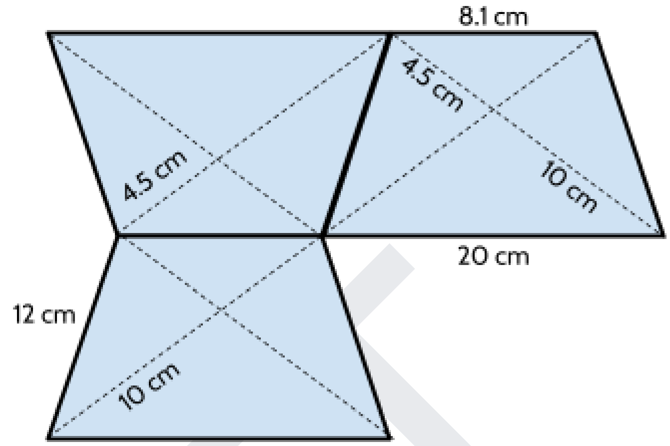
20) The design below was made with three identical trapezoids. What is the perimeter of the design?

A. 40.1 cm

B. 69.1 cm

C. 116.1 cm

D. 145.1 cm



21) The circumference of a circular cake is 50.24 inches. What is the approximate radius of the cake?

A. 4 inches

B. 8 inches

C. 16 inches

D. 32 inches

22) The area of a rectangle is 140 m^2. The perimeter of the rectangle is 48 m. What are the dimensions of the rectangle?

A. 14 m and 10 m

B. 20 m and 7 m

C. 28 m and 5 m

D. 45 m and 3 m

23) What is the perimeter of a square with an area of 324 ft^2?

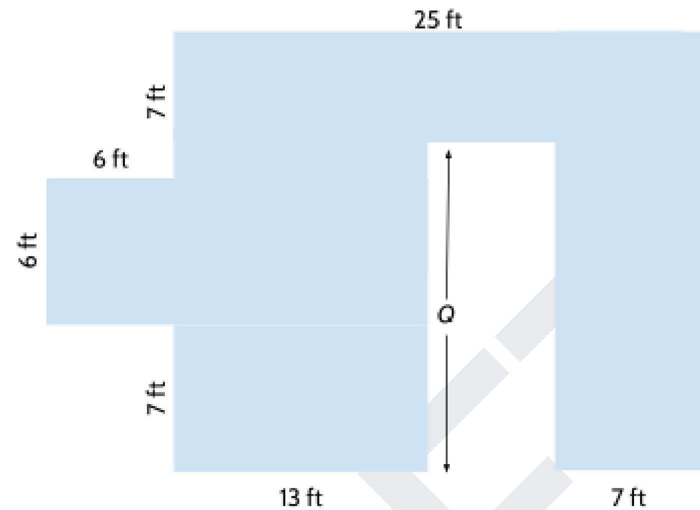
A. 18 ft

B. 36 ft

C. 72 ft

D. 81 ft

24) All angles in the polygon below are right angles. If the perimeter of this polygon is 132 feet, what is the length of side *Q*?



A. 15 ft

B. 30 ft

C. 61 ft

D. 71 ft

25) Jay and Tracy are installing rectangular solar panels on the roof of their garage. The solar panels are 6.5 feet long. If the area of each panel is 22.75 square inches, what is the width of each panel?

A. 3.5 feet

B. 4.875 feet

C. 7 feet

D. 9.75 feet

26) The legs of a right triangle are 8 yards and 12 yards. Which of the following statements is true about the length of the missing side?

A. The length of the missing side is between 14 and 15 yards.

B. The length of the missing side is between 15 and 16 yards.

C. The length of the missing side is between 16 and 17 yards.

D. The length of the missing side is between 17 and 18 yards.

27) Each side of a stop sign is equal in length. If one side measures 12 ½ inches, what is the perimeter of the stop sign?

28) Maggie is planning a rectangular garden that is 8 feet by 12 feet. If Maggie wants one half the area of the garden to be strawberries, which of the following could be the dimensions of the strawberry patch?

A. 2 feet by 3 feet

B. 4 feet by 6 feet

C. 6 feet by 8 feet

D. 6 feet by 10 feet

29) At a mattress store, mattresses are available in the following sizes:

- Twin mattress: 38 inches by 75 inches.
- Full mattress: 53 inches by 75 inches.
- Queen mattress: 60 inches by 80 inches.
- King mattress: 76 inches by 80 inches.

Jay has a twin mattress and replaces it with a full mattress. How much larger is the area of their new mattress?

A. 15 square inches

B. 30 square inches

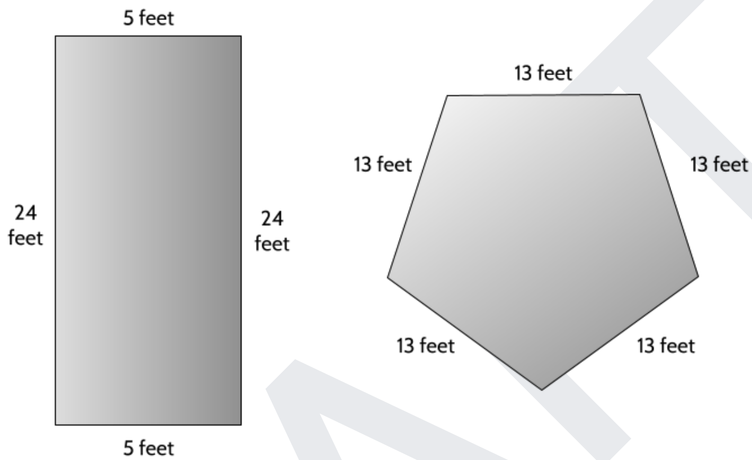
C. 1125 square inches

D. 2850 square inches

Test Practice Questions - Answer Key

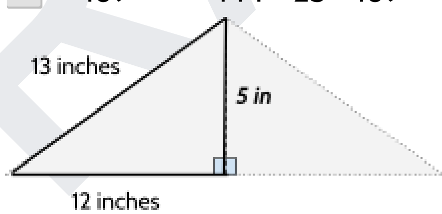
Note: *The explanations given are not the only way to solve these problems. They are just an example of one way. If you did it differently, and got the correct answer, then you probably used an effective strategy.*

1) Choice A. The perimeter of the pentagon is 65 feet and the perimeter of the rectangle is 58 feet. Choice D is what you get if you forget to add the unlabeled sides of the rectangle.



2) Choice A. You can imagine the dotted line dividing this into two right triangles. The length of the hypotenuse is 13 inches. The length of one of the legs is 12 inches.

$12^2 + \square^2 = 13^2$ $\quad$ $144 + \square^2 = 169$ $\quad$ $144 + 25 = 169$ $\quad$ $5^2 = 25$



3) Choice C

4) Choice B. The question is asking for a rectangle with the same area and different perimeter. A 12 × 2 rectangle has an area of 24 square inches and a perimeter of 28 inches. The 8 by 3 rectangle is the only other rectangle with an area of 24 square inches. Be careful: The rectangle in Choice C has an area of 28 sq. in. The rectangle in Choice D has the same perimeter but a different area.

5) Choice C. First you need to determine the scale factor. From the corresponding sides of 3 and 9, we know the larger triangle is three times larger than the smaller triangle. The missing side length, *b*, of the larger triangle corresponds to the side length of 5 in the smaller triangle, so the length of *b* is 15. The perimeter of the larger triangle is 45 (9 + 21 + 15).

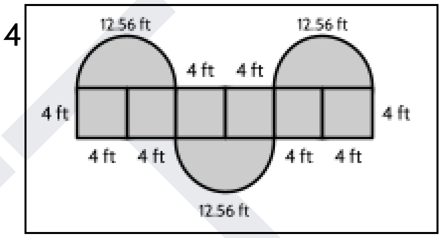
6) Choice C. 14 m. The corresponding sides have each been enlarged by a scale factor of 2. 7 m is the side that corresponds to side *x*, and 7 × 2 = 14.

7) Choice D. $24^2 + 32^2 = 40^2$. The Pythagorean Theorem doesn't work for the other options.

8) Choice D. To find the perimeter, we first need to find the length of the missing side. We are given the hypotenuse and one of the legs of a right triangle.

   $41^2 = 1681$ $9^2 = 81$ 1681 - 81 = 1600 $\sqrt{1600} = 40$ 41 + 9 + 40 = 90cm

9) Choice B. Eight sides of the squares are included in the perimeter. Each of those sides measures 4 feet and 8 × 4 = 32 feet. Each circle has a diameter of 8. If we multiply the diameter by π, we get the circumference. 8 × 3.14 = 25.12 feet. Since we only have half of the circle as part of the perimeter, we divide 25.12 by 2, which is 12.56. Each semicircle has a circumference of 12.56 and there are three of them, so 12.65 × 3 = 37.68 feet.32 + 37.68 = 69.68 feet.



10) **Part One:** Choice D. The area is four times as big in the larger rectangle. The area of the small rectangle is 18 **(**3 × 6). The area of the large rectangle is 72 (6 × 12).

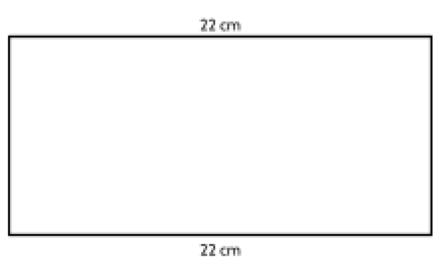
    Part Two: Choice B. The perimeter is twice as long in the larger rectangle. The perimeter of the small rectangle is 18 (3 + 6 + 3 + 6). The perimeter of the large rectangle is 36 (6 + 12 + 6 + 12).

11) **Part One:** Choice C. 2.5

    Part Two: Choice A. One side of the small triangle is 8 in. The corresponding side of the large triangle is 20 in (8 + 12). 20 ÷ 8 = 2.5 (scale factor). 2.5*x* = 15 --> *x* = 6

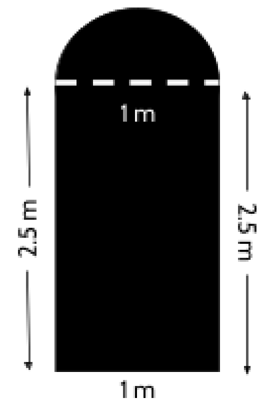
12) Choice D. 52 + 22 = 29. The length of the hypotenuse is the square root of 29. For this problem, the correct answer choice is written as a square root.

13) Choice A. We know one side measures 22 cm and the perimeter is 72 cm. It can help to sketch the rectangle:



We have 44 of the 72 cm we need for the perimeter. We need 28 more cm to make a perimeter of 72 cm. Since we have two sides, we need that 28 to be divided between those 2 sides. That means each shorter side is 14 cm. 14 cm × 22 cm is 308 sq cm.

14) Choice B.



The perimeter of the rectangular part of the door is

2.5 m + 2.5 m + 1 m = 6 m

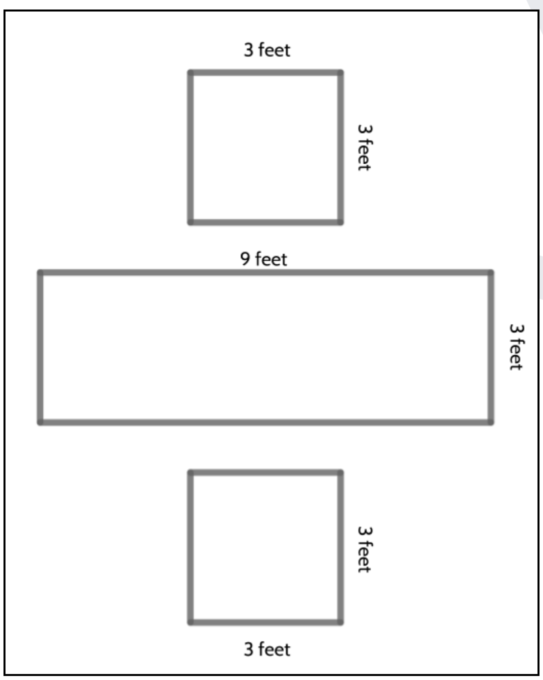
The circumference of the circular part of the doorway is half the circumference of a circle with a diameter of 1 m.

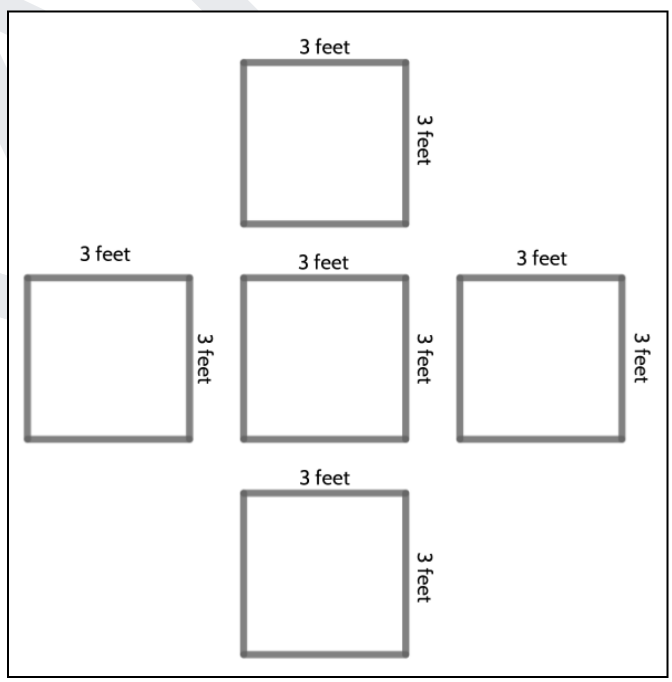
3.14 × 1 = 3.14 m   3.14/2 = 1.57 m

6 m + 1.57 m = 7.57 m

15) Choice B. If the circumference of a basketball hoop is 56.52 inches, then its diameter is 18 inches. 18 - 9.4 = 8.6 inches.

16) Choice C. There are a few different ways you might break up this shape. Here are two examples:





9 sq ft + 27 sq ft + 9 sq ft = 45 sq ft

9 sq ft + 9 sq ft + 9 sq ft + 9 sq ft + 9 = 45 sq ft

17) Choice D.

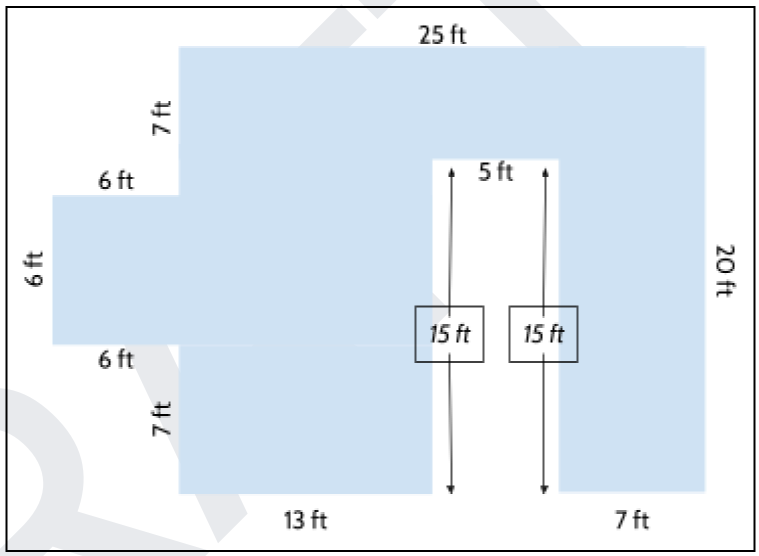
18) Choice C. The area of the rectangle is 160 square yards. The area of each triangle is 10 square yds. The area of all the triangles is 30 square yards. 160 - 30 = 130 square yds.

19) Choice B is the correct answer. Choice A, 15 inches, is the radius of the shield. Choice C 94.2 inches, is the circumference of the shield.

20) Choice C. 12 cm + 12 cm + 12 cm + 12 cm + 20 cm + 20 cm + 20 cm + 8.1 cm = 116.1 cm

21) Choice B. If the circumference is 50.24 in, the diameter would be approximately 16 in, so the radius would be 8 in.

22) Choice A.
23) Choice C.
24) Choice A. This diagram shows all sides labeled. To find the lengths of the two sides marked with arrows, you need to use the fact that the total perimeter is 132 feet. All the other sides add up to 102 ft, so the two remaining sides must add up to 30 feet. Because they are the same length as each other, they are 15 feet each.



25) Choice A.
26) Choice A. The relationship between the length of the sides of a right triangle can be expressed as $a^2 + b^2 = c^2$, so in this case, $8^2 + 12^2 = 208$. If $c^2 = 208$, then $c = \sqrt{208}$. Since $\sqrt{208} = 14.42...$, then c is between 14 and 15 yards.

27) 100 inches. Each side measures 12 ½ in length and there are 8 sides. We can add 12 ½ + 12 ½ + 12 ½ + 12 ½ + 12 ½ + 12 ½ + 12 ½ + 12 ½ = 100. Or we can multiply 12 ½ × 8.

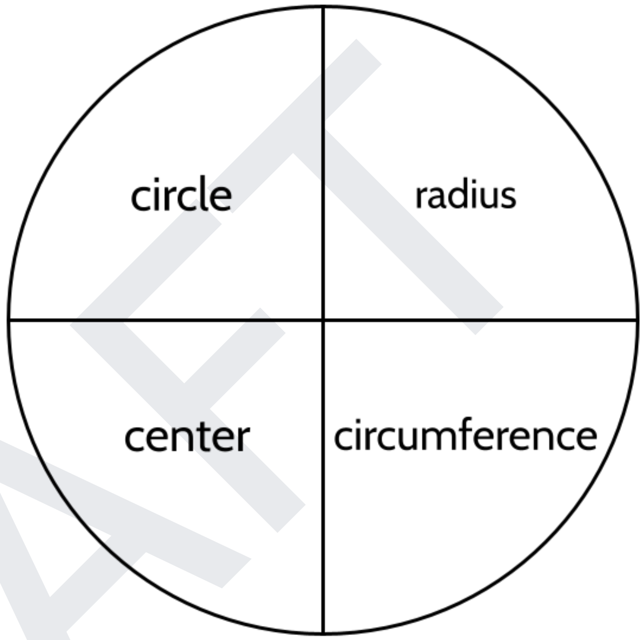
28) Choice C. The area of the entire garden is 96 square feet. A 6 by 8 ft garden has an area of 48 sq feet, which is half of 96 square feet.

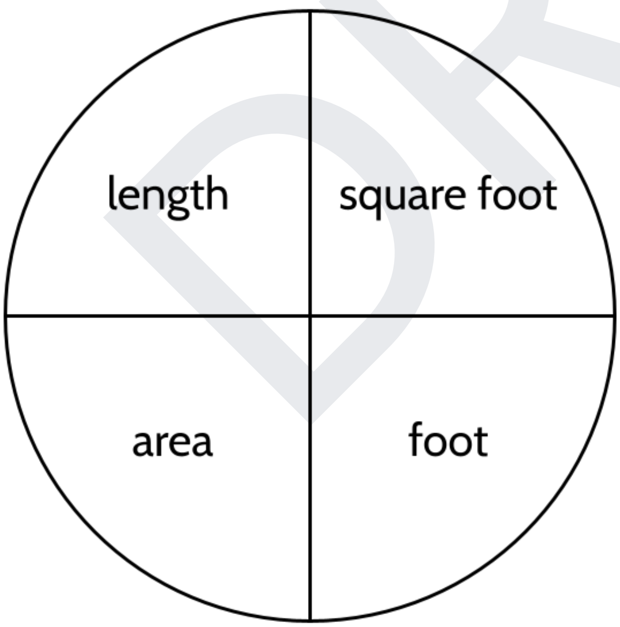
29) Choice C. The area of a twin mattress is 2850 sq. in. The area of a full mattress is 3975 sq. in. The difference is an area of 1125 square inches.

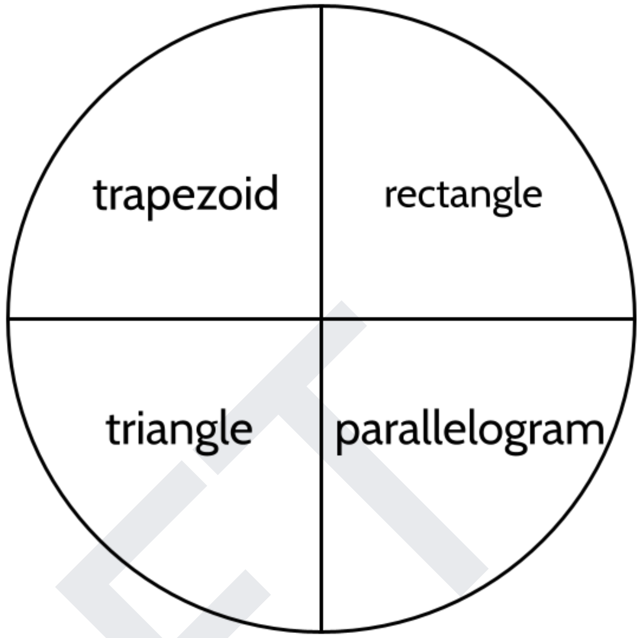
The Language of Geometry

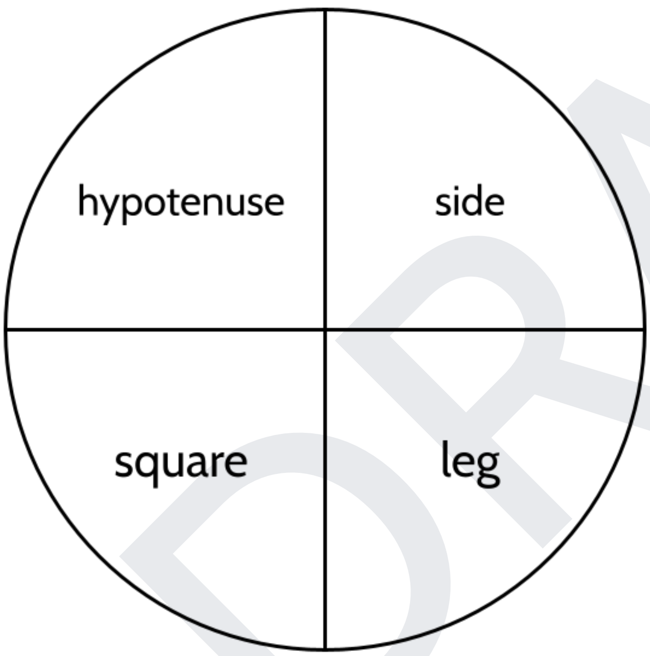
Concept Circle

Explain these words and the connections you see between them.






trapezoid
rectangle
triangle
parallelogram


hypotenuse
side
square
leg

Geometry in Your Life

Look around you. Where do you see geometry? Describe the world you see using as many of the geometric vocabulary words included in the glossary of this packet.

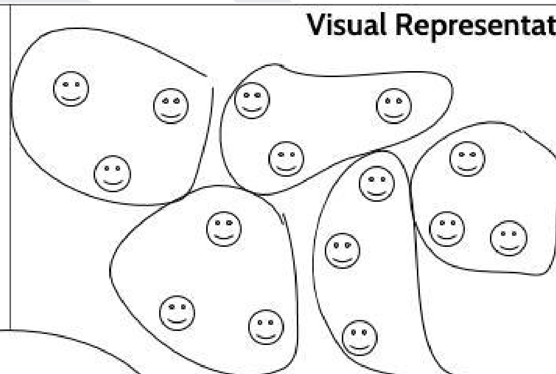
What questions do you still have about geometry?

Graphic Organizers Help People Learn Vocabulary

In order to learn math vocabulary, we need practice using it in different ways. As you work through this packet, choose words that you want to practice. Complete one of the graphic organizers on the next few pages for each word. To complete the graphic organizer, you will choose a word and then answer four questions:

- What is the definition of the word? You can look at the glossary and the vocabulary review at the end of the packet for help. Try to write the definition in your own words.
- Make a visual representation. You can make a drawing or diagram that will help you remember what the word means.
- What are some examples of the word you're studying?
- What are some non-examples of this word? These are things that are **not** examples of the word you're studying.

Look at the sample for the word *quotient* below.

What is it?	**Visual Representation**
A quotient is the result of dividing one number by another. It is the answer to a division question.	
What are some examples?	**What are some non-examples?**
15 divided by 3 equals **5** 66 ÷ 6 = **11** 63/18 = **3.5** 5, 11 and 3.5 are quotients in these calculations. population ÷ area = **density**	4 times 6 equals 24 18 + 5 = 23 17 – 2.5 = 14.5 3.5 × 18 = 63

quotient

There are four graphic organizers on the next few pages. To study more words, you can create as many graphic organizers as you need on paper or in a notebook.

What is it?

Visual Representation

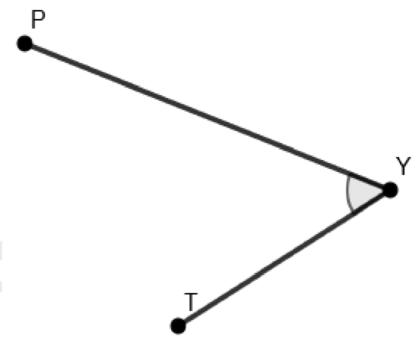
What are some examples?

What are some non-examples?

Glossary and Vocabulary Review

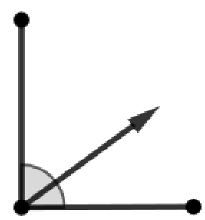
acute angle (noun): an angle that is less than 90°. (See ANGLE)

angle (noun): the opening between two rays or line segments that have a common endpoint. We call that endpoint the vertex. In the angle on the right, point Y is the vertex. We can name the angle ∠PYT or ∠TYP. Note that the vertex has to be the middle letter. We measure angles using degrees.



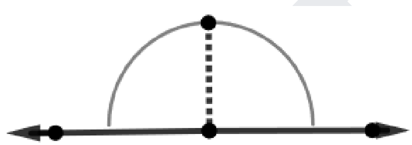
(See OBTUSE ANGLE, ACUTE ANGLE, RIGHT ANGLE, STRAIGHT ANGLE, VERTEX)

Right Angle (90°)

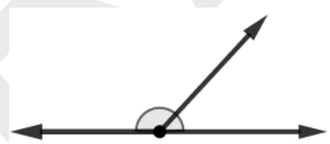


Complementary Angles

Acute Angle



Straight Angle (180°)



Supplementary Angles

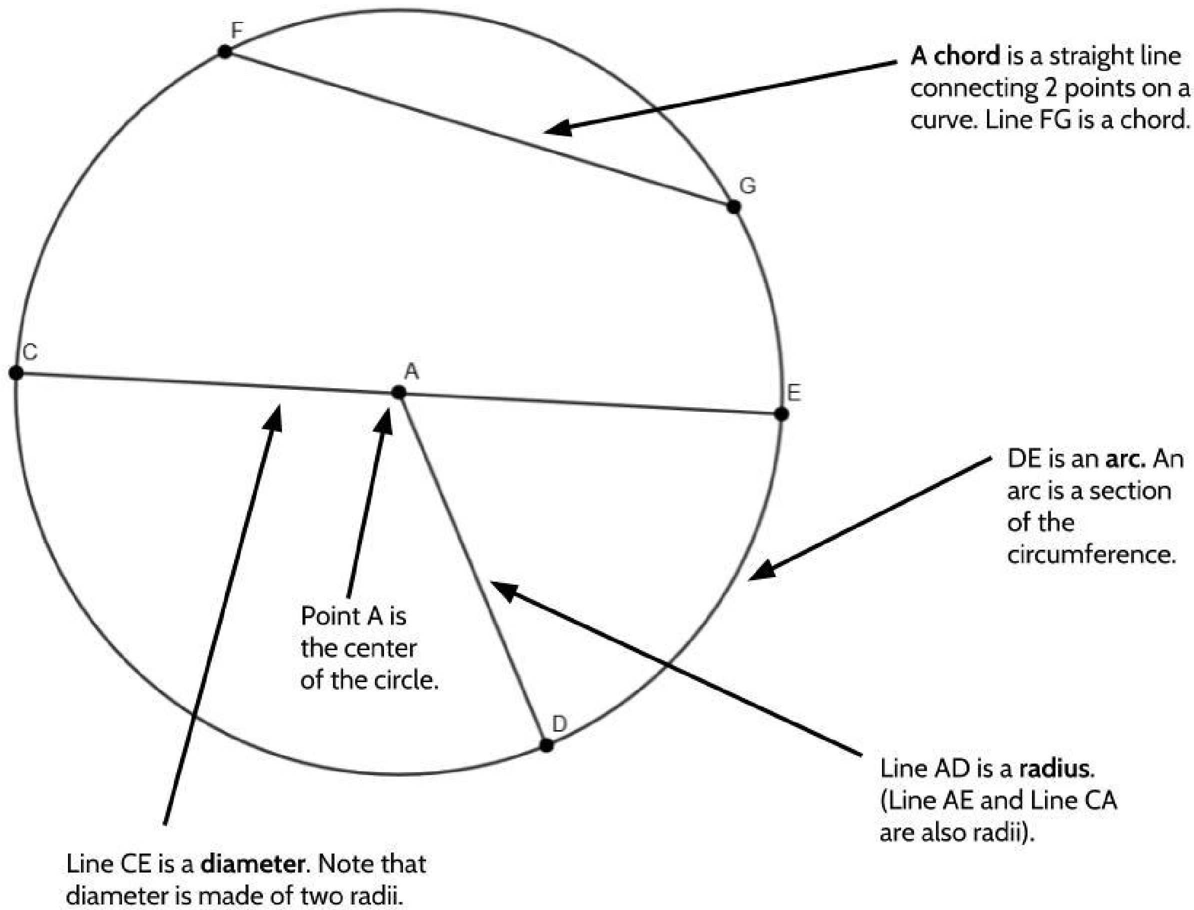
Obtuse Angle

approximate (adjective): close to the actual, but not completely accurate or exact. We can use this symbol≅to show that two things are *approximately* equal to. For example, $\pi \cong 3.14$

area (noun): The size of a surface. When we talk about area, we are talking about how many square units it would take to cover a surface.

attribute (noun): An attribute is an aspect of an object that can be measured. Other words that have a similar meaning are: quality, trait, characteristic, feature, property.

circle (noun): a two-dimensional figure in which every point is the same distance from a point called the center. (See CIRCUMFERENCE, DIAMETER, RADIUS)



circumference (noun): The distance around a circle. You may think of the circumference as the perimeter of a circle. The circumference of every circle is a little more than three times the length of the diameter of the circle. To be more exact, the circumference of a circle is equal to pi (π) times the length of the diameter. In the circle above, the circumference is the distance from point C all the way around the circle back to point C. That distance is a little more than three times the length of line CE.

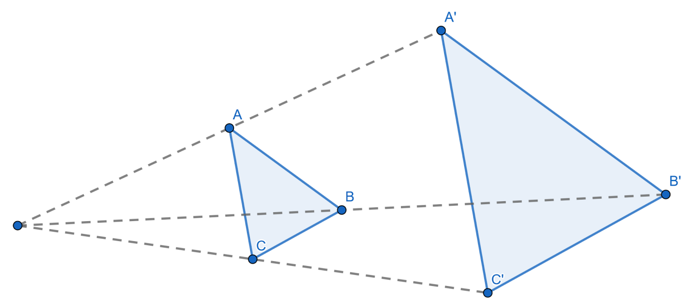
compare (verb): to describe how two or more things are alike or different.

corresponding sides (noun): Corresponding sides have the same basic position in similar figures. Corresponding sides are related to each other by a scale factor. (See SCALE FACTOR, DILATION)

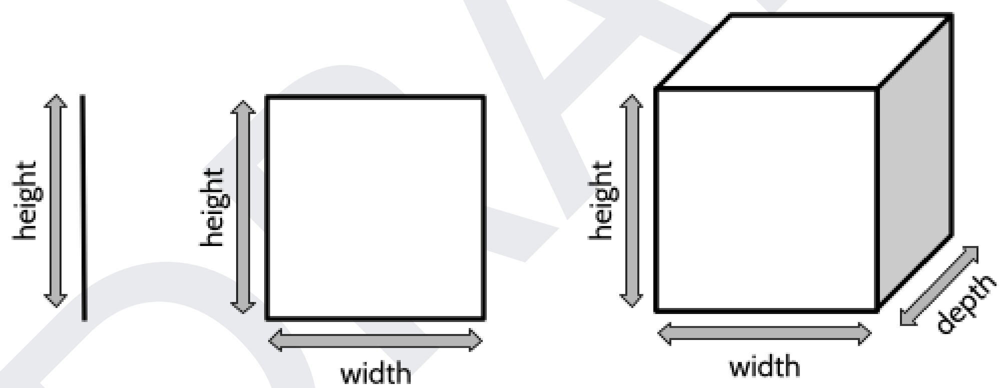
degree (noun): a degree is the unit used to measure angles. Degrees describe the opening in an angle.

diameter (noun): a line segment that goes from one point on a circle, through the center of the circle, to another point on the circle. *See Circle*

dilation (noun): dilation makes a figure larger or smaller without changing its shape. Dilations can be described by the term *scale factor*. In this diagram, the smaller triangle is enlarged by a scale factor of 2. That means the image is 2 times larger than the original figure. A scale factor of ½ means that the dilated image is half the size of the original figure.



dimensions (noun): A dimension is a measurement of length in one direction. Height, width, depth are all examples of dimensions. A line has one dimension (1D), a square has two dimensions (2D), and a cube has three dimensions (3D).



equilateral triangle (noun): a triangle with <u>3</u> sides equal to the same length. All three angles in an equilateral triangle will be 60°.

estimate (verb): to make a rough guess at a number, usually without making written calculations. To estimate is to look for an answer that is close enough to the right answer.

exponent (noun): an exponent tells us how many times to multiply a number by itself. Exponents are written next to the number, but raised. For example, 8^2. 8^2 means 8×8. 2 is the exponent that tells us to multiply 8 by itself twice.

hypotenuse (noun):The side opposite the right angle in a right-angled triangle.It is also the longest side of the right-angled triangle.

isosceles triangle (noun): a triangle with <u>two</u> sides of equal length.

length (noun): The measurement of something from one end to the other. The distance between two points. *Height, base, depth, width, side, perimeter* are all words used to describe length.

line (noun): A line is an infinite set of points continuing in opposite directions without end.

line segment (noun): a line segment is part of a line, defined by two endpoints and all the points between them. You can think of the rungs of a ladder as physical examples of line segments.

obtuse angle (noun): an angle that is larger than 90° and smaller than 180° (See ANGLE.)

parallel lines (noun): parallel lines are lines on a plane that never meet.

parallelogram (noun): a quadrilateral with opposite sides parallel. Opposite angles are also parallel. A rectangle, rhombus, and square are all examples of special parallelograms.

perfect square (noun): a number made by squaring a whole number.

perimeter (noun): the whole length of the border around an area or shape.

perpendicular lines (noun): perpendicular lines are lines that intersect to form 90° angles.

pi (π): Pi is the 16th letter in the Greek alphabet. It is used in math to represent a special relationship found in any circle in the world. The circumference of any circle in the world is about 3 times greater than the diameter of that circle. If you divide the length around any circle (circumference) by the length across it (diameter), you will get something close to 3.14. (See CIRCLE, CIRCUMFERENCE, DIAMETER.)

plane (noun): a completely flat surface

point (noun): a point has no size, it is only a location and it is named with a capital letter.

polygon (noun): polygons are shapes that are formed by straight line segments so that each segment meets exactly two other segments.

radius (noun): a line segment from the center of a circle to any point on the circle. The radius is half of the diameter. The way to describe more than one radius is “radii” (pronounced ray-dee-i). Every radii in a circle is the same length.

ray (noun): a ray is a part of a line, starting with one endpoint, and made up of all the points on one side of that endpoint.

rectangle (noun): a parallelogram with all right angles. Squares are a special kind of rectangle.

rhombus (noun): a quadrilateral that has all sides the same length.

right angle (noun): a right angle is an angle that measures 90°. All four angles in a square or rectangle are right angles. (See ANGLE, COMPLEMENTARY ANGLES)

right triangle (noun): a right triangle is a triangle with one 90° angle. In a right triangle, the side opposite the right angle is called the *hypotenuse*. The two sides that form the right angle are called *legs*.

scale factor (noun): The number used to multiply the lengths of a figure to enlarge or shrink the image. If we have a scale factor of 2, all of the lengths in the image are 2 times longer than in the original. When you are given two corresponding lengths, if you divide them, you get the scale factor. (See DILATION)

scalene triangle (noun): a triangle with no sides of equal length.

semicircle (noun): half of a circle. The length of a semicircle is ½ the length of the circumference.

similar triangles (noun): similar triangles are triangles that are the same shape, but different sizes.

square (noun): a quadrilateral with all four sides the same length and four right angles.

square units (noun): Square units are used to measure area. When we describe the area of something we are imaging how many square units it would take to cover its surface. A square unit is a square that measures one unit on each side. For example, a square foot is a square that measures 1 foot on each side. Square units include *square foot, square inch, square mile, square yard, square centimeter, square meter, etc.*

surface area (noun): The area on the surface of a 3-dimensional figure. For example, the amount of wrapping paper on a gift would be measured in surface area. (See AREA.)

theorem (noun): A statement that can be shown to be true, especially in math.

trapezoid (noun): a trapezoid is a quadrilateral with 4 straight sides and has at least one pair of opposite sides parallel. Note: If you went to school that followed the UK model, you might know this shape as a trapezium. Trapezium and trapezoids are the same, but trapezoid is the more common name in the US.

triangle (noun): a flat shape with three straight sides and three angles. The sum of all three angles in any triangle is 180°

unit square (noun): a square whose sides are 1 unit in length. A square unit is the basic unit of measure we use for area. (Examples of unit squares: square feet, square inch, square mile, etc.)

vertex (noun): The vertex is the point where the segments, or lines intersect to form an angle. (See ANGLE)

volume (noun): the amount of space inside a three-dimensional object or the amount of space that object takes up. For example, when we talk about the volume of a cereal box, we would be talking about the amount of space inside the box. If we were talking about the volume of an apple, we would be talking about how much space the apple takes up. When we talk about volume, we are talking about how many unit cubes it could take to fill the object.

Sources

Academo. Area of a circle (Rearrangement Method). https://academo.org/demos/circle-area-rearrangement-method (Section: Circles)

Carter, S. & Inaba, N. Sankaku Puzzles. https://mathequalslove.net/sankaku-puzzles (Section: Area of Other Polygons)

Chapin, S. 2000. Math Matters: Understanding the Math You Teach. Grades K-6. Math Solutions Publications. (Section: Introduction to Geometry)

Stadel, A. Estimation 180. https://estimation180.com (Sections: Estimation & Precision, Circles)

Haun, L. Framing with Larry Huan. FineHomebuilding YouTube Channel. https://www.youtube.com/playlist?list=PLv8J8XldbK3_l5e8jGe5CCuFuvVGtkgYO (Section: Triangles)

JMAP Regents Tests. http://www.jmap.org (Sections: Area and Perimeter of Rectangles, Test Practice Questions)

Krieg, P. 2023. Finding the Center of a Circle. https://bookzoompa.wordpress.com/2023/03/05/the-center-of-a-circle (Section: Circles)

Math is Fun. https://www.mathsisfun.com (Sections: Estimation & Precision, Glossary)

Mathigon Timeline of Mathematics. https://mathigon.org/timeline (Sections: Introduction to Geometry, Triangles)

Open Middle. http://www.openmiddle.com. (Section: Scale and Dilation)

Tanton, J. 2019. Two Key - but Ignored - Steps to Solving Any Math Problem. https://medium.com/q-e-d/two-key-but-ignored-steps-to-solving-any-math-problem-8cd927bf60a0 (Section: Introduction to Geometry)

Van de Walle, J. A. 2013. Elementary And Middle School Mathematics: Teaching Developmentally. New York. (Sections: Polygons, Length and Area, Area and Perimeter of Rectangles, Area of Other Polygons, Triangles)