

Teachers' Guide to the NYSED/CUNY Fast Track GRASP Math Packets

Updated 11/15/2023



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All packets are available for free download at <http://www.collectedny.org/ftgmp>.

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If you have any questions about the administration of a Fast Track Program or a GRASP Distance Learning Program, please contact Rosemary Matt, New York State Director of Accountability at Rosemary.Matt@cayuga-cc.edu.

Introduction

The NYSED/CUNY Fast Track GRASP Math Packets are a set of study materials for math topics for high school equivalency. The packets provide an introduction to a range of topics, such as proportional reasoning, geometry, algebra, and probability. The packets provide practice in the high-priority topic areas as identified by subject experts from the New York State Department of Education.

This teacher's guide was created to help teachers understand the scope and content of each of the packets. It consists of a compilation of the overviews, assessment questions, and placement suggestions from each of the packets.

The Fast Track GRASP Math Packets were designed to be used in any of the following program models:

- Distance Learning: For only those EPE programs with NYSED approval: 24 contact hours per packet
 - May also include 6 hours of in-person class time
 - As an additional supplement to classroom instruction providing the students in class have not been given the packets for independent work
- NYSED-approved Fast Track classes (may be used as lessons in math classes)
- ABE/HSE math classes (in class or as additional independent work)

Fast Track GRASP Math Packets, as of November 2023:

- **Number Lines and the Coordinate Grid** - Part 1 & Part 2 - published July 2023 (New)
- **Proportional Reasoning** - Part 1 & Part 2 - available January 2024
- **Two-Dimensional Geometry** - Part 1 & Part 2 - published June 2023 (New)
- **Three-Dimensional Geometry** - Part 1 & Part 2 - available January 2024
- **Being Counted: Probability & Statistics** - Part 1 & Part 2
- **The Power of Exponents** - Part 1 & Part 2
- **Tools of Algebra: Expressions, Equations, Inequalities** - Part 1 & Part 2
- **Tools of Algebra: Linear Functions** - Part 1 & Part 2
- **Tools of Algebra: Nonlinear Functions** - Part 1 & Part 2

Each packet includes:

- An overview of topics and recommended prerequisites
- Pre-assessment questions to help determine if the packet is a good fit for students
- Scaffolded activities for students to develop conceptual understanding, followed by application of those concepts in context
- High school equivalency practice test questions
- Language activities, helpful for all students
- Answer keys

Scope of Fast Track Grasp Math Packets

Quantitative Reasoning & Problem Solving

Number and Quantity

Number Lines and the Coordinate Grid Part 1 & Part 2

- ordering numbers (including fractions and decimals) on the number line
- signed numbers
- absolute value
- coordinate grid

Proportional Reasoning - Part 1 & Part 2 (available January 2024)

- ratios
- scaling
- fractions, decimals, percentages
- visual representations of fractions
- financial math

Geometry

Two-Dimensional Geometry - Part 1 & Part 2

- perimeter and circumference
- measures and units of area
- geometric formulas
- pythagorean theorem
- scale factors
- unit rate: per = “for every”
- population density (area in a social studies context)

Three-Dimensional Geometry - Part 1 & Part 2 (available January 2024)

- three-dimensional figures
- surface area
- volume
- geometric formulas
- density of matter (volume in a science context)
- measurement conversion

Probability And Statistics

Being Counted: Probability & Statistics - Part 1 & Part 2

- ratios, fractions, and percents
- measures of central tendency (mean, median, mode) and range
- data visualizations: histograms, dot plots, two-way relative frequency tables, etc.
- basic probability (sample space, tree diagrams)
- random sampling & experimental design

Algebraic Reasoning & Problem Solving

The Power of Exponents - Part 1 & Part 2

- factors/multiples, least common multiple, greatest common factor
- squares/square roots, cubes/cube roots
- fractional ($\frac{1}{2}$ & $\frac{1}{3}$) and negative exponents
- rules for exponents
- exponential growth & decay

Tools of Algebra: Expressions, Equations, Inequalities - Part 1 & Part 2

- match expressions/equations with context/situations
- area models for combining like terms & the distributive property
- explain steps when solving simple equations
- inequality notation
- systems of equations (guess and check, using a graph)

Tools of Algebra: Linear Functions - Part 1 & Part 2

- what is and what is not a function
- four views of linear functions (written description, rule/equation, graph, table)
- rate of change/slope and starting amount/y-intercept (in context)
- function notation
- creating and interpreting linear functions

Tools of Algebra: Nonlinear Functions - Part 1 & Part 2

- features of graphs of quadratic and exponential functions
- matching situation to graph/graph to situation
- comparing linear and nonlinear functions
- matching graph to function

NOTE: The following packets have been retired from the Fast Track GRASP Math Packet series because they are not aligned with the GED exam: **Rigid Transformations: Shapes on a Plane**¹ and **Lines, Angles, and Shapes: Measuring Our World**². For teachers wanting to teach rigid transformations and/or angles, those sections from these retired packets are still available on CollectEdNY.org for classroom use.

¹ Some materials from this packet have been re-written and included in **Number Lines and the Coordinate Grid**, Parts 1 and 2.

² Some materials from this packet have been re-written and included in **Two-Dimensional Geometry**, Part 1.

Number Lines and the Coordinate Grid

Overview

Prerequisites	There are no prerequisites to <i>Number Lines and Coordinate Planes, Part 1</i> . As long as you are able to read this packet independently, you don't have to study any other math packets first. Students should complete <i>Number Lines and Coordinate Planes, Part 1</i> before working on <i>Number Lines and Coordinate Planes, Part 2</i> .
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In this packet, you will explore concepts in

In Part 1, you will study the following topics:

- Plotting points on a number line, including fractions, decimals, and signed numbers
- Measurement and distance on a number line
- Absolute value

In Part 2, you will build on what you learned in Part 1, and study the following topics:

- Plotting points and interpreting points on the coordinate grid
- Drawing lines and shapes on the coordinate grid
- Data on the coordinate grid (including scatter plots) and correlation

In addition to the learning the topics above, you will find the following materials to help you:

- High School Equivalency Test Practice Questions. You will practice the concepts you have learned from this packet to work on these questions. The answer key for this section explains the correct answers, and also some of the wrong answers.
- A graphic organizer to study vocabulary is included, along with a vocabulary activity to review concepts. A glossary with important terms from this packet is also included for your study.
- Concept Circles can help you make connections between the concepts you have learned and help you remember those connections.

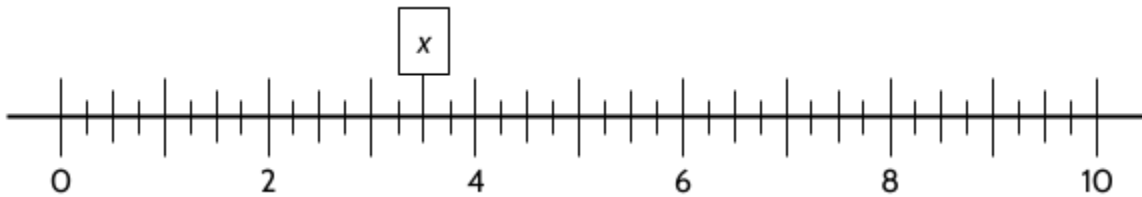
Assessment Questions

Calculator allowed

The following questions will help to see if this packet is right for you. Do your best to answer each question. If you can't answer, don't worry—this packet will help you answer questions like these and more. When you are finished with the questions, read our recommendations.

Question 1

Look at the number line below.



Which of the following numbers is not at position x on the number line above?

A. $2\frac{3}{4}$

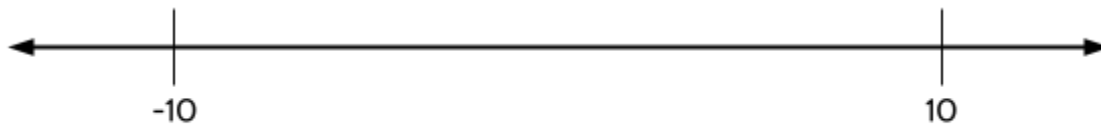
C. $3\frac{1}{2}$

B. $\frac{7}{2}$

D. $\frac{14}{4}$

Question 2

Place the following numbers on the number line below: 0 $-2\frac{1}{2}$ $|-5|$



Question 3

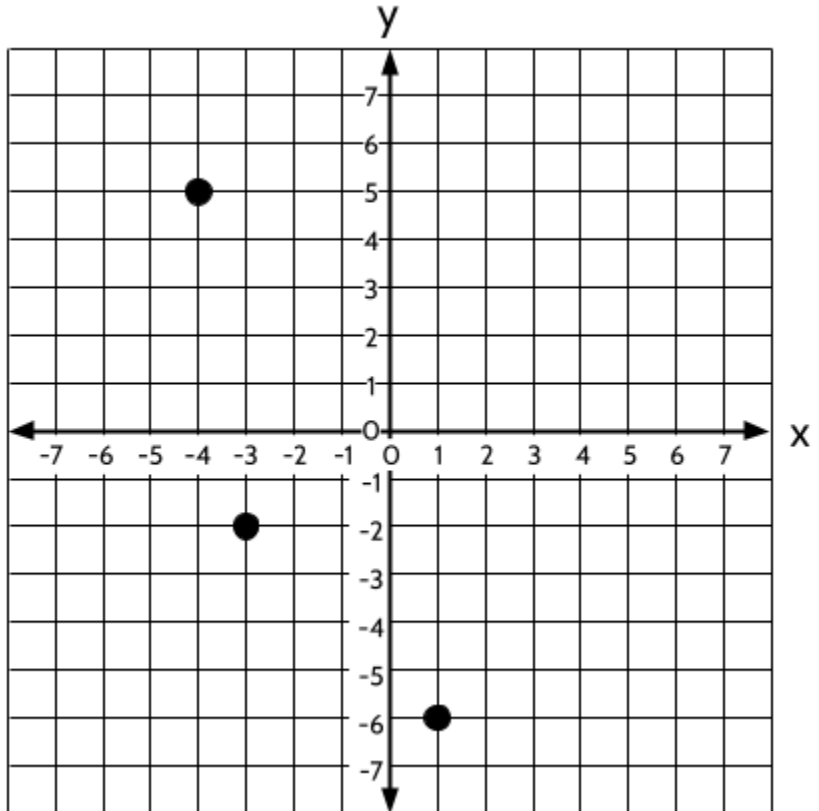
Imagine the numbers $-4\frac{3}{4}$ and $1\frac{1}{4}$ plotted on a number line. What would the distance be between the two points?

- A. $-4\frac{3}{16}$ C. $3\frac{1}{2}$
 B. $-3\frac{1}{2}$ D. 6

Question 4

Look at the coordinate plane below. What are the coordinates of the three locations on the graph?

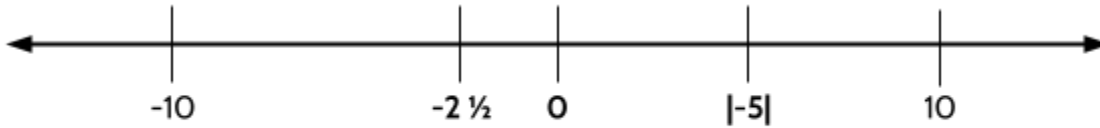
- A. $(-4, 5), (-2, 3), (-6, 1)$
 B. $(-4, 5), (-3, -2), (1, -6)$
 C. $(4, 5), (3, 2), (1, 6)$
 D. $(5, -4), (-2, -3), (-6, 1)$



Answer Key

Question 1. Choice A. $2\frac{3}{4}$ is at a different position on the number line.

Question 2. The exact positions of the numbers are shown on the number line below.



Question 3. Choice D. 6.

Question 4. Choice B. $(-4, 5)$, $(-3, -2)$, $(1, -6)$

Recommendations

Consider the following when making a decision about working through this packet:

- Student has some difficulty with Question 1, 2, or 3: The student may choose to work through *Number Lines and Coordinate Planes, Part 1*.
- Student has some difficulty with Question 4: If a student comfortably answers Questions 1, 2 & 3, but has some difficulty with Question 4, the student may feel confident enough to skip *Number Lines and Coordinate Planes, Part 1* and go directly to *Number Lines and Coordinate Planes, Part 2*.
- Student comfortably answers all four questions: The student may choose to work on a different packet. However, it is recommended that students complete the Test Practice Questions in *Number Lines and Coordinate Planes, Part 2*, for questions that require students to interpret a variety of data representations before they take the HSE exam.

This assessment asks students to demonstrate understanding of:

Question 1 (*from Number Lines and Coordinate Planes, Part 1*): Ordering rational numbers (GED Quantitative Problem Solving Assessment Targets Content Indicator: Q.1.a).

Question 2 (*from Number Lines and Coordinate Planes, Part 1*): Ordering rational numbers, absolute value (GED Quantitative Problem Solving Assessment Targets Content Indicators: Q.1.a, Q.1.d).

Question 3 (*from Number Lines and Coordinate Planes, Part 1*): Determine the distance between two rational numbers on the number line, including using the absolute value of their difference (GED Quantitative Problem Solving Assessment Targets Content Indicators: Q.1.d).

Question 4 (*from Number Lines and Coordinate Planes, Part 2*): Locate points in the coordinate plane (GED Algebraic Problem Solving Assessment Targets Content Indicators: A.5.a.)

Being Counted: Probability and Statistics

Overview

Prerequisites	There are no prerequisites to this packet. As long as you are able to read this packet independently, you don't have to study any other math packets first.
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In this packet, you will explore basic concepts in probability (the math of studying chance) and statistics (the math of collecting, analyzing, and visually representing data).

In Part 1, you will study the following topics:

- Understanding part-to-whole and part-to-part ratios
- How to use equivalent ratios and ratio tables
- Expressing ratios with fractions, percents, and decimals
- An introduction to probability (experimental probability and theoretical probability)

In Part 2, you will build on what you learned in Part 1, and study the following topics:

- Gathering data with statistical questions
- Analyzing data with measures of center and spread (mean, median, mode, range)
- Comparing data using two-way frequency tables
- Displaying data (dot plots, tables, histograms, bar graphs, scatter plots, line graphs)
- Statistics in the world

In addition to the learning the topics above, you will find the following materials to help you:

- High School Equivalency Test Practice Questions. You will practice all the concepts you've learned from this packet (parts 1 and 2) to work on these questions. The answer key for this section explains the correct answers, and also some of the wrong answers.
- A graphic organizer to study vocabulary is included, along with a vocabulary activity to review concepts. A glossary with important terms from this packet is also included for your study.
- Concept Circles can help you make connections between the concepts you have learned and help you remember those connections.

Assessment Questions

Calculator allowed

The following questions will help to see if this packet is right for you. Do your best to answer each question below. If you can't answer, don't worry—this packet will help you answer questions like these and more. When you are finished with the questions, read our recommendations.

Question 1

The Centers for Disease Control and Prevention (CDC) project that 2 out of 5 Americans will develop diabetes in their lifetime. Which of the values below is equivalent to the ratio 2 out of 5? Select all that apply and show your work.

- A. 40%
- B. 6:15
- C. $\frac{8}{20}$
- D. 2.5
- E. 0.4

Question 2

Calculate the probability of the following events, when rolling a single six-sided die:

- What is the probability of not rolling a 4?
- What is the probability of rolling a number greater than 2?
- What is the probability of rolling an odd number?
- What is the probability of rolling a 7?



Question 3

A street vendor offers customers the choice of a vegan burger or beef burger combo meal that comes with one side: french fries, a small salad, or coleslaw. The combo also includes a soda, ice tea, or bottled water. How many different combo meals result from choosing one type of burger, one side, and a drink? In the empty space on the right, draw a diagram or create an organized list to show all possible combinations.

Question 4

Students in a class tracked the number of miles they walked in three days. The dot plot below represents the data they collected.



What is the median?

What is the mean?

What is the mode?

What is the range?

Question 5

The table shows the gender and color of 10 puppies in a litter.

Gender and Color of Puppies		
	Male	Female
Black	4	2
Brown	1	3

Part I: If a puppy selected at random from the group is brown, what is the probability it is a male?

A. $\frac{1}{10}$

C. $\frac{1}{3}$

B. $\frac{1}{4}$

D. $\frac{1}{2}$

Part II: Which of the following statements is not true?

- A. Most of the female puppies are black.
- B. 50% of the puppies are female.
- C. Most of the puppies are black.
- D. 40% of the puppies are brown.

Answer Key

Question 1

Choices A, B, C, & E are equivalent to the ratio 2 out of 5. This part-to-whole relationship can be represented as 2:5. If both numbers of the ratio are multiplied by 3, the equivalent ratio of 6:15 is created. The fraction form of this ratio is $\frac{2}{5}$ and can be converted to a percent, 40%, a decimal, 0.4, or the equivalent fraction of $\frac{8}{20}$.

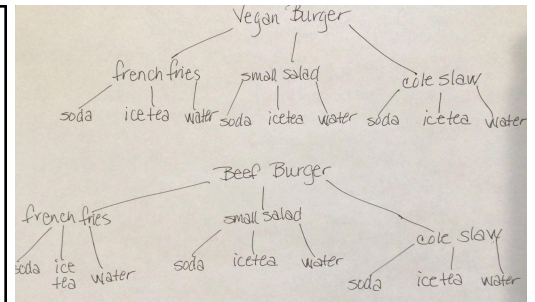
Question 2

- What is the probability of not rolling a 4? **Five out of 6 or $\frac{5}{6}$**
- What is the probability of rolling a number greater than 2? **Four out of six or $\frac{4}{6}$ or two out of three or $\frac{2}{3}$**
- What is the probability of rolling an odd number? **Three out of six or $\frac{3}{6}$ or one out of two or $\frac{1}{2}$**
- What is the probability of rolling a 7? **Zero. There is no chance of rolling a 7.**

Question 3

This problem can be solved with the equation $2 \times 3 \times 3 = 18$ (2 choices of burgers x 3 choices of sides x 3 choices of drinks).

Vegan burger, french fries, soda Vegan burger, french fries, ice tea Vegan burger, french fries, water	Beef burger, french fries, soda Beef burger, french fries, ice tea Beef burger, french fries, water
Vegan burger, small salad, soda Vegan burger, small salad, ice tea Vegan burger, small salad, water	Beef burger, small salad, soda Beef burger, small salad, ice tea Beef burger, small salad, water
Vegan burger, cole slaw, soda Vegan burger, cole slaw, ice tea Vegan burger, cole slaw, water	Beef burger, cole slaw, soda Beef burger, cole slaw, ice tea Beef burger, cole slaw, water



Question 4

The median is 5. If all of the numbers in the data set are placed in order from smallest to greatest, 5 would be the number in the middle.

The mode is 5. It is the number that appears most often.

The mean is 4.85. If all the numbers in the data set are added together and divided by the number of total values (20), the result is 4.85.

The range is 6. This is the difference between the greatest number of miles walked, 9, and the smallest number of miles walked, 3.

Question 5

Part I: Choice B. The total number of brown puppies is 4 but only 1 is a male. If a puppy is selected at random from only the brown puppies, there would be a one-in-four chance of selecting a male puppy. This can be represented as a fraction: $\frac{1}{4}$.

Part II: Choice A. Of the female puppies, 3 are brown and 2 are black so it is not true to say that "Most of the female puppies are black." B is true because 5 out of the 10 puppies are female, C is true because 6 out of 10 puppies are black, and D is true because 4 out of 10 of the puppies are brown.

Recommendations

Consider the following when making a decision about working through this packet:

- Student has some difficulty with Question 1, 2, or 3: The student may choose to work through the whole packet, both Part 1 and Part 2.
- Student has some difficulty with Question 4 or 5: If a student comfortably answers Questions 1 & 2, but has some difficulty with Question 3 or 4, the student may feel confident enough to skip Part 1 of the packet and go directly to Part 2.
- Student comfortably answers all five questions: The student may choose to work on a different packet. However, it is recommended that students complete the Test Practice Questions in *Being Counted: Probability & Statistics, Part 2*, for test questions that require students to interpret a variety of data representations before they take the GED exam.

This assessment asks students to demonstrate understanding of:

Question 1 (*from Being Counted: Probability & Statistics, Part 1*): Proportional reasoning and equivalent ratios, fractions, decimals, percents (GED Algebraic Problem Solving Assessment Targets Content Indicators: Q.3.c)

Question 2 (*from Being Counted: Probability & Statistics, Part 1*): Finding the probability of an event will occur (GED Algebraic Problem Solving Assessment Targets Content Indicators: Q.8.b)

Question 3 (*from Being Counted: Probability & Statistics, Part 1*): Sample space and combinations (GED Algebraic Problem Solving Assessment Targets Content Indicators: Q.8.a)

Question 4 (*from Being Counted: Probability & Statistics, Part 2*): Measures of central tendency and interpreting data on a dot plot (GED Algebraic Problem Solving Assessment Targets Content Indicators: Q.7.a, and Q.6.b)

Question 5 (*from Being Counted: Probability & Statistics, Part 2*): Using a two-way frequency table to determine probability and part-whole relationships (GED Algebraic Problem Solving Assessment Targets Content Indicators: Q.8.b)

Two-Dimensional Geometry

Overview

Prerequisites	<p>There are no prerequisites to <i>Two-Dimensional Geometry, Part 1</i>. As long as you are able to read this packet independently, you don't have to study any other math packets first.</p> <p>Students should complete <i>Two-Dimensional Geometry, Part 1</i> before working on <i>Two-Dimensional Geometry, Part 2</i>.</p>
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In this packet, you will explore concepts in geometry and measurement. Geometry is the part of math that involves lines, shapes and space.

In Part 1, you will study the following topics:

- Two-dimensional shapes
- Measures and units of length (perimeter and circumference)
- Measures and units of area
- Geometric formulas
- The Pythagorean Theorem
- Scale factors

In Part 2, you will build on what you learned in Part 1, and study the following topics:

- Review of area
- Density
- Rates
- Population density (math in a social studies context)

In addition to the learning the topics above, you will find the following materials to help you:

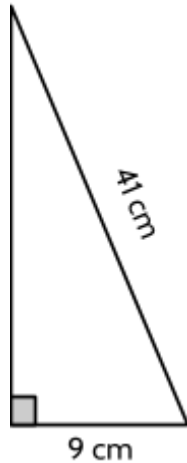
- High School Equivalency Test Practice Questions. You will practice all the concepts you've learned from this packet to work on these questions. The answer key for this section explains the correct answers, and also some of the wrong answers.
- A graphic organizer to study vocabulary is included, along with a vocabulary activity to review concepts. A glossary with important terms from this packet is also included for your study.
- Concept Circles can help you make connections between the concepts you have learned and help you remember those connections.

Assessment Questions

Calculator allowed

The following questions will help to see if this packet is right for you. Do your best to answer each question. If you can't answer, don't worry—this packet will help you answer questions like these and more. When you are finished with the questions, read our recommendations.

Question 1

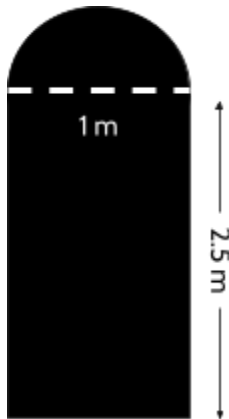


What is the **perimeter** of this triangle?

- A. 40 cm
- B. 42 cm
- C. 50 cm
- D. 90 cm

Question 2

Loretta is putting weather stripping around her door frame and needs to figure out the perimeter. The diagram below represents the dimensions of the doorway.



What is the approximate perimeter of Loretta's doorway?

- A. 4.07 m
- B. 7.57 m
- C. 9.14 m
- D. 10.14 m

Question 3

The length of one side of a rectangle is 22 cm and its perimeter is 72 cm. What is the area of the rectangle?

- A. 308 sq. cm.
- B. 528 sq. cm.
- C. 616 sq. cm.
- D. 1584 sq. cm.

Question 4

There are about 20 million people in New York State, with a total land area of about 47,000 square miles.

What is the population density of New York State?

- A. 0.0004 people per sq. mile
- B. 426 people per sq. mile
- C. 2,350 people per sq. mile
- D. 940,000 people per sq. mile

Answer Key

Question 1. Choice D. *90 cm*

Question 2. Choice B. *7.57 m*

Question 3. Choice A. *308 sq. cm.*

Question 4. Choice B. *426 people per sq. mile*

Recommendations

Consider the following when making a decision about working through this packet:

- Student has some difficulty with Question 1, 2, or 3: The student may choose to work through *Two-Dimensional Geometry, Part 1*.
- Student has some difficulty with Question 4: If a student comfortably answers Questions 1, 2 & 3, but has some difficulty with Question 4, the student may feel confident enough to skip *Two-Dimensional Geometry, Part 1* and go directly to *Two-Dimensional Geometry, Part 2*.
- Student comfortably answers all four questions: The student may choose to work on a different packet. However, it is recommended that students complete the Test Practice Questions in *Two-Dimensional Geometry, Part 1* and *Two-Dimensional Geometry, Part 2* for test questions that require students to interpret a variety of data representations before they take the HSE exam.

This assessment asks students to demonstrate understanding of:

Question 1 (*from Two-Dimensional Geometry, Part 1*): Pythagorean Theorem and perimeter of triangles (GED Algebraic Problem Solving Assessment Targets Content Indicators: Q.4.c and Q.4.e).

Question 2 (*from Two-Dimensional Geometry, Part 1*): Finding perimeter of composite shapes, including a semicircle. (GED Algebraic Problem Solving Assessment Targets Content Indicators: Q.4.b, Q.4.c and Q.4.d).

Question 3 (*from Two-Dimensional Geometry, Part 1*): Perimeter and area of rectangles. (GED Algebraic Problem Solving Assessment Targets Content Indicators: Q.4.a)

Question 4 (*from Two-Dimensional Geometry, Part 2*): Calculating population density (GED Algebraic Problem Solving Assessment Targets Content Indicators: Q.3.a)

The Power of Exponents

Overview

Prerequisites	There are no prerequisites for this packet. As long as you are able to read this packet independently, you don't have to study any other math packets first.
Optional	You may find it helpful to complete the <i>Tools of Algebra: Expressions, Equations, & Inequalities</i> for an in-depth introduction to variables.

In this packet you will learn about exponents and roots as well as how to break numbers down and examine how they work.

In Part 1, you will study the following topics:

- Multiplication, including factors, multiples, and arrays
- Characteristics of numbers, including factors, prime factorization, and finding common factors between numbers
- An introduction to exponents, square roots and cube roots, with connections to geometry
- What it means when an exponent is 1 or less (fractional exponents, to the power of zero, and negative exponents)

In Part 2, you will build on what you learned in Part 1, and study the following topics:

- Place value, powers of ten, and scientific notation
- Powers of two and exponential growth
- Variables and exponents
- Operations with exponents (multiplication, division, and raising a power to a power)

In addition to the learning the topics above, you will find the following materials to help you:

- A review of the big ideas at the end of Part 2.
- High School Equivalency Test Practice Questions. You will practice all the concepts you have learned from this packet (parts 1 and 2) to work on these questions.
- A graphic organizer to study vocabulary is included, along with a vocabulary activity to review concepts. A glossary with important terms from this packet is also included for your study.
- Concept Circles can help you make connections between the concepts you have learned and help you remember those connections.

Assessment Questions

Calculator allowed

The following questions will help to see if this packet is right for you. Do your best to answer each question below. If you can't answer, don't worry—this packet will help you answer questions like these and more. When you are finished with the questions, read our recommendations.

Question 1

Part I: What are all the factors of 36?

Part II: What is the prime factorization of 72? Use a factor tree or your own method to show how you know.

Question 2

Mark each of the following statements as True or False. Show your work.

A. $2^4 3^2 = (2 \times 4)(3 \times 2)$ _____

B. $\sqrt{144} = 2 \cdot 2 \cdot 3$ _____

C. $9 = (\sqrt[3]{27})^2$ _____

D. $(\sqrt{16})^3 = 48$ _____

Question 3

Which equations below are true? Choose two and show how you know.

A. $(5^2)(5^3) = (5 \times 2)(5 \times 3)$

B. $2^3 5^0 = 0$

C. $2x^2 \cdot 3x^3 = 6x^5$

D. $4^2 = 4^5 \div 4^3$

Question 4

Which of the following is equal to $\frac{(x^5)(x^4)}{x^3}$?

A. $\frac{1}{3}$

B. x^3

C. $x^{20/3}$

D. x^6

Answer Key

Question 1

Part I: The factors of 36 are: 36, 18, 12, 9, 6, 4, 3, 2, 1

$$1 \times 36 = 36$$

$$2 \times 18 = 36$$

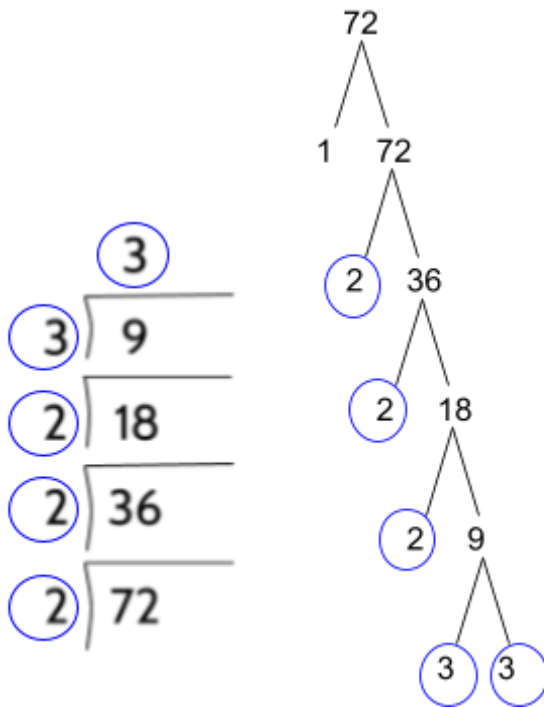
$$3 \times 12 = 36$$

$$4 \times 9 = 36$$

$$6 \times 6 = 36$$

Part II: The prime factorization of $72 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$

Students may show their work using a factor tree, the cake method, or their own method.



Question 2

Choice B & C: B is true because both sides of the equal sign are equivalent to 12. The square root of 144 = 12 and $2 \cdot 2 \cdot 3 = 12$. C is true because the cube root of 27 is 3 and 3 squared is 9.

Question 3

Choices C & D.

C. $2x^2 \cdot 3x^3$ is the same as: $2 \cdot x \cdot x \cdot 3 \cdot x \cdot x \cdot x$. Simplified that would be $2 \cdot 3 \cdot x^6$ or $6x^6$

D. $4^5 \div 4^3$ is the same as $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$ divided by $4 \cdot 4 \cdot 4$. If we set up this division using a fraction bar we see that $4 \cdot 4 \cdot 4$ divided by $4 \cdot 4 \cdot 4$ equals 1. That leaves us with $4 \cdot 4$ or 4^2 .

$$\frac{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4}{4 \cdot 4 \cdot 4}$$

Question 4

Choice D. x^6

Recommendations

Consider the following when making a decision about working through this packet:

- Student has some difficulty with Question 1 or 2: The student may choose to work through the whole packet, both Part 1 and Part 2.
- Student has some difficulty with Question 3 or 4: If a student comfortably answers Questions 1 & 2, but has some difficulty with Question 3 or 4, the student may feel confident enough to skip Part 1 of the packet and go directly to Part 2.
- Student comfortably answers all four questions: The student may choose to work on a different packet. However, it is recommended that students who skip this packet complete the Test Practice Questions in The Power of Exponents, Part 2 for additional test practice before they take the GED exam.

This assessment asks students to demonstrate understanding of:

Question 1 (from *The Power of Exponents, Part 1*): Prime factorization and rewriting a number in exponential form (GED Algebraic Problem Solving Assessment Targets Content Indicators: Q.1.b)

Question 2 (from *The Power of Exponents, Part 1*): Square and cube roots, rules of exponents (GED Algebraic Problem Solving Assessment Targets Content Indicators: Q.2.b, Q.2.c)

Question 3 (from *The Power of Exponents, Part 2*): Order of operations, rules of exponents, multiplying and dividing exponents with the same base (GED Algebraic Problem Solving Assessment Targets Content Indicators: Q.2.a, Q.2.b, Q.2.c)

Question 4 (from *The Power of Exponents, Part 2*): Variable based rules of exponents (GED Algebraic Problem Solving Assessment Targets Content Indicators: A.1.h)

Tools of Algebra: Expressions, Equations, and Inequalities

Overview

Prerequisites	There are no prerequisites to this packet. As long as you are able to read this packet independently, you don't have to study any other math packets first.
Optional	You may find it helpful to complete the Fast Track GRASP Math Packets, <i>Two-Dimensional Figures, Part 1: Area & Perimeter</i> and <i>Three-Dimensional Figures, Part 1: Surface Area and Volume</i> for an in-depth introduction to some of the geometry concepts explored in Part 2 of this packet.

In this packet you will develop your algebraic reasoning and learn how to use algebra as a tool in problem-solving. You will also learn about mathematical symbols and how they are used.

In Part 1, you will study the following topics:

- Mathematical equality
- How to evaluate equations and expressions and solve equations using visual models like area models, hanger diagrams, and balance scales
- The distributive property of multiplication and the order of operations
- How to write equations to describe real-life situations

In Part 2, you will build on what you learned in Part 1, and study the following topics:

- Four different uses for variables in math
- How to combine like terms and solve equations using tape diagrams
- Using variables in scientific and geometric formulas, including the Pythagorean Theorem and those for finding the volume of three-dimensional figures
- Evaluating systems of equations through guess and check and matching systems of equations to real-life situations
- Understanding, solving, evaluating, and graphing inequalities

In addition to the learning the topics above, you will find the following materials to help you:

- High School Equivalency Test Practice Questions. You will practice all the concepts you have learned from this packet (parts 1 and 2) to work on these questions.
- A glossary with important terms from this packet for you to study.
- Concept Circles can help you make connections between the concepts you have learned and to help you remember those connections.

Assessment Questions

Calculator allowed

The following questions will help to see if this packet is right for you. Do your best to answer each question below. If you can't answer, don't worry—this packet will help you answer questions like these and more. When you are finished with the questions, read our recommendations.

Question 1

If $x = 3$, what is the value of the expression $21 - 2(4 + x)$?

- A. 7
- B. 10
- C. 16
- D. 26

Question 2

Pietro took the following steps to solve for p in the equation $5p + 7 = 2p + 28$.

Equation: $5p + 7 = 2p + 28$

Step 1: $5p = 2p + 21$

Step 2: $3p = 21$

Step 3: $p = 7$

In which step, if any, did Pietro make an error?

- A. Step 1
- B. Step 2
- C. Step 3
- D. Pietro did not make an error.

Question 3

Liz and Selvija go to the movie theater and purchase refreshments for their friends. Liz spends a total of \$17.50 on two bags of popcorn and three drinks. Selvija spends a total of \$25.00 for four bags of popcorn and two drinks. Let p be the price of popcorn and d be the price of a drink. Which system of equations could be used to find the price of a bag of popcorn and the price of a drink?

- | | |
|---|---|
| <p>A. $p + d = 17.50$
$2p + 2d = 25.00$</p> | <p>C. $2p + 3d = 17.50$
$4p + 2d = 25.00$</p> |
| <p>B. $2p + 3d = 25.00$
$4p + 2d = 17.50$</p> | <p>D. $17.50p + 25.00d = 5$
$25.00p + 17.50d = 6$</p> |

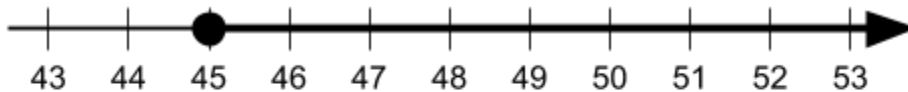
Question 4

Which value of x does not make $5x \geq 400$ true?

- | | |
|--------------|---------------|
| <p>A. 80</p> | <p>C. 100</p> |
| <p>B. 81</p> | <p>D. 70</p> |

Question 5

Which inequality does this number line represent?



- A. $n < 45$
 B. $n \leq 45$
 C. $n > 45$
 D. $n \geq 45$

Answer Key

Question 1

Choice A. 7

Question 2

Choice D. Pietro did not make an error.

Question 3

Choice C. Liz bought 2 bags of popcorn + 3 drinks that cost \$17.50, which can be represented by the equation $2p + 3d = 17.50$. Selvija bought 4 bags of popcorn + 2 drinks that cost \$25.00, which can be represented by the equation $4p + 2d = 25.00$.

Question 4

Choice D. 70

Question 5

Choice D is the inequality represented by the given graph.

The graphs of the other inequalities would be represented as follows:

$n < 45$	
$n \leq 45$	
$n > 45$	

Recommendations

Consider the following when making a decision about working through this packet:

- Student has some difficulty with Question 1 or 2: The student may choose to work through the whole packet, both Part 1 and Part 2.
- Student has some difficulty with Question 3, 4, 5: If a student comfortably answers Questions 1 & 2, but has some difficulty with Question 3 or 4, the student may feel confident enough to skip Part 1 of the packet and go directly to Part 2.
- Student comfortably answers all five questions: The student may choose to work on a different packet. However, it is recommended that students complete the Test Practice Questions in Tools of Algebra: Expressions, Equations, and Inequalities, Part 2, for additional practice in using formulas to solve equations before they take the GED exam.

This assessment asks students to demonstrate understanding of:

Question 1 (*from Tools of Algebra: Equations, Expressions, & Inequalities, Part 1*): Evaluate linear expressions by substituting integers for unknown quantities (GED Algebraic Problem Solving Assessment Targets Content Indicators: A.1.b)

Question 2 (*from Tools of Algebra: Equations, Expressions, & Inequalities, Part 2*): Equivalent expressions, solving equations, distributive property (GED Algebraic Problem Solving Assessment Targets Content Indicators: A.2.a)

Question 3 (*from Tools of Algebra: Equations, Expressions, & Inequalities, Part 2*): Systems of equations (GED Algebraic Problem Solving Assessment Targets Content Indicators: A.1.c and A.2.d)

Question 4 (*from Tools of Algebra: Equations, Expressions, & Inequalities, Part 2*): Solve a linear inequality (GED Algebraic Problem Solving Assessment Targets Content Indicators: A.3.a)

Question 5 (*from Tools of Algebra: Equations, Expressions, & Inequalities, Part 2*): Evaluate and graph an inequality (GED Algebraic Problem Solving Assessment Targets Content Indicators: A.3.b)

Tools of Algebra: Linear Functions

Overview

Prerequisites	Before working on this packet, you should complete the <i>Number Lines to Coordinate Planes</i> Fast Track GRASP Math Packet or feel confident locating and plotting points on the coordinate plane.
Optional	You may find it helpful to complete the <i>Tools of Algebra: Equations, Expressions, and Inequalities</i> Fast Track GRASP Math Packet for an in-depth introduction to variables.

In this packet you will learn the concepts to flexibly create, interpret, and use linear functions.

In Part 1, you will study the following topics:

- Number patterns, repeating patterns, growing patterns, and visual patterns
- How to use input/output machines to understand the basic structure of functions.
- Functions can be represented in 4 connected ways - (1) in words, (2) using tables, (3) using graphs, and (4) using equations. You will learn how each of those forms work together to express linear relationships.

In Part 2, you will build on what you learned in Part 1, and study the following topics:

- Rate of change (also known as slope) in graphs, tables, and equations.
- Starting amount (also known as the y-intercept) in graphs, tables, and equations.
- Linear functions can be used to help us understand our world. You will apply everything you have learned about linear functions to some real-life situations, including paying bills and tracking medical antibodies.

In addition to the learning the topics above, you will find the following materials to help you:

- A review of some of the big ideas at the end of Part 1 and Part 2.
- High School Equivalency Test Practice Questions. You will practice all the concepts you've learned from this packet (parts 1 and 2) to work on these questions.
- A graphic organizer to help you study vocabulary and a glossary with important terms from this packet for you to study.
- Concept Circles can help you make connections between the concepts you have learned and to help you remember those connections.

Assessment Questions

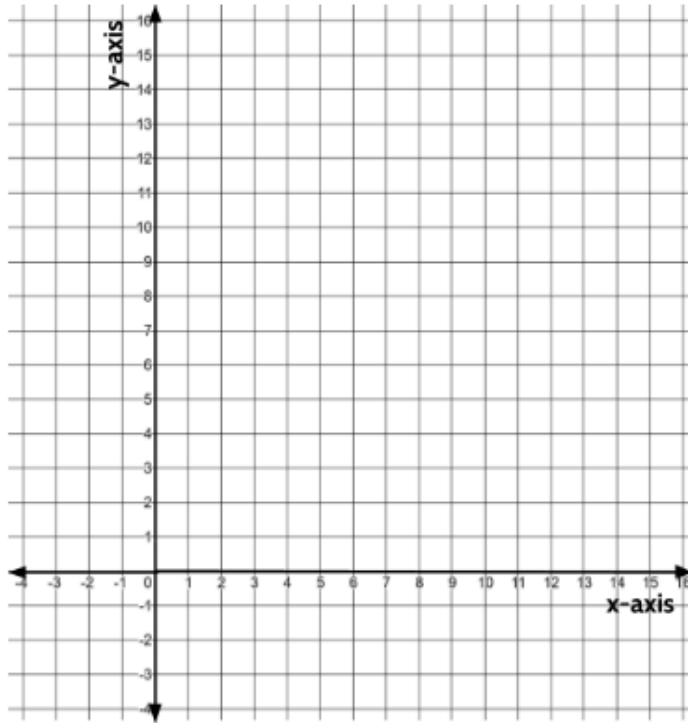
Calculator allowed

The following questions will help to see if this packet is right for you. Do your best to answer each question below. If you can't answer, don't worry—this packet will help you answer questions like these and more. When you are finished with the questions, read our recommendations.

Question 1

Complete the table based on the function equation. Then, plot the points on the coordinate plane.

$y = 3x + 4$		
x	y	Ordered pairs
0		(,)
1		(,)
2		(,)
3		(,)
4		(,)



Question 2

Consider the following function description:

Excelsior Gym charges customers a one-time joining fee of \$35 and then \$50 a month.

Which function equation can be used to represent this situation?

- A. $y = 35x + 50$
- C. $y = x + 85$
- B. $y = 50x + 35$
- D. $y = 85x$

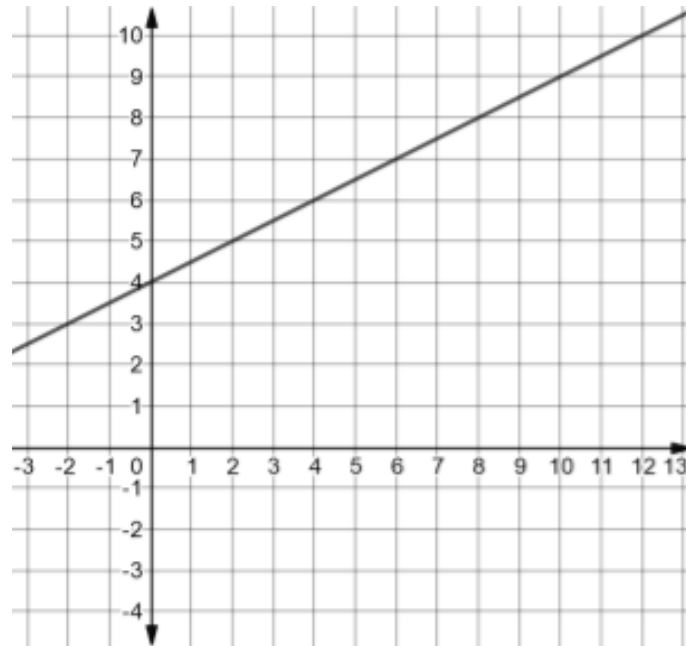
Question 3

Below are two different functions. Function A is represented by a table. Function B is represented by a graph.

Function A

x	y
0	1
2	1.5
4	2
6	2.5
8	3

Function B

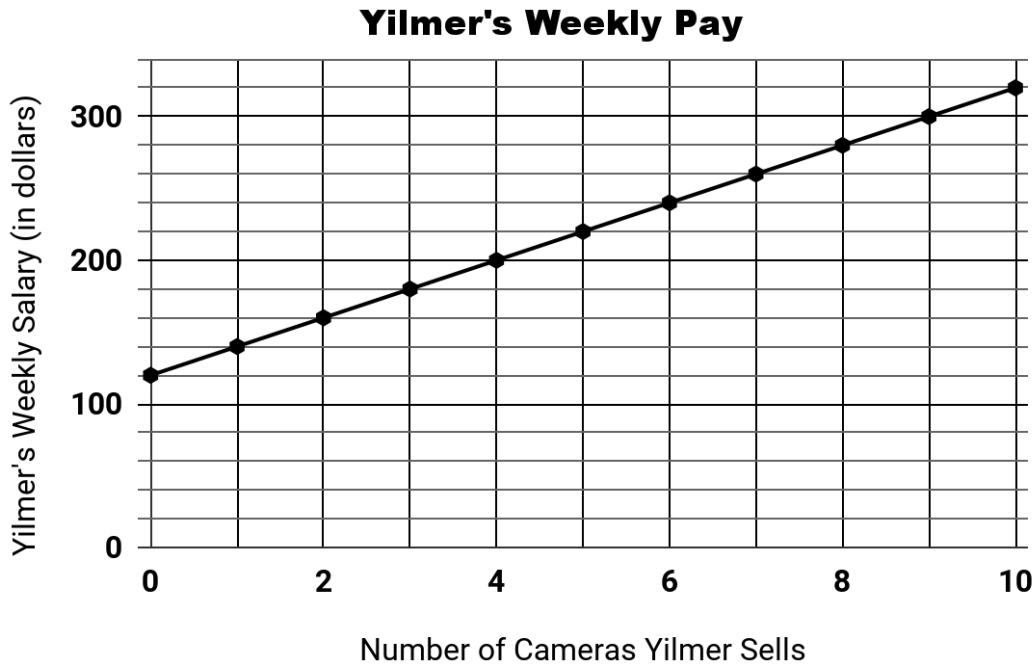


Which statement is true?

- A. The slope of Function B is greater than the slope of Function A in the table.
- B. The slope of Function B is less than the slope of the Function A in the table.
- C. The slope of both functions is the same.
- D. There is not enough information given to determine the slope of the two functions.

Question 4

Yilmer works at a camera store. He earns a guaranteed salary each week. In addition to his guaranteed salary, he earns a commission for each camera he sells. The graph below shows his weekly pay as a function of how many cameras he sells during the week.



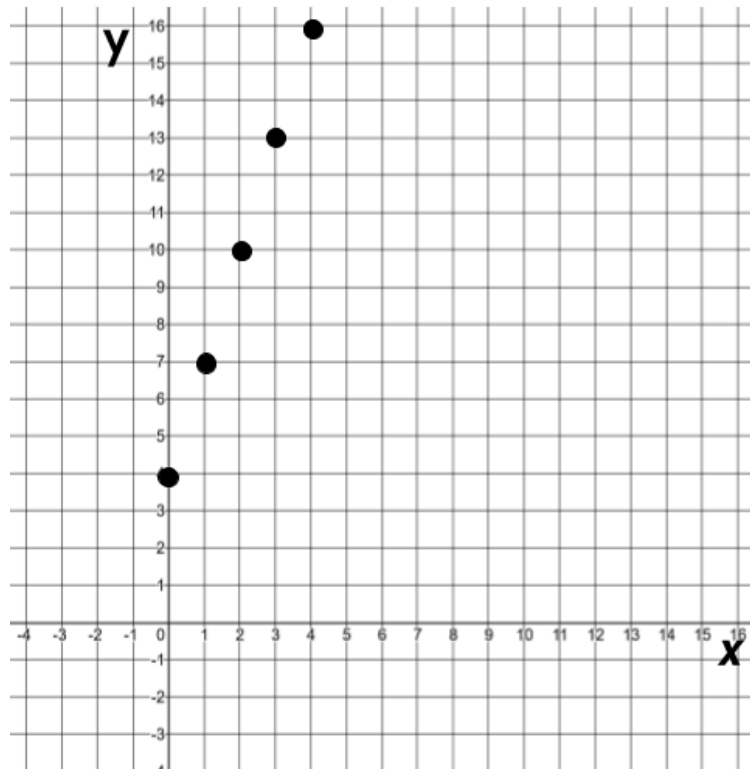
- a. What would Yilmer's salary be if he sold 13 cameras in one week?

- b. Which equation can be used to calculate how much Yilmer earns each week? Let P represent his weekly pay and let c represent the number of cameras he sells.
 - A. $P = 140c$
 - B. $P = 120c$
 - C. $P = 20c + 120$
 - D. $P = 120c + 20$

Answer Key

Question 1

$y = 3x + 4$		
x	y	Ordered pairs
0	4	(0,4)
1	7	(1,7)
2	10	(2,10)
3	13	(3,13)
4	16	(4,16)



Question 2

Choice B. $y = 50x + 35$. The cost of membership, y , can be represented as $50x$ (\$50 dollars a month) plus a one-time joining fee of \$35.

Question 3

Choice A. The rate of change (slope) for the function represented by the graph is 0.5 or $\frac{1}{2}$. The rate of change (slope) for the function in the table is 0.25.

Question 4

- \$380
- Choice 3.** The graph shows that the y -intercept (starting amount) of this function is 120 and Choice C is the only equation that has 120 as the starting amount. Another strategy is to pick a point and put it into each answer choice. Point (4, 200) appears on the line of this function as $20(4) + 120 = 200$.

Recommendations

Consider the following when making a decision about working through this packet:

- Student has some difficulty with Question 1 or 2: The student may choose to work through the whole packet, both Part 1 and Part 2.
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- Student comfortably answers all four questions: The student may choose to work on a different packet. However, it is recommended that students complete the Test Practice Questions in *Tools of Algebra: Linear Functions, Part 2*, for additional practice before they take the GED exam.

This assessment asks students to demonstrate understanding of:

Question 1 (from *Tools of Algebra: Linear Functions, Part 1*): Function notation, locating points on a coordinate grid, functions in table, graph and equations (GED Algebraic Problem Solving Assessment Targets Content Indicators: A.5.a, A.7.b, A.7.c)

Question 2 (from *Tools of Algebra: Linear Functions, Part 1*): Linear functions that represent real-world context (GED Algebraic Problem Solving Assessment Targets Content Indicators: A.1.c, A.2.c)

Question 3 (from *Tools of Algebra: Linear Functions, Part 2*): The slope of two linear functions, each represented in a different view of a function (GED Algebraic Problem Solving Assessment Targets Content Indicators: A.5.b, A.7.d)

Question 4 (from *Tools of Algebra: Linear Functions, Part 2*): Key features of a graph, an equation that represents the graph, word to symbol translation (GED Algebraic Problem Solving Assessment Targets Content Indicators: A.1.c, A.2.c, A.5.e)

Tools of Algebra: Nonlinear Functions

Overview

Prerequisites	<p>Before working on this packet, you should complete the following packets:</p> <ul style="list-style-type: none"> ● <i>Number Lines to Coordinate Planes</i> Fast Track GRASP Math Packet, or feel confident locating and plotting points on the coordinate plane. ● <i>Tools of Algebra: Linear Functions</i> Fast Track GRASP Math Packet ● <i>The Power of Exponents</i> Fast Track GRASP Math Packet
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In this packet you will explore nonlinear functions, which are functions that represent nonlinear growth and result in graphs that are not straight lines.

In Part 1, you will study the following topics:

- Comparing linear and nonlinear growth
- Area models and quadratic functions
- Three views of a quadratic functions: tables, equations, and graphs
- Exploring quadratic growth in visual patterns

In Part 2, you will build on what you learned in Part 1, and study the following topics:

- Exploring quadratic growth in real-world situations
- Exponential growth in number patterns and visual patterns*
- Three views of exponential functions: tables, equations, and graphs*
- Exploring exponential growth in real-world situations*

In addition to the learning the topics above, you will find the following materials to help you:

- A review of the big ideas at the end of Part 1 and Part 2.
- High School Equivalency Test Practice Questions. You will practice all the concepts you have learned from this packet (parts 1 and 2) to work on these questions.
- A graphic organizer to help you study vocabulary and a glossary with important terms from this packet for you to study.
- Concept Circles can help you make connections between the concepts you have learned and to help you remember those connections.

** Exponents and scientific notations are part of the GED exam. While exponential functions are an important math topic (especially in science), you will probably not see them on the test.*

Assessment Questions

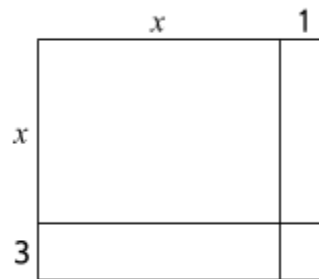
Calculator allowed

The following questions will help to see if this packet is right for you. Do your best to answer each question below. If you can't answer, don't worry—this packet will help you answer questions like these and more. When you are finished with the questions, read our recommendations.

Question 1

Which expression shows the area of the rectangle below?

- A. $x^2 + 3x + 4$
- B. $x^2 + 3x$
- C. $x^2 + 4x + 3$
- D. $3x^2 + x$

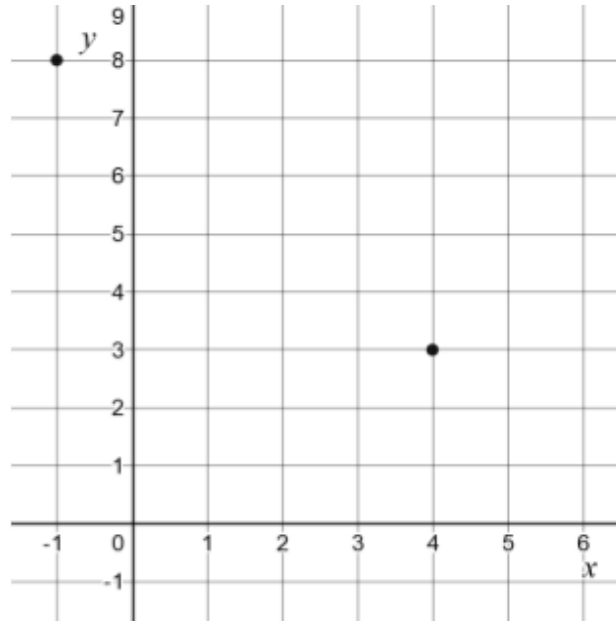


Question 2

Complete the function table for $y = x^2 - 4x + 3$ and plot the rest of the points on the graph. Two points have been plotted for you.

Note: The ordered pair shows the location of a point on a graph. The first number is the x value and the second number is the y value.

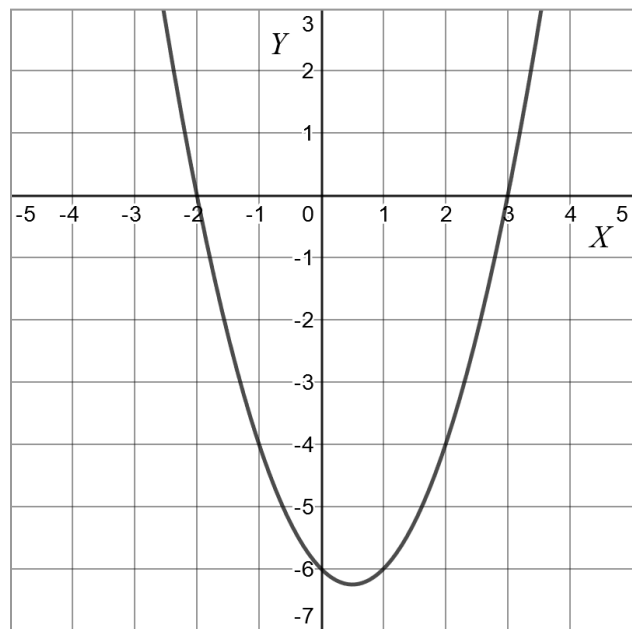
Equation: $y = x^2 - 4x + 3$		
x	y	Ordered Pair
-1	8	(-1, 8)
0		
1		
2	-1	
3		
4	3	(4, 3)
5		



Question 3

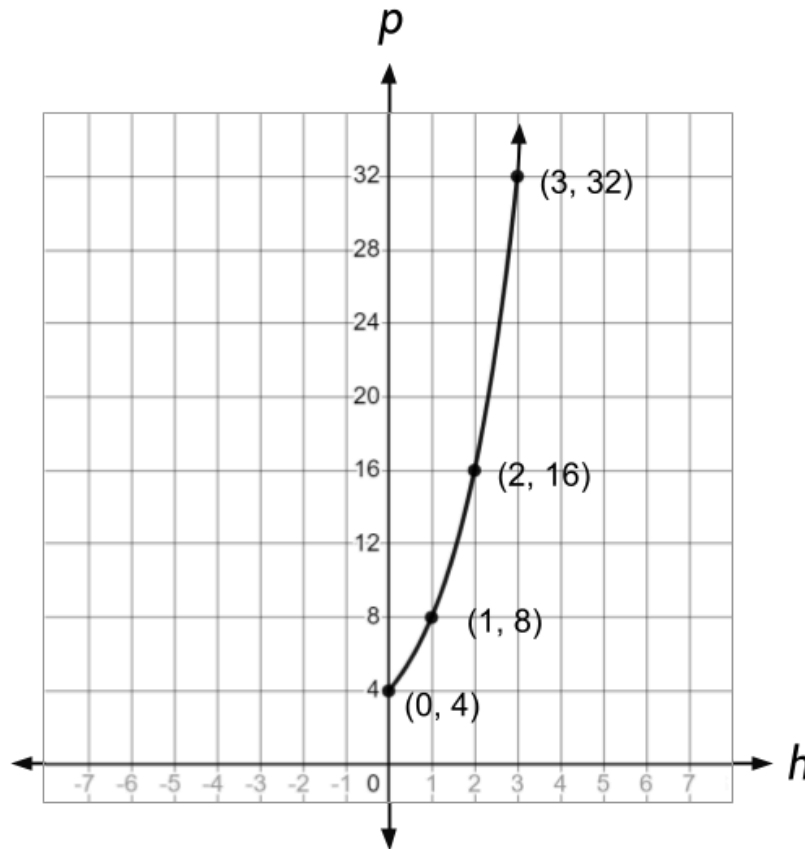
What are the roots of the quadratic equation associated with the graph?

- A. -6 and 3
- B. -6 and 0
- C. -3 and 2
- D. -2 and 3



Question 4

Liz collected population data, p , from a colony of *E. coli* bacteria over time in hours, h , as shown in the graph.



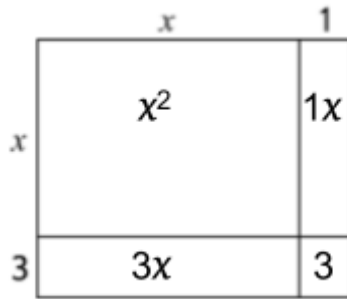
Which equation matches the data in the graph?

- A. $p = 4h + 4$
- B. $p = 4h^2$
- C. $p = 4 \cdot 2^h$
- D. $p = 2 \cdot 4^h$

Answer Key

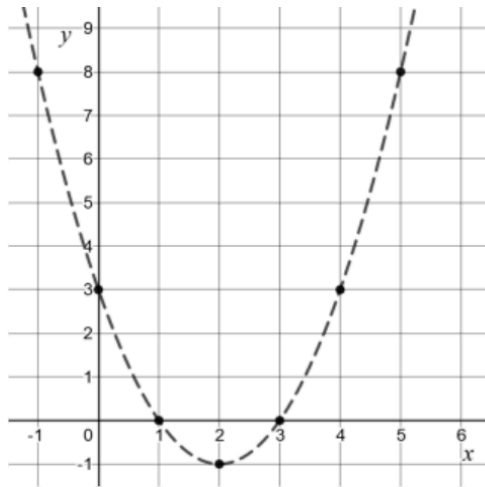
Question 1

Choice C. $x^2 + 4x + 3$



Question 2

Equation: $y = x^2 - 4x + 3$		
x	y	Ordered Pair
-1	8	(-1, 8)
0	3	(0, 3)
1	0	(1, 0)
2	-1	(2, -1)
3	0	(3, 0)
4	3	(4, 3)
5	8	(5, 8)



Question 3

Choice D. -2 and 3

Question 4

Choice C.

Hours (h)	$4 \cdot 2^n$	Bacteria (p)
0	$4 \cdot 2^0$	4
1	$4 \cdot 2^1$	8
2	$4 \cdot 2^2$	16
3	$4 \cdot 2^3$	32

Recommendations

Consider the following when making a decision about working through this packet:

- Student has some difficulty with Question 1 or 2: The student may choose to work through the whole packet, both Part 1 and Part 2.
- Student has some difficulty with Question 3 or 4: If a student comfortably answers Questions 1 & 2, but has some difficulty with Question 3 or 4, the student may feel confident enough to skip Part 1 of the packet and go directly to Part 2.
- Student comfortably answers all four questions: The student may choose to work on a different packet. However, it is recommended that students complete the Test Practice Questions in Tools of Algebra: Nonlinear Functions, Part 2, for additional practice before they take the GED exam.

This assessment asks students to demonstrate understanding of:

Question 1: (*from Tools of Algebra: Nonlinear Functions, Part 1*): Multiplying polynomials using an area model (GED Algebraic Problem Solving Assessment Targets Content Indicators: A.4.a)

Question 2 (*from Tools of Algebra: Nonlinear Functions, Part 1*): Function notation, calculating outputs of a function, locating points on the coordinate grid (GED Algebraic Problem Solving Assessment Targets Content Indicators: A.5.a and A.7.b)

Question 3 (*from Tools of Algebra: Nonlinear Functions, Part 1*): Key features of linear, quadratic, and exponential graphs (GED Algebraic Problem Solving Assessment Targets Content Indicators: A.5.e)

Question 4: (*from Tools of Algebra: Nonlinear Functions, Part 1*): The relationship between an exponential function and the graph that represents the function (GED Algebraic Problem Solving Assessment Targets Content Indicators: A.1.i, A.4.a, A.5.a)

Contact Us

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Please contact the authors with suggestions, corrections, questions, etc.

Additionally, if you use the Fast Track GRASP Math Packets with your students, we welcome feedback on the experience. We have both a [Teacher survey](#) and a [Student survey](#). Your feedback on the packets will help us improve the materials, and support our students' success.