Tools of Algebra:
Linear Functions

Fast Track GRASP Math Packet

Part 1

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Table of Contents

Welcome!

Congratulations on deciding to continue your studies! We are happy to share this study packet on linear functions. We hope that these materials are helpful in your efforts to earn your high school equivalency diploma. This packet is part of a larger group of math study materials that cover the mathematics topics that we often see on high school equivalency exams. If you study these topics carefully, while also practicing other basic math skills, you will increase your chances of passing the exam.

Please take your time as you go through the packet. You will find plenty of practice here, but it's useful to make extra notes for yourself to help you remember. You will probably want to have a separate notebook where you can recopy problems, write questions and include information that you want to remember. Writing is thinking and will help you learn the math.

At the end of the packet, you will find an answer key. Try to answer all the questions and then look at the answer key. It's not cheating to look at the answer key, but do your best on your own first. If you find that you got the right answer, congratulations! If you didn't, it's okay. This is how we learn. Look back and try to understand the reason for the answer. Please read the answer key even if you feel confident. We added some extra explanation and examples that may be helpful. If you see a word that you don't understand, try looking at the *Vocabulary Review* at the end of the packet.

We also hope you will share what you learn with your friends and family. If you find something interesting here, tell someone about it! If you find a section challenging, look for support. If you are in a class, talk to your teacher and your classmates. If you are studying on your own, talk to people you know or try searching for a phrase online. Your local library should have information about adult education classes or other support. You can also find classes listed here: **http://www.acces.nysed.gov/hse/hse-prep-programs-maps**

You are doing a wonderful thing by investing in your own education right now. You have our utmost respect for continuing to learn as an adult.

Please feel free to contact us with questions or suggestions.

Best of luck!

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Language and Skills Practice

Vocabulary

It is important to understand mathematical words when you are learning new topics. The following vocabulary will be used a lot in this study packet:

change · constant · function · generalization · pattern · rate · slope · variable

In this first activity, you will think about each word and decide how familiar you are with it. For example, think about the word "cube." Which of these statements is true for you?

- I know the word "cube" and use it in conversation or writing.

- I know the word "cube," but I don't use it.

- I have heard the word "cube," but I'm not sure what it means.

- I have never heard the word "cube" at all.

In the chart on the next page, read each word and then choose one of the four categories and mark your answer with a ✔ (checkmark). Then write your best guess at the meaning of the word in the right column. If it's easier, you can also just use the word in a sentence.

Here's an example of how the row for "cube" might look when you're done:

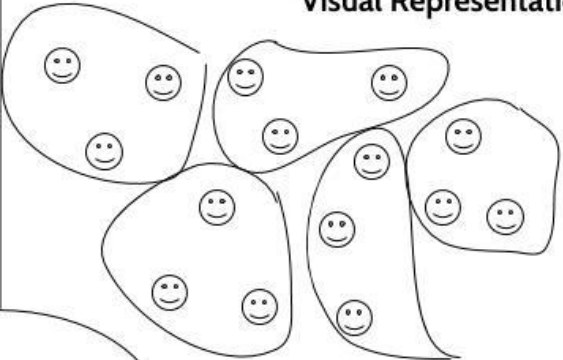| Word	I know the word and use the word	I know the word but don't use it	I have heard the word, but I'm not sure what it means	I have never heard the word	My best guess at the meaning of the word (or use the word in a sentence)
cube	✓				like a box, all the sides are the same length

Complete the table on the next page.

| Word	I know the word and use the word	I know the word but don't use it	I have heard the word, but I'm not sure what it means	I have never heard the word	My best guess at the meaning of the word (or use the word in a sentence)
change					
constant					
function					
generalization					
pattern					
rate					
slope					
variable					

Using Graphic Organizers to Learn Vocabulary

In order to learn math vocabulary, we need to practice using words in different ways. In this activity, you will choose a few words from this packet that you want to practice, then you will complete a graphic organizer for each word. Look at the sample for the word *quotient* below.

To start, choose a word from the packet and complete the graphic organizer:

- **What is the definition of the word?** You can look at the vocabulary review on page 87 for help. Write the definition in your own words to really make the word yours.

- **Make a visual representation.** You can make a drawing or diagram that will help you remember what the word means.

- **What are some examples of the word you're studying?** Below you can see that there are examples of *quotients*, which are the answers to division problems.

- **What are some non-examples of this word?** These are things that are **not** the word you're studying. For example, 24 is **not** the quotient of 4 divided by 6.

What is it?

A quotient is the result of dividing one number by another. It is the answer to a division question.

Visual Representation

quotient

What are some examples?

15 divided by 3 equals **5**

$66 \div 6 = 11$

$63/18 = 3.5$

5, 11 and 3.5 are quotients in these calculations.

population ÷ area = **density**

What are some non-examples?

4 times 6 equals 24

$18 + 5 = 23$

$17 - 2.5 = 14.5$

$3.5 \times 18 = 63$

What is it?

What are some examples?

Visual Representation

What are some non-examples?

What is it?

What are some examples?

Visual Representation

What are some non-examples?

What is it?

What are some examples?

Visual Representation

What are some non-examples?

What is it?

What are some examples?

Visual Representation

What are some non-examples?
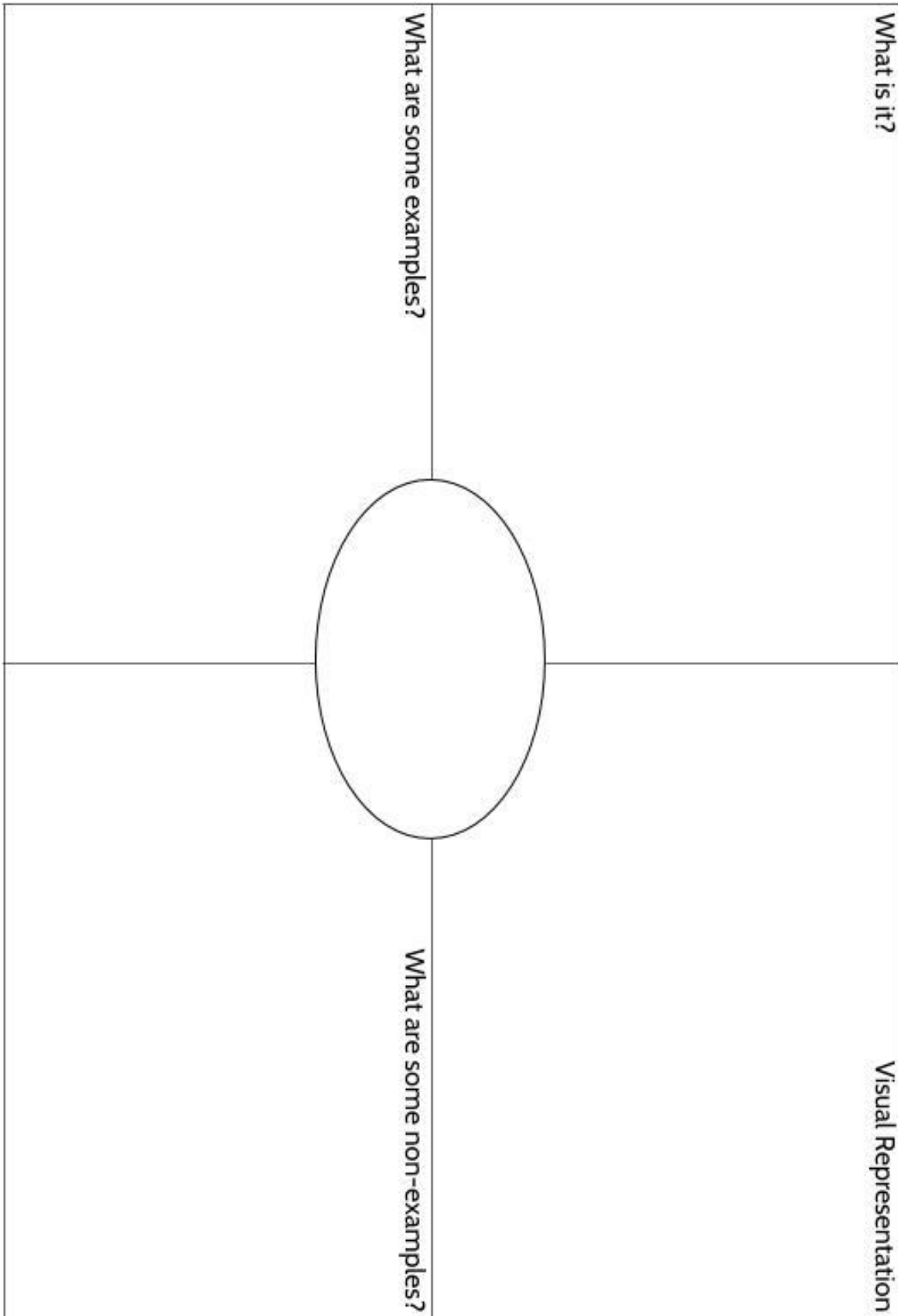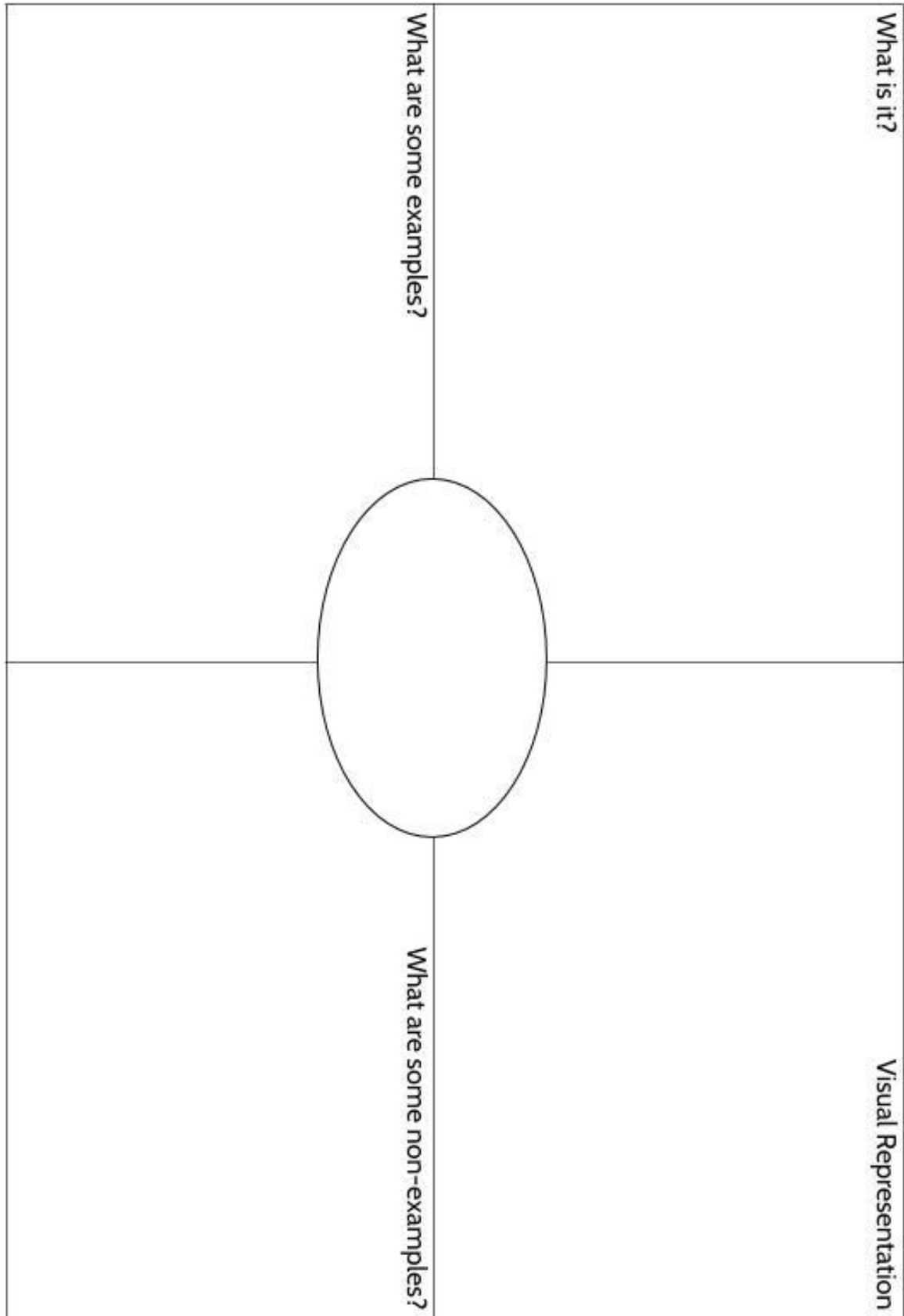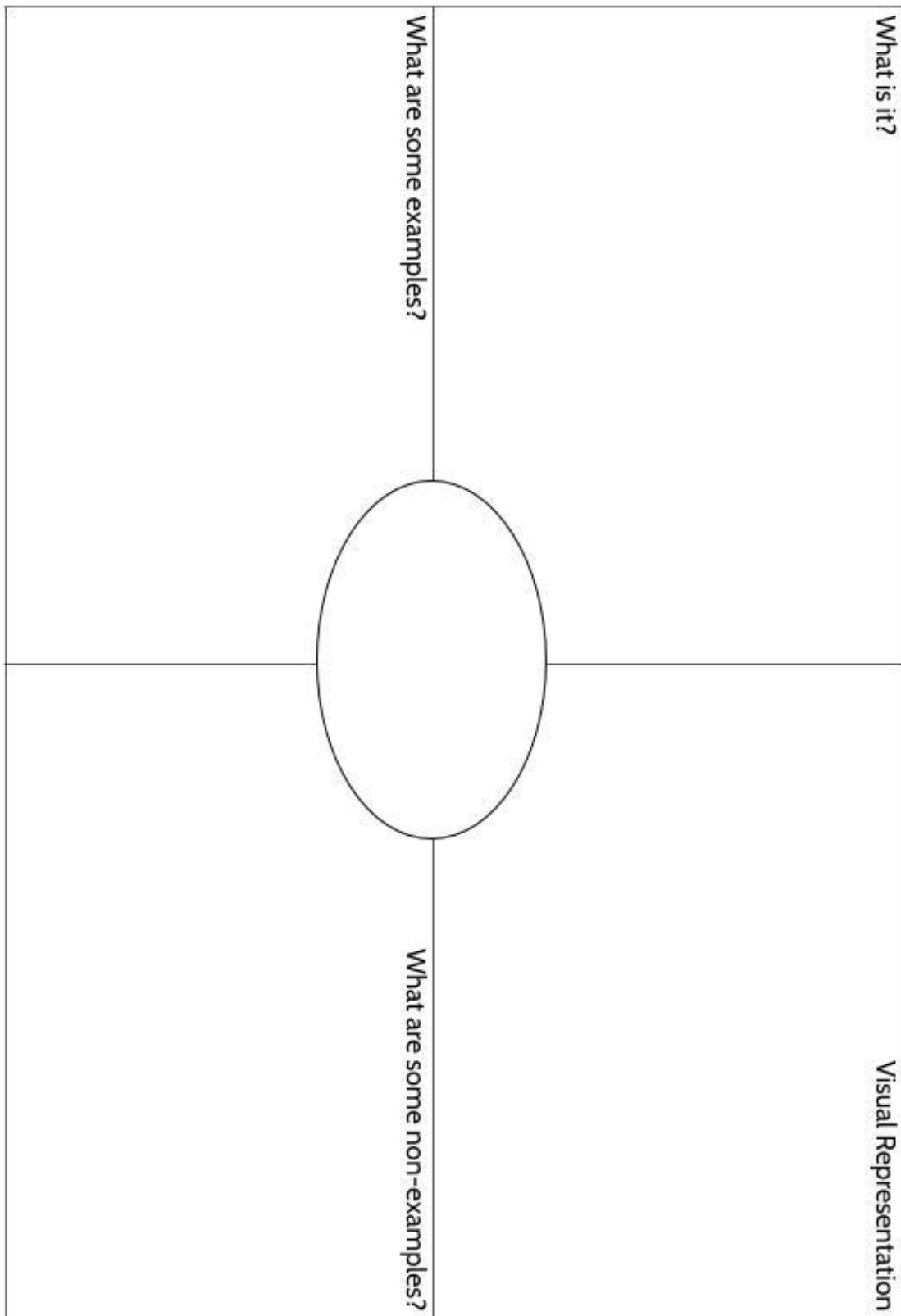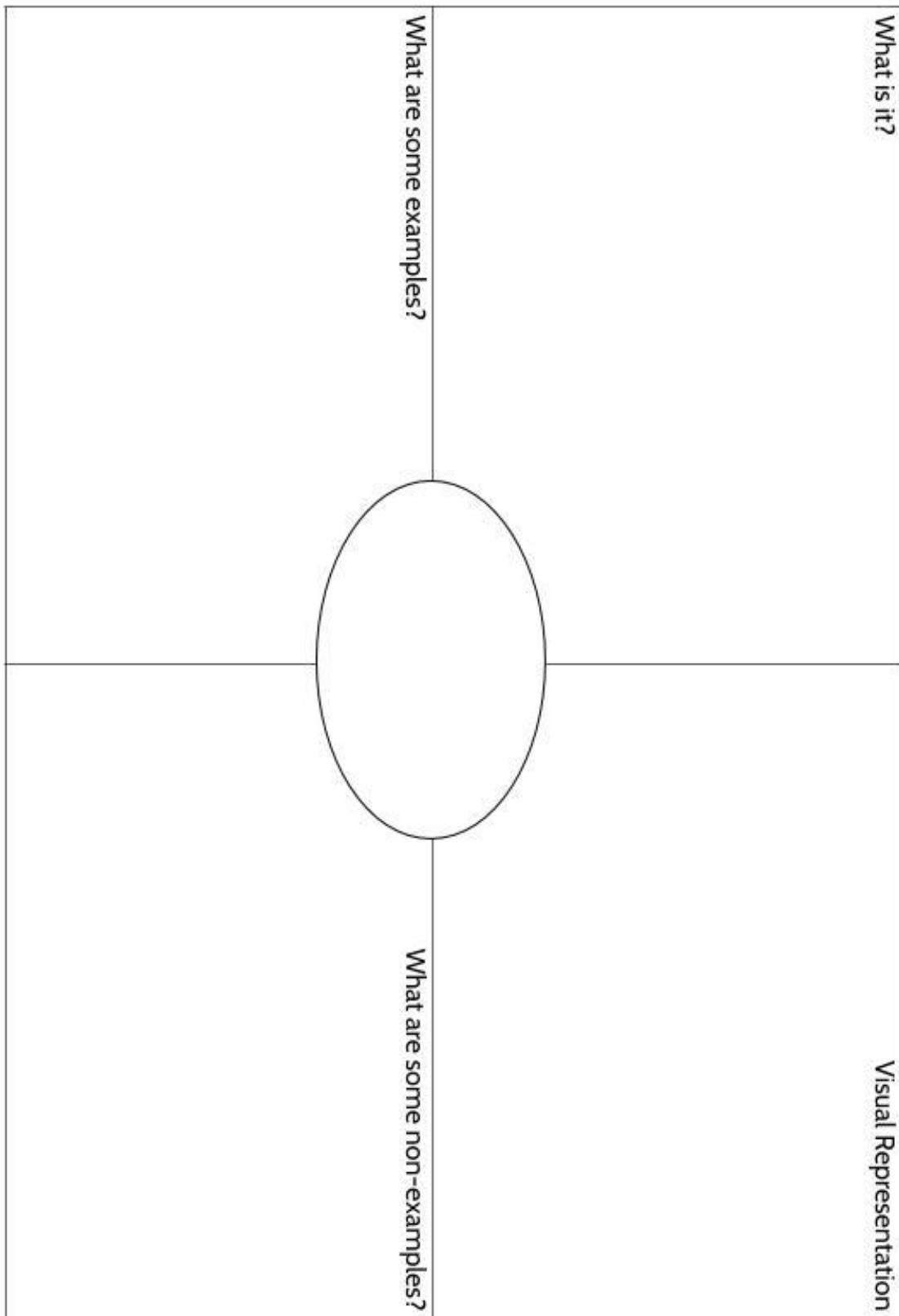
Patterns

The heart of mathematics is the study of patterns. In this packet, you will develop your ability to notice, identify, describe and continue patterns. You will learn how to do all of these things with a powerful tool in math called a *linear function.*

So, what exactly is a pattern?

There are patterns all around us. There are patterns in nature, patterns we create, even patterns within other patterns. Consider a few examples of patterns:

- rhythm and rhyme in a song
- dancing
- the migration patterns of animals
- the seasons
- the patterns on clothing
- how we spend our money
- our behavior patterns (the way we act)
- our daily routine – the things we do every day
- how children often look like their parents
- weather patterns
- the movement of the stars

It's interesting that we use one word to describe so many different situations.

What do dancing, weather, clothing design, and human behavior have in common? What is a pattern?

I think a pattern is _____

_____

_____

_____

_____

For your work in this packet, here is a helpful way to think about patterns:

> Patterns are things we can **observe**. We **gather information** about what has come before to **make predictions** about what will come next.

We are able to dance to a song because we recognize patterns. When we listen to music, we hear the beat, observe what repeats and what changes, and we can move our bodies even if we've never heard the song before.

Weather is another interesting example. How can we possibly know what the weather will be like tomorrow? Do meteorologists have time machines or crystal balls? The simple version behind the weather reports is that humans have been observing weather patterns for over 200,000 years. Some humans long ago noticed that before it rained, the clouds in the sky turned dark and the air felt different and was able to plan their day accordingly. Today we use technology and computers; we have more sophisticated ways of gathering information but the idea is the same. We collect information and use it to make predictions, based on what came before.

We are going to focus on two types of patterns: repeating patterns and growing patterns.

Repeating patterns have something that repeats. The four seasons, the days of the week, and the months of the year are all examples of repeating patterns in our lives. The seasons are spring, summer, fall, winter, and then the pattern repeats again. The days are Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday, and then the pattern repeats again.

A **growing pattern** is a special order of numbers or objects that grows in a way that can be described and continued. A special order of numbers is called a *sequence*. To understand growing patterns, we will analyze sequences to see how they change as they grow.

Repeating Patterns

One example of a repeating pattern is the Chinese Zodiac. Every year is assigned one of twelve animals: the pig, the rat, the ox, the tiger, the rabbit, the dragon, the snake, the horse, the goat, the monkey, the rooster, and the dog. Those are the 12 signs. After that, the signs repeat again, always in the same order. 2019 is the Year of the Pig. That means that 2018 is the Year of the Dog and 2020 is the Year of the Rat.

1) Use the pattern to complete the chart on the right.

2) After 2019, when is the next Year of the Pig?

3) If someone was born in 2012, what is their Chinese Zodiac sign?

Year	Chinese Zodiac Sign
2018	Dog
2019	Pig
2020	Rat
2021	
2022	
2023	
2024	
2025	
2026	

4) What was the Chinese Zodiac sign for the year you were born? Explain how you know.

We can also explore number patterns that repeat. Consider the following repeating number pattern:

$$1, 2, 3, 4, 1, 2, 3, 4, \ldots$$

This pattern is made up of a core that repeats. The core of this pattern is 1, 2, 3, 4.

5)   What would the next three numbers in the pattern be?

Organizing the numbers into a table can sometimes help us see more patterns.

The table below shows the first 16 numbers in our repeating pattern.
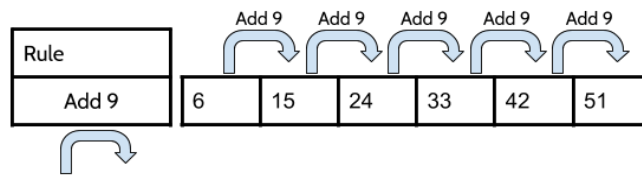
6)   What other patterns do you notice in the table?

7)   What would the 28th number in the pattern be? Explain your answer.

Number in Pattern (Term)	Number
1st	1
2nd	2
3rd	3
4th	4
5th	1
6th	2
7th	3
8th	4
9th	1
10th	2
11th	3
12th	4
13th	1
14th	2
15th	3
16th	4
17th	1
18th	2
19th	3
20th	4

8)   Is the 37th number in the pattern a 4? How do you know?

9)   What would the 51st number in the pattern be? Explain how you got your answer.

10)   What would the 89st number in the pattern be? Explain how you got your answer.

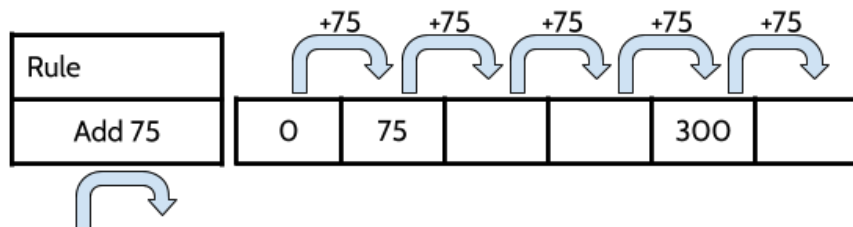11)   What would the 234th number in the pattern be? Explain how you got your answer.
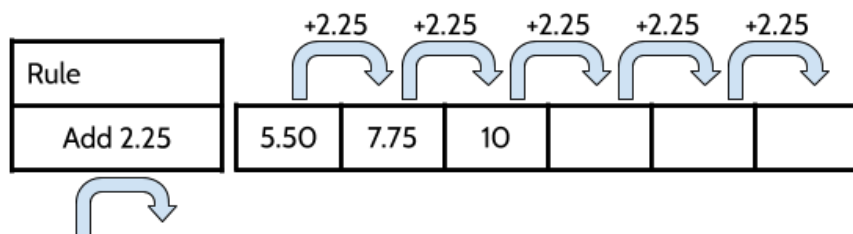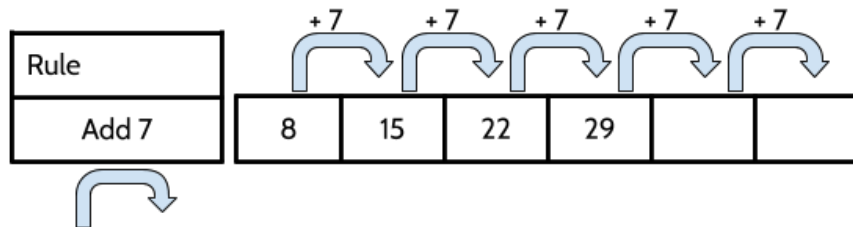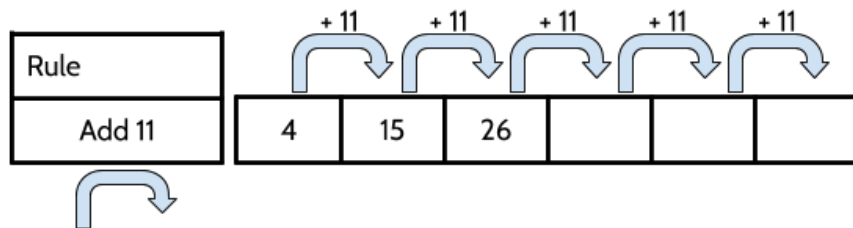
Growing Patterns

A growing pattern is a particular type of pattern where numbers or objects are in a certain order. When we study patterns, it can be helpful to think about change. In the number patterns below, we can ask how the numbers change as we go from one number to the next number.

Here is an example of a sequence where the numbers change by adding 9 to each term in the sequence.
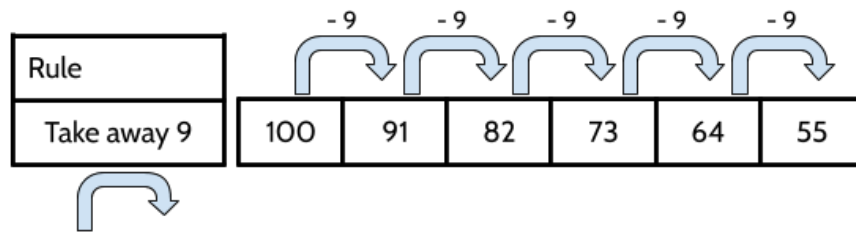
	Add 9	Add 9	Add 9	Add 9	Add 9
Rule					
Add 9	6	15	24	33	42	51

12) Complete the sequences for the following growing patterns:

	+ 11	+ 11	+ 11	+ 11	+ 11
Rule					
Add 11	4	15	26			

	+ 7	+ 7	+ 7	+ 7	+ 7
Rule					
Add 7	8	15	22	29		

	+2.25	+2.25	+2.25	+2.25	+2.25
Rule					
Add 2.25	5.50	7.75	10			

	+75	+75	+75	+75	+75
Rule					
Add 75	0	75			300	

Let's look at some examples of growing patterns that are decreasing. They are still changing according to a pattern that we can identify, describe, and continue.

Rule		-9	-9	-9	-9	-9
Take away 9	100	91	82	73	64	55

13) Complete the sequences for the following growing patterns:

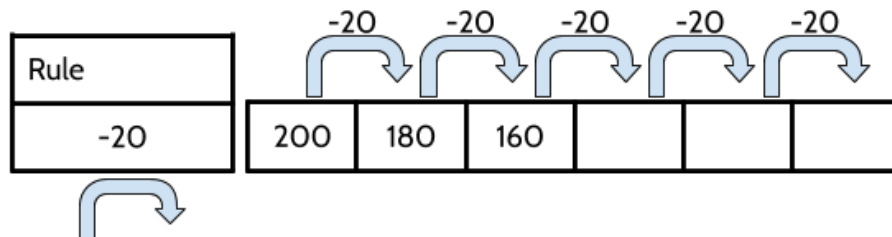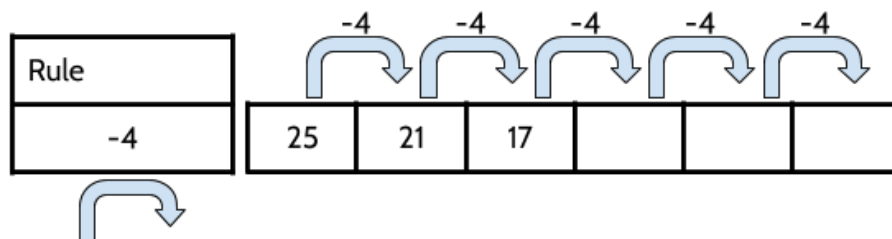| Rule		-4	-4	-4	-4	-4
-4	25	21	17			

Rule		-20	-20	-20	-20	-20
-20	200	180	160			

Rule		-3	-3	-3	-3	-3
-3	10	7	4			

Rule		-6	-6	-6	-6	-6
-6		36				
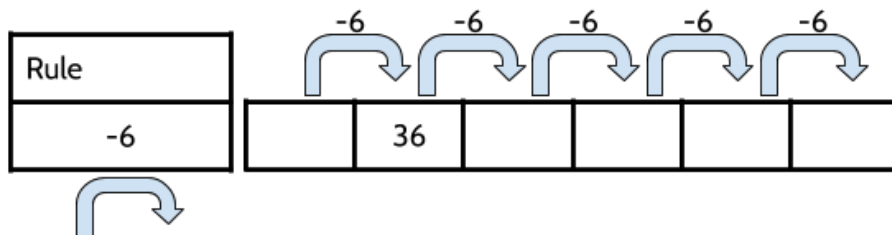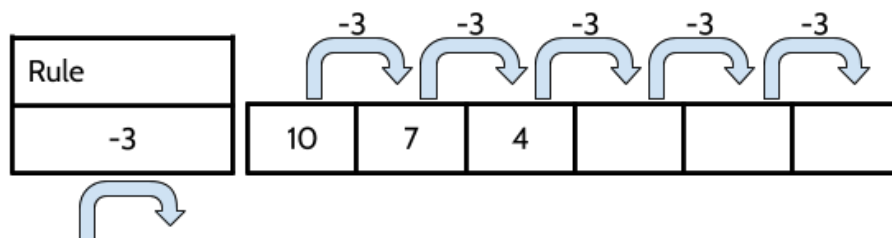
In the growing patterns on the last few pages, you were given a rule that described how the numbers were changing. You used that rule to continue and extend the pattern. In the patterns below, you will figure out the rule by looking at how the numbers change.

14) Identify the pattern in each sequence. How are the numbers changing? Above each arrow, describe the change as the pattern goes from one number to the next. Then add the rule to the rule box.
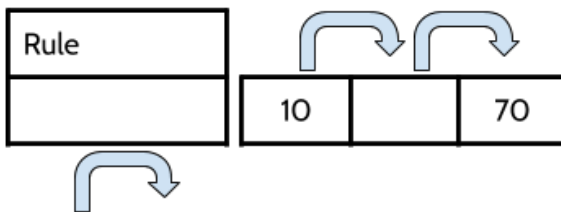
Rule		–7	–7			
Minus 7	49	42	35			

Rule						
		14	18			30

Rule						
			38		34	

Rule						
			42	32		12

Rule						
		19				27

15) Below are five sequences that go from 10 to 70. Each one does it in a different number of moves. Figure out the rule for each growth pattern.

Rule		10						70

Rule		10				70

Rule		10			70

Rule		10		70

Rule		10		70
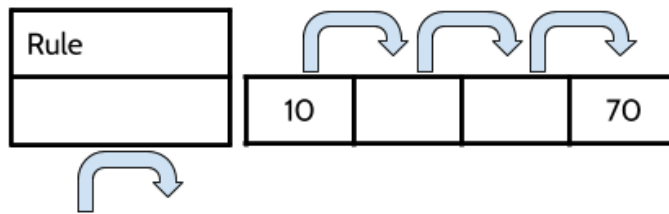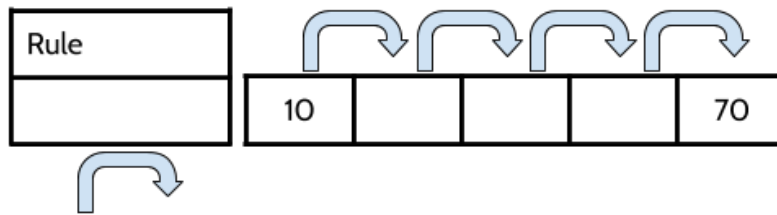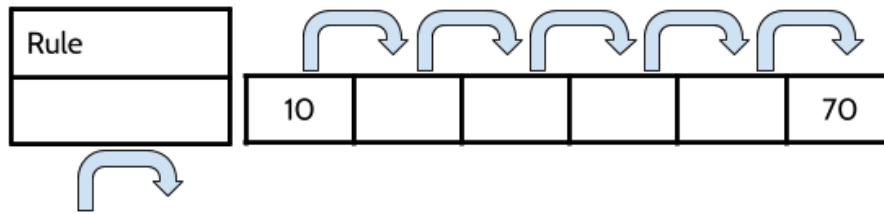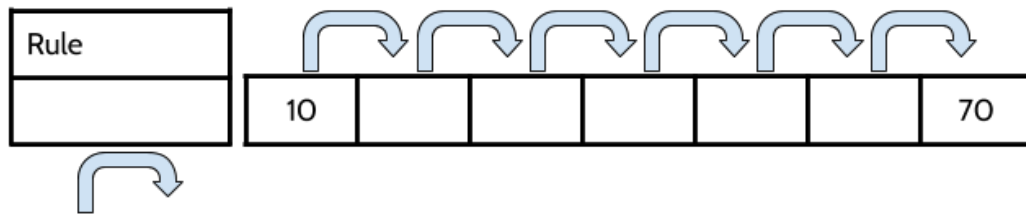
16) Which of the 10 to 70 sequences above was the most challenging for you? Explain your answer.

Growth patterns can be shown in other ways. We can use arrows to show that numbers change by the same amount from one number to the next. However, the pattern can also be shown as a simple series of numbers separated by a comma. The *ellipse* punctuation symbol (...) is often used to show that the pattern continues.

For example:

15, 30, 45, 60, 75, 90, 105, ...

We can use the word "term" to describe where a number is in a pattern. The first term, or Term 1, is the first number in the pattern, The second term, or Term 2, is the second number in the pattern and so on.

In the pattern above, 15 is the first term, 30 is the second term, 45 is the third term, and so on.

Term	1	2	3	4	5	6	7
Number	15	30	45	60	75	90	105

Find the rule and write the next three terms in each pattern. Notice that each term is separated by a comma and that each missing term is represented by a blank line.

17)   20, 18, 16, 14, _____, _____, _____   What is the rule for this pattern? _____

18)   7, 11, 15, 19, _____, _____, _____   What is the rule for this pattern? _____

19)   15, 10, 5, 0, _____, _____, _____   What is the rule for this pattern? _____

20)   36, 42, 48, 54, _____, _____, _____   What is the rule for this pattern? _____

21)   60, 51, 42, _____, _____, _____   What is the rule for this pattern? _____

22)   Create your own pattern that has 24 as its 5th term.

_____, _____, _____, _____, 24, _____, _____, _____, ...

The power of patterns lets us extend the sequence beyond the next few numbers. Below are the first 4 numbers in several different growing patterns. Find the 8th term for each pattern.

23)   1.5, 3, 4.5, 6, ...   The 8th term is _____

24)   12, 20, 28, 36, ...   The 8th term is _____

25)   Consider the following sequence. What is the 8th term of the sequence? Record your answer on the answer grid.

75, 72, 69, 66, ...

26)   5, 8, 11, 14, ...   The 8th term is _____

27)   34, 27, 20, 13, ...   The 8th term is _____

28)   2012, 2008, 2004, ...   The 8th term is _____

This packet focuses on growing patterns that have a *constant* change. A constant change is when you add or subtract the same value to each number in the sequence. These are called *linear patterns*.

There are other types of growing patterns in mathematics where the amount of growth changes as the pattern grows. The change is not constant in this type of growing pattern.

29) For the two sequences below, identify how each pattern is growing from one number to the next, then extend the pattern.
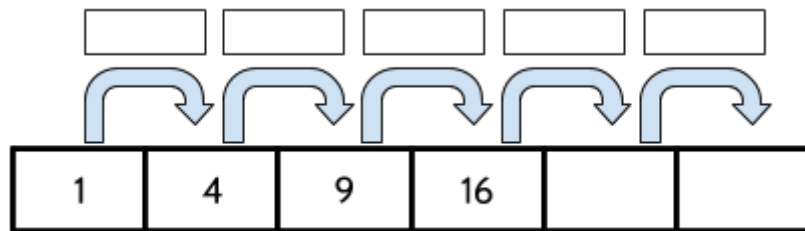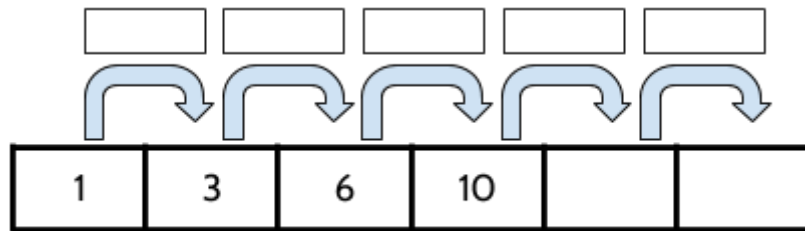
1	3	6	10		

1	4	9	16		

30) Find the 10th term for the two patterns above.

  a) 1, 3, 6, 10, …

  b) 1, 4, 9, 16, …

The growing pattern 1, 3, 6, 10, … and 1, 4, 9, 16, … are called *nonlinear* patterns. You can find more practice with this type of growing pattern in **Tools of Algebra: Nonlinear Functions**.

Visual Patterns

We can also identify, describe, and continue visual patterns using the same skills we use when we are looking at number patterns.

A visual pattern is similar to a number pattern. Instead of numbers, we have shapes that change as we move from term to term. Below are the first 3 terms of a visual pattern.

| Term 1 | Term 2 | Term 3 |
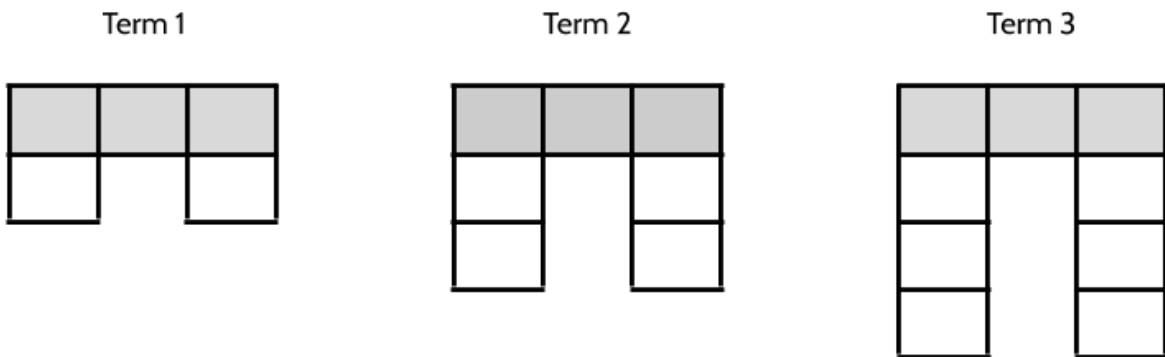
31)   What is changing as the pattern goes from term 1 to term 3?

32)   What stays the same as the pattern goes from term to term?

Thinking about the next term can help us identify different patterns in this sequence.

33) How many shaded squares will there be in Term 4 of this sequence? How do you know?

34) How many unshaded squares will there be in Term 4? Explain your answer.

35) How many total squares (shaded and unshaded) will there be in Term 4?

36) Draw Term 4.

37) In a few sentences, describe what the 10th term would look like. (It's okay if you are not sure. Make a guess and then continue reading).

When we looked at number patterns, you were able to make predictions and extend patterns beyond just the next few terms. You can do the same thing with visual patterns. Tables can be a helpful tool in understanding the pattern.

38)   Continue the pattern to complete the chart below.

Term Number	Total Number of Squares
1	5
2	7
3	9
4	
5	
6	
7	
8	
9	
10	

39)   Draw what you think the 10th term would look like.

40)   How many shaded squares would there be in Term 20? Explain how you know.

41)   How many total squares would there be in the 20th term? Explain how you know.

Here is another visual pattern. Notice that instead of referring to each object in the sequence as Term 1, Term 2, and Term 3, in this pattern they are called Figure 1, Figure 2, and Figure 3. You might also see the objects called *steps,* as in Step 1, Step 2, and Step 3. Terms, Figures, and Steps are all different words that mean the same thing. They can all be used to describe a visual pattern and any of them may be used on your high school equivalency exam.

Figure 1   Figure 2   Figure 3
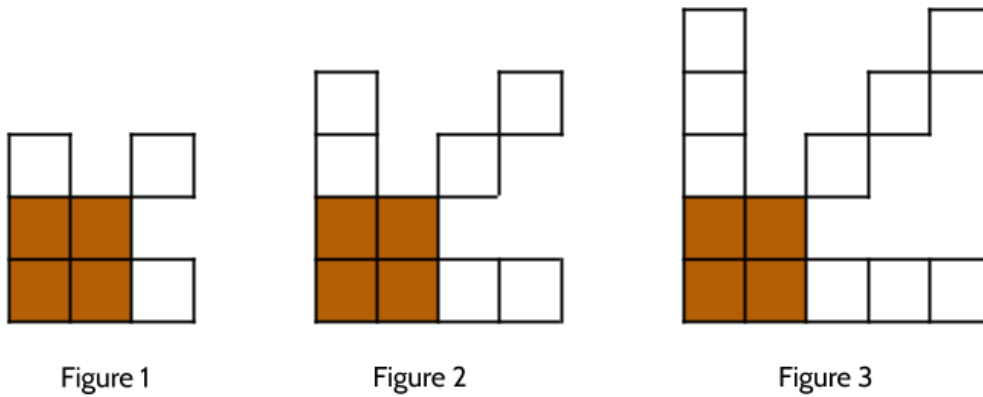
42) Fill in the blanks.

As the figure number changes, _____ also changes.

As the figure number changes, _____ also changes.

As the figure number changes, _____ also changes.

As the figure number changes, _____ also changes.

43) What stays the same in all three figures?

44) How many squares will there be in the next figure? Explain how you know.

45) Complete the chart.

Figure Number	Number of Squares
1	
2	
3	
4	
5	

46) How many squares will there be in the 8th figure? Explain your answer.

47) Describe how you would draw the 10th figure.

48) How many squares would there be in the 20th figure? Explain your answer.

You have identified patterns and used them to make predictions. But what if you wanted to make a prediction further along the pattern. What if you wanted to know how many squares there are in the 46th figure? What about the 99th figure? What if we wanted to predict the number of squares in any figure number? In the next section, you will begin to understand how linear functions can help us describe patterns.

Answer Key - Patterns

1)

Year	Chinese Zodiac Sign
2019	Pig
2020	Rat
2021	Ox
2022	Tiger
2023	Rabbit
2024	Dragon
2025	Snake
2026	Horse

2) The next Year of the Pig will be 2031. The Chinese zodiac repeats every 12 years. Since 2019 is the Year of the Pig, the next Year of the Pig will happen 12 years later
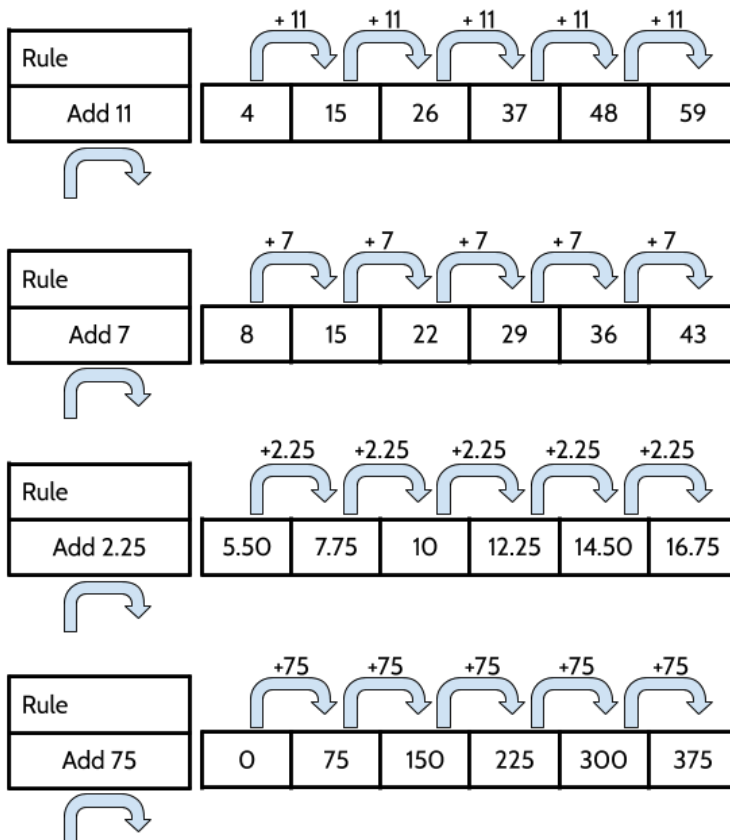
3) The Year of the Dragon.

4)

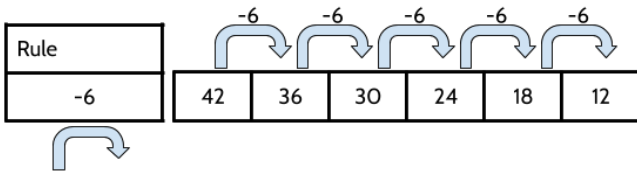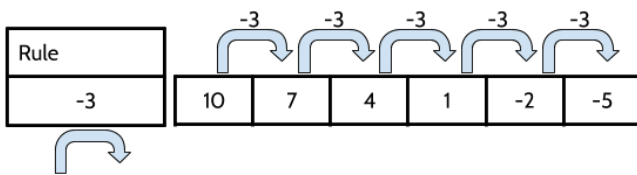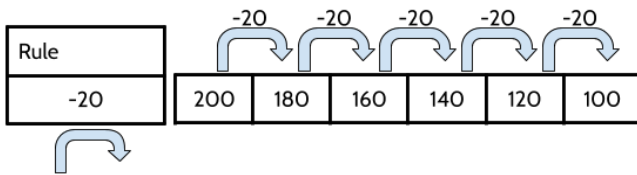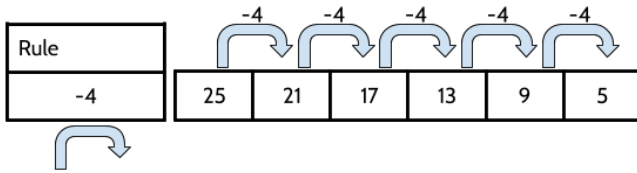|			
Pig	1935, 1947, 1959, 1971, 1983, 1995, 2007, 2019, 2031	**Snake**	1929, 1941, 1953, 1965, 1977, 1989, 2001, 2013, 2025
Rat	1924, 1936, 1948, 1960, 1972, 1984, 1996, 2008, 2020	**Horse**	1930, 1942, 1954, 1966, 1978, 1990, 2002, 2014, 2026
Ox	1925, 1937, 1949, 1961, 1973, 1985, 1997, 2009, 2021	**Goat**	1931, 1943, 1955, 1967, 1979, 1991, 2003, 2015, 2027
Tiger	1926, 1938, 1950, 1962, 1974, 1986, 1998, 2010, 2022	**Monkey**	1932, 1944, 1956, 1968, 1980, 1992, 2004, 2016, 2028
Rabbit	1927, 1939, 1951, 1963, 1975, 1987, 1999, 2011, 2023	**Rooster**	1933, 1945, 1957, 1969, 1981, 1993, 2005, 2017, 2029
Dragon	1928, 1940, 1952, 1964, 1976, 1988, 2000, 2012, 2024	**Dog**	1934, 1946, 1958, 1970, 1982, 1994, 2006, 2018, 2030

5) The next three numbers would be 1, 2, 3

6) There are several different patterns you might notice. Here are three examples: (1) If the number in the pattern (the term) is in the 4 times table (4, 8, 12, 16, 20, etc), the number will be 4. We can predict that the 24th number will also be 4. (2) The 5th number is 1, the 10th number is 2, the 15th number is 3, the 20th number is 4. We can predict that the 25th number will be 1. (3) The 6th number is 2, the 12th number is 4, the 18th number is 2, the 24th number is 4. We can predict that the 30th number will be 2.

7) The 28th number would be a 4.

8) The 37th number in the pattern would <u>not</u> be 4. There are several ways to know. You could continue the table. You could write the pattern out for 37 terms. You could count, 1, 2, 3, 4, 1, 2, 3, 4... until you got to the 37th number. You might also notice that if the 28th number is a 4, so the 32nd would be a 4, and the 36th would be a 4. The 37th would be a 1.

9) 3. There are different ways to figure this out. The 48th number would be 4, so the 49th number is 1, the 50th is 2 and the 51st number is 3.
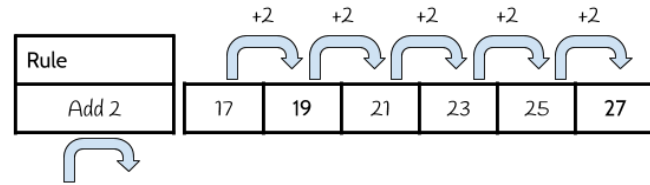
10) 1

11) 2

12)

13)

Rule						
-4	25	21	17	13	9	5

-4 -4 -4 -4 -4

Rule						
-20	200	180	160	140	120	100

-20 -20 -20 -20 -20

Rule						
-3	10	7	4	1	-2	-5

-3 -3 -3 -3 -3

Rule						
-6	42	36	30	24	18	12

-6 -6 -6 -6 -6

14)

Rule						
Minus 7	49	42	35	28	21	14

-7 -7 -7 -7 -7

Rule						
Add 4	10	14	18	22	26	30

+4 +4 +4 +4 +4

Rule						
Minus 2	42	40	38	36	34	32

-2 -2 -2 -2 -2

Rule						
Minus 10	62	52	42	32	22	12

-10 -10 -10 -10 -10

Rule						
Add 2	17	19	21	23	25	27

+2 +2 +2 +2 +2
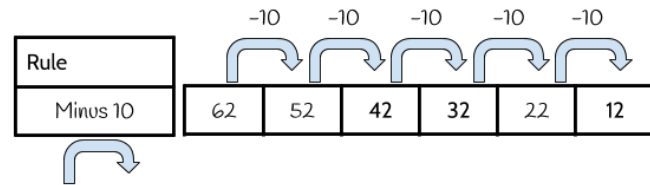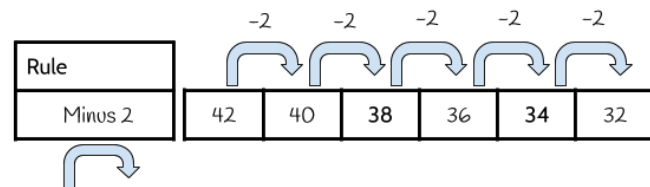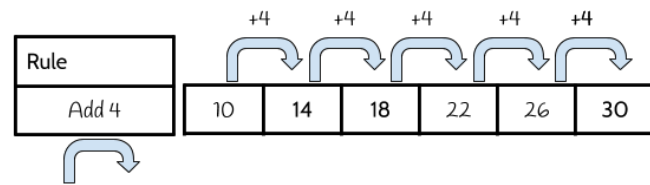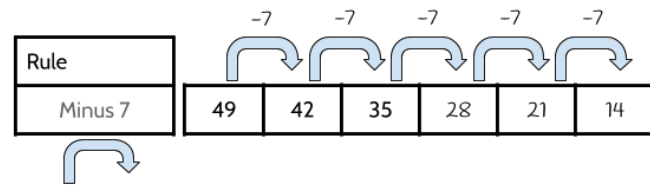
15)
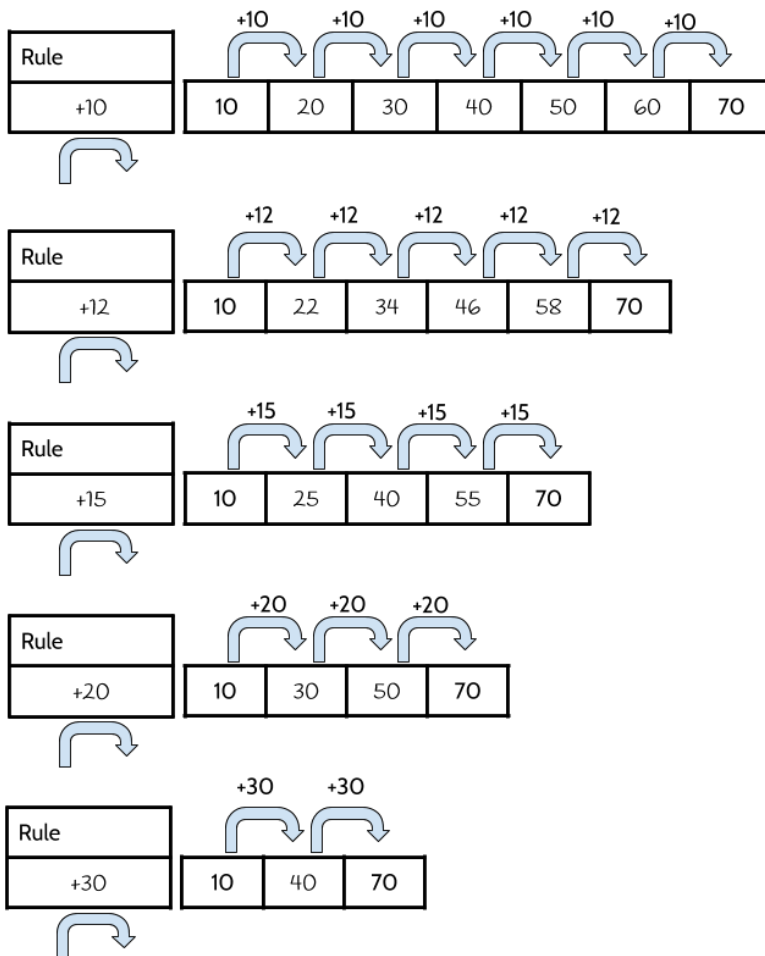
Trial and error is one effective strategy for solving this activity. Here is another way. For each of these patterns, you need to add 60 to the 10 to get to 70. The difference between each pattern is you have a different number of jumps. In the pattern on the bottom, there are 2 equal jumps to add 60, so each jump has to be 30. In the second pattern from the bottom, you have 3 equal jumps to add 60, so each jump is 20. So if you divide the 60 into however many equal jumps you have, you have the size of each jump. Does that work for the other patterns?

16) The answer to this question will be different for everyone. Why do you think it might be helpful to go back and think about the challenges we face?

17) The next three terms are 12, 10, 8. The rule for this pattern is Minus 2.

18) The next three terms are 23, 27, 31. The rule for this pattern is Add 4.

19) The next three terms are –5, –10, –15. The rule for this pattern is Minus 5.

20) The next three terms are 60, 66, 72. The rule for this pattern is Add 6.

21) The next three terms are <u>33</u>, <u>24</u>, <u>15</u>. The rule for this pattern is <u>Minus 9.</u>

22) Everyone will create a different pattern for this question. It exercises a different part of your brain to create a pattern, doesn't it?

23) The 8th term is 12.

24) The 8th term is 68.

25)
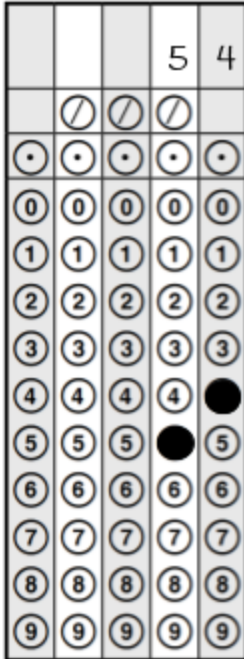
26) The 8th term is 26.

27) The 8th term is –15.

28) The 8th term is 1984.

29)
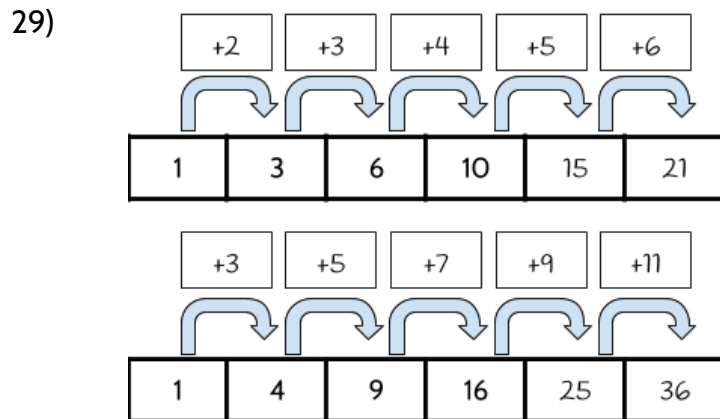
30)
   a) The 10th term in this pattern is 55.
   b) The 10th term in this pattern is 100.

31)   There is not one correct answer. There are several different things you might notice. Here are a few examples:
   ● The number of squares in each term is increasing.
   ● The height of each term is increasing.
   ● The number of squares in each "leg" is increasing.
   ● The number of unshaded (white) squares is increasing.

32)   There is not one correct answer. Here are a few examples of things that stay the same:
   ● The number of shaded (grey) squares.
   ● The number of "legs".
   ● There is always the same number of squares in each "leg".

33)   3. Every term has 3 shaded squares across the top.

34)   8. There will be 4 in each "leg".

35)   Term 4 will have 11 squares. There are many ways to answer this question. Here are 3 examples:
   ● You can identify the pattern with the number of squares in each term. The number of squares in the first 3 terms is 5, 7, 9, ____
   ● You can identify a pattern in the pictures. Each term has 2 more squares than the term before it. Term 4 will look like term 3, with an extra square and the end of each "leg".
   ● You can focus on the unshaded squares. We know there are 3 shaded squares across the top. Term 4 will have 4 unshaded squares in each "leg". Since there are 2 legs, that makes 8 unshaded squares. 8 + 3 is 11.

36)
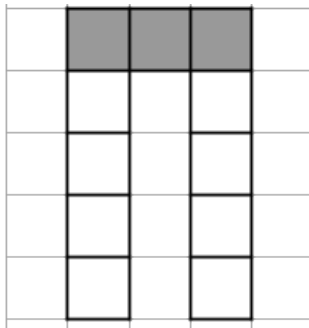
37)   The 10th term would have 3 shaded squares across the top. Each "leg" would have 10 unshaded squares.

38)

Term Number	Total Number of Squares
1	5
2	7
3	9
4	11
5	13
6	15
7	17
8	19
9	21
10	23

39) 10th Term

40)   There would be 3 shaded squares across the top.

41)   There are 43 squares in the 20th term. There are 20 unshaded squares in *each* "leg" and 3 shaded squares across the top.

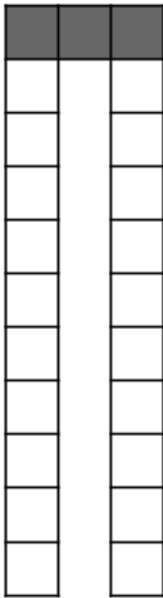42)   There are many correct answers. Here are a few possible answers:
   - As the figure number changes, <u>the number of squares</u> also changes.
   - As the figure number changes, <u>the number of unshaded squares</u> also changes.
   - As the figure number changes, <u>the height of each figure</u> also changes.
   - As the figure number changes, <u>the number of squares across the bottom</u> also changes.

43)   There are several correct answers. Here are a few possibilities:
   - There are always four shaded squares.
   - There are three "arms" of unshaded squares growing out of the 4 shaded squares.
   - The number of unshaded squares is always 3 times more than the figure number (for example: Figure 1 has 3 unshaded squares. Figure 2 has 6 unshaded squares. Figure 3 has 9 unshaded squares.

44)   The next figure will have 16 total squares. 4 shaded squares and 12 unshaded squares.

45)

Figure Number	Number of Squares
1	7
2	10
3	13
4	16
5	19

46)   There would be 28 squares in the 8th figure. 4 shaded and 24 unshaded squares.

47)   You could start with 4 shaded squares. Then there would be 10 unshaded squares going up, 10 unshaded squares across the bottom and 10 unshaded squares in the diagonal.

48)   There would be 64 squares in the 20th figure. 4 unshaded squares and 20 unshaded squares in each "arm".

What is a Function?

You might already be familiar with the word *function* in everyday use. For example, you might talk about the *functions* of a new smartphone. Or you might go to dress up to go to a *function* with a friend. These are two different everyday uses of the word *function*. In mathematics, *function* has a different meaning.

Functions are one of the biggest ideas in all of mathematics. **A function is a description of the relationship between two quantities.** The number of squares in a visual pattern is a function of the figure number. The distance around a circle (called the *circumference*) is a function of the distance across the circle (called the *diameter*). If you get paid by the hour, the amount of money you earn is a function of the number of hours you work. The number of servings a recipe makes is a function of the amount of ingredients you use. The amount you owe on your electric bill is a function of the amount of electricity you use.

A function is a tool we can use to *model* real-world situations. In this case, modelling doesn't mean wearing nice clothes. In math, to model a real-world situation means to create a mathematical representation of that situation. Mathematical representations of a situation help us analyze and understand that situation.

We will focus on four mathematical representations for function relationships: language, tables, graphs, and equations. In this packet, you will learn how to use each representation.

We will analyze and understand how numbers change. There are different kinds of functions, but all of them describe change. Quadratic functions, exponential functions, and linear functions each describe different kinds of changes in the relationship between two quantities. In this packet you are going to explore linear functions. Linear functions are functions that increase or decrease in a <u>constant</u> manner.

In mathematics, quantities that change are called *variables.* The word *variable* comes from the word *vary*, which means "to change." When people see *variable,* they often think of problems that ask them to solve for x. In those situations, the variable x represents a specific, unknown number for us to find.

For example, there is only one value of x that makes the following equation true:

$$x + 5 = 15$$

When we work with functions, we use variables a little differently. With functions, we use variables to model the relationship between certain numbers. For example, there are many values of x and y that make the following equation true:

$$x + 5 = y$$

To understand how the two variables work together, we describe them as *inputs* and *outputs*.

Function Machines

A function machine is a diagram that gives us a basic idea about how functions work. A function machine works like an automobile factory. Raw materials go into a factory—glass, steel, rubber, plastic. Inside the factory those raw materials are welded, painted, and shaped. And then a finished car comes out of the factory.
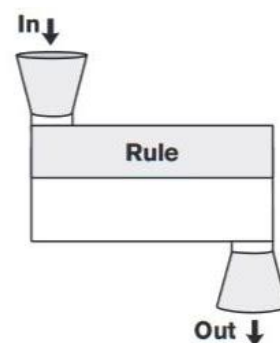
Functions work in a similar way. A number goes into the function machine, we do something to that number, and then a number comes out of the function machine. Instead of raw materials, numbers go into the function machine. We call these numbers *inputs*. Instead of welding, painting and shaping them, the numbers can be changed using addition, subtraction, multiplication, division, or other methods (and sometimes combinations of these). Finally, another number comes out of the machine, which we call an *output*.

Look at the table below.
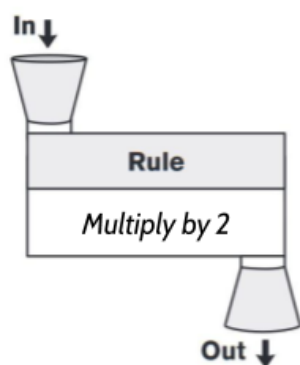
Input	Output
8	16
50	100
23	46
75	150
300	600
450	

Rule
Multiply by 2

What do you notice?

How does this table work?

The relationship between each input and output is described by the rule. For this function, the rule is *Multiply by 2*. This means that we will multiply any input into the function by 2 to get the output.

- When an input of 8 goes into the machine, we multiply by 2, and the output is 16.

- When an input of 50 goes in the machine, we multiply by 2, and the output is 100.

- When a 23 goes in, we multiply by 2, and a 46 comes out.

1) What is the output of this function when the input is 450?

2) Choose your own input to put into the function. Write that number in the table above under the 450. What is the output of the function when you put your number in?

Let's look at another function machine. In this example, you have some of the inputs that went into the function. For each input, the table tells you the number that comes out. You are given inputs and outputs but not the rule.

3) What is happening to each input to result in the given outputs? Write the function rule in the function machine.

In	Out
0	0
1	3
30	90
40	120
60	180
7	21
10	30

In ↓

Rule

Out ↓
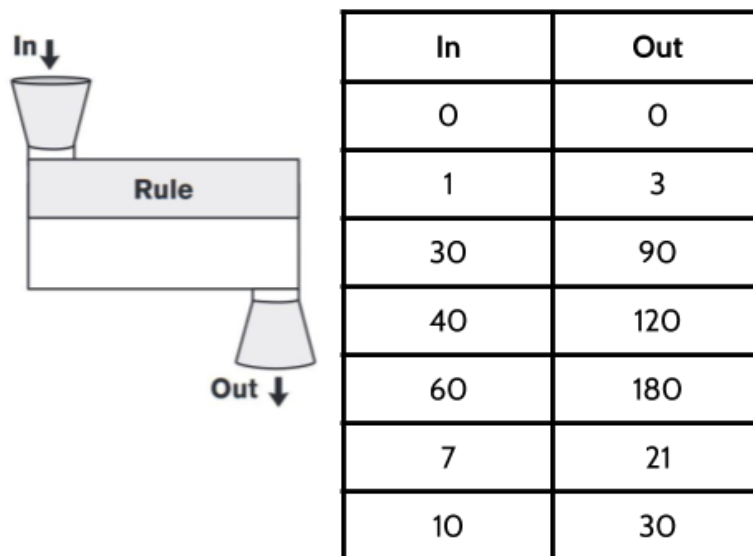
Function Machine Puzzles 1

Use the function rule and the inputs to complete the function machines below:

4)

In ↓

Rule

Multiply by 15

Out ↓

In	Out
1	
2	
3	
4	
15	

5)

In ↓

Rule

Divide by 2

Out ↓
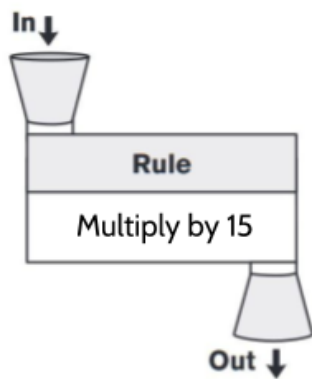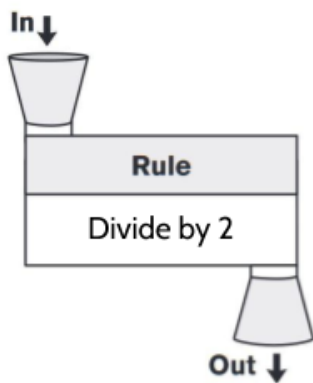
In	Out
10	
14	
24	
90	
30	

6)

In ↓

Rule

Add 6

Out ↓
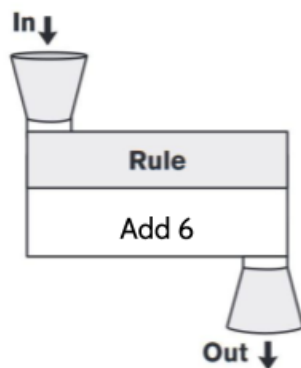
In	Out
2	
3	
5	
7	
21	

Function Machine Puzzles 2

Use the inputs and outputs to find the function rule for each function machine below.

7)
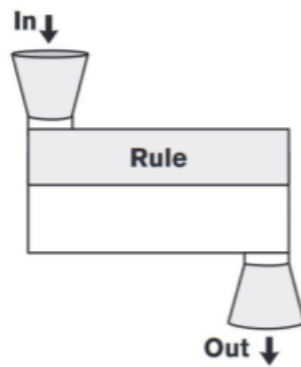
In	Out
16	8
10	2
20	12
8	0
100	92

8)
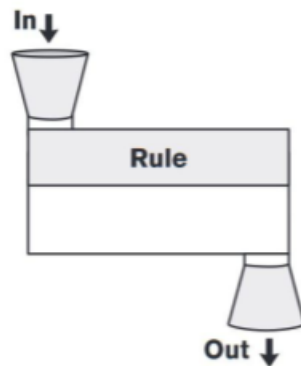
In	Out
3	33
6	66
9	99
11	121
13	143

9)
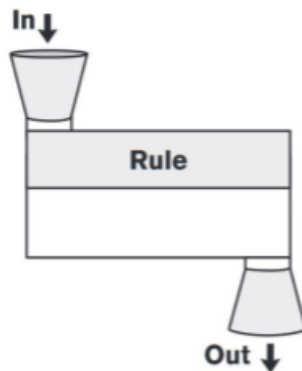
In	Out
.50	2
.75	3
1.25	5
2.5	10
3.25	13

Function Machine Puzzles 3

In Function Machine Puzzles 1, you were given the rule and the inputs and asked to find the outputs. In Function Machine Puzzles 2, you were given the inputs and outputs and had to figure out the rules. For Function Machine Puzzles 3, each machine will give you different pieces of information. Use whatever information is given to figure out what is missing, then fill in each blank.

10)

In ↓

Rule

Add 2.25

Out ↓

In	Out
6	
1.50	
5	
10.75	
	15

11)

In ↓

Rule

Multiply 3 then add 6

Out ↓

In	Out
10	
1	
2	12
12	
8	
	24

12)

In ↓

Rule

Multiply by 1.50

Out ↓

In	Out
2	
8	
10	
16	
20	

13)

In ↓

Rule

Increase by 50%

Out ↓
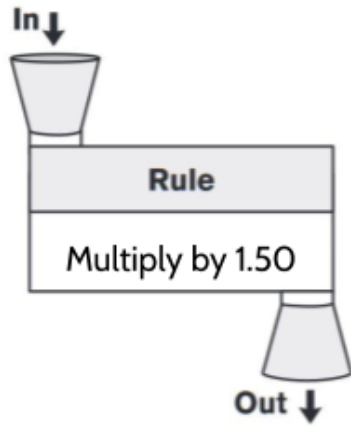
In	Out
12	
16	
20	
11	
	36

14)

In ↓

Rule

Out ↓
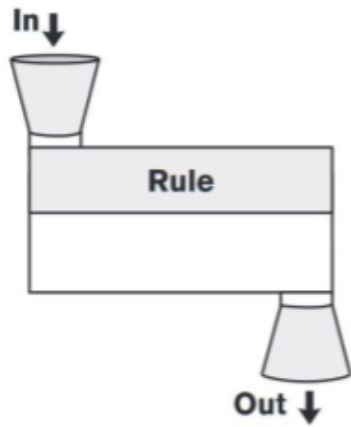
In	Out
10	40
2	8
6	
	60
	320

15)

Rule: Subtract 12	
In	Out
20	
35	
	6.50
	40
12.50	
	11

16)

Rule:	
In	Out
2	13
4	15
	23
	30
	40
	100

17)

Rule: Multiply by itself	
In	Out
x	y
2	
4	
5	25
	49
	81
	144

18)

Rule:	
In	Out
3	9
6	18
9	27
	36
	81
	99

Answer Key - Function Machines

1) 900

2) This answer will be different for everyone. The important thing is whatever number you put into this function machine, the output will be double that number.

3) Times 3 (You could also write this as Triple or Multiply by 3)

4)

In ↓

Rule
Multiply by 15

Out ↓

In	Out
1	15
2	30
3	45
4	60
15	225

5)

In ↓

Rule
Divide by 2

Out ↓

In	Out
10	5
14	7
24	12
90	45
30	15

6)

In ↓

Rule
Add 6

Out ↓

In	Out
2	8
3	9
5	11
7	13
21	27
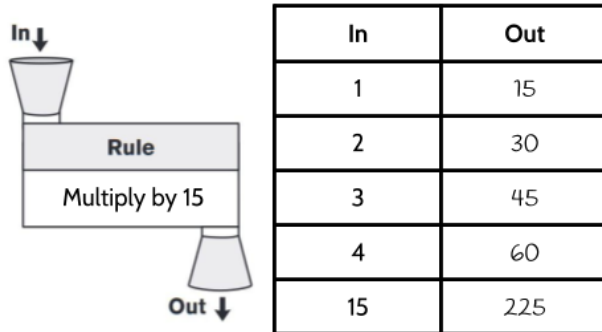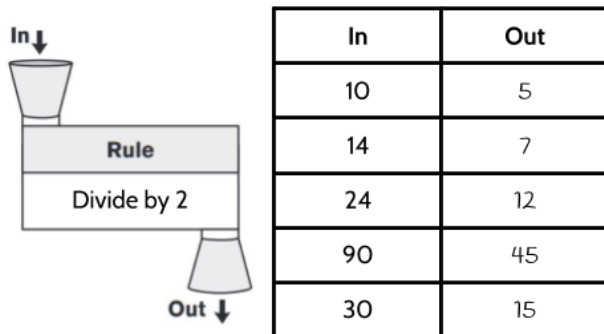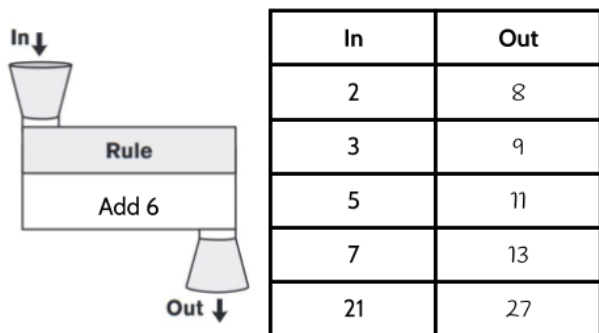
7) Minus 8

8) Times 11

9) Times 4

10)

In	Out
6	8.25
1.50	3.75
5	7.25
10.75	13
12.75	15

In ↓

Rule

Add 2.25

Out ↓

This is a little different from the other function machines you've seen so far. For this machine, you had the *output* and were asked to find the *input*. Did you write 17.25 as the input on the bottom row? This is a common mistake and good to think about even if you didn't make it. If you put a 15 into this machine, you will get 17.25 out, but the question in the bottom row is, "What number went in, was increased by 2.25 to result in an output of 15?"

11)

In	Out
10	36
1	9
2	12
12	42
8	30
6	24

In ↓

Rule

Multiply 3 then add 6

Out ↓
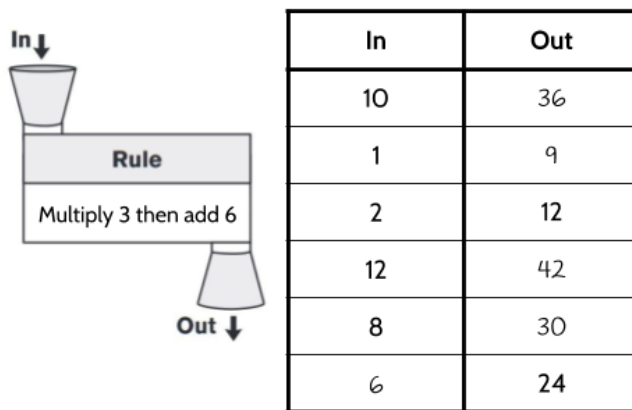
This function machine has a two-step rule. That means there are two operations. Again, in the bottom row we are given the output and asked to find the input. There are a few different strategies you can use. One effective strategy is to use trial and error. Make a guess and put that number into the function machine. If the output you are looking for comes out, great! If not, ask yourself, "is the number too high or too low?" and then try another input.

12)

In	Out
2	3
8	12
10	15
16	24
20	30

In ↓

Rule
Multiply by 1.50

Out ↓
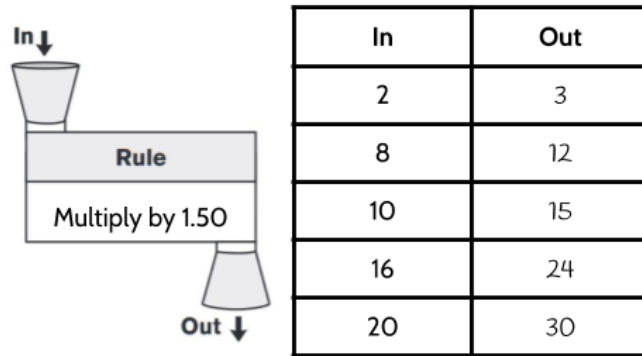
The rule for this function machine is to multiply the input by 1.50. It can help to think of multiplication as adding groups. 2 x 1.50 means 2 groups of 1.50. 8 x 1.50 means 8 groups of 1.50...

13)

In	Out
12	18
16	24
20	30
11	16.5
24	36

In ↓

Rule
Increase by 50%

Out ↓
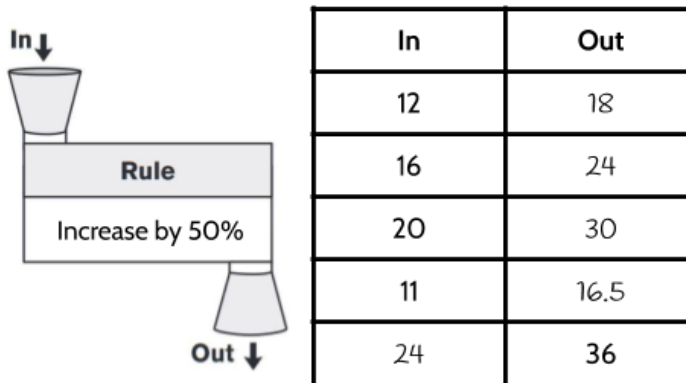
Increase by 50% means to take half of a number (50%) and increase the input by that amount. For example: The first input in our table is 12. Half of 12 is 6. So if we increase 12 by 6, we get 18. The second input is 16. Half of 16 is 8. If we increase 16 by 8, we get 24. The third input is 20. Half of 20 is 10. If we increase 20 by 10, we get 30. Do you notice any connection between this function machine and the previous function machine?

14)

In	Out
10	40
2	8
6	24
15	60
80	320

In ↓

Rule
Multiply by 4

Out ↓

15)
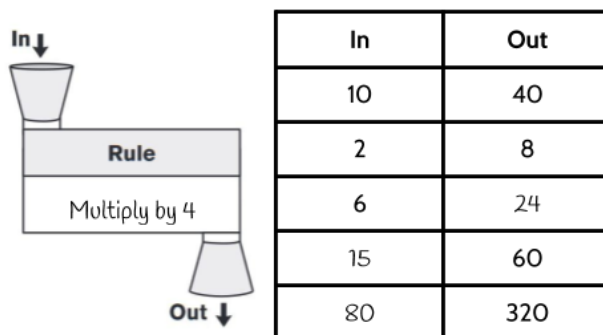
Rule: Subtract 12	
In	Out
20	8
35	23
18.5	6.50
52	40
12.50	.50
23	11

16)

Rule: Add 11	
In	Out
2	13
4	15
12	23
19	30
29	40
89	100

17)

Rule: Multiply by itself	
In	Out
x	y
2	4
4	16
5	25
7	49
9	81
12	144

18)

Rule: Multiply by 3	
In	Out
3	9
6	18
9	27
12	36
27	81
33	99

Four Views of a Function

Functions in Words

We can represent functions that are connected to the world by describing the function with words. Here's an example:

> *The Apple Taxi company charges a base fare of $3 and then charges $2 for every mile traveled.*

1) What if you traveled 4 miles in the taxi? Use the description to figure out how much the ride would cost.

2) How much would it cost if you traveled 6 miles in the taxi?

3) How much would a taxi ride cost if you traveled 2 and a half miles? Explain your answer.

4) If your taxi ride cost $25, how many miles did you travel? Explain your answer.

Using Tables to Represent Functions

A written description of the function relationship allows us to try some numbers and calculate the cost of riding in a taxi for different distances. We can use tables to show some particular inputs and outputs.

Function tables have at least two columns. The left column is where we put the input variable and the right column is for the output variable. Since the cost of the ride depends on the miles traveled, we put the miles traveled on the left and the cost on the right.

5) Complete the table. Calculate the costs of each taxi ride for the given numbers of miles traveled.

Miles Traveled	Calculation	Cost of Taxi Ride
Input		Output
0	2(0) + 3	3
1	2(1) + 3	
2	2(2) +3	
3		
4		
5		
10		
11		

Notice that the inputs and outputs change but what we do to the inputs stays the same. Whatever number we put into this function, we always multiply it by 2 and then add 3. In an equation, a *constant* is a value that stays the same even though other things change. In this function, no matter the mileage cost, the $3 added to the cost stays the same. The 3 is called the constant.

The input of 0 and the output of 3 might seem strange. You might be asking yourself, "How can it cost $3 if I travel 0 miles?" This is because $3 is the base fare, or flat fee, for your trip. As soon as you get in the taxi, the cost is $3, even before the driver starts to drive. We'll come back to this idea when we learn about *starting amounts* in functions.

Using Graphs to Represent Functions

A graph is another way of looking at a function. The graph shows us the inputs and outputs for a function in a visual way. We use the coordinate grid to graph functions.[1]

The Coordinate Grid
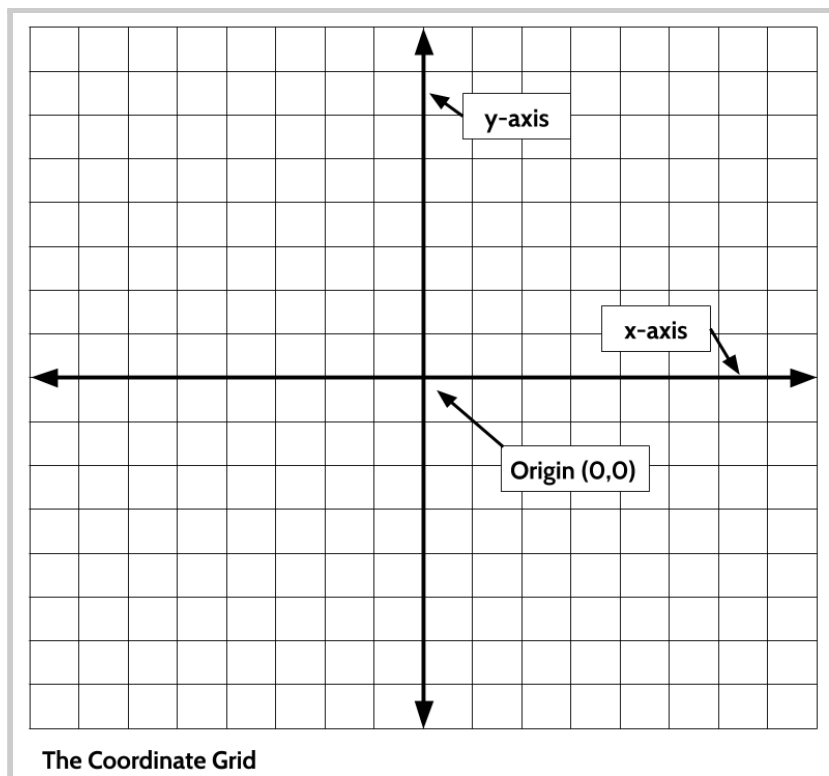
6)   Write down everything you know about coordinate grids.

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[1] For more practice working with the coordinate grid, see **Rigid Transformations: Shapes on a Plane, Part 1**.

The coordinate grid is formed by the intersection of two straight number lines. These lines are called the x-axis (the horizontal line ↔) and the y-axis (the vertical line ↕). The point where the two lines intersect is called the *origin*. The origin is the point with the ordered pair (0, 0).

We can use function tables as a guide to create a graph of a function.

In the table below, there are different pairs of inputs and outputs that fit the function rule. If we travel 1 mile, the ride costs $5. If we travel 2 miles, the ride costs $7 and so on. The pairs of inputs and outputs can be written like this: (1, 5) and (2, 7). We call each of those input/output pairs an *ordered pair*.

7) Complete the table by adding the missing ordered pairs.

Miles Traveled	Cost of Taxi Ride	Ordered Pairs
Input	Output	(Input, Output)
0	3	(0, 3)
1	5	(1, 5)
2	7	
3	9	
5	13	
10	23	

Note: In ordered pairs, the input always comes first and the output comes second. Ordered pairs are almost always written using parentheses like this (1, 5).

Ordered pairs allow us to add inputs and outputs to the coordinate grid. The numbers in an ordered pair work as a set of directions to tell us where to add points to the graph. For example, this is how we add the ordered pair (1, 5) to the coordinate grid.

Step 1: Always start in the origin (0, 0).

Step 2: The first number in the ordered pair tells you how many spaces to move to the right or left. A positive number means to move to the right and a negative number means a move to the left. In (1, 5) the first number is positive 1, so we move 1 space to the right.

Step 3: The second number in the ordered pair tells you how many spaces to move up or down. A positive number means to move up and a negative number means a move down. In (1, 5), the second number is a positive 5, so we move 5 spaces up.

Here is our coordinate grid with the ordered pair (1, 5) shown as a point:
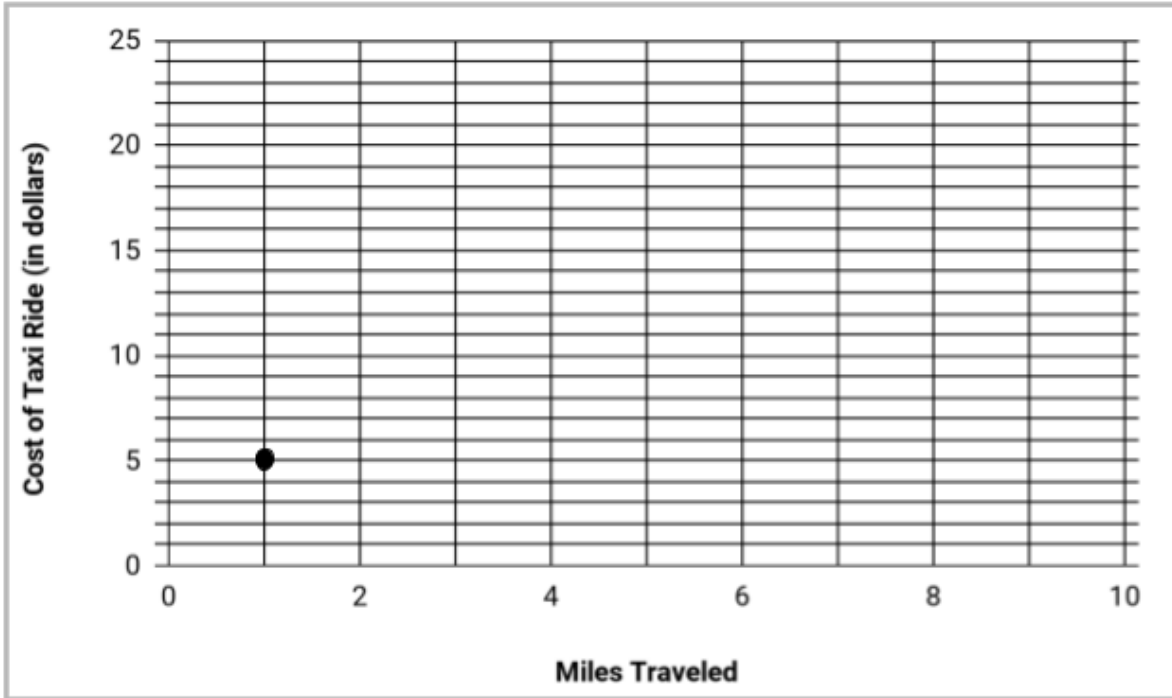
8) Plot the other ordered pairs from the table onto the graph above.

9) Once you have added all the ordered pairs from the table onto the graph, what do you notice?

The graph of this function forms a straight line. When the graph of a function makes a straight line, it is called a *linear* function. Linear functions are functions that grow in a constant way. All of the functions you will be studying in this packet will be linear, which means their graphs will form straight lines.

<div style="border:1px dashed">

<u>Core Concept</u>: The graphs of all linear functions form a straight line.

</div>

Graphs of functions can help us make general statements about the relationship between the two variables.

10) Which of the following statements is true?

    A. As the number of miles traveled increases, the cost of the ride increases.

    B. As the number of miles traveled decreases, the cost of the ride increases.

    C. As the cost of the ride decreases, the number of miles traveled increases.

Graphs can also help us answer specific questions about the function relationship.

Use the graph to answer the following questions.

11) How many miles did you travel if the ride cost you $15?

12) According to the graph, how much it would cost to travel...

    A. 7 miles?

    B. 8 miles?

    C. 9 miles?

    D. 6.5 miles?

Using Equations to Represent Functions

Tables and graphs allow us to see a limited set of inputs and outputs, just the ones that fit in a table or on a graph. Using equations to represent functions helps us calculate the output for *any* input.

The taxi function shows the relationship between two variables.

- The input → the number of miles traveled

- The output → the cost of a taxi ride

We know that it costs $2 for every mile traveled and we have to pay a base fare of an additional $3. We could write an equation with words to represent this function:

The cost of the taxi ride = 2 multiplied by *the number of miles traveled* + 3

To shorten this equation, we can use letters to represent each of the variables. If we let x represent the number of miles we travel and y represent the cost of the taxi ride, we can rewrite the function equation using symbols:

$$y = 2x + 3$$

where *y* represents the total cost of the ride and *x* represents the number of miles traveled.

When variables are used in a function equation, there will be a sentence like the one above that explains what each variable represents.

Equations can help us quickly see the relationship between the variables. Since linear functions show the relationship between two variables, linear equations have two variables—one to represent the input and one to represent the output. Remember that inputs are the variables that we put <u>*into*</u> the function. In this function, the input is the number of miles we travel. When we put that number into the function equation we get the total cost of the ride as an output.

Another way to understand the relationship between the two variables is to think of the input as the *independent variable* and the output as the *dependent variable.* The output is the *dependent* variable because the output *depends* on the input. The cost of the ride depends on the number of miles we travel. If someone asked how much the taxi cost, you might answer that it depends on how far you need to go.

Look at the variables in the function equation.

$$y = 2x + 3$$

The y in this equation is the
- output
- dependent variable

The x in this equation is the
- input
- independent variable
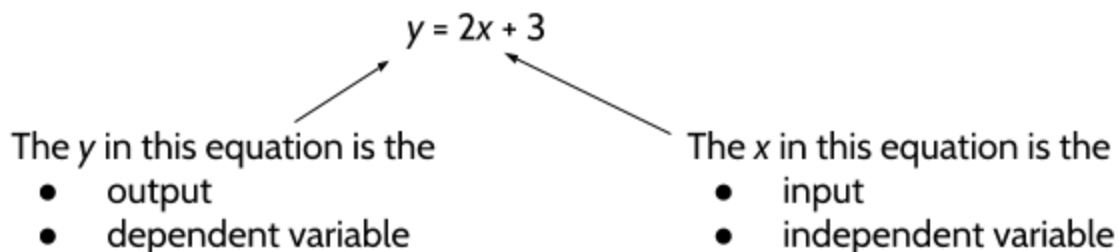
What do you notice?

One thing to notice is that the dependent variable is by itself on one side of the equal sign. The independent variable is the one we are performing operations on. This is the same idea as we saw with function machines. The input goes into a function, we do something to it, and it gives us the output. The independent variable is the variable that we "do something to." In this equation, we multiply the independent variable by 2 and then add 3.

The equation $y = 2x + 3$ uses x and y to represent the two variables, but as you work through this packet you will see that <u>any letters can be used as variables</u>. For example, the taxi company function can be represented by the equation,

$$c = 2m + 3$$

where c represents the total cost of the ride and m represents the number of miles traveled.

You try it.

13) Use any two letters to complete the equation and complete the sentence below.

$$\boxed{} = 2\boxed{} + 3$$

where $\boxed{}$ represents the total cost of the ride and $\boxed{}$ represents the number of miles traveled

Function Notation

The word *notation* means the way we write something. Tests, teachers, and professors may use different notations to write function rules and tables. Below you will see four ways of writing the same function. Even though some will look very different from the rules and tables you have seen up until now, the basic principle of functions you saw in the Input/Output machines is the same.

Look at these four ways of writing the same function. What do you notice?

Multiply by 2, then add 3	
Input	*Output*
1	5
2	7
3	9
10	23

$y = 2x+3$	
x	*y*
1	5
2	7
3	9
10	23

$c = 2m + 3$	
m	*c*
1	5
2	7
3	9
10	23

$f(x) = 2x+3$	
x	*f(x)*
1	5
2	7
3	9
10	23

14)   What is different?

15)   What is the same?

The notation on the bottom right is probably the least familiar to you, and because you will see it on the high school equivalency exam, it is important to understand how it is related to the other ways of writing the function.

$$f(x) = 2x + 3$$

The x still represents the input. The $f(x)$ represents the output.

When we read $f(x)$ as "The function of x".

We read the whole equation as:

The function of x is equal to two multiplied by x plus three

or

f of x is equal to two multiplied by x plus three. In this example, "f of x" is short for a function of x.

According to the table, a ride of 10 miles costs $23. You might see that written using this notation:

$$f(10) = 23$$

We read this as "The function of 10 is 23," which means when we put a 10 in, we get 23 out.

Consider this question:

What is $f(16)$ for $f(x) = 2x + 3$?

We read this question as "What is the function of 16 if $f(x) = 2x + 3$?"
This question is asking, "If you put a 16 into the function, what will come out?"

16) What is $f(12)$ for the function $f(x) = 2x + 3$?

17) What is $f(15)$ for the function $f(x) = 2x + 3$?

18) What is $f(24)$ for the function $f(x) = 2x + 3$?

$f(x)$can be confusing the first few times you see it. Here are a few things to remember:

- The *f* in function notation $f(x)$ is <u>not</u> a variable. It stands for "The function of."

- Often in math, parentheses are used in multiplication. For example, 4(5) means 4 ✕ 5. In function notation, $f(x)$<u>does not</u> mean you should multiply *f* and x. This is because *f* *is not a variable.*

- The most important thing to remember is that $f(x)$is just another way of writing "output". Everything else works the same way.

Try it with a different function.

19)   Complete the table and graph the function.

f(x) = 3x +4		
x	*f(x)*	Ordered pairs
O		(,)
1		(,)
2		(,)
3		(,)
4		(,)
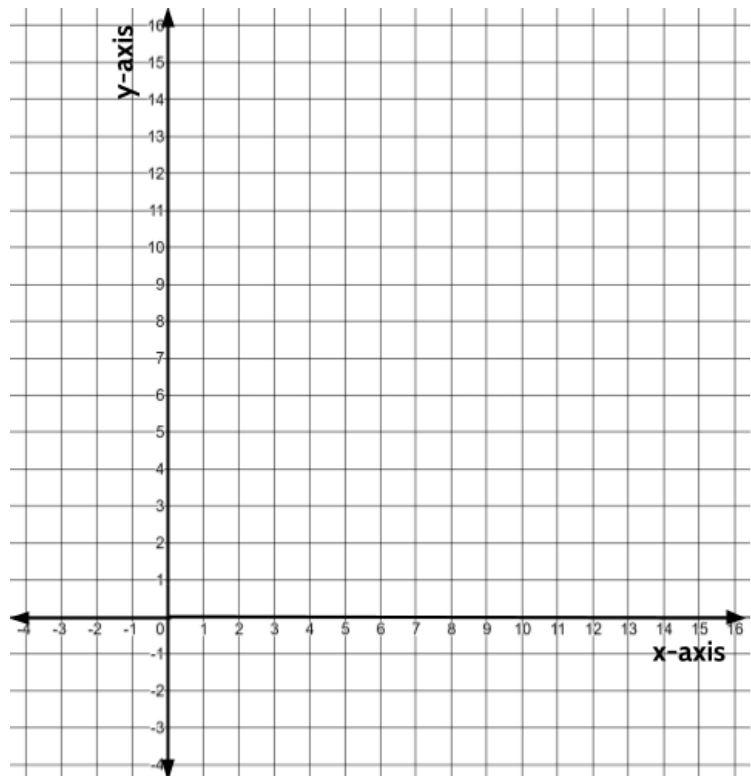
Each Input Has Exactly One Output

When we put different inputs into a function equation, we apply the same rule to each input. This means that if we put the same input into a function more than once, we must get the same output each time.

> **Core Concept: If the same input results in two different outputs, the relationship is <u>not</u> a function.**

On high school equivalency exams, you will probably see at least one question where you will need to identify the function that follows this core concept.

20) According to the rule above, only one of the three tables below can represent a function. Which one represents a function: A, B or C?

In	Out
x	*y*
1	2
2	4
3	6
6	12
2	3

A

In	Out
x	*y*
3	22
4	26
6	34
7	38
4	26

B

In	Out
x	*y*
4	28
5	35
3	21
2	14
5	42

C

The question above gives you inputs and outputs in a table. You may see a similar question where the inputs and outputs are written as ordered pairs.

21) Which of the following sets of ordered pairs represent a function?[2]

A. {(3,26), (5,38), (3,62), (8,44)}

B. {(5,10), (5,15), (5,20), (5,25)}

C. {(8,47), (15,82), (3,22), (8,47)}

D. {(2,17), (2,14), (4,19), (9,24)}

---

[2] We can use brackets like these { } to group things together. Here they are used around the ordered pairs in each answer choice.

Answer Key – Four Views of a Function

1) It would cost $11 to travel 4 miles. That includes the base fare ($3) and $2 for each mile ($8).

2) It would cost $15 to travel 6 miles.

3) A ride of 2.5 miles would cost $8. After the base fare, each mile costs $2. If you travel half a mile, it would cost $1.

4) If your ride cost $25, you rode in the taxi for 11 miles.

5)

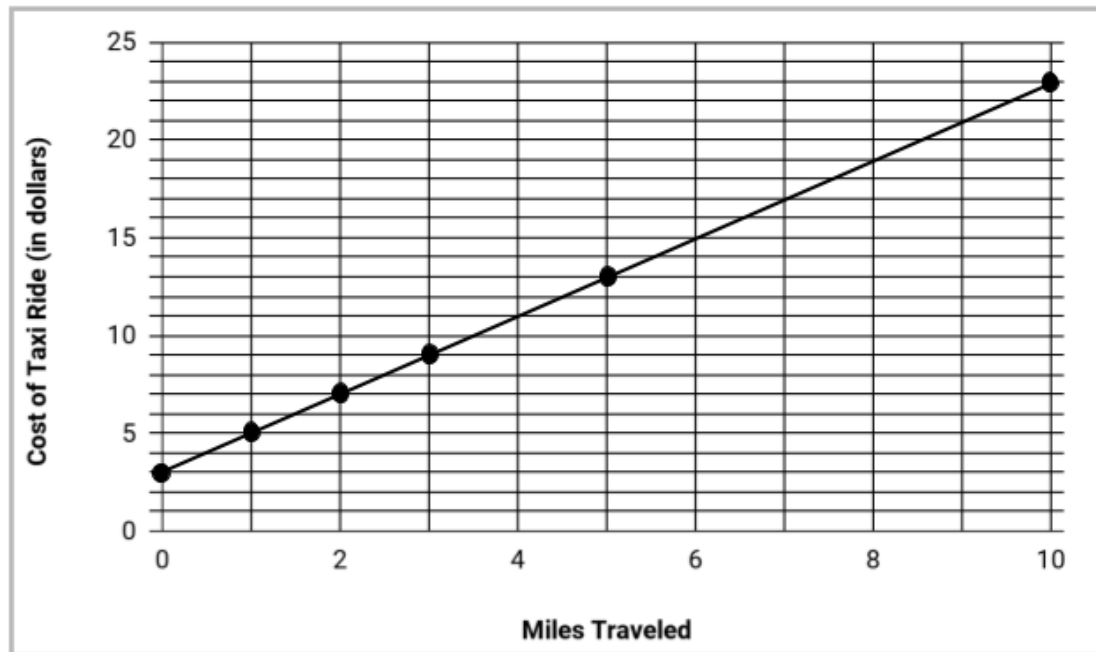| Miles Traveled	Calculation	Cost of Taxi Ride
Input		Output
0	2(0) + 3	3
1	2(1) + 3	5
2	2(2) +3	7
3	2(3) +3	9
4	2(4) +3	11
5	2(5) +3	13
10	2(10) +3	23
11	2(11) +3	25

6) Since this question asks about your own personal understanding of the coordinate grid, it will be different for everyone.

Miles Travelled	Cost of Taxi Ride	Ordered Pairs
Input	Output	(Input, Output)
0	3	(0, 3)
1	5	(1, 5)
2	7	(2,7)
3	9	(3,9)
5	13	(5,13)
10	23	(10,23)

7)

8)

A line graph titled with y-axis "Cost of Taxi Ride (in dollars)" ranging from 0 to 25, and x-axis "Miles Traveled" ranging from 0 to 10.

9) There are many things you may have noticed.
   - The numbers across the bottom are going up by 2. The numbers going up the left side of the graph are going up by 5.
   - All of the points from the table form a straight line.
   - The line intersects the y-axis at (0,3).
   - There are 6 points on the line. It looks like there are points missing. The line is moving up.
   - Another thing to notice is that the straight line can help us identify other ordered pairs. For example, do you notice how the line goes through (4,11)?

How much does it cost to ride 6 miles? 7 miles? 8 miles? 9 miles?
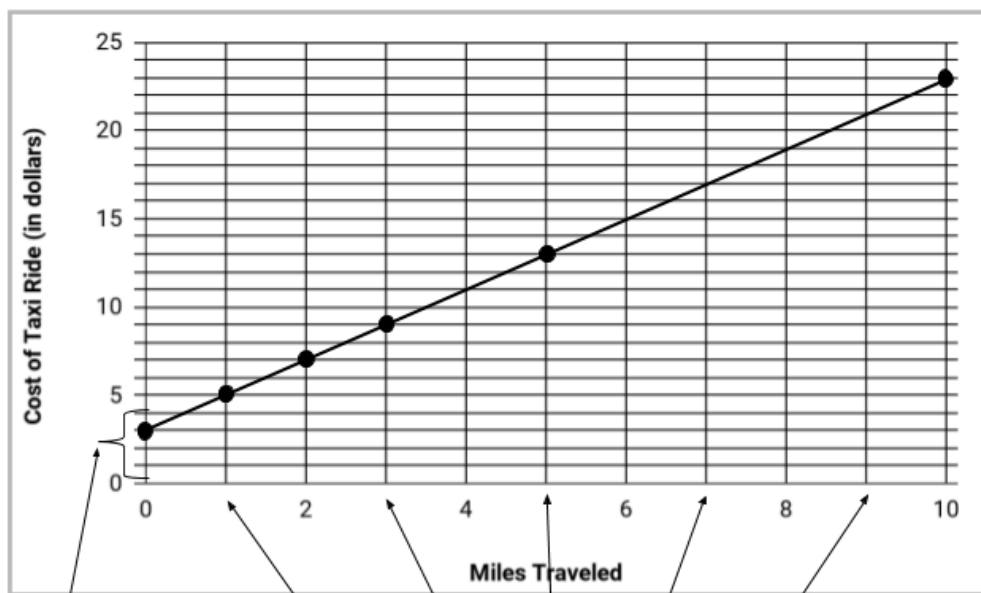
On the y-axis, every 5 lines is labeled. There are 4 lines between each labeled line.

On the x-axis, every 2 lines is labeled. There is 1 line between each labeled line

10) Choice A

11) 6 miles.

12)
   a) $17
   b) $19
   c) $21
   d) $16

13) There are multiple correct answers.

14) There are multiple correct answers. <u>Some possible answers</u>:
   - Sometimes the input is labeled "*input*", sometimes the input is labeled *x*, sometimes the input is labeled *m*.
   - Sometimes the output is labeled "*output*", sometimes the output is labeled *y*, sometimes the output is labeled *f(x)*.

15) There are multiple correct answers. <u>Some possible answers</u>:
   - No matter what notation is used, the input is always on the left side of the function table and the output is always on the right side of the function table.
   - We perform the operations on the input.
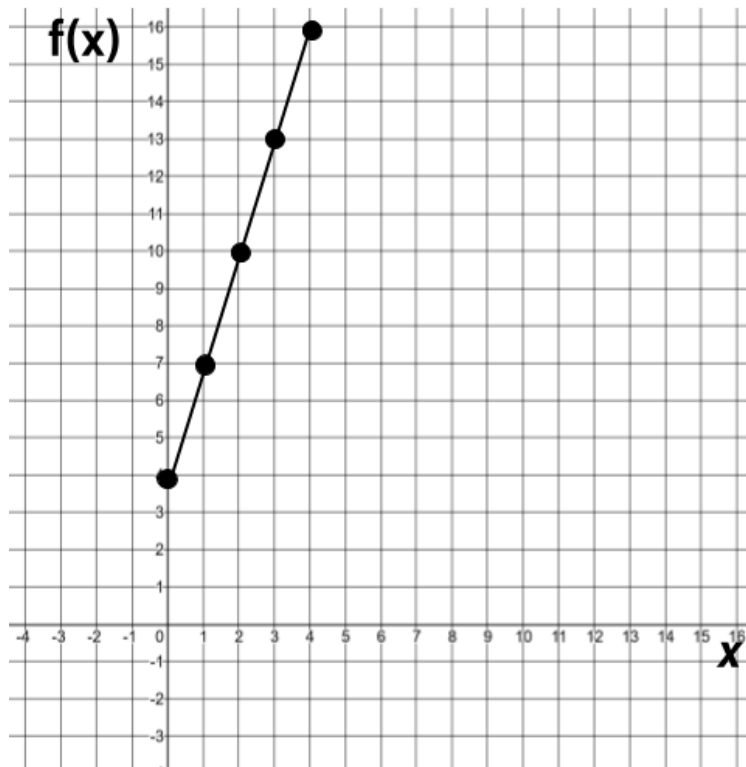   - We get the same inputs and outputs no matter which function notation is used

16) 27

17) 33

18) 51

19)

f(x) = 3x +4		
x	f(x)	Ordered pairs
0	4	(0,4)
1	7	(1,7)
2	10	(2,10)
3	13	(3,13)
4	16	(4,16)

20) Choice B is the answer. Choice A has two different outputs for an input of 2. No function can have a different output for the same input. Choice C has two different outputs for an input of 5. Choice B does have 4 as an input more than one, but each time has the same output.

21) Choice C is the only option that does not have different outputs for the same input.

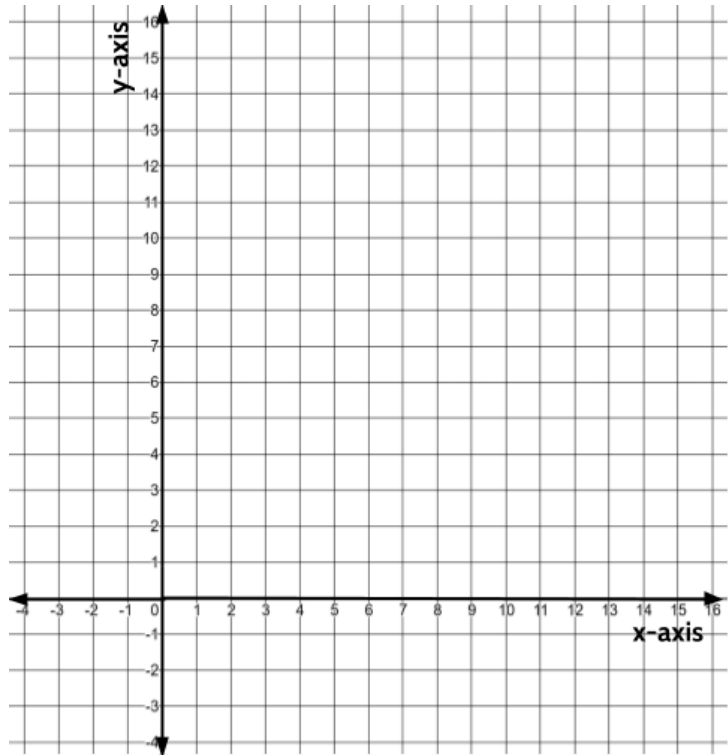## Reviewing What We've Learned So Far: Linear Functions

1) Complete the table and graph the function.

f(x) = 2x		
x	f(x)	Ordered pair
-1		(,)
0		(,)
1		(,)
2		(,)
3		(,)
4		(,)
5		(,)
6		(,)
7		(,)
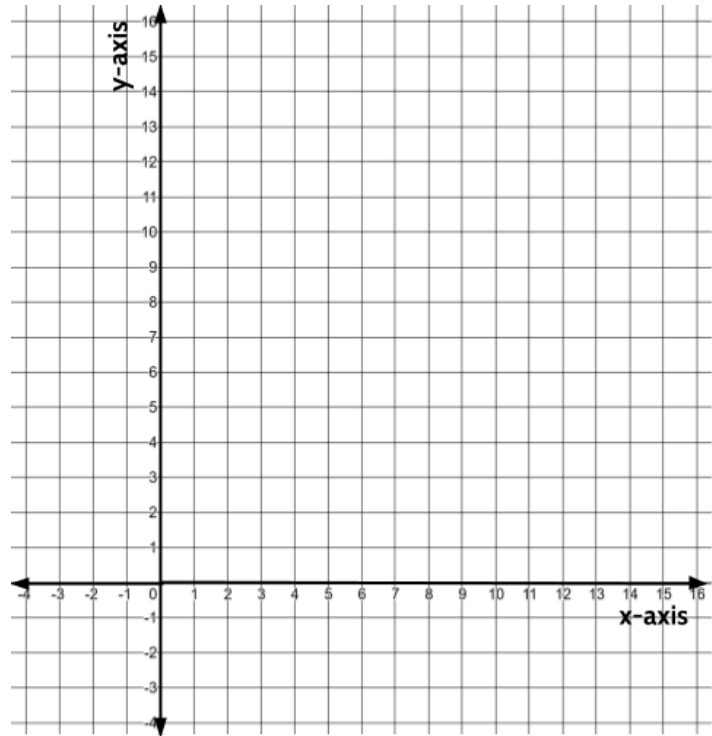
2) Complete the table and graph the function.

y = 2x + 2		
x	y	Ordered pair
-1		(,)
0		(,)
1		(,)
2		(,)
3		(,)
4		(,)
5		(,)
6		(,)
7		(,)

3) Complete the table and graph the function. Then write the rule above the table.

x	y	Ordered pair
0	0	(,)
1	1.5	(,)
2	3	(,)
3	4.5	(,)
4		(,)
5		(,)
6		(,)
7		(,)
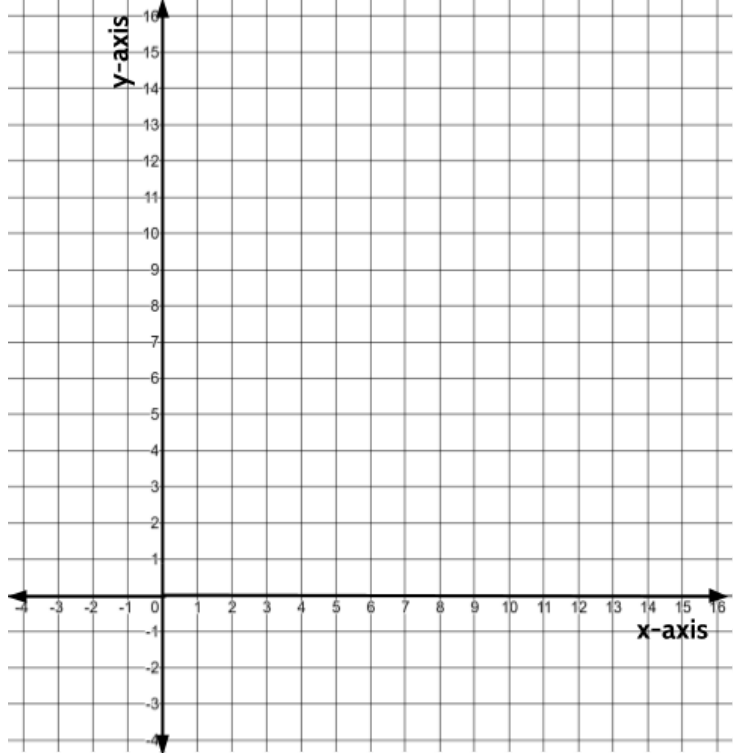
4) Fill in the table and find the rule.
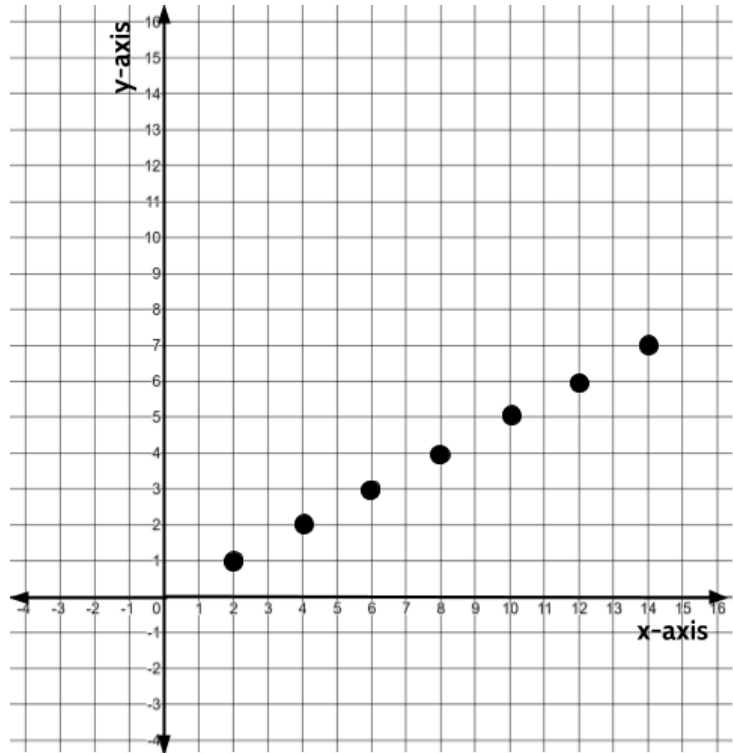
m	d	Ordered pair
		(,)
		(,)
		(,)
		(,)
		(,)
		(,)
		(,)

5)   Fill in the table and find the rule.

m	d	Ordered pair
		(,)
		(,)
		(,)
		(,)
		(,)
		(,)
		(,)
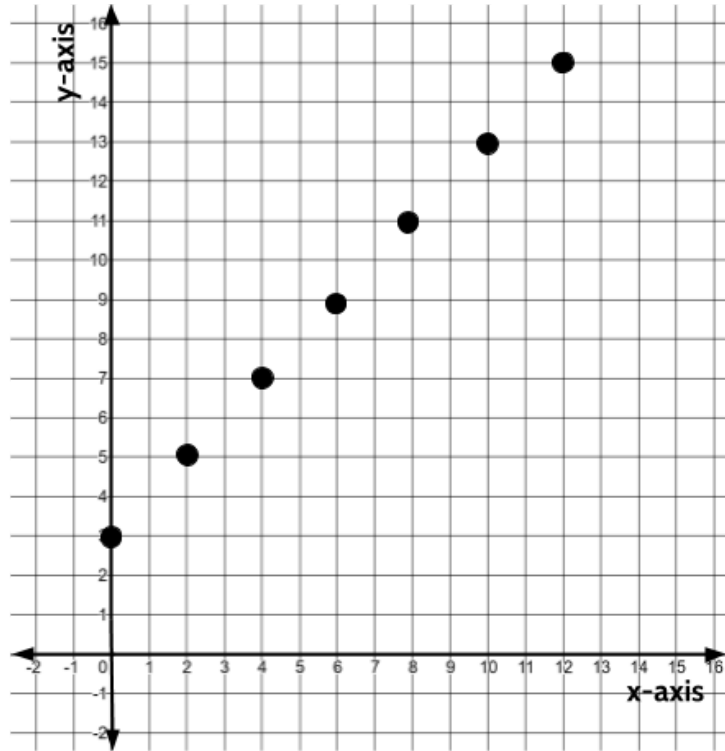
6)   Consider the following function description:

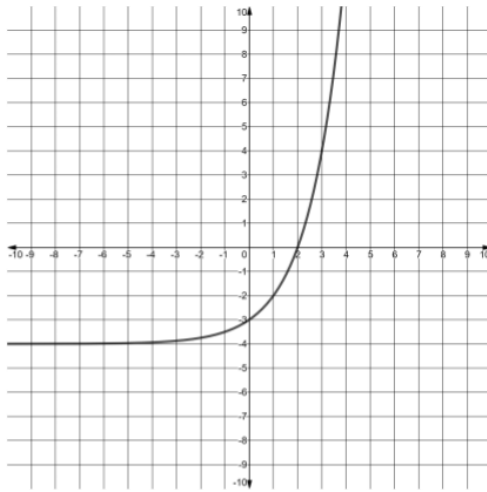*Excelsior Gym charges customers a one-time joining fee of $35 and then $50 a month.*

Which function equation can be used to represent this situation?

A. $f(x) = 35x + 50$

B. $f(x) = 50x + 35$

C. $f(x) = x + 85$

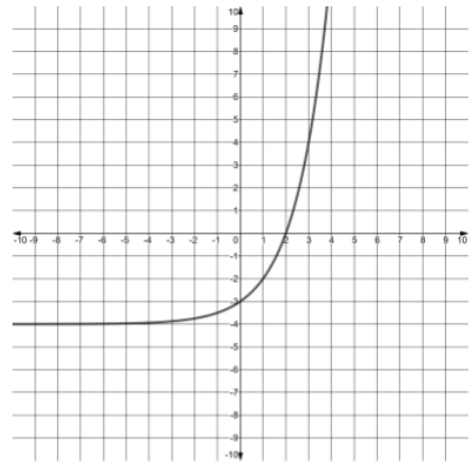D. $f(x) = 85x$

Explain your answer.

7) Which graph represents a linear function?

A.

B.

C.

D.

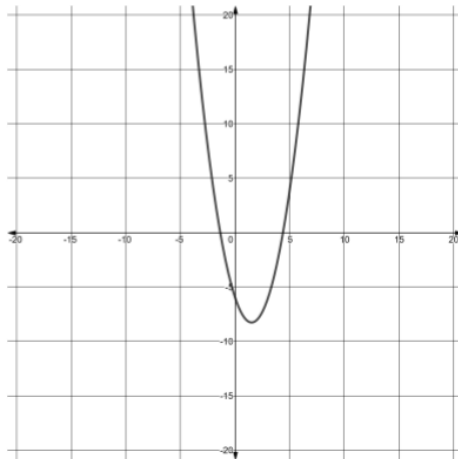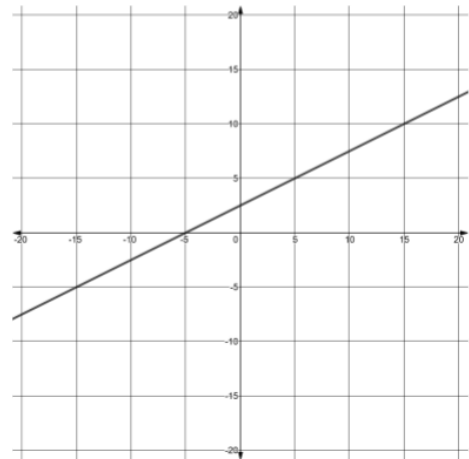Explain your answer.

8) For the function equation, $y = 4x + 12$, which of the following statements is true?

    A. x is the *dependent* variable and y is the *independent* variable

    B. x is the *independent* variable and y is the *dependent* variable

    C. 4 is the *independent* variable and 12 is the *dependent* variable

    D. 12 is the *independent* variable and 4 the *dependent* variable

9) Which of the following equations is <u>not</u> the same as the others?

    A. $f(x) = 8x + 14$

    B. $g = 8p + 14$

    C. $y = 14x + 8$

    D. $y = 8x + 14$

10) Use the function equation, $f(x) = 3x + 5$, to complete the table:

x	f(x)
15	
18	
24	
46	
2.25	
6.75	

11) For the function, $y = 5x + 2$, what is the value of y when x is 14?

    A. 16

    B. 33

    C. 70

    D. 72

12) Which of the following sets of ordered pairs represent a function?

    A. {(2,19), (5,28), (2,19), (15,58)}   Explain your answer.

    B. {(4,10), (5,16), (9,40), (5,61)}

    C. {(2,0), (4,30), (3,15), (3,4)}

   D. {(0,7), (3,19), (7,27), (7,35)}

Use the following situation to answer the next 4 questions.

Four friends used Taksio Taxi to meet at a restaurant for lunch. When they arrived at the restaurant, they tried to figure out the rule that the Taksio Taxi uses to calculate the cost.

a) Stella rode for 1 mile and paid $4.50.

b) Gigi traveled 6 miles and her total cost was $12.00.

c) Mira's trip was 3 miles and her total cost was $7.50.

d) Lusa rode for 8 miles and paid $15.00.

13) Use the description above to complete the table.

Passenger	Distance Traveled (in miles)	Cost (in dollars)	Ordered Pair
Stella			
Gigi			
Mira			
Lusa			

14) Graph the ordered pairs below. Then draw a straight line through the points.

Distance Traveled (in miles)

Cost (in dollars)
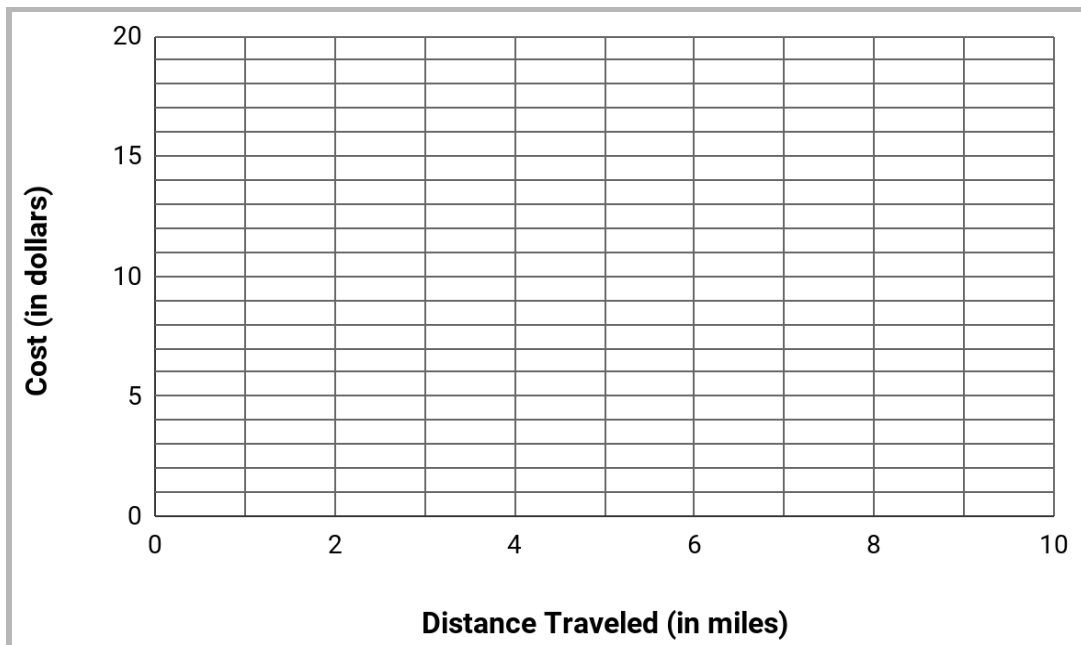
15) What would the taxi company charge someone who travels 10 miles?

Explain how you got your answer.

16) Taksio Taxi charges a base fare and a certain amount per mile. What is the base fare and what is the cost per mile? Explain how you got your answer.

Use the following function situation to answer the next five questions.

Jacinta has a new job working as a salesperson at a camera store. Each week she is paid a guaranteed salary of $250. She also gets a commission of $20 for each camera she sells.

17) Which of the following statements is true?

A. Jacinta's weekly salary is a function of the number of cameras she sells.

B. The number of cameras Jacinta sells is a function of her weekly salary.

Explain your answer.

18) Create a table that shows the total amount Jacinta earns in a week if she sells 0, 1, 2, 3, 4, 5, or 6 cameras. Label the columns using "Number of Cameras Sold" and "Weekly Pay".

19) Write a function equation that represents your table. Let *c* represent the number of cameras Jacinta sells in a week. Let *p* represent her weekly pay.

20) Match each question to the correct answer:

- If Jacinta earned $450 for the week, how many cameras did she sell?

- If Jacinta sells 20 cameras, how much would she earn?

- If Jacinta needs to earn at least $1000 for the week, how many cameras does she need to sell?

- How much does Jacinta earn if she doesn't sell any cameras?

- 38

- $250

- $650

- 10

21) After working at the camera store for 1 year, Jacinta got a raise. Her base salary increased $125 and her commission went up $5 per camera. Which of the following function equations represents her new weekly pay?

A. $f(x) = 5x + 125$

B. $f(x) = 125x + 5$

C. $f(x) = 25x + 375$

D. $f(x) = 375x + 25$

Use the following function situation to answer the next three questions.

> When making rice, Barbara uses 2 cups of water for 1 cup of rice.

22) Complete the table.

Rice	Water	Does this follow the correct ratio? YES or NO	Too Much Water, Not Enough Water, or The Correct Amount of Water
1	2	Yes	The correct amount of water
2	3	No	Not enough water
2.5	5		
3	6		
2	7		
4.5	9		
4	12		
10	13		
7	14		

23) Graph the data from the Rice and Water function table, including measurements that don't follow the correct ratio.
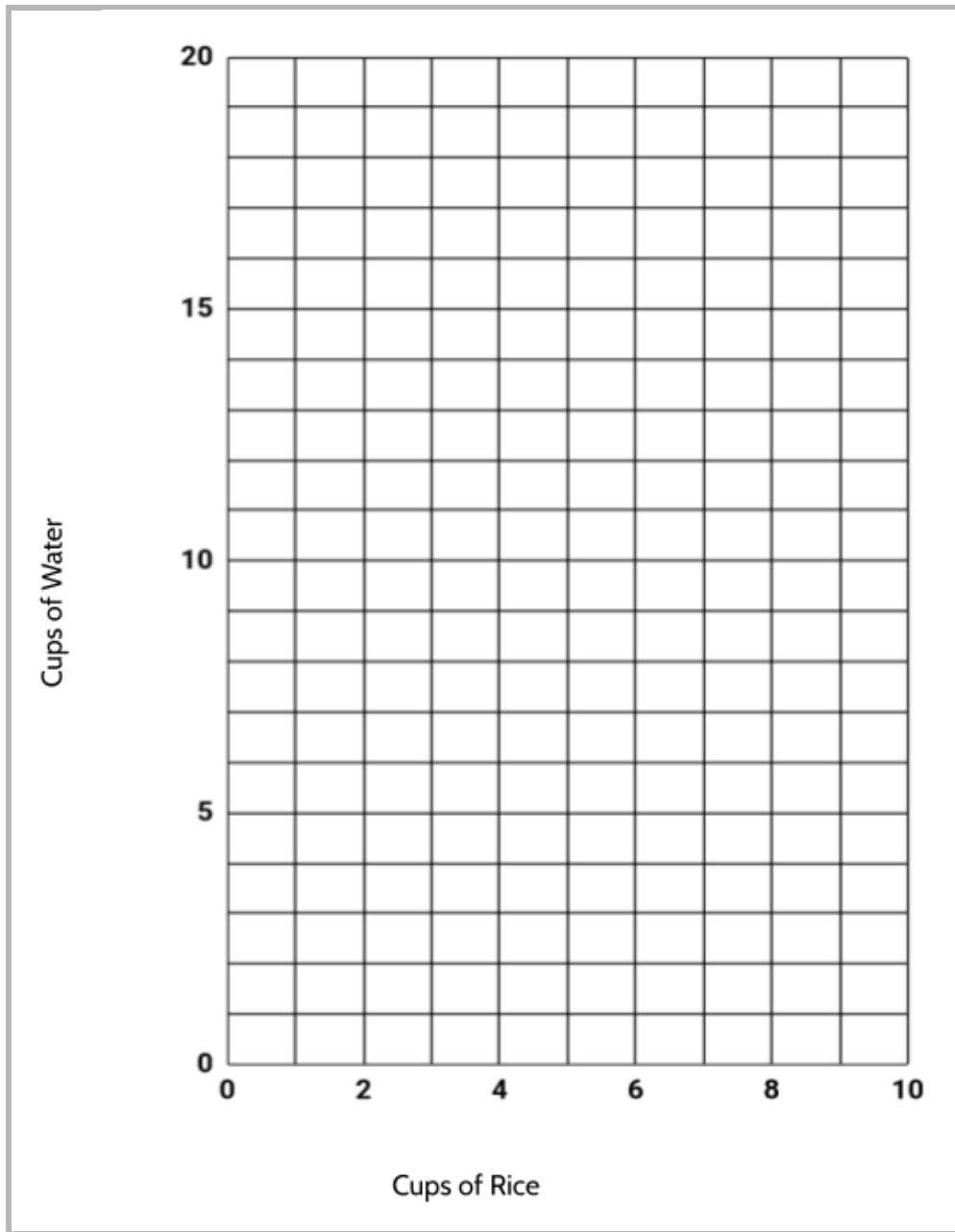
24) What do you notice?

Answer Key – Reviewing What We've Learned So Far: Introduction to s

1)

f(x) = 2x		
x	f(x)	Ordered pair
-1	-2	(-1,-2)
0	0	(0,0)
1	2	(1,2)
2	4	(2,4)
3	6	(3,6)
4	8	(4,8)
5	10	(5,10)
6	12	(6,12)
7	14	(7,14)
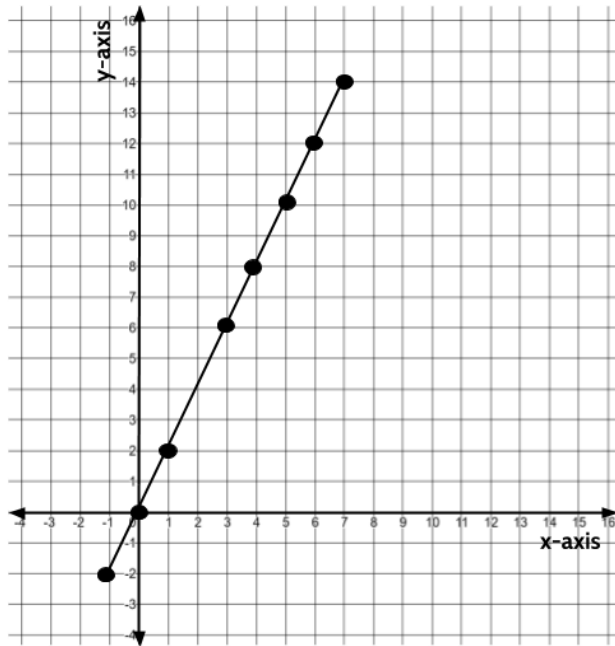
2)

y = 2x + 2		
x	y	Ordered pair
-1	0	(-1,0)
0	2	(0.2)
1	4	(1,4)
2	6	(2,6)
3	8	(3,8)
4	10	(4,10)
5	12	(5,12)
6	14	(6,14)
7	16	(7,16)
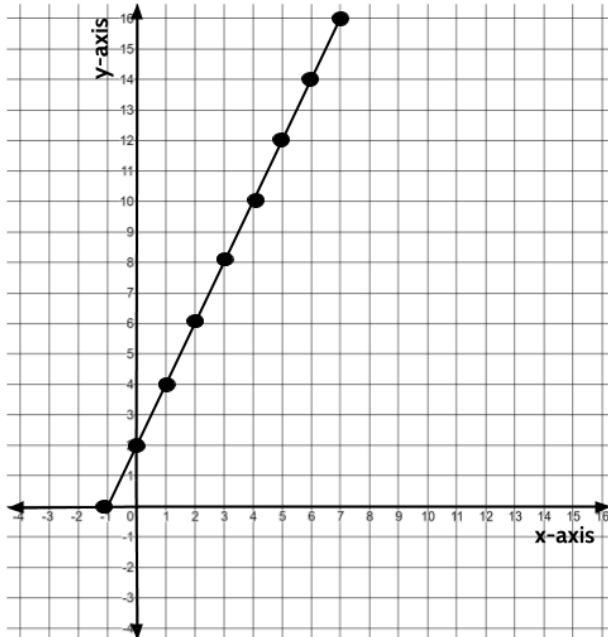
3)

y=1.5x		
x	y	Ordered pair
0	0	(0,0)
1	1.5	(1,1.5)
2	3	(2,3)
3	4.5	(3,4.5)
4	6	(4,6)
5	7.5	(5,7.5)
6	9	(6,9)
7	10.5	(7,10.5)
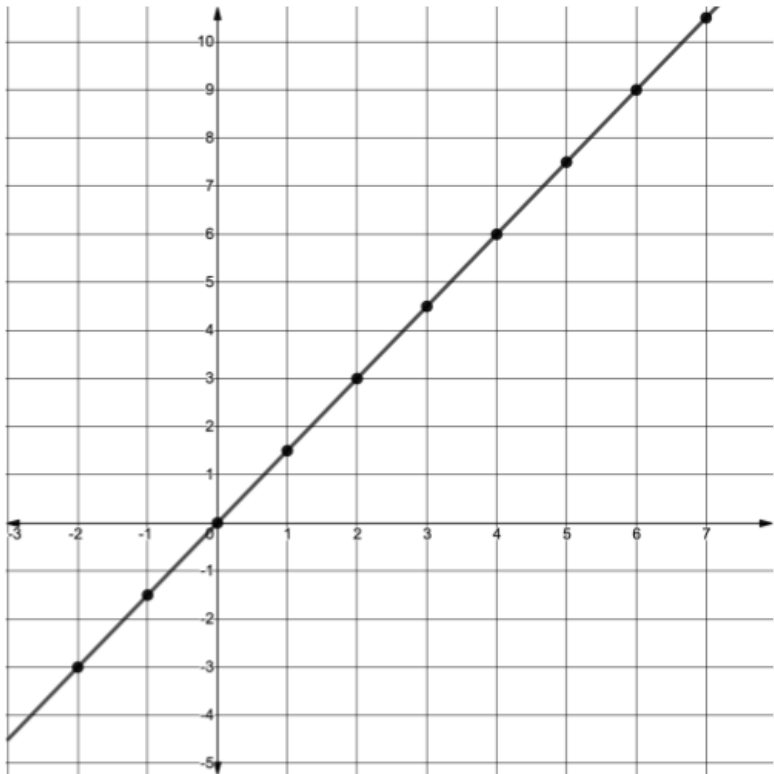
4)

$d = 1/2m$ or $d = m/2$		
m	d	Ordered pair
2	1	(2,1)
4	2	(4,2)
6	3	(6,3)
8	4	(8,4)
10	5	(10,5)
12	6	(12,6)
14	7	(14,7)

The outputs in this function are half of the inputs. For any number that goes in, we divide that number in half to get the output. You may have written:

$$d = m \div 2$$

This is correct and would allow you to calculate any input and output for this function.

The most common way of writing this function however is:

$$d = \tfrac{1}{2}\,m \quad \text{or} \quad d = \frac{m}{2}$$

5)

d = m+3		
m	d	Ordered pair
0	3	(0,3)
2	5	(2,5)
4	7	(4,7)
6	9	(6,9)
8	11	(8,11)
10	13	(10,13)
12	15	(12,15)

6) Choice B.

7) Choice D. The graph of a linear function always forms a straight line.

8) Choice B.

9) Choice C.

10)

x	f(x)
15	50
18	59
24	77
46	143
2.25	11.75
6.75	25.25

11) Choice D. 5(14) + 2 = 72

12) Choice A.

13)

Passenger	Distance Traveled (in miles)	Cost (in dollars)	Ordered Pair
Stella	1	4.5	(1,4.5)
Gigi	6	12	(6,12)
Mira	3	7.5	(3,7.5)
Lusa	8	15	(8,15)

14)
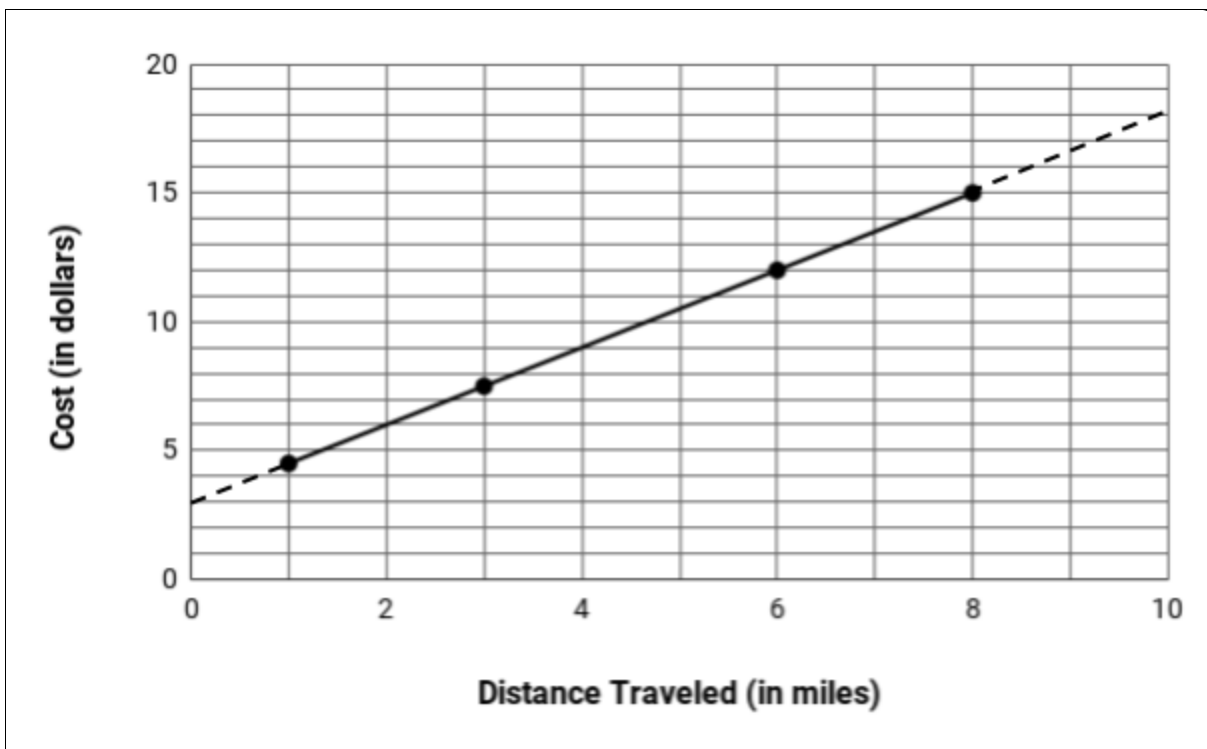
15) $18. A taxi ride of 10 miles would cost $18. There are several strategies you could use. One strategy is to extend the graph with a straight line to see what output for the ordered pair, when the input is 10.

16) Taksio Taxi charges a base fare of $3 and then $1.50 per mile.

17) Choice A.

18)

Number of Cameras Sold	Weekly Pay
0	250
1	270
2	290
3	310
4	330
5	350
6	370

19) $p = 20c + 250$

20)

- If Jacinta earned $450 for the week, how many cameras did she sell?

- If Jacinta sells 20 cameras, how much would she earn?

- If Jacinta needs to earn at least $1000 for the week, how many cameras does she need to sell?

- How much does Jacinta earn if she doesn't sell any cameras?

38

$250

$650

10

21) Choice C. Her original salary could be calculated by the function $p = 20c + 250$, where she received 20 per camera sold and a base salary of $250. Her raise included $5 more per camera and $125 more as a base salary.

22)

| Rice	Water	Does this follow the correct ratio? YES or NO	Too much water, Not enough water, or The correct amount of water
1	2	Yes	The correct amount of water
2	3	No	Not enough water
2.5	5	Yes	The correct amount of water
3	6	Yes	The correct amount of water
2	7	No	Too much water
4.5	9	Yes	The correct amount of water
4	12	No	Too much water
10	13	No	Not enough water
7	14	Yes	The correct amount of water

23)
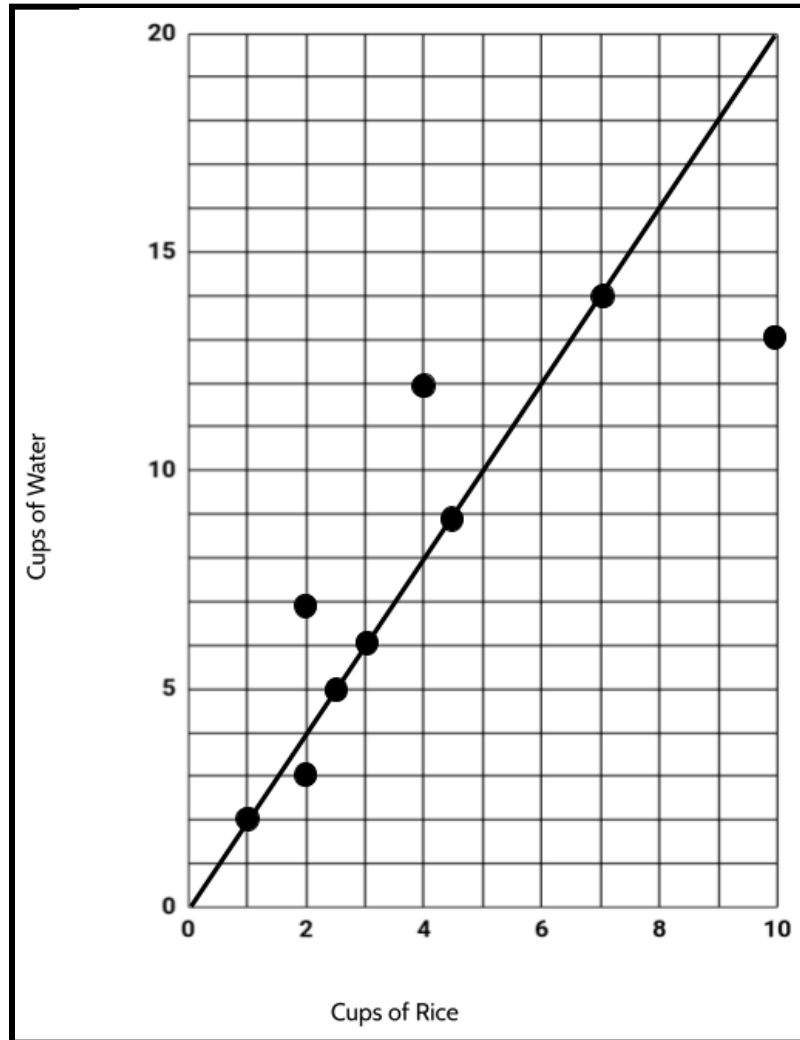
24) When the ratio is correct, the points form a straight line. The points above the line are combinations that have too much water. The points below the line are combinations that have not enough water.

Vocabulary Review

You can use this section to look up words used in this math packet.

coefficient (noun): a number used to multiply a variable. In the linear equation y=7x+12, 7 is a coefficient. <see *RATE OF CHANGE, SLOPE*>

coordinate plane (noun): a completely flat surface formed when two straight number lines intersect each other at right angles. The point where the lines intersect is called the origin and represents the zero on each axis. An exact position on the grid can be described using coordinates. The coordinate plane is also called the coordinate grid. <see *ORIGIN, X-AXIS, Y-AXIS*>
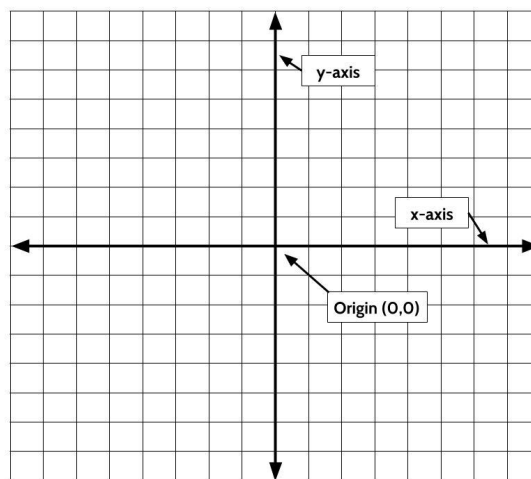
constant (noun): A value in an expression or equation that doesn't change. For example, in the function *y = 4x + 6* the 6 is the constant.*<see STARTING AMOUNT, Y-INTERCEPT>*

constant (adjective): Something that stays the same, that doesn't change. A *constant* rate of growth means that the rate stays the same over time. *<see RATE of CHANGE, SLOPE>*

dependent variable (noun): The variable in a function that represents the output. <see IN*DEPENDENT VARIABLE*>

domain (noun): the set of possible values for the independent variable. <see *RANGE*>

equation (noun): A number sentence that shows two expressions are equal by using the equal sign.

evaluate (verb): to calculate the value of something.

function (noun): a relationship between two quantities. For every input into a function, there can only be one output. A function can be represented by an equation, a table, a graph, and a verbal or written description.

function notation (noun): a way of writing a function that emphasizes the relationship between the independent variable and the dependent variable used in the function. For example we can read f*(x)* = 2x + 5 as "the function of x is 2x +5."

graph (noun): a visual representation of data.

horizontal (adjective): going side to side. On the coordinate grid, the x-axis is horizontal.

independent variable (noun): The variable in a function that represents the input.<see *DEPENDENT VARIABLE*>

input (noun): the number that goes into a function.

linear (adjective): linear means "a straight line". A Linear function is a function that grows in a constant way. The graph of a linear function is a straight line.

linear function (noun): A function that has a constant rate of change. The graph of a linear function is a straight line.

line of best fit / trend line (noun): a line on a graph that shows the general direction and trend that a set of points seem to follow.

model (verb): In math, to model a real-world situation means to create a mathematical representation of that situation.

origin (noun): the point where the x-axis and the y-axis intersect on the coordinate grid. The origin has the coordinates (0,0).

ordered pair (noun): Ordered pairs are used to show a position on a graph. An ordered pair is made up of an input and an output that make a function true. They are usually written in parentheses like this (14, 7), where the first number shows the x value (horizontal) and the second number shows the y value (vertical). <see *FUNCTION SOLUTIONS, COORDINATE GRID*>

output (noun): the unique value that comes out of a function when each input goes in. The output is dependent on the input.

pattern (noun): Patterns are things we can observe. We gather information about what has come before to make predictions about what will come next.

parallel (adjective): parallel lines are lines on a plane that never meet. The graphs of linear functions that have the same slope are parallel. For example: The line graph of *y = 4x+3* is parallel to the line graph of *y = 4x*

per (preposition): for each or for every
- The car was traveling 40 miles *per* hour.
- Most showers use 2.1 gallons of water *per* minute.
- The taxi charges $2.75 *per* mile.
- The air pressure in your car tires should be 32-35 pounds *per* square inch.

plot (verb): to draw a point on the coordinate grid or a graph that represents an ordered pair.

quantitative (adjective): something related to amounts or numbers.

quantity (noun): a measurement that is (or can be) specified by a number and a unit. For example 3 mangos, 16 feet, "the amount of money in a savings account", "the number of times a heart beats in a minute"

range (noun): the set of outputs for a function. the set of possible values for the dependent variable. <see *DOMAIN*>

rate (noun): A rate compares two different quantities.

- The sink was leaking at a rate of ½ an ounce of water per minute
- The train traveled at 80 miles per hour. For every hour, the train traveled 80 miles.
- The hybrid car gets a gas mileage rate of 50 miles per gallon. For every 1 gallon of gas, the car can travel 50 miles.

rate of change (noun): the rate of change in the outputs when the inputs are consecutive. <see *SLOPE*>

sequence (noun): a list of numbers, objects, or events in a special order.

slope (noun): Slope refers to how steep a line is. The slope of a line is also known as the rate of change. The slope is how the rate of change <see *RATE of CHANGE*>

solution (noun): A solution to a function is any ordered pair (x, y) that can be used to make a function equation true. For example, (3, 14) and (4, 18) are both solutions to the function $y=4x+2$ because if you an input of 3 results in an output of 14 and an input of 4 results in an output of 18.

starting amount (noun): the value of the output when the input is zero. <see *Y-INTERCEPT*>

table (noun): information arranged in rows and columns. A function table can be used to list some of the input and output values for a function.

term (noun): Term is used to describe where a number is in a pattern. The first number in a pattern is Term 1, the second number is Term 2, and so on. Term 8 would be the eighth number in a pattern.

variable (noun): A letter or symbol that represents another value, either any number, a specific number, or a set of numbers.

vertical (adjective): Going up and down. On the coordinate grid, the y-axis is vertical.

x-axis (noun)**:** the line that goes from left to right in a graph. The horizontal axis on the coordinate grid.

x-intercept (noun): the point where a line crosses the x-axis of a graph.

y-axis (noun)**:** the line that goes from top to bottom of a graph. The vertical axis on the coordinate grid.

y-intercept (noun): the point where a line crosses the y-axis of a graph. <see *STARTING AMOUNT*>

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