Tools of Algebra:
Linear Functions

Fast Track GRASP Math Packet

Part 2

Photo credit: Image by SethInk on Pixabay

Version 1.3

Released 8/12/2020

http://www.collectedny.org/ftgmp

TABLE OF CONTENTS

Introduction to Rates

What is a Rate?

A *rate* is a comparison between two related quantities. Here are a few examples of rates:

- When we go shopping, we can think about <u>price</u> *per* <u>pound</u>.
- When we talk about a pay rate, we can look at <u>dollars</u> *per* <u>hour</u>.
- When checking our heart rate, we can count <u>heartbeats</u> *per* <u>minute</u>.

We often use the word *per* when we are working with rates. The word *per* means *for each* or *for every*.

Speed is a common rate that we encounter in our daily lives. When we drive, we measure our speed in *miles per hour* (mph). When we say we were driving 60 miles per hour, it means the car is traveling 60 miles *for each* hour of driving.

1) Complete the following table for a car traveling at an average speed of 60 mph.

Miles per hour (mph)	
Hours Driven	Distance Traveled (in miles)
1	60
2	120
3	
4	
5	
10	
	1440

2) How far would you travel if you drove 60 mph for 15 minutes?

Another example of a rate is the cost of gas. If you pay $3.08 a gallon, that means for each gallon of gas you put in your car, you pay $3.08.

3)   Complete the following chart showing cost per gallon.

Cost per gallon	
Gallons	Cost
x	y
1	$3.08
2	
3	
4	$12.32
5	
8	
	$30.80

We also work with rates when we do hourly work. Let's say you work 32 hours a week and for every 32 hours you work, you get $704.

4)   Complete the following table showing your pay rate.

Pay Rate	
Hours Worked	Pay
32	$704
16	
8	
4	
2	
1	
128	

*You can find more practice with rates, in the math packet **Population Density.***

What is a Rate? – Answer Key

1)

Miles per hour (mph)	
Hours Driven	**Distance Traveled (in miles)**
1	60
2	120
3	180
4	240
6	360
10	600
24	1440

2) 15 miles. If you travel 60 miles in 1 hour (60 minutes), you can travel 30 miles in 30 minutes and 15 miles in 15 minutes.

3)

Cost per gallon	
Gallons	**Cost**
x	*y*
1	$3.08
2	$6.16
3	$9.24
4	$12.32
5	$15.40
8	$24.64
10	$30.80

4)

Pay Rate	
Hours Worked	Pay
32	$704
16	$352
8	$176
4	$88
2	$44
1	$22
128	$2816

Rate of Change and Starting Amount

Introduction to Rate of Change

Functions allow us to analyze change. In this section you will learn about the *rate of change* of a function. The rate of change describes how fast (or how slow) something is changing. Rates of change can be found in tables, graphs, and equations.

1) Fill in the missing values for the two functions below.

$y = 2x + 1$	
x	**y**
1	
2	
3	
4	
5	
6	

$y = 2x + 4$	
x	**y**
1	
2	
3	
4	
5	
6	

2) How are the two function tables similar?

3) How are the two function tables different?

Did you notice a pattern in the inputs and outputs for both functions?

The inputs for both functions go up by 1. The outputs of both functions go up by 2.

y = 2x + 1	
x	y
1	3
2	5
3	7
4	9
5	11
6	13

y = 2x + 4	
x	y
1	6
2	8
3	10
4	12
5	14
6	16

The inputs increase by 1. In other words, they are *consecutive*. Consecutive numbers follow each other in order without skipping. As the inputs increase by 1, the outputs increase by 2. The *rate of change* for both of the functions above is 2.

> Core Concept: The rate of change is the *constant* change in the *outputs* when the inputs are consecutive.

The rate of change can be found in the table of a function by looking at the difference between the outputs as the inputs increase by 1.

The rate of change for a function can also be found in the function equation. If you look back at the equations, you'll notice that 2 appears in both of them.

$$y = 2x + 1 \qquad y = 2x + 4$$

The rate of change in the equation is the number we multiply by the input. A number multiplied by a variable is called a *coefficient*. In the equations above, 2 is the coefficient of x. 2 is also the rate of change in both functions.

4) What is the rate of change for the following functions?

a)

x	y
12	66
13	71
14	76
15	81

b)

x	y
0	16
1	17
2	18
3	19

c) $f(x) = 12x - 56$

d) $y = 34x + 17$

e) $p = 0.8x + 60$

Introduction to Starting Amount

5) Fill in the two function tables below. Identify the rate of change in the equation and table for both functions.

$y = 5x + 4$	
x	y
0	
1	
2	
3	
4	
5	

Rate of Change _____

$y = 3x + 4$	
x	y
0	
1	
2	
3	
4	
5	

Rate of Change _____

6) How are the two function tables similar?

7) How are the two function tables different?

Both functions share an ordered pair: (0,4). How is it possible for two functions with different rates of change to both have an output of 4 for an input of 0?

The clue is the input of 0. When we multiply any number by 0, the result is 0. If we multiply an input of 0 by 5 or by 3, the product will be 0.

y = 5x + 4		
x	Calculation	y
0 ⟶	5(0) + 4 ⟶	4

0 + 4

y = 3x + 4		
x	Calculation	y
0 ⟶	3(0) + 4 ⟶	4

0 + 4

An input of 0 knocks out the first part of the equation, leaving us to add 4 in both functions. So when a 0 goes in, a 4 comes out.

We call this value *the starting amount.* The starting amount for both of these functions is 4.

Core Concept: The starting amount is the value of the output when the input is zero.

8) What is the starting amount for the following functions?

a)

x	f(x)
0	13
2	17
4	21
6	25

b)

x	y
1	12
12	67
13	72
14	77

c) $f(x) = 1.5x + 125$

Let's look at function equations using rate of change and starting amount. Consider the function equation $y = 6x + 3$.

9) What is the rate of change for this function? Explain how you know.

10) What is the starting amount for this function? Explain how you know.

11) Complete the table. Then show the rate of change and the starting amount in the table.

x	y
0	
1	
2	
3	
4	

Rate of Change _____

Starting Amount _____

12) Find the rates of change, starting amounts and equations for the four functions below.

Equation:

x	y
0	6
1	10
2	14
3	18
4	22
5	26

Rate of Change _____

Starting Amount _____

Equation:

x	y
0	7
1	10
2	13
3	16
4	19
5	22

Rate of Change _____

Starting Amount _____

Equation:

x	y
0	6
2	10
4	14
6	18
8	22
10	26

Rate of Change _____

Starting Amount _____

Equation:

x	y
1	13
2	17
3	21
5	29
6	33

Rate of Change _____

Starting Amount _____

Rate of Change and Starting Amount in Graphs

The rate of change and starting amount of a function can also be observed in the graph of that function.

Finding the Rate of Change in Graphs

Consider the following situation:

> Two friends, Sam and Chris, plan to race 15 blocks on their bicycles. Sam rides at a speed of 3 blocks per minute and Chris rides 5 blocks per minute.

13) Complete the tables.

Sam			Chris		
Time (minutes)	Block	Ordered Pair	Time (minutes)	Block	Ordered Pair
x	y	(x, y)	x	y	(x, y)
0	0	(0, 0)	0	0	(0, 0)
1			1		
2			2		
3			3		
4			4		
5			5		

14) What is the rate of change for Sam's function? _____

15) What is the rate of change for Chris' function? _____

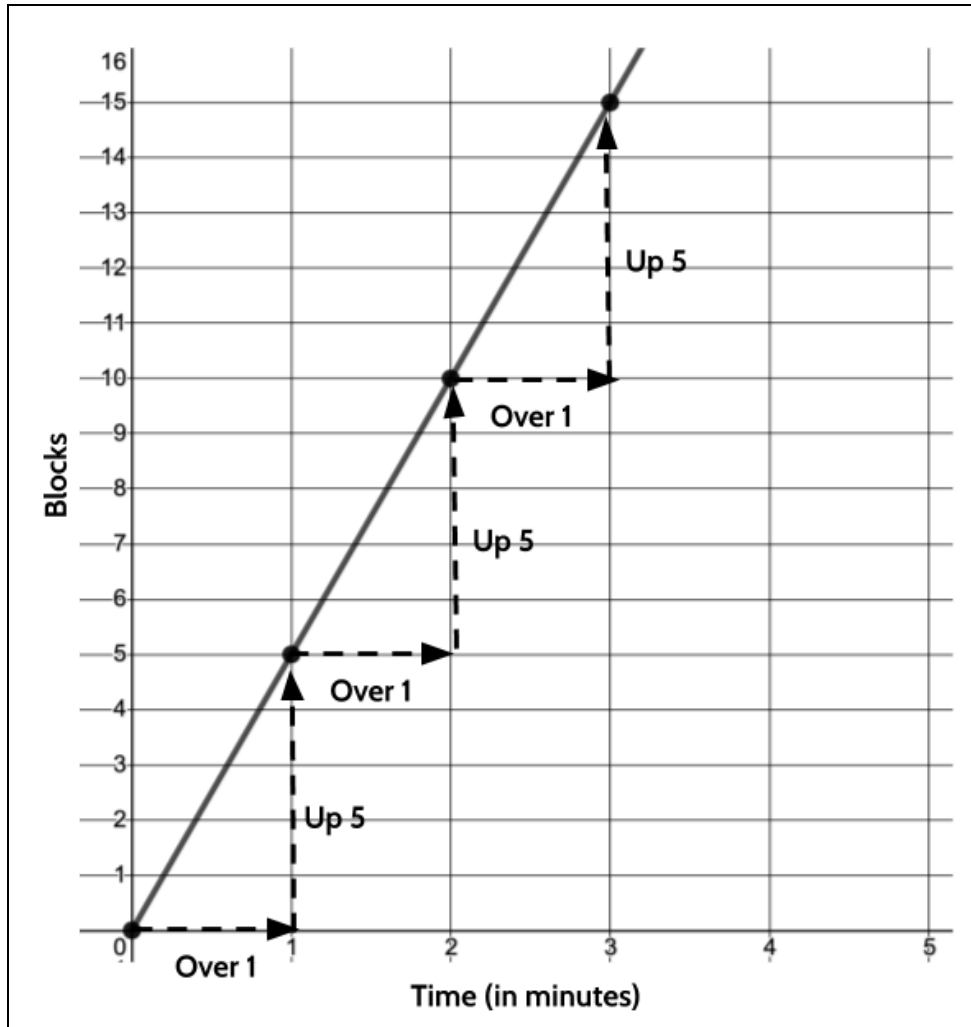16) Who won the race? How do you know?

Here is the graph of both functions. Each line represents one of the riders.
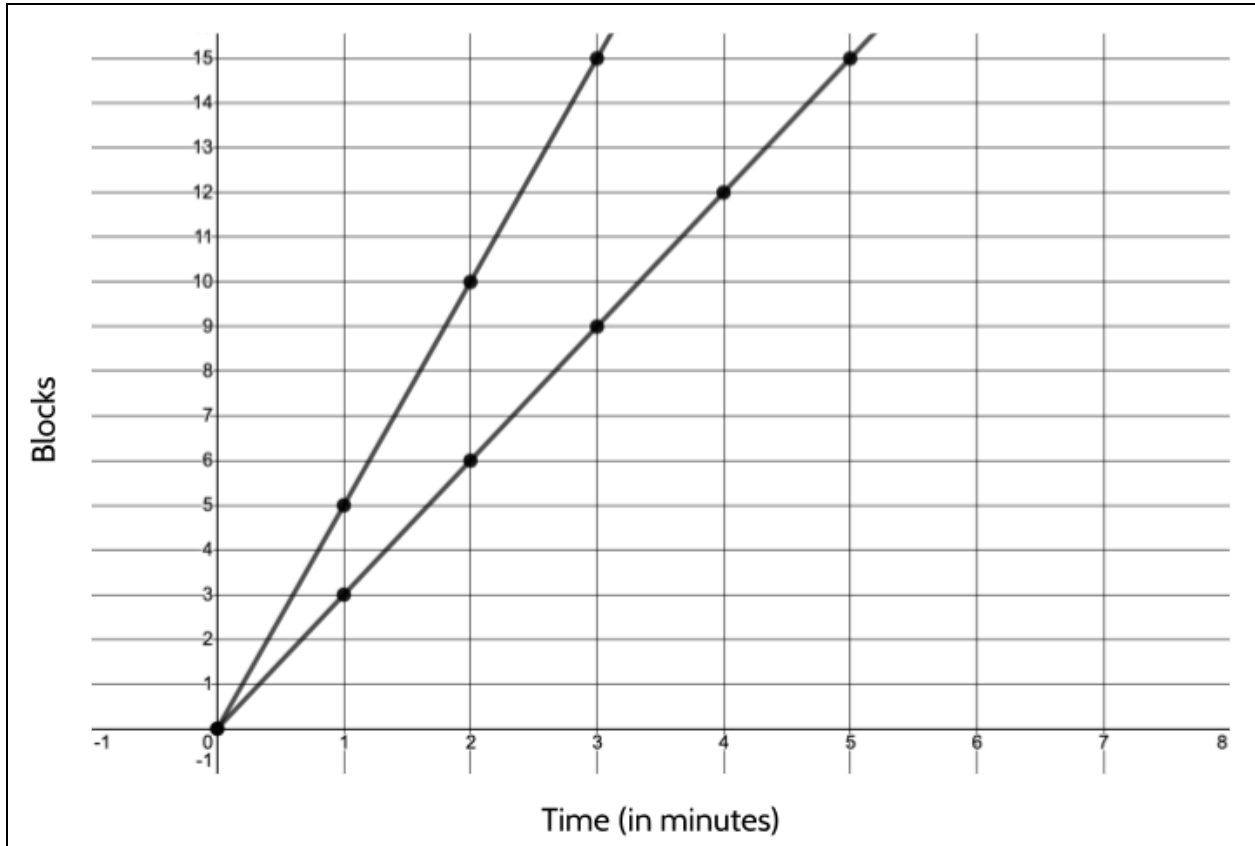
17) Label each line with the name of the rider. How did you figure out which line goes with which rider?

18) What do you notice about the two lines?

Here is the graph of the function for Chris' ride.


19) What do you notice?

Remember that the rate of change is the constant change in the outputs when the inputs are consecutive. In this function, the input is the number of minutes and the output is the number of blocks traveled.

The rate of change for the function of Chris' ride is 5. The graph representing Chris' ride shows that for each minute that goes by, Chris travels 5 blocks. We can identify the rate of change from the graph of this function by looking at the change in the outputs as the inputs increase by 1.

20) The rate of change for the function of Sam's ride is 3. In the graph of Sam's function below, show the rate of change.
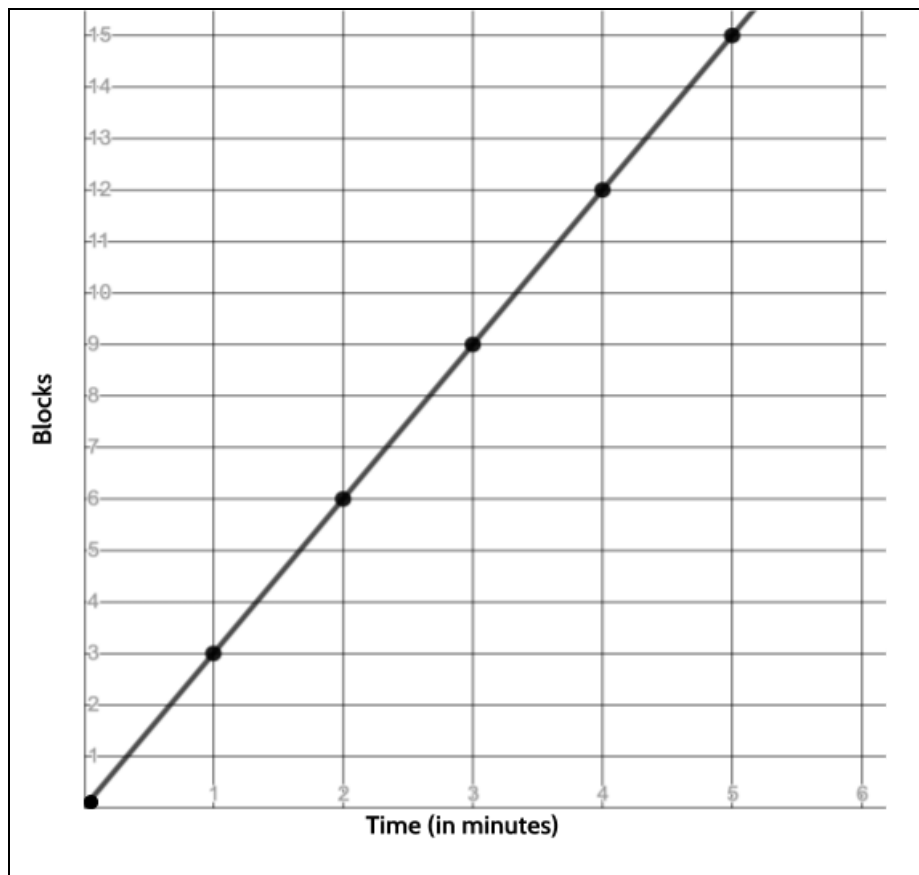
Chris rides at a rate of 5 blocks per minute. If we want to know how many blocks Chris rides in a certain time period, we can multiply the number of minutes by 5. For example, if Chris rides for 6 minutes, then Chris will travel 30 blocks. Since Sam rides 3 blocks for every minute, we can multiply the number of minutes Sam rides by 3 to figure out the number of blocks. For example, if Sam rides for 6 minutes, Sam will travel 18 blocks.
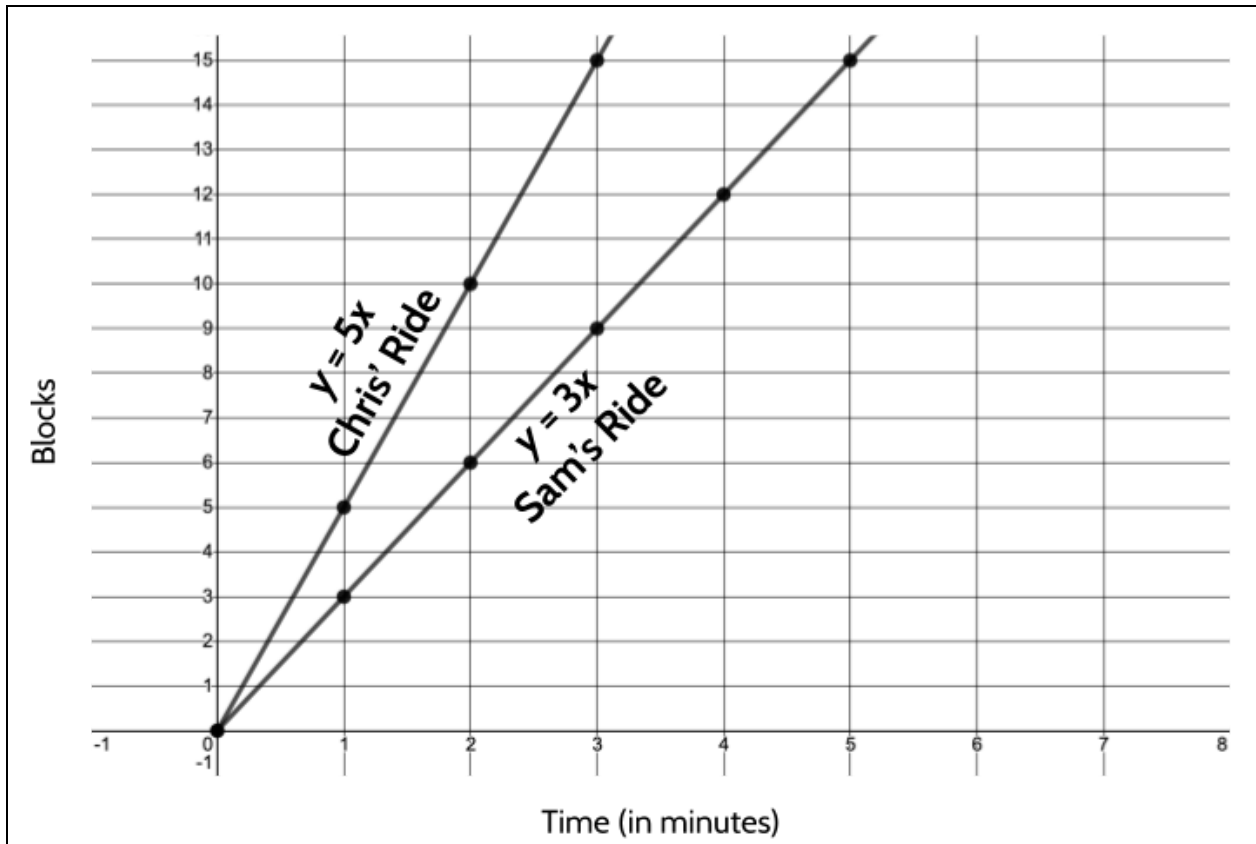
Both riders start at the same place. At 0 minutes, they have both traveled 0 blocks. After 1 minute of riding, Chris has traveled 5 blocks and Sam has traveled 3 blocks. After 1 minute, Chris is two blocks ahead of Sam.

21) After two minutes, how big is Chris' lead?

22) After three minutes, how long is Chris' lead?

Every minute that they race, Chris will continue to get further and further ahead of Sam. When we compare different linear functions on a graph, we can tell which one has a larger rate of change. A function with a larger rate of change means that it changes faster. In this bike example, the line representing Chris's ride is *steeper* than the line that represents Sam's ride. It is steeper because it is changing faster.

> **Core Concept:** The rate of change for any function is sometimes referred to as the *slope*. This is because the rate of change affects the steepness of the graph. The larger the slope, the steeper the line of the function will be. The rate of change of a function is the same thing as its slope.

Finding the Starting Amount in Graphs

Remember that the starting amount is the value of the output when the input is zero. In the race between Chris and Sam, the input was the number of minutes and the output was the number of blocks traveled. In that race, the starting amount for both functions was 0. Let's see what happens if they have a different starting amount.

After Chris wins their first race, Sam wants a head start, since Chris is the faster rider. Chris agrees to let Sam start 3 blocks closer to the finish line.

Who do you think will win the race?

23) Complete the table for Sam's ride. Remember that Sam bikes 3 blocks per minute.

Second Race	
Minutes	Number of Blocks (Sam)
0	3
1	
2	
3	
4	
5	

Notice that at 0 minutes, Sam is already on the 3rd block. That is his head start.

24) The coordinate grid below shows the graph of Chris' ride. Use the ordered pairs from the table above to plot the points for Sam's ride.
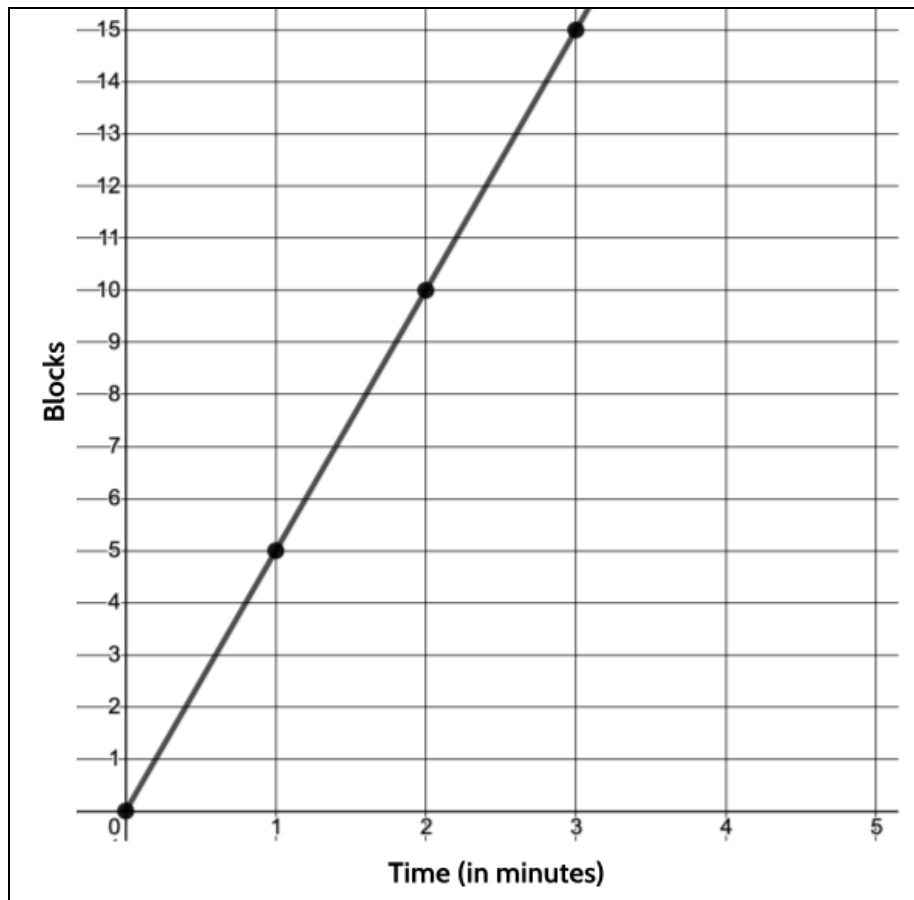
25) How is the graph of Sam's second race *different* from Sam's first race?

26) How is the graph of Sam's second race *similar* to Sam's first race?

27) Who wins the second race? Explain how you know.

28) Sam's ride in the first race can be represented by the function $y = 3x$, where x is the number of minutes and y is the number of blocks traveled. Which of the following function equations represent Sam's ride in the second race?

   A. $y = 6x$
   B. $y = 3x + 3$
   C. $y = 9x$

29) At 1 and a half minutes into the race, the two lines intersect. What do you think happened at that moment during the race?

30) Bonus Question: Even with a 3-block head start, Chris still wins the race. How big of a headstart would Sam need in order for the race to be a tie? Explain your answer.

Since the starting amount is the value of the output when the input is zero, the x value of the starting amount is 0.

31) Plot the following points: (3, 0), (3, 4), (0,5), (0, 0) and (0, -7).
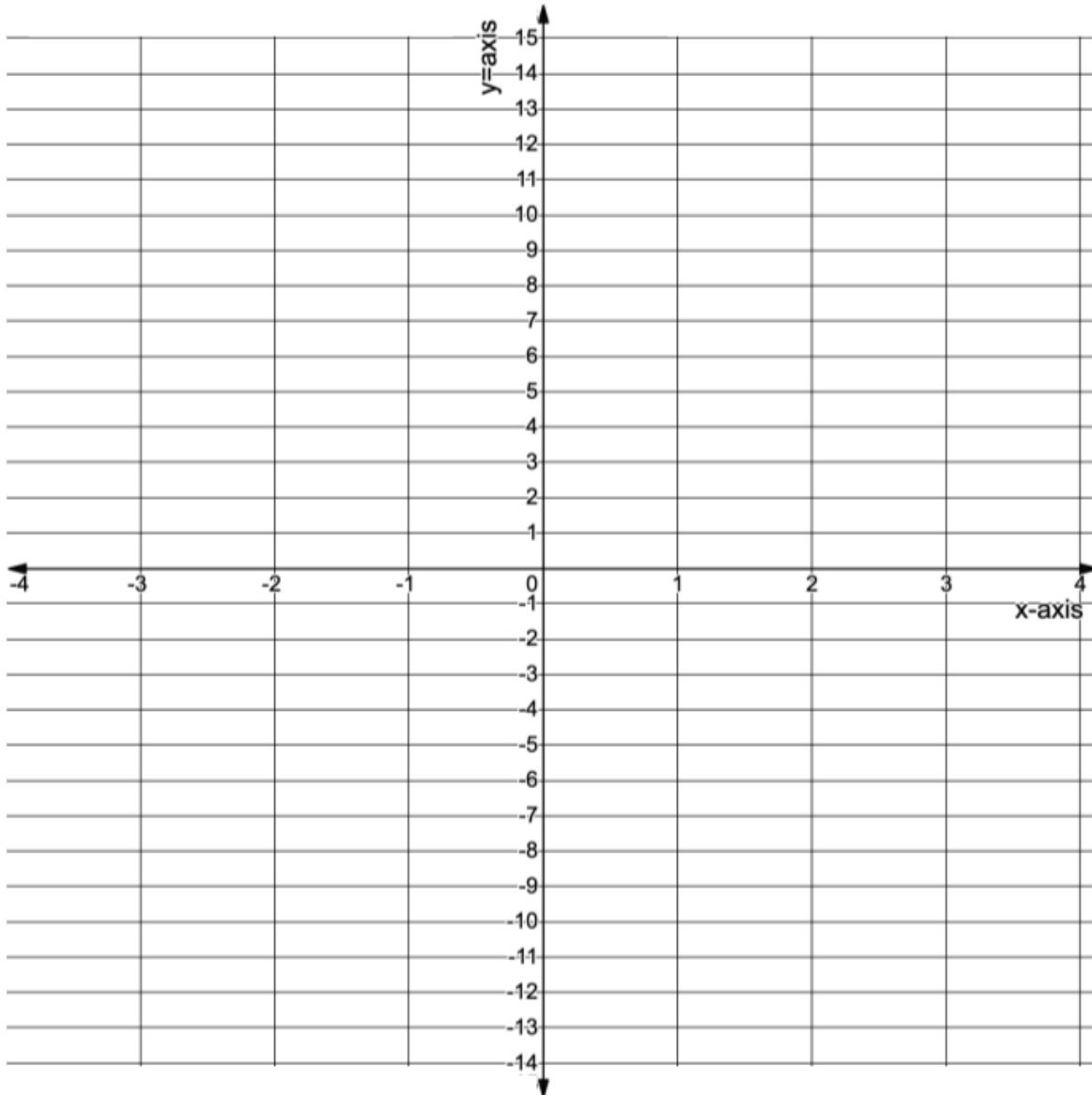
32) What do you notice about the points that have inputs of 0?

When the input, or the x value, of a function is 0, the point will fall on the y-axis. The starting amount for a function is also known as the *y-intercept* because that is the point where the line of the function will cross the y-axis.
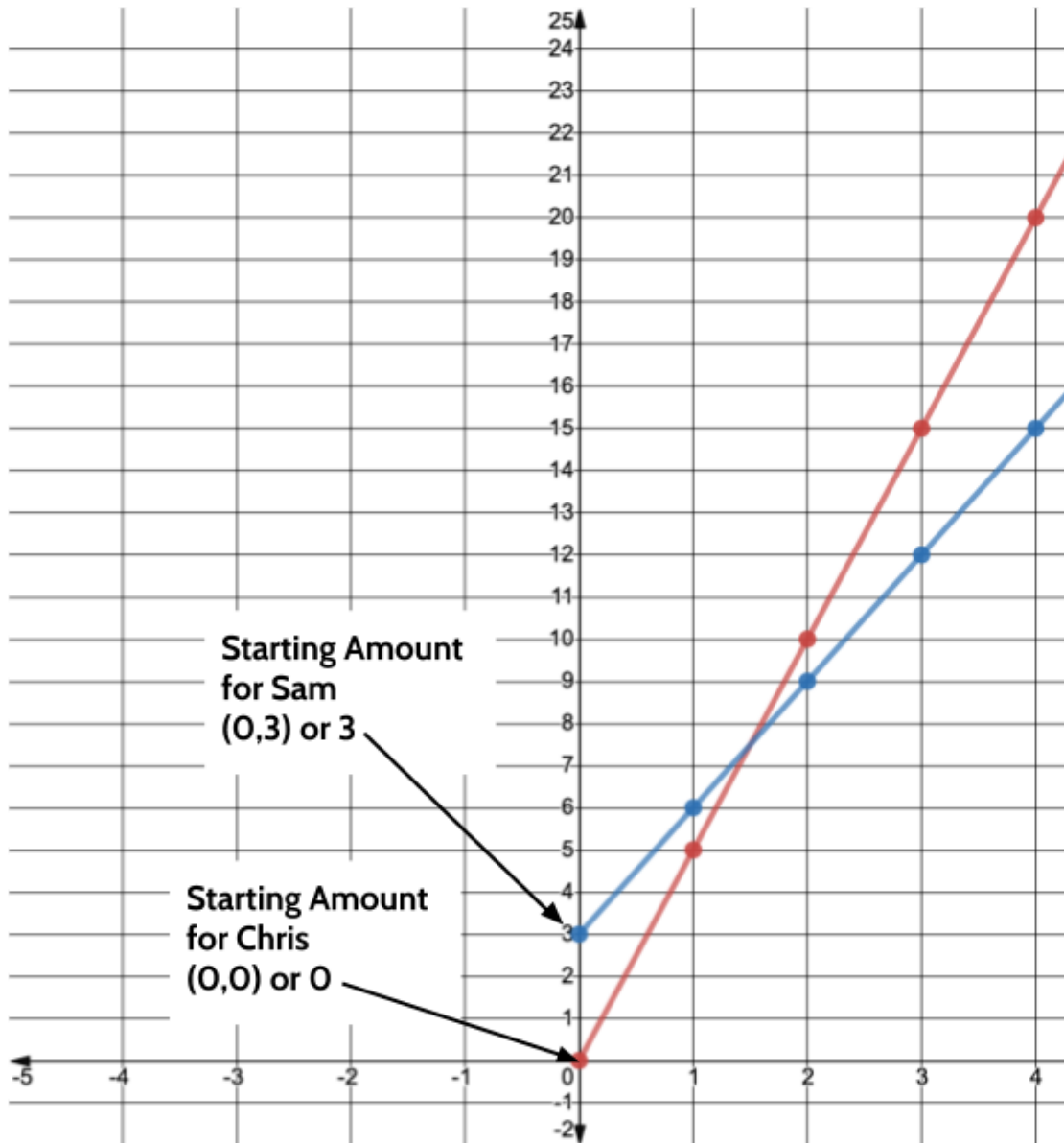
Core Concept: The starting amount of a function is sometimes called the *y-intercept*. This is because in the graph of a linear function, the starting amount is the point where the function intersects with the y-axis.

Linear Functions with the Same Slope

In the example of Sam and Chris' bike race, you compared the graphs of two functions with different slopes (rates of change). The following functions have the same slope but different y-intercepts (starting amounts).

$y = 4x$ $\qquad\qquad\qquad\qquad$ $y = 4x + 2$

$y = 4x + 6$ $\qquad\qquad\qquad\qquad$ $y = 4x - 3$

33) Fill out the following tables.

y = 4x			y = 4x + 2		
x	y	(x,y)	x	y	(x,y)
3			3		
2			2		
1			1		
0			0		
-1			-1		
-2			-2		
-3			-3		

y = 4x + 6			y = 4x - 3		
x	y	(x,y)	x	y	(x,y)
3			3		
2			2		
1			1		
0			0		
-1			-1		
-2			-2		
-3			-3		

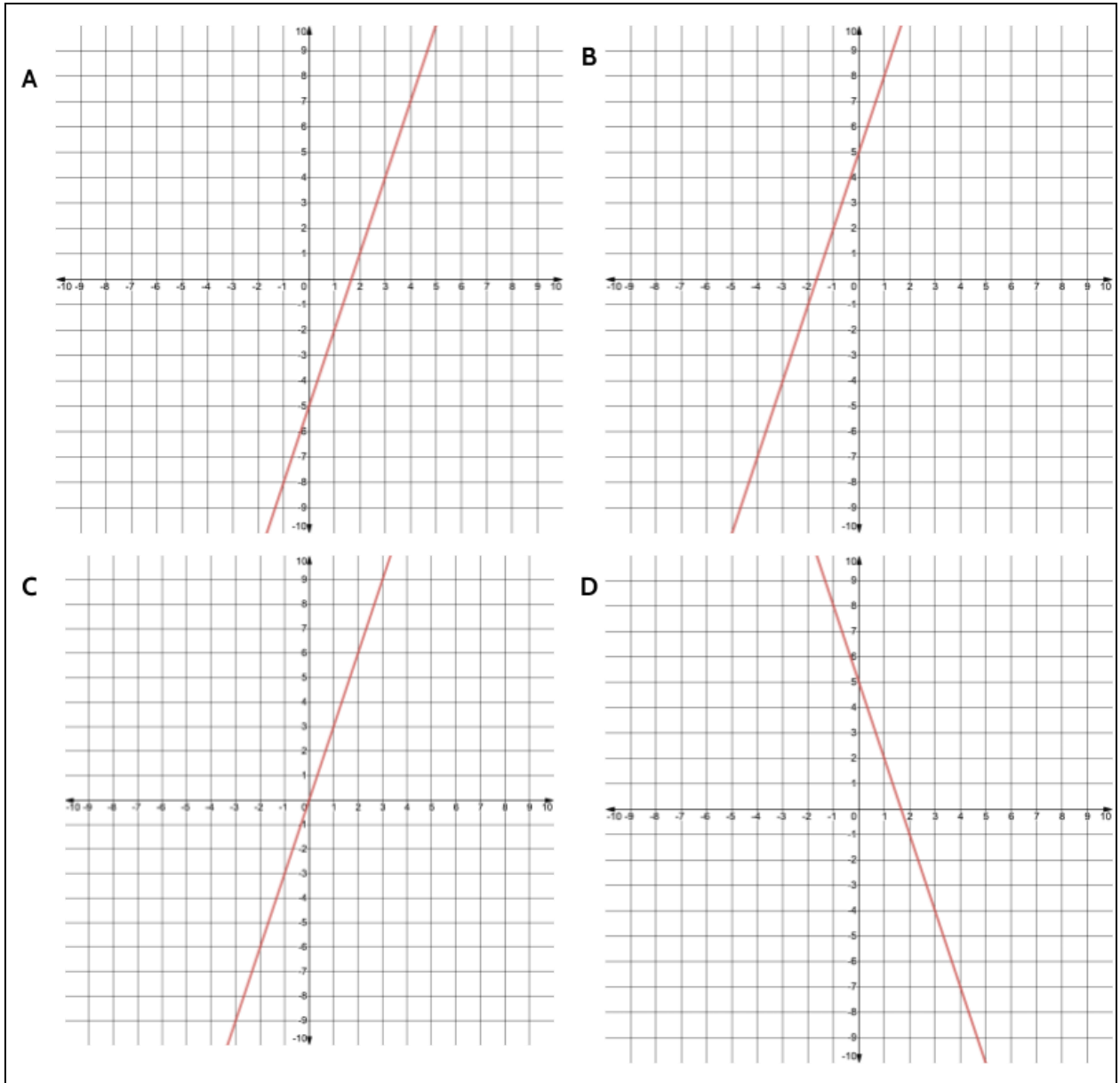34) What is the y-intercept for each of the functions above?

35) Using the tables on the previous page, plot as many points from each function as will fit on the graph. Draw a line connecting the points for each linear function.
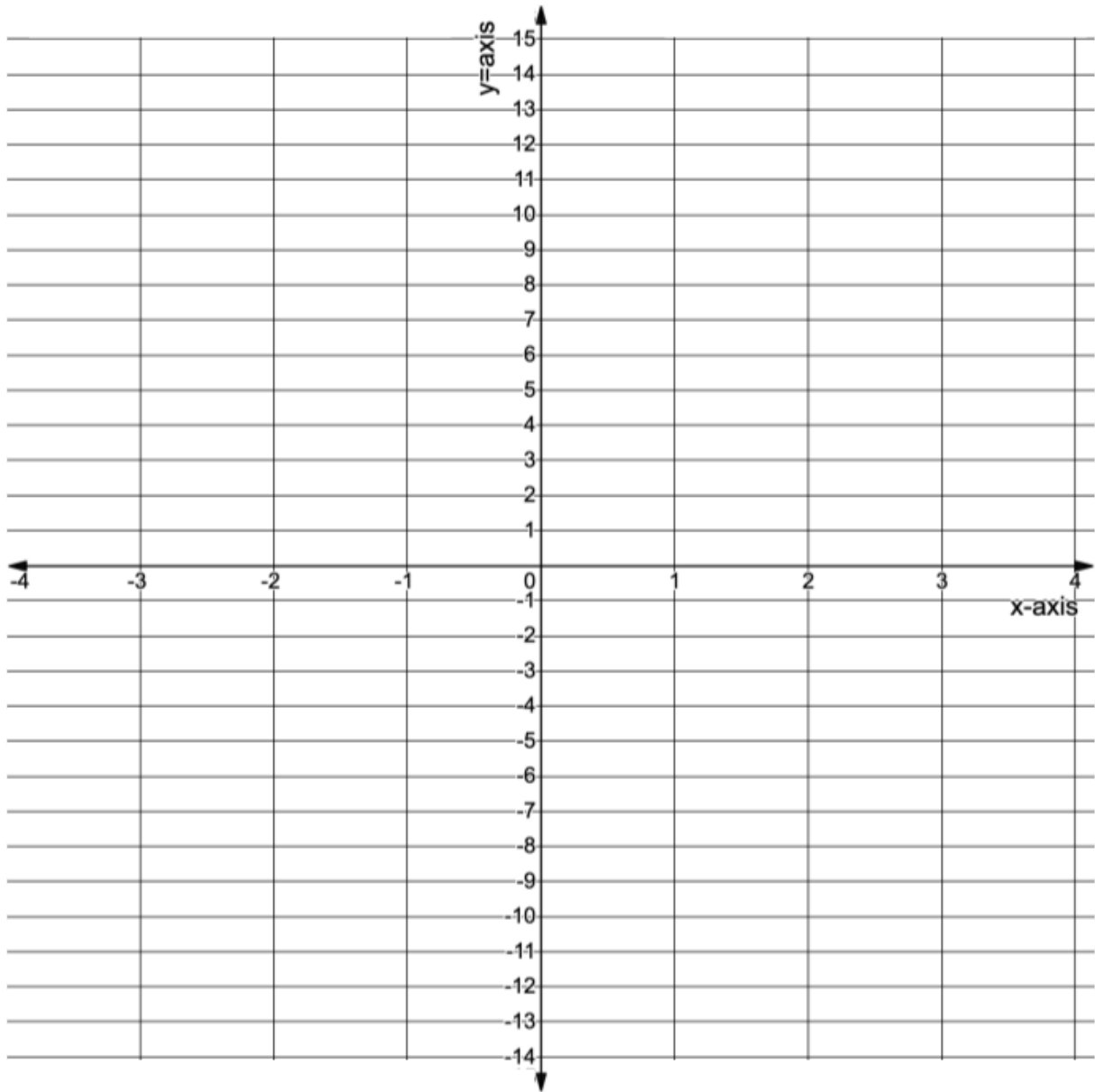
36) What do you notice?

37) Which of the following graphs goes with the function $y = 3x + 5$?

38) Write one thing that makes each graph above different from the other three graphs.

Graph A is the only one that _____.

Graph B is the only one that _____.

Graph C is the only one that _____.

Graph D is the only one that _____.

Rate of Change and Starting Amount in Equations

The rate of change and starting amount can also be found in function equations.

Let's imagine we are going apple picking and we are trying to choose between two orchards: Redding Farm and Cooke Orchard.

Both orchards have a wide selection of quality apples.

- At Redding Farm, they charge $3.00 for each pound of apples.

- At Cooke Orchard, they charge a $5.00 entry fee and $2.00 for each pound of apples.

At Redding Farm there is no entrance fee. The price is $3.00 for each pound of apples you buy.

- If we buy 1 pound of apples, we pay $3.

- If we buy 2 pounds of apples, we pay $6 → $3 + **$3**

- If we buy 3 pounds of apples, we pay $9 → $3 + $3 + **$3**

- If we buy 4 pounds of apples, we pay $12 → $3 + $3 + $3 + **$3**

Here's another way to look at the cost of the first 4 pounds of apples at Redding Farm:

1 pound	2 pounds	3 pounds	4 pounds
$3	$3 + $3	$3 + $3 + $3	$3 + $3 + $3 + $3
1 x 3	2 x 3	3 x 3	4 x 3

The number of pounds changes depending on how many apples we buy, but the price per pound stays the same. It is always $3 per pound. If we multiply the number of pounds by $3, we get the total cost of apples.

We can write an equation with words to represent this relationship:

The total cost = Multiply the number of pounds by 3

We can use variables to represent that same relationship:

$$c = 3p$$

where c represents the total cost and p represents the number of pounds of apples.

At Cooke Orchard, the cost is also determined by the number of pounds of apples you buy.

1 pound	2 pounds	3 pounds	4 pounds
$2	$2 + $2	$2 + $2 + $2	$2 + $2 + $2 + $2
1 x 2	2 x 2	3 x 2	4 x 2
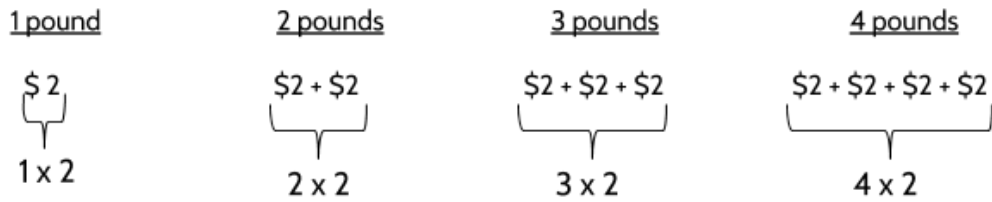
But at Cooke Orchard there is also an entry fee of $5.

39) Complete the chart.

| INPUT | Cost per pound ($2/pound) | Cost for the Pounds of Apples | Entry Fee ($5) | OUTPUT |
Number of Pounds of Apples				Total Cost
0	x 2		+$5.00	
1	x 2		+$5.00	
2	x 2		+$5.00	
3	x 2		+$5.00	
4	x 2		+$5.00	
10	x 2		+$5.00	

Whatever number of pounds you put into the function, you are going to multiply that number by 2 and then add 5.

If you buy 20 pounds of apples → 20(2) + 5 = $45

If you buy 15 pounds of apples → 15(2) + 5 = $35

If you buy 100 pounds of apples → 100(2) + 5 = $205

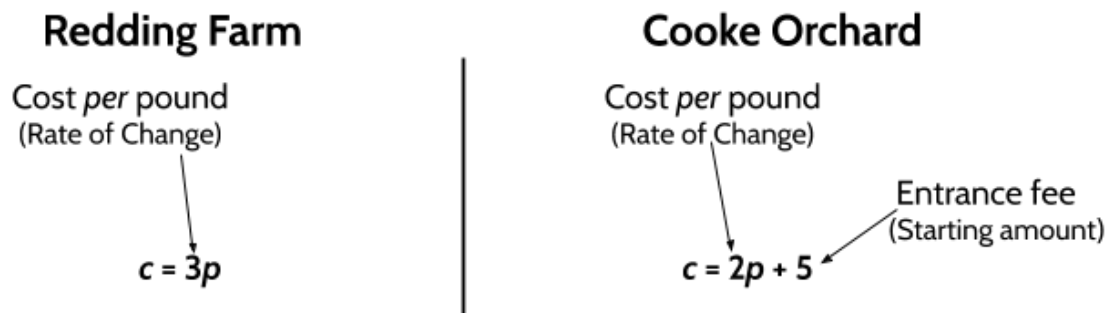We can write an equation with words to represent this relationship:

the total cost = multiply the number of pounds of apples by 2 + $5 for the entry fee.

We can also use variables to represent the input and the output.

$$c = 2p + 5$$

where p represents the number of pounds of apples and c represents the total cost.

Let's look at both function equations.

Redding Farm

Cost *per* pound
(Rate of Change)

$c = 3p$

Cooke Orchard

Cost *per* pound
(Rate of Change)

Entrance fee
(Starting amount)

$c = 2p + 5$

In the function equations for both orchards, the rate of change is the cost per pound. The starting amount is the entrance fee—for Redding Farm it is 0 and at Cooke Orchard it is 5.

40) If you wanted to buy 3 pounds of apples, which orchard is cheaper? Explain your answer.

41) If you wanted to buy 8 pounds of apples, which orchard is cheaper? Explain your answer.
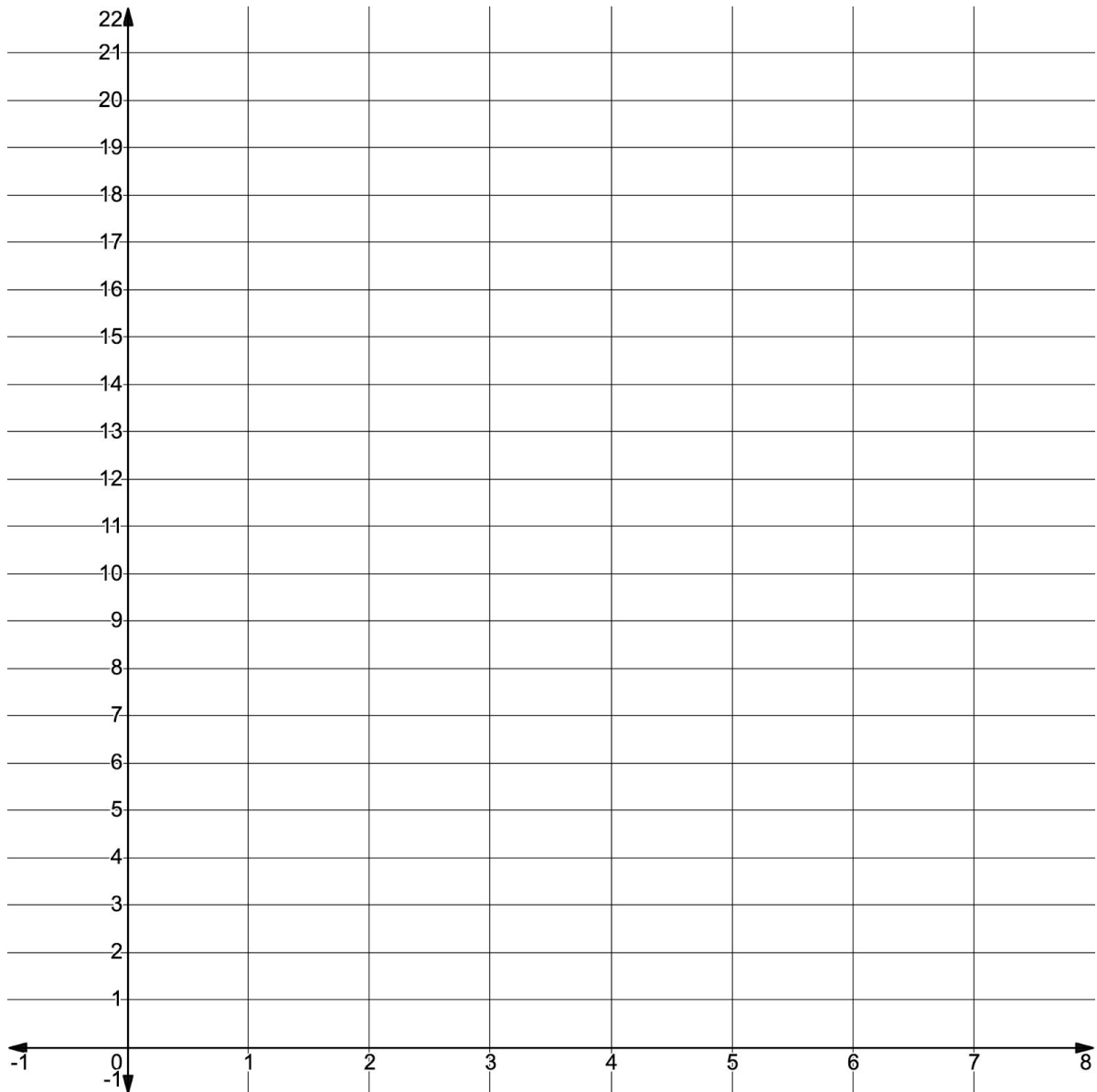

Sometimes it is cheaper to go to Redding Farm and sometimes it is cheaper to go to Cooke Orchard.

42) Complete the tables.

Redding Farm		Cooke's Orchard	
$c = 3p$		$c = 2p + 5$	
Pounds of Apples	Cost	Pounds of Apples	Cost
0		0	5
1		1	
2		2	
3		3	
4		4	
5		5	
6		6	
7		7	
8		8	

What do you notice?

43) Use the tables to graph the functions for Redding Farm and Cooke Orchard below.

44) If a friend wanted to buy several pounds of apples and asked you which orchard is cheaper, what would you tell them?

1)

y = 2x + 1	
x	*y*
1	3
2	5
3	7
4	9
5	11
6	13

y = 2x + 4	
x	*y*
1	6
2	8
3	10
4	12
5	14
6	16

2) Some things the tables have in common: They both have inputs 1, 2, 3, 4, 5, 6. They both have a 2 in their equation. The outputs are going up by 2. Both equations use *x* for the inputs and *y* for the outputs.

3) Some ways the tables are different: The table on the left has odd outputs, and the table on the right has even outputs. The function on the left has an equation with a +1, and the one on the right has an equation with a +4.

4)
   a) The rate of change is 5
   b) The rate of change is 1
   c) The rate of change is 12
   d) The rate of change is 34
   e) The rate of change is .8

5)

y = 5x + 4	
x	y
0	4
1	9
2	14
3	19
4	24
5	29

Rate of Change is 5

y = 3x + 4	
x	y
0	4
1	7
2	10
3	13
4	16
5	19

Rate of Change is 3

6) Some ways the tables are similar: Both equations have a +4. The ordered pair (0, 4) works for both functions. The inputs are 0, 1, 2, 3, 4, 5.

7) Some ways the tables are different: They have different rates of change. They have different outputs (except 4 and 19).

8)
   a) The starting amount is 13

   b) The starting amount is 7. The table does not have an input of 0, but it does have consecutive inputs. We can use those inputs to find the rate of change, which is 5. If an input of 1 results in an output of 12, we can use the rate of change to work backwards.

   c) The starting amount is 125

9) The rate of change for this function is 6. You can tell because the 6 is multiplying the input (x).

10) The starting amount is 3. If you put a 0 into the function, a 3 comes out.

11) The Rate of Change is 6. The Starting Amount is 3.

x	y
0	3
1	9
2	15
3	21
4	27

12)

    <u>Top Left</u>: Rate of Change: 4   Starting amount: 6   Equation: $y = 4x + 6$

    <u>Bottom Left</u>: Rate of Change: 2   Starting amount: 6   Equation: $y = 2x + 6$

    Did you think that both functions on the left were the same? That is a very common mistake to make. Look back at the two tables and see if you can see why they are different.

    <u>Top Right</u>: Rate of Change: 3   Starting amount: 7   Equation: $y = 3x + 7$

    <u>Bottom Right</u>: Rate of Change: 4   Starting amount: 9   Equation: $y = 4x + 9$

13)

Sam			Chris		
Time (in minutes)	Block	Ordered Pair	Time (in minutes)	Block	Ordered Pair
x	y	(x, y)	x	y	(x, y)
0	0	(0, 0)	0	0	(0, 0)
1	3	(1, 3)	1	5	(1, 5)
2	6	(2, 6)	2	10	(2, 10)
3	9	(3, 9)	3	15	(3, 15)
4	12	(4, 12)	4	20	(4, 20)
5	15	(5, 15)	5	25	(5, 25)

14) 3

15) 5

16) Chris won the race. He bikes at a faster speed. The race was 15 blocks long. Sam finished in 5 minutes and Chris finished in 3 minutes.

17)
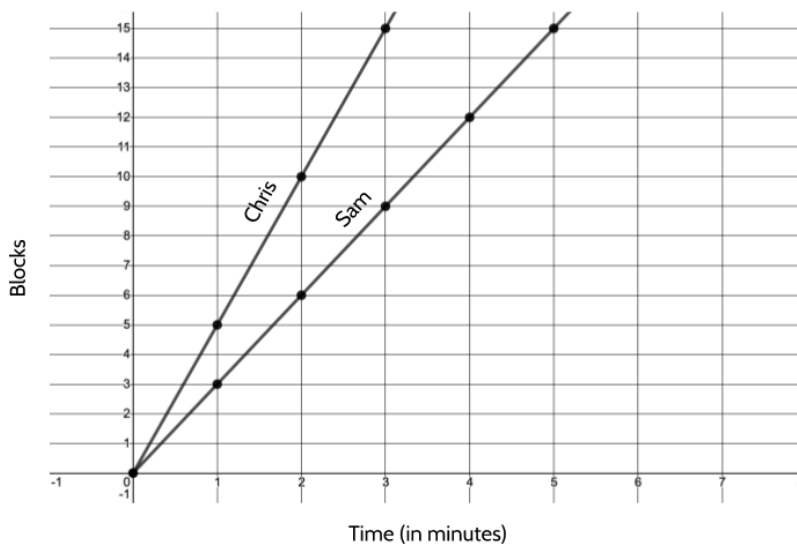
18) Some things you might have noticed: Both lines start in the origin. Chris' line is going up faster than Sam's line. Sam finished 2 minutes after Chris. Sam was at the 9th block when Chris finished the race. Sam's line has 6 points and Chris' line has 4 points.

19) Some things you may have noticed: To go from point to point, you can move 1 space to the right and 5 spaces up. The points are spread out evenly. For every 1 minute in the race, Chris traveled 5 blocks.
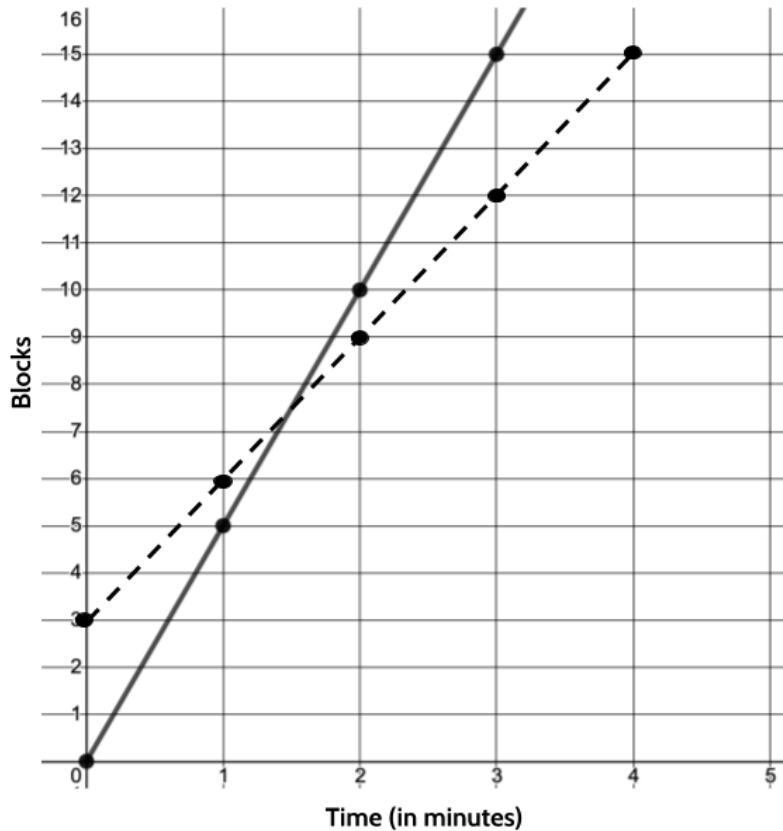
20)
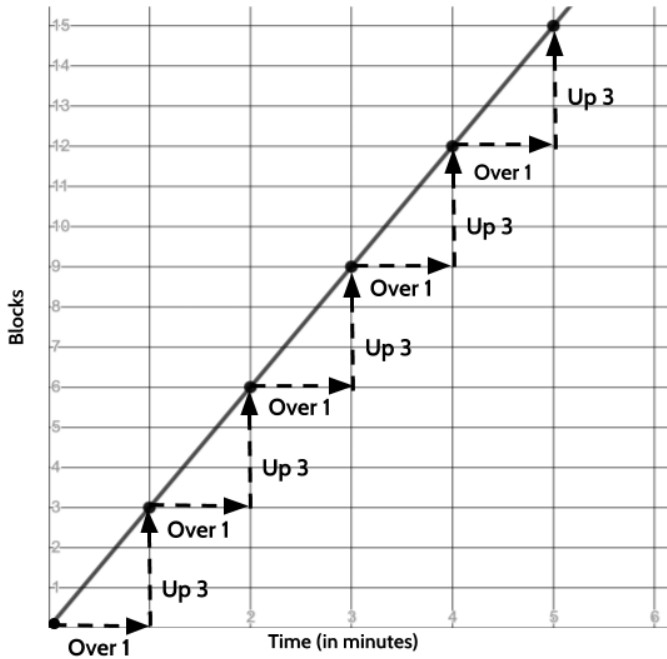
21) 4 blocks

22) 6 blocks

23)

Second Race	
Minutes	Number of Blocks (Sam)
0	3
1	6
2	9
3	12
4	15
5	18

24)

25) Some differences: The line doesn't go through the origin. The line starts at (0, 3). Sam's line and Chris' line intersect at 1 and a half minutes. Sam finishes the second race in 4 minutes.

26) Some similarities: For every minute, Sam travels 3 blocks.

27) Chris wins the second race. Chris rides 15 blocks in 3 minutes. Even with a head start, Sam still takes 4 minutes to finish the race.

28) Choice B.

29) Sam started off with a head start so he was 3 blocks ahead when the race began. After 1 minute, Sam was only 1 block ahead of Chris. At 1 and a half minutes, Chris caught up and passed Sam. 2 minutes into the race and Chris was 1 block ahead of Sam.

30) Chris finishes the race in 3 minutes. Sam would need to start on the 6th block to be able to tie Chris.

31)
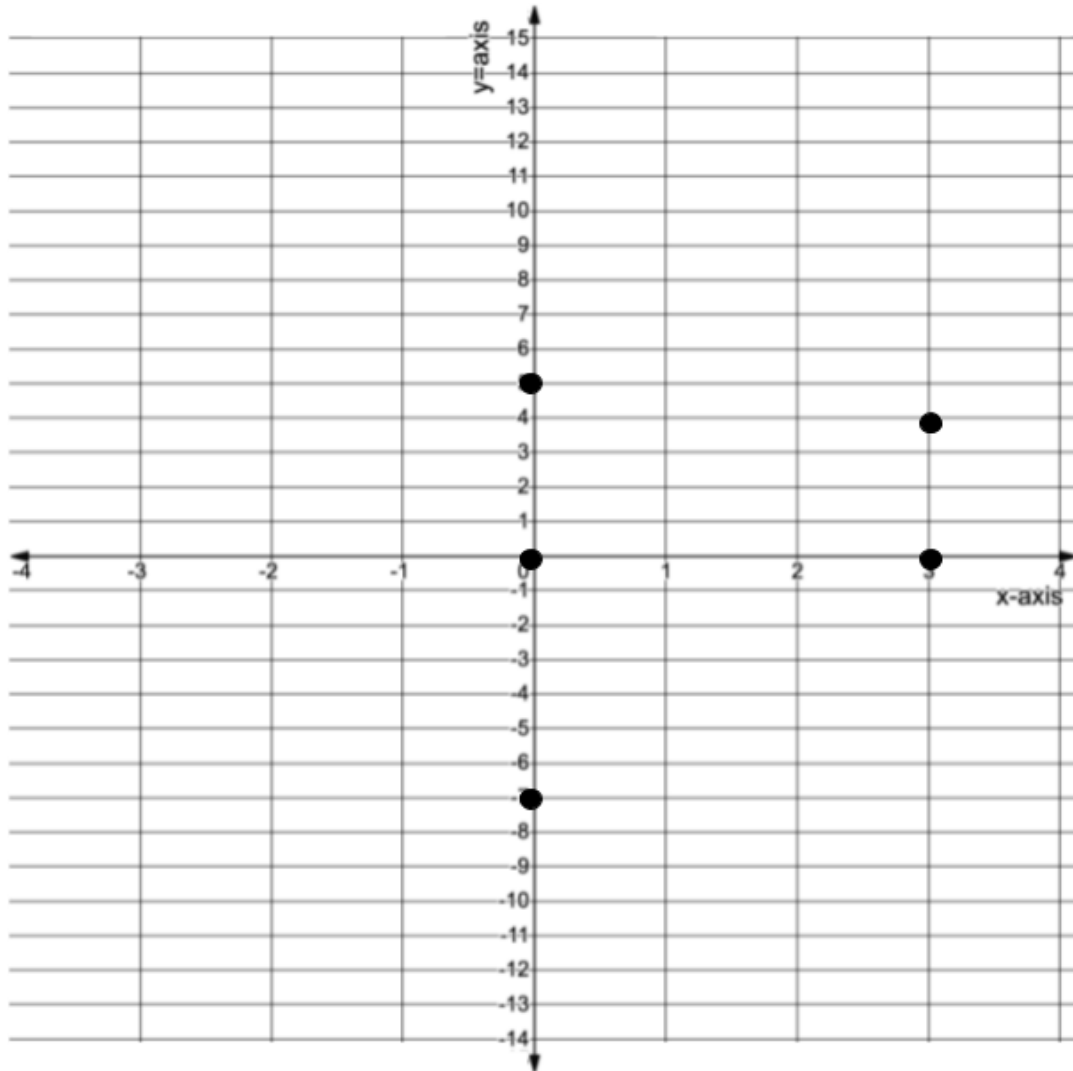
32) The three points with inputs of 0 are (0, 5), (0, 0) and (0, -7).
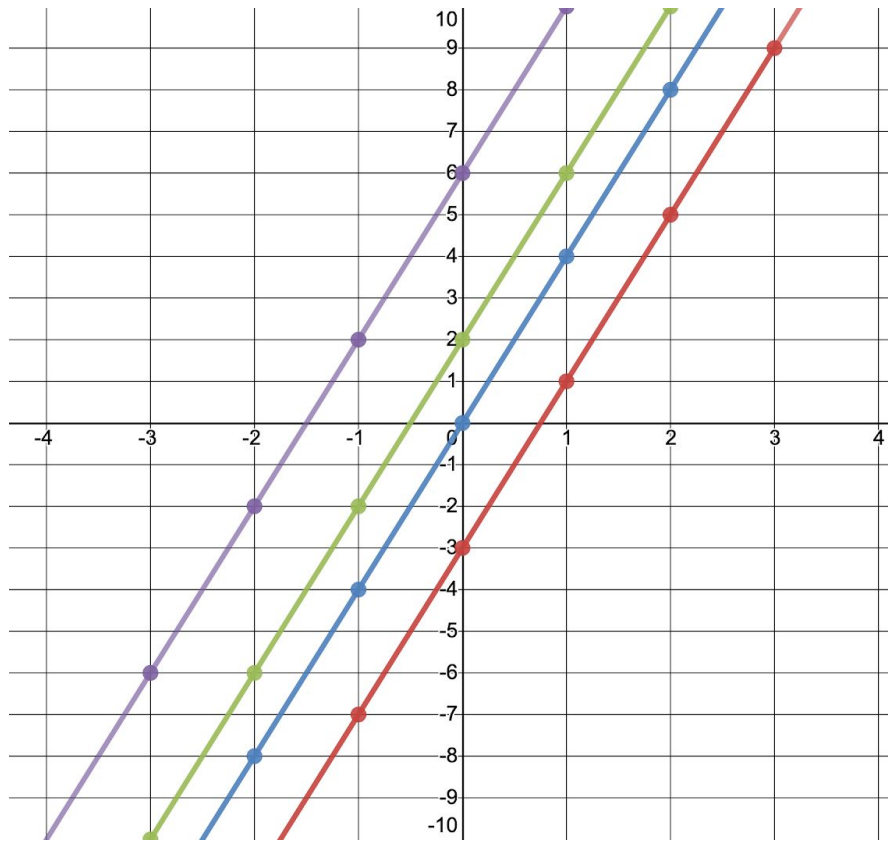All three points are on the y-axis.

33)

y=4x		
x	y	(x,y)
3	12	(3, 12)
2	8	(2, 8)
1	4	(1, 4)
0	0	(0, 0)
-1	-4	(-1, -4)
-2	-8	(-2, -8)
-3	-12	(-3, -12)

y=4x+2		
x	y	(x,y)
3	14	(3, 14)
2	10	(2, 10)
1	6	(1, 6)
0	2	(0, 2)
-1	-2	(-1, -2)
-2	-6	(-2, -6)
-3	-10	(-3, -10)

y=4x+6		
x	y	(x,y)
3	18	(3, 18)
2	14	(2, 14)
1	10	(1, 10)
0	6	(0, 6)
-1	2	(-1, 2)
-2	-2	(-2, -2)
-3	-6	(-3, -6)

y=4x-3		
x	y	(x,y)
3	9	(3, 9)
2	5	(2, 5)
1	1	(1, 1)
0	-3	(0, -3)
-1	-7	(-1, -7)
-2	-11	(-2, -11)
-3	-15	(-3, -15)

34) The y-intercept for $y = 4x$ is (0, 0)
The y-intercept for $y = 4x + 2$ is (0, 2)
The y-intercept for $y = 4x + 6$ is (0, 6)
The y-intercept for $y = 4x - 3$ is (0, -3)

35)

36) The lines are all *parallel* to each other - if you continue the lines, they will never intersect. All of the lines have the same rate of change (4). All of the lines cross the y-axis in different places.

37) Choice B. If you put a 0 into the function, you learn that the starting amount (also known as the y-intercept) is 5. That means the line of the function will cross through the point (0, 5). That eliminates Choices A and C. If you put a 1 into the function, the result is an output of 8, so the point (1, 8) must also appear on the line. That eliminates Choice D.

38) There are many correct ways to complete the statements. Here are a few possibilities:

Graph A is the only one that has a negative y-intercept.

Graph B is the only one with a positive slope and a y-intercept of 5.

Graph C is the only one that passes through the origin.

Graph D is the only one that has a negative slope.

39)

| INPUT	Cost per pound ($2/pound)	Cost for the Pounds of Apples	Entry Fee ($5)	OUTPUT
Number of Pounds of Apples				Total Cost
0	x 2	0	+$5.00	5
1	x 2	2	+$5.00	7
2	x 2	4	+$5.00	9
3	x 2	6	+$5.00	11
4	x 2	8	+$5.00	13
10	x 2	20	+$5.00	25

40)   3 pounds of apples are cheaper at Redding Farm.

41) 8 pounds of apples are cheaper at Cooke Orchard.

42)

Redding Farm		Cooke's Orchard	
$c = 3p$		$c = 2p + 5$	
Pounds of Apples	Cost	Pounds of Apples	Cost
0	0	0	5
1	3	1	7
2	6	2	9
3	9	3	11
4	12	4	13
5	15	5	15
6	18	6	17
7	21	7	19
8	24	8	21

43)
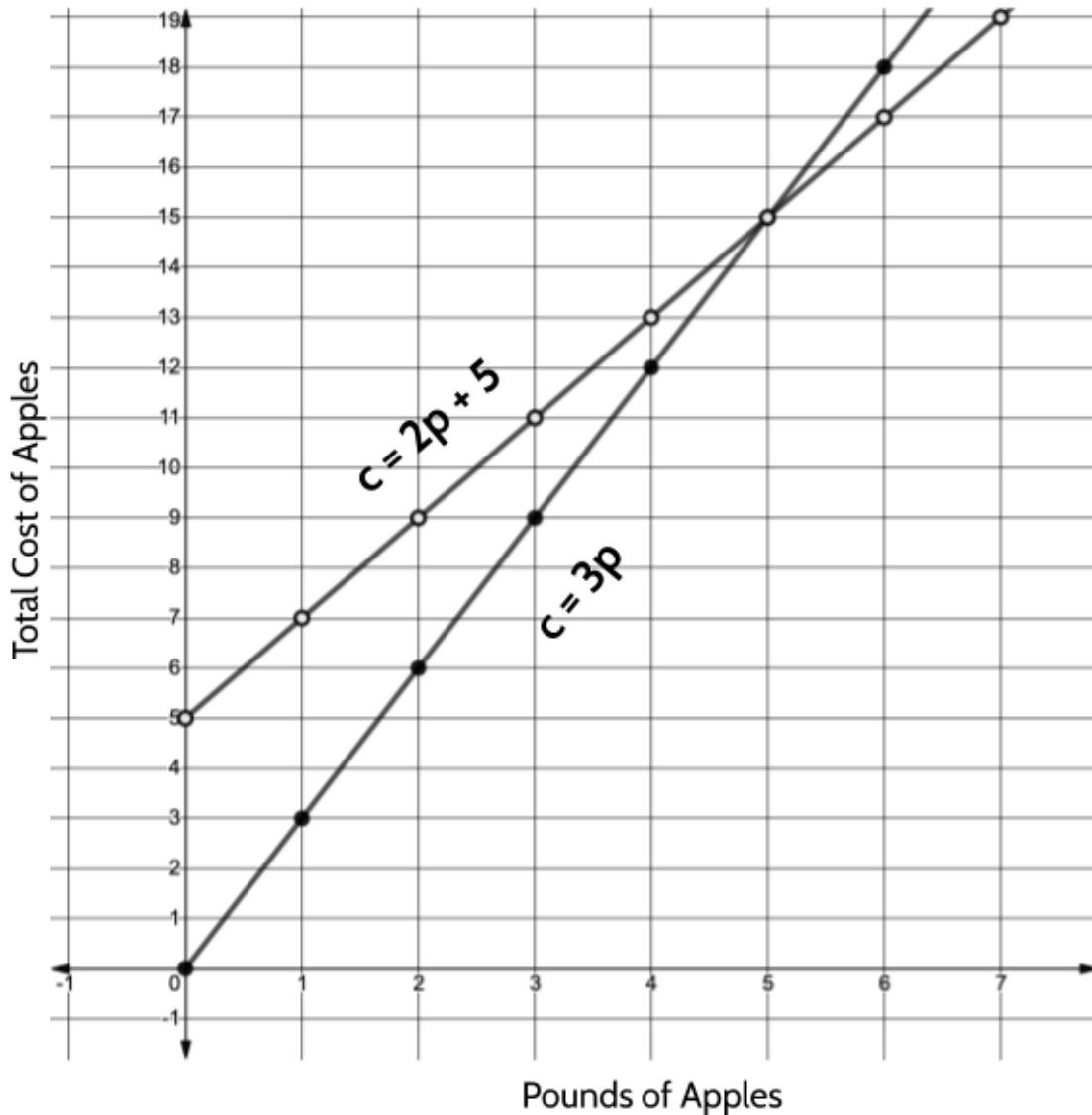
Pounds of Apples

44) There are many possible answers to this question. You could tell your friend that if they buy 5 pounds it doesn't matter where they go, because 5 pounds of apples cost the same at both. If they want to buy less than 5 pounds, they should go to Redding Farm. If they want to buy more than 5 pounds, they should go to Cooke Orchard.

Functions in Our World

In this section, you will explore several real-world situations using linear functions. You will use the skills you have developed to explore temperature conversion, an electric bill, thunderstorms, and a model medical trial.

Temperature

This table shows the highest recorded temperatures for several cities across New York State.

CITY	HIGHEST RECORDED TEMPERATURE
Rochester, NY	38°
Buffalo, NY	37°
Syracuse, NY	38°
Binghamton, NY	34°
Albany, NY	40°
Newburgh, NY	39°
New York City, NY	41°
Montauk, NY	32°

1) What do you notice?

One thing you might have noticed is that, for someone from the United States, the highest recorded temperatures in each of those cities seem very low! That is because the high temperatures are given in the temperature scale called *Celsius*. The way to write a temperature in Celsius is to write the measurement followed by °C. The way we read 24° C is *"24 degrees Celsius."*

The Celsius scale was developed around 1744 and is the measure of temperature used in every country in the world except for the United States, the Bahamas, Belize, the Cayman Islands, and Palau. Those countries use an older temperature scale called *Fahrenheit*. The Fahrenheit scale was created in 1724 by Daniel Gabriel Fahrenheit. Twenty-four degrees Fahrenheit would be abbreviated as 24° F. The temperatures 24° C and 24° F are very different!

The table shows the highest temperatures for the same cities across New York State, given in Fahrenheit.

Most thermometers use both scales when displaying the temperature. We can make observations to learn about the relationship between Celsius and Fahrenheit.

CITY	HIGHEST RECORDED TEMPERATURE (in °Fahrenheit)
Rochester, NY	100°
Buffalo, NY	99°
Syracuse, NY	100°
Binghamton, NY	93°
Albany, NY	104°
Newburgh, NY	102°
New York City, NY	106°
Montauk, NY	90°

2)   Look closely at this thermometer. What do you notice?

One important thing to notice is that, for every temperature in Fahrenheit, there is an equivalent temperature in Celsius. For example, 10° C is the same temperature as 50° F. There is one equivalent temperature in Fahrenheit for every temperature in Celsius.

You may be familiar with a few equivalent temperatures. For example, 0° C and 32° F are the same temperature. This is the temperature at which water freezes. 100° C and 212° F are another example of two temperatures that are equivalent. This is the temperature of boiling water[1].

Let's take a look at some other equivalent temperatures.

CELSIUS	FAHRENHEIT
-20° C	-4° F
-10° C	14° F
0° C	32° F
10° C°	50° F
20° C	68° F
30° C	86° F
40° C	104° F
50° C	122° F
60° C	140° F
70° C	158° F
80° C	176° F
90° C	194° F
100° C	212° F

3) What patterns do you notice?

---

[1] 100° C and 212° F is the temperature at which water boils <u>at sea level</u>. At higher altitudes, water boils at lower temperatures. For example, at an altitude of 6,000 feet, water boils at 94° C or 201.1° F.

As the Celsius temperature goes up by *10* degrees, the temperature in Fahrenheit goes up by 18 degrees.

	CELSIUS	FAHRENHEIT	
	0° C	32° F	
+10	10° C	50° F	+18
+10	20° C	68° F	+18
+10	30° C	86° F	+18

4) Is 18 the rate of change in the Celsius to Fahrenheit function? Explain your answer.

Up to now, when we have looked at the rate of change in function tables, we have focused on *consecutive* inputs. But in this table, the inputs (the Celsius temperatures) are *not* consecutive since they go up by 10 instead of 1.

Even when the inputs are not consecutive, we can still determine the rate of change. To see how this works, let's try making the inputs consecutive, by filling in the missing inputs and outputs.

In the table, we can see that 0 degrees Celsius is equivalent to 32 degrees Fahrenheit. And if the temperature in Celsius goes up by 10 degrees, the temperature in Fahrenheit goes up 18 degrees, from 32 to 50 degrees.

5) Fill in the missing outputs in the table below.

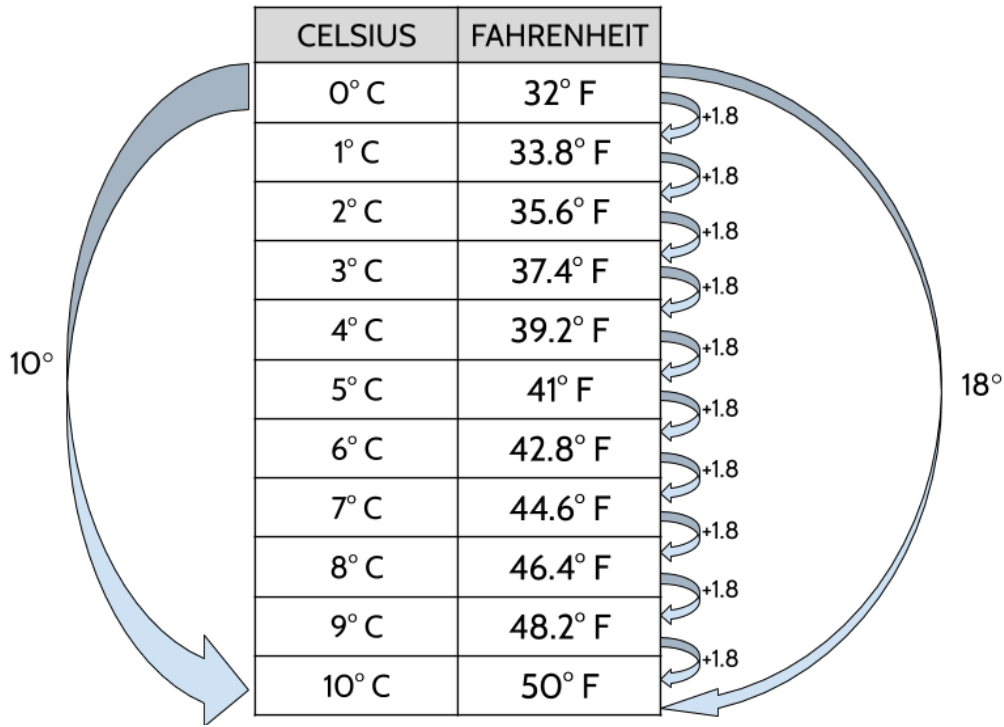| CELSIUS	FAHRENHEIT
0° C	32° F
1° C	
2° C	
3° C	
4° C	
5° C	
6° C	
7° C	
8° C	
9° C	
10° C	50° F

<Hint: If you start at 32 and go up by 1 degree, you won't reach 50. If you start at 32 and the outputs go up by 2 degrees, you will end up higher than 50.>

The approach below may remind you of the 0 to 70 number patterns you worked on in **Tools of Algebra: Linear Functions, Part 1**. To go from an output of 32 to an output of 50, we need to go up 18 degrees in total. The inputs increase by 10, so we have 10 "jumps" to go up 18.

CELSIUS	FAHRENHEIT
0° C	32° F
1° C	
2° C	
3° C	
4° C	
5° C	
6° C	
7° C	
8° C	
9° C	
10° C	50° F

10°   18°
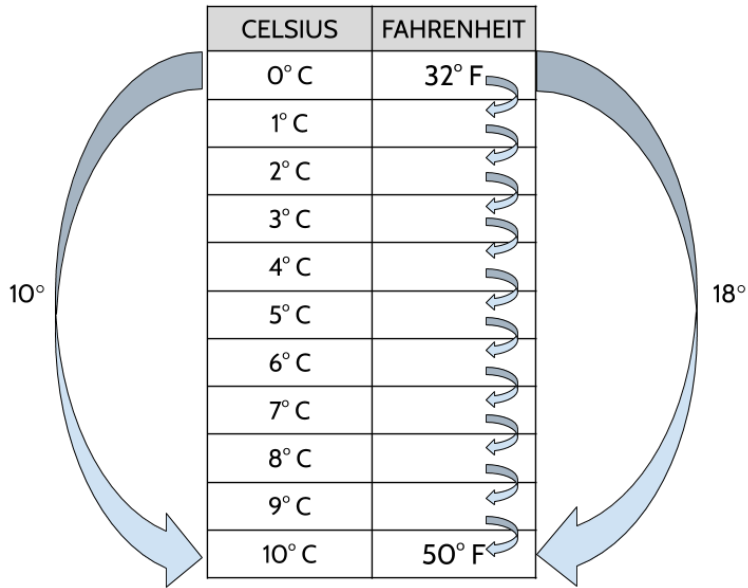
So how big is each jump?

If we divide 18 up into 10 equal jumps, we get 1.8 for each jump.

CELSIUS	FAHRENHEIT	
0° C	32° F	+1.8
1° C	33.8° F	+1.8
2° C	35.6° F	+1.8
3° C	37.4° F	+1.8
4° C	39.2° F	+1.8
5° C	41° F	+1.8
6° C	42.8° F	+1.8
7° C	44.6° F	+1.8
8° C	46.4° F	+1.8
9° C	48.2° F	+1.8
10° C	50° F	

10°   18°

We can use this strategy to find the rate of change in any function table, even if the inputs are not consecutive.

Finding the Rate of Change when the Inputs are Not Consecutive

Step 1: **Find the difference between two outputs.** (This will tell you how much the outputs have changed.)

Step 2: **Find the difference in the inputs.** (This will tell you the number of "jumps")

Step 3: **Divide the difference between the outputs by the difference between the inputs.**

6) Find the rate of change for the following functions:

a)

x	f(x)
20	195
30	285
40	375
60	555

b)

x	y
20	325
26	430
22	360
30	500

c)

x	y
16	138
29	242
42	346
68	554

7) We can create a function equation to find equivalent temperatures in Celsius and Fahrenheit. In each of the following equations, F represents the temperature in Fahrenheit and C represents the temperature in Celsius.

Which of the following function equations can be used to convert a temperature in Celsius to a temperature in Fahrenheit?

   A. $F = 1.8C + 32$

   B. $F = \frac{1}{8}C + 32$

   C. $F = 32C + 1.8$

   D. $F = 1.8C - 32$

   E. $C = 1.8F + 32$

8) The temperature one day last fall in Santiago, Chile was 25 degrees Celsius. What is the equivalent temperature in Fahrenheit?

Santiago, Chile
Monday
Partly Cloudy

25 °C

Electricity Bills

The monthly bills we receive for basic utilities like telephone, gas, and power often involve functions. For example, the electricity bill from your power company is calculated using a function rule. Here's an example of how your bill might look:

Elektro Power

This month's electricity charges

These charges are based on a 30-day period of electricity usage. Your power usage is measured in the number of kilowatt hours (kWh) you used in the past month.
For reference, one kWh powers a 10-watt light bulb for 100 hours.

Usage this month

June 15th meter reading	2600 kWh
May 15th meter reading	- 2450 kWh
Electricity usage	150 kWh

Electricity usage rate $.20/kWh

Calculating your bill

Service charges *This includes electricity service, meter reading, and taxes.*	$18.00
Usage charges @ $.20/kWh *This is the charge for the electricity supplied by Elektro Power.*	$30.00
Total electricity charges	**$48.00**

9) Look carefully at the bill above. What do you notice?

The monthly bill from Elektro Power is calculated with the following formula:

Total electricity charges = Electricity usage × Electricity usage rate + Service charges

The formula above uses words so that we can see how the calculations are done. We will substitute numbers and variables later to make it look like other algebraic formulas.

We know that the electricity usage rate is $0.20/kWh (20 cents per kilowatt hour). This rate will be the same no matter how much electricity we use. We also know that the service charges of $18.00 will be the same for different amounts of electricity usage.

10) Let's use a table to see how the charges are calculated for different amounts of electricity usage. Fill in the blanks.

Electricity usage	Electricity usage rate (per kWh)	Usage charges	Service charges	Total electricity charges
0	0.20		18	
50	0.20		18	
100	0.20	20	18	
150	0.20	30	18	$48.00
200	0.20		18	
125	0.20		18	
175	0.20		18	
	0.20		18	$24.00
210	0.20		18	
	0.20	36	18	
	0.20		18	82

11) Which quantities change in the table above? Check off the numbers that change.

- ❏ Electricity usage
- ❏ Electricity usage rate
- ❏ Usage charges
- ❏ Service charges
- ❏ Total electricity charges

12) Which quantities do <u>not</u> change in the table above?

- ❏ Electricity usage
- ❏ Electricity usage rate
- ❏ Usage charges
- ❏ Service charges
- ❏ Total electricity charges

The *Electricity usage rate* and *Service charges* are called *constants* since they don't change. The word *constant* means to stay the same. A *constant companion* is a friend who is always there for you. In mathematics, constants are the part of an equation that don't change when different numbers are substituted for the variables. In this situation, the constants stay the same for different amounts of electricity used.

We could rewrite the formula with the constants 0.20 and 18 included as numbers:

$$Total\ electricity\ charges = Electricity\ usage \cdot 0.20 + 18$$

Total electricity charges and *Electricity usage* are different for each electricity bill and are considered variables, since they *vary* or change. *Usage charges* also change with each bill, but isn't a separate variable since it is a combination of *Electricity usage* and *Electricity usage rate*.

For a simplified algebraic formula, let's use c instead of *Total electricity charges* and replace *Electricity usage* with u:

$$c = 0.20u + 18$$

13) Carthian used 250 kWh of electricity. How much would her user charges be?

14) Mauricio used 72 kWh of electricity. How much would his total bill be?

15) Simone was charged $42.60 in total electricity charges. How many kilowatt-hours of power did she use?

16) What does u represent in the formula $c = 0.20u + 18$?
    A. The total bill
    B. The amount of power used
    C. The cost per kilowatt hour
    D. The service charges

17) What does 0.20 represent in the formula above?

    A. The total bill
    B. The amount of power used
    C. The cost per kilowatt hour
    D. The service charges

18) Which equation matches the situation below?

    Amy was charged $55 for her electricity bill last month. She used 185 kilowatt hours of electricity.

    A. $55 = .20u + 18$

    B. $c = .20(55) + 18$

19) Write a situation to match the other equation above.

20) Write a situation to explain each equation, then solve for the missing variable.

a) $c = (.20)(165) + 18$

b) $65.50 = .20u + 18$

c) $41.10 = .22u + 18$

d) $c = .20(153) + 23$

The Speed of Light and the Speed of Sound

Have you ever looked up and seen a plane, only to hear the sound of the plane a minute or two later? This happens because light travels faster than sound. How much faster? Light travels at a rate of about 186,000 miles per second. Sound travels much slower. Sound travels at a rate of about 1 mile for every 5 seconds. We can observe the difference between the speed of light and the speed of sound in the world around us.

Thunder is the sound produced by lightning[2], but you may have noticed that you can always see the lightning before you can hear the thunder. The delay between the lightning and the thunder happens because light from the lightning travels to your eyes faster than sound travels to your ears.

When you see a lightning bolt, you can figure out how far away it is by counting the seconds between when you see the lightning and when you hear the thunder. The farther away the storm, the more time there will be between seeing lightning and hearing the thunder.

21) Fill out the table below.

Time after the lightning strike, before you hear the thunder (in seconds)	Distance between lightning strike and observer (in miles)
5	1
10	
15	
20	
1	
2	
3	
	6

---

[2] Image credit: Ron Rev Fenomeno from Pixabay

22) What is the rate of change for this function? Explain your answer.

23) What does the rate of change mean in this context? (Describe the rate of change using the words *"seconds"*, *"miles"* and *"for every"*).

24) Which of the following functions can be used to calculate the distance of a thunderstorm?

    A. Time in seconds x 5 = distance in miles

    B. Time in seconds/5 = distance in miles

    C. Time in seconds + 5 = distance in miles

    D. Time in seconds – 5 = distance in miles

25) What is the approximate speed of sound in miles per hour?

Counting Antibodies

A scientist tested two different asthma medications to see how effective each one was in encouraging the production of antibodies.

The following function can be used to determine the number of antibodies in a blood sample for a patient using Medication A:

$$a = 20d + 100$$

Antibodies are substances produced by your body to fight disease.

where *d* represents the number of days that passed in the experiment, and *a* represents the number of antibodies in a sample of the patient's blood.

26) Complete the following table for the function.

Data for Medication A	
Days Passed	Antibodies in the Sample of Blood
d	*a*
0	
1	
2	
3	
4	
5	

27) How many antibodies were there in the sample after ten days? Show your work.

28) How many days would it take for 360 antibodies to appear in the patient's blood sample? Show your work.

29) How many antibodies were in the patient's blood sample before taking medication A?

30) What is the rate of change for this function?

31) Describe the rate of change for Medication A using the words, "*days*" and "*number of antibodies*".

The scientist tested Medication B and recorded data after following a <u>different</u> patient for several days.

Data for Medication B	
Days Passed	Antibodies in the Sample of Blood
d	*a*
0	60
1	90
2	120
3	150
4	180
5	210
6	240

32) If the growth of antibodies continues at the rate shown in the table, predict the number of antibodies in the blood sample after 12 days.

33) How many days would be needed to produce 600 antibodies in the second patient's blood sample at this same rate?

34) Which function rule represents the data for Medication B, where *d* represents the number of days that passed in the experiment, and *a* represents the number of antibodies in a sample of blood from the second patient.

   A. *a* = 30*d* + 60
   B. *a* = 60*d* + 30
   C. *a* = 3*d* + 60
   D. *a* = 30*d*

35) What is the starting amount for this function?

36) How can the starting amount be interpreted using the context of the problem? In other words, describe the starting amount for this function using the words, "days", "second patient" and "antibodies."

37) What is the rate of change for the function for Medication B?

38) Describe the rate of change for Medication B using the words, "*days*" and "*number of antibodies*".

39) If you had asthma and had both medications to choose from, which one would you choose? Explain your answer.

Answer Key - Functions in Our World

1) Some things you may have noticed: Different cities across NY. It looks like a function table, but there are cities in the input column. The lowest temperature is 32° and the highest is 41°. The temperatures are in a column labeled "Highest Recorded Temperature." These temperatures seem too low to be high temperatures.

2) Some things you may have noticed: The temperatures are in Fahrenheit on the left and Celsius on the right. The Celsius temperatures are labeled by 10s and the Fahrenheit temperatures are labeled by 20s. There are numbers below 0. The Fahrenheit temperatures go from -40° to 120°. The Celsius temperatures go from -40° to 50°. They both start at -40° but the Fahrenheit goes all the way up to 120°.

3) Some patterns you may have noticed: As the Celsius temperatures go up by 10, the Fahrenheit temperatures go up by 18. The difference between each Celsius and its equivalent Fahrenheit is 8 more than the one before it (For example, The difference between 0 and 32 is **32**. The difference between 10 and 50 is **40**. The difference between 20 and 68 is **48**.) The ones digit in the Fahrenheit temperatures repeats the pattern 4, 2, 0, 8, 6, 4, 2, 0, 8, 6,...

4) Even though the outputs are going up by 18, 18 is not the rate of change for this function, because the inputs are not consecutive.

5)

CELSIUS	FAHRENHEIT
0° C	32° F
1° C	33.8° F
2° C	35.6° F
3° C	37.4° F
4° C	39.2° F
5° C	41° F
6° C	42.8° F
7° C	44.6° F
8° C	46.4° F
9° C	48.2° F
10° C	50° F

6)
   a) The rate of change is 9
   b) The rate of change is 17.5
   c) The rate of change is 8

7) Choice A.

8) 77°. If we put 25 into the equation, we see 25°C is equivalent to 77°F →1.8(25)+32 = 77

9) Some of the things you may have noticed: The total charges are $48. More kWh were used in June than in May. There is a rate $0.20/kWh. There is a service charge of $18. One kWh is enough to power a 10-watt light bulb for 100 hours. 2600 kWh is enough to power a 10-watt bulb for 260,000 hours.

10)

Electricity usage	Electricity usage rate	Usage charges	Service charges	Total electricity charges
0	.20	0	18	18
50	.20	10	18	28
100	.20	20	18	38
150	.20	30	18	48
200	.20	40	18	58
125	.20	25	18	43
175	.20	35	18	53
30	.20	6	18	24
210	.20	42	18	60
180	.20	36	18	54
320	.20	64	18	82

11)
- ✔ Electricity usage
- ✔ Electricity usage rate
- ❏ Usage charges
- ✔ Service charges
- ❏ Total electricity charges

12)
- ❏ Electricity usage
- ✔ Electricity usage rate
- ❏ Usage charges
- ✔ Service charges
- ❏ Total electricity charges

13) .20(250) + 18 = **$68**

14) .20(72) + 18 = **$32.40**

15) .20(**123**) + 18 = $42.60. Simone used 123 kWh.

16) Choice B.

17) Choice C.

18) Choice A. The output is the total bill.

19) $c = .20(55) + 18$ would describe a situation where someone used 55 kWh of power.

20)
- A person used 165 kWh of power. Their total bill would be $51.

- A person who had a total bill of $65.50. The amount of power they used was 237.5 kWh.

- A person whose total bill has $41.10. Also, their cost per kWh is $0.22/kWh. The amount of power they used was 105 kWh.

- A person who used 123 kWh of power. Their plan charges $0.20 per kWh and $23 in service charges. Their bill would be $53.60

21)

Time after the lightning strike, before you hear the thunder (in seconds)	Distance between lightning strike and observer (in miles)
5	1
10	2
15	3
20	4
1	.2 or 1/5
2	.4 or 2/5
3	.6 or 3/5
30	6

22) .20 or ⅕.

23) There are different ways to explain the rate of change in this context. Here are two possible explanations:

- The lighting is .20 of a mile away for every second between when you see the lightning and hear the thunder.

- After you see lightning, every second before you hear the thunder means the sound is traveling ⅕ of a mile.

24) Choice B.

25) If sound travels 6 miles in 30 seconds, then it could travel 12 miles in 1 minute. 1 hour is 60 minutes. If sound travels 12 miles for each of those 60 minutes, it would travel 720 miles. So the speed of sound is approximately 720 miles per hour. (The actual speed of sound is 767 miles per hour).

26)

Data for Medication A	
Days Passed	Antibodies in the Sample of Blood
d	a
0	100
1	120
2	140
3	160
4	180
5	200

27) There would be 300 antibodies in the patient after 10 days. 20(10) + 100 = 300

28) There would be 360 antibodies in the patient's blood after 13 days.

29) There were 100 antibodies in the patient's blood. That is the starting amount for this function.

30)   20

31) For every day, the number of antibodies in the patient's blood sample increased by 20.

32) After 12 days, the patient's blood sample would have 420 antibodies in it.

33) 18 days

34) Choice A.

35) 60

36) There were 60 antibodies in the second patient's blood sample at 0 days.

37) 30

38) For every day, the number of antibodies in the second patient's blood sample increased by 30.

39) There are different arguments you could make. One argument in favor of Medicine B is that it has a higher rate of change. Every patient will have a different starting amount of antibodies in their blood sample, but Medicine B increases 30 antibodies a day and Medicine A only increases 20 antibodies a day.

Reviewing What We Know: Many Views of a Function

You have seen four different views of a function: a function described in words, function tables, graphs, and equations. It is important to remember that each of these views of a function is just a different way of representing the same idea. Each representation gives us a different way to look at the function. Each of the representations is useful in its own way. The value of any of them depends on what you are trying to do. The representations are all connected—if you have one of the representations, you can create all the others.

Function described in words	To join the EMPower Gym you pay a one-time $40 membership fee, and then $20 per month.
Function Table	<table><tr><td>Number of Months</td><td>Input</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>Cost of Membership</td><td>Output</td><td>40</td><td>60</td><td>80</td><td>100</td></tr></table>
Graph	
Function Written as an Equation	$y = 20x + 40$

Reviewing What We Know: The Parts of a Function Equation

$$f(x) = 3x + 4$$

- The starting amount
- The y-intercept

- Input
- The independent variable

- Output
- The dependent variable
- "The function of x is…"
- "The f of x is…"
- y means the same thing as $f(x)$.

- The rate of change
- The slope of the line
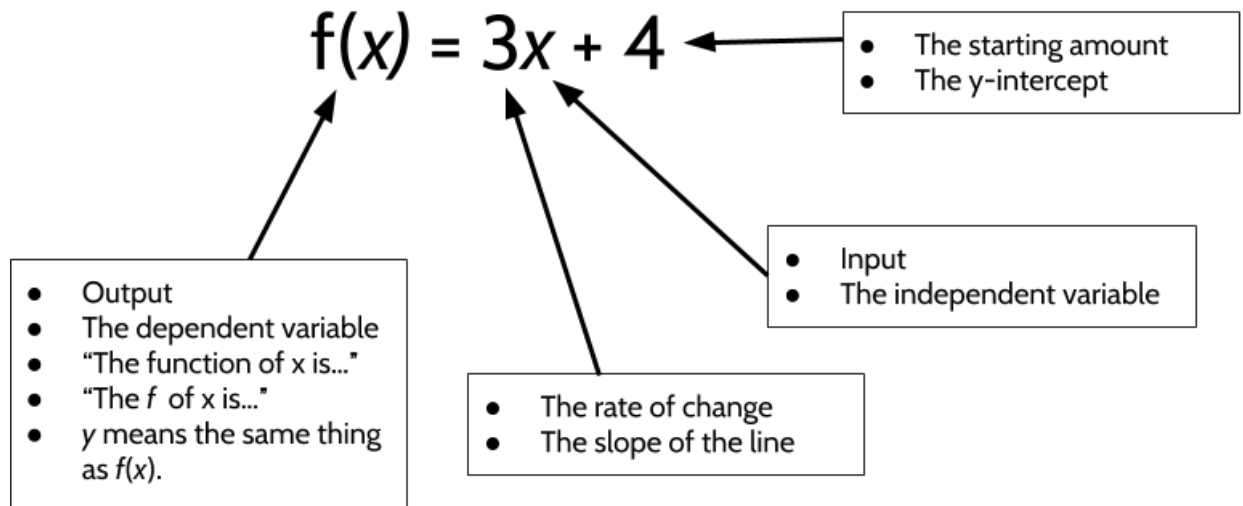
Write everything you know about the following function equation:

$$f(x) = 5x + 8$$

Test Practice Questions

1) Last week, Samantha sold iced tea at the park. Her profit can be represented by the function P(c) = .75c – 9.46, where P(c) represents her profits, and c represents the number of cups of iced tea she sold.

   Part One: Which statement correctly describes the meaning of .75 and 9.46 in the function?

   A. Samantha charged $0.75 per cup of ice tea and it cost her $9.46 to make the ice tea she sold.

   B. Samantha charged $0.75 per cup of ice tea and she earned $9.46 selling iced tea.

   C. It cost Samantha $0.75 to make each cup of iced tea, and she earned $9.46 selling iced tea.

   D. It cost Samantha $0.75 to make each cup of iced tea, and she charged $9.46 per cup.

   Part Two: This week Samantha is planning on raising the price by 50 cents per cup. Which function can be used to determine her profit for this week?

   A. $P(c) = .75c - 9.96$

   B. $P(c) = 1.25c - 9.46$

   C. $P(c) = .25c - 9.46$

   D. $P(c) = .75c - 8.96$

2) The cost of membership at a new gym can be modeled by the function $C(m) = 50m + 75$. Which of the following statements is true?

   A. Membership costs $125 per month.

   B. It costs $50 to join the gym and then you pay $75 each month.

   C. Being a member for two years would cost $1350.

   D. It costs $75 dollars to join the gym and then you pay $50 each month.

3) Which of the following methods could *not* be used to calculate the rate of change of the function described in the table below?

x	f(x)
3	52
6	100
7	116
21	340
42	676

A. $(100-52) \div 3$

B. $116-100$

C. $(340-116) \div 3$

D. $(676-340) \div 21$

4) In 2018, the United States Postal Service (USPS) raised its rates for mailing letters. The new charge was $0.50 to mail a letter weighing up to 1 oz. and $0.21 for each additional ounce. Which function would determine the cost of sending a letter through the USPS?

A. $y = 0.5x + 0.21$

B. $y = 0.21x + 0.50$

C. $y = 0.5(x-1) + 0.21$

D. $y = 0.21(x-1) + 0.50$

5) Alexis has $50 in a loose change jar and Melvin has $10 in his own jar. They each decide to save a certain amount of money each week. Alexis adds $5 to her jar each week. Melvin adds $15 every week. Which statement is not true?

A. If they graphed these two functions, Melvin's graph would have a steeper line.

B. After 2 weeks, Melvin would have $40 in his jar.

C. After 4 weeks, Alexis and Melvin have the same amount of money in their jar.

D. If they graphed these two functions, Alexis's graph would have a steeper line.

6) A function is shown in the table.

   If included in the table, which ordered pair would result in the relationship no longer being a function?

   A.   (-2, 24)

   B.   (2, 36)

   C.   (3, 42)

   D.   (7, 51)

x	f(x)
-4	18
-1	27
0	30
3	39
10	60

7) A typical shower in the United uses about 2 gallons of water per minute. If 1 gallon equals 16 cups, how many cups of drinking water are used in a 15-minute shower?

   A. 30

   B. 60

   C. 240

   D. 480

8) Which set of related x and y values represent a function?

   A. {(1,9), (2,15), (3,21), (2,13), (4,27)}

   B. {(5,31), (2,13), (0,1), (4,25), (1,7)}

   C. {(7,25), (1,7), (9,31), (3,13), (7,21)}

   D. {(0,2), (4,22), (2,12), (2,17), (5,27)}

9) A community center planted a tree to honor the opening of a new childcare center. The height of the tree as it grows can be represented by the function $h = 15t + 36$ where h represents the height of the tree (in inches) and t represents the number of years. Which interpretation of the function is correct?

A. The tree was 3 feet tall when it was planted and grows 15 inches each year.

B. The tree was 15 inches tall when it was planted and grows 36 inches each year.

C. The tree grows 51 inches each year.

D. Every five years, the tree grows 111 inches.

10) Brenda and Eddie agreed to save money, consistently adding to their separate savings accounts every month for the same number of months. The amount of money Eddie saved can be determined by the following function, where x represents the number of months:

$$f(x) = 200x + 4000$$

Brenda deposited $300 each month into her account.

If Eddie and Brenda had the same amount of money in their accounts after 10 months, how much did Brenda have in her savings account before she and Eddie started the savings plan? Enter your answer in the grid below.

11) Consider the line drawn on the coordinate plane below.
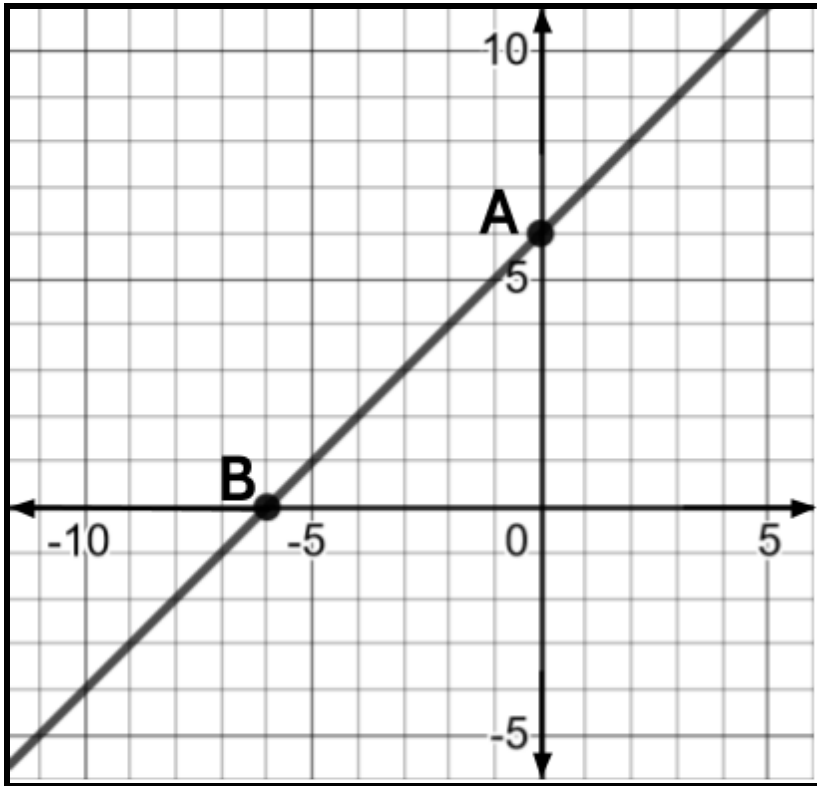
Which of the following can be used to refer to Point A? Choose all possible answers.

A. y-intercept

B. x-intercept

C. the starting amount

D. the rate of change

E. the slope

F. (0, 6)

G. (6, 0)

12) The pictures below represent the first three terms of a sequence.
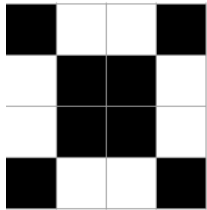
Term 1
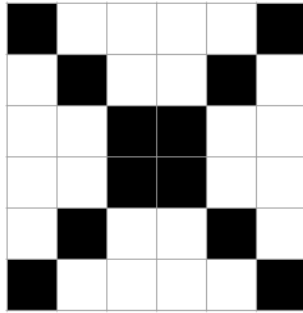
Term 2
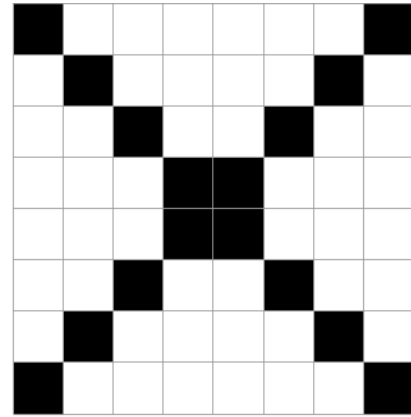
Term 3

Assuming the pattern continues, how many shaded squares would there be in the 10th term?

Enter your answer in the grid to the right.

13) Sal has been working as a nutritionist since 2004. She gets paid by the hour and has increased her hourly charges each year since she started.

The equation $y = 2.5x + 16.00$ can be used to model Sal's hourly charge, y, where x represents the number of years since 2004.

Which statement describes the nutritionist's hourly rate?

A. Her hourly charge was $2.50 in 2004, and it is $16.00 now.

B. Her hourly charge was $16.00 in 2004, and it has increased $2.50 each year.

C. Her hourly charge was $2.50 in 2004, and it has increased $16.00 per year.

D. Her hourly charge was $16.00 in 2004, and has gotten 2.5 times bigger each year.

14) The first four numbers in a sequence of numbers are

-12, -9, -6, -3,...

What would the 9th number in the sequence be? Enter your answer in the grid.

15) Consider the two functions below.

x	y
0	1
2	1.5
4	2
6	2.5
8	3
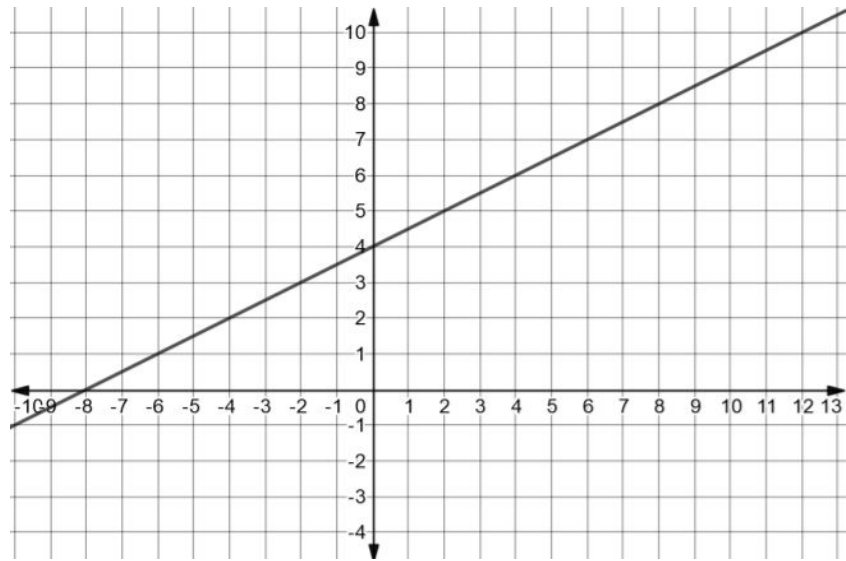
Which statement is true?

A. The rate of change of the function on the graph is greater than the rate of change of the function in the table.

B. The rate of change of the function on the graph is less than the rate of change of the function in the table.

C. The rate of change of both functions is the same.

D. There is not enough information given to determine the rate of change of the two functions.

16) A car rental company charges $80 to rent a car, plus .20 per mile. They also offer their customers the option of buying insurance for a flat fee of an additional $40. Which function can be used to calculate the total cost of renting a car for a customer who decides to buy the optional insurance?

A. $y = .20x + 80$

B. $y = .20x + 120$

C. $y = 80x + .20$

D. $y = 120x + .20$

17) The first three figures in a visual pattern can be seen below.
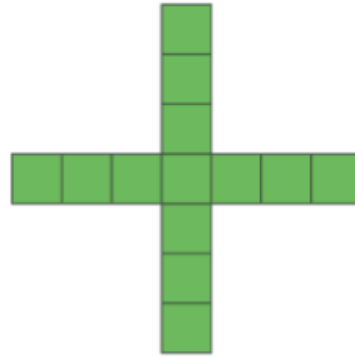
Figure 1   Figure 2   Figure 3

If the pattern continues, which figure would have 89 squares in it?

Please record your answer in the grid to the right.

18) Which equation represents the same function as the table?

A. $d = 1.50c$

B. $d = 1.50c + 20.00$

C. $d = 20.00c + 1.50$

D. $d = 21.50c$

c	d
0	20.00
1	21.50
3	24.50
24	56

19) Yilmer works at a camera store. He earns a guaranteed salary each week. In addition to his guaranteed salary, he earns a commission for each camera he sells. The graph below shows his weekly pay as a function of how many cameras he sells during the week.
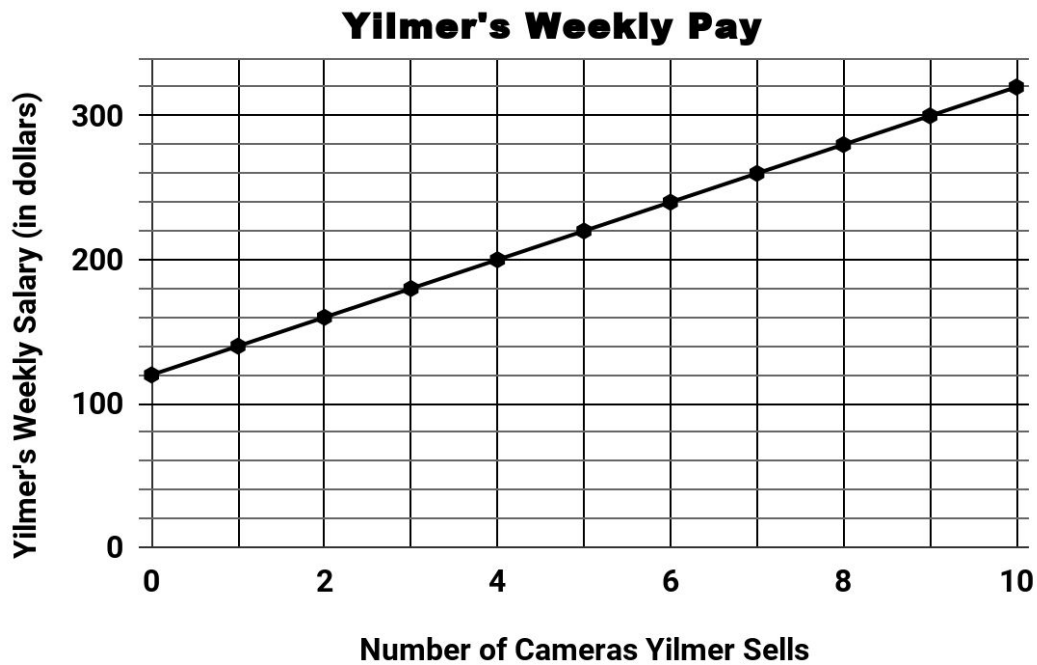
Yilmer's Weekly Pay

Which equation can be used to calculate how much Yilmer earns each week? Let P represent his weekly pay and let c represent the number of cameras he sells.

A. $P = 140c$

B. $P = 120c$

C. $P = 20c + 120$

D. $P = 120c + 20$

20) Which situation does <u>not</u> represent a linear function?

    A. An online streaming service charges a membership fee of $50 and $10 per month.

    B. A cab company charges $2.50 initially and $3.00 per mile.

    C. A restaurant employee earns $12.50 per hour.

    D. A population of bacteria doubles every 20 minutes.

21) Which equation expresses the relationship between *x* and *f(x)*, as shown in the table below?

x	1	3	5	7	9
y	5	9	13	17	21

    A. $f(x) = 4x + 1$

    B. $f(x) = x + 4$

    C. $f(x) = 3x - 2$

    D. $f(x) = 2x + 3$

22) As time goes by, a car depreciates in value. The value of a car is a function of how many years it has been on the road. An example of this relationship can be seen in the table.

Why is it correct to describe this relationship as linear?

Years of Ownership	Value of Car
1	$22,500
2	$19,500
3	$16,500
4	$13,500
5	$10,500

    A. The value of the car is increasing at a constant rate.

    B. The value of the car is decreasing at a constant rate.

    C. The value of the car is not increasing at a constant rate.

    D. The value of the car is not decreasing at a constant rate.

23) The scatter plot below shows the winning race times of the Olympic gold medal winners in the 100-meter sprint as a function of time. Tom Burke won the gold medal for the United States at the first ever Olympic Games in 1896. In 2012, Usain Bolt completed the race in 9.63 seconds to win the gold medal for Jamaica.
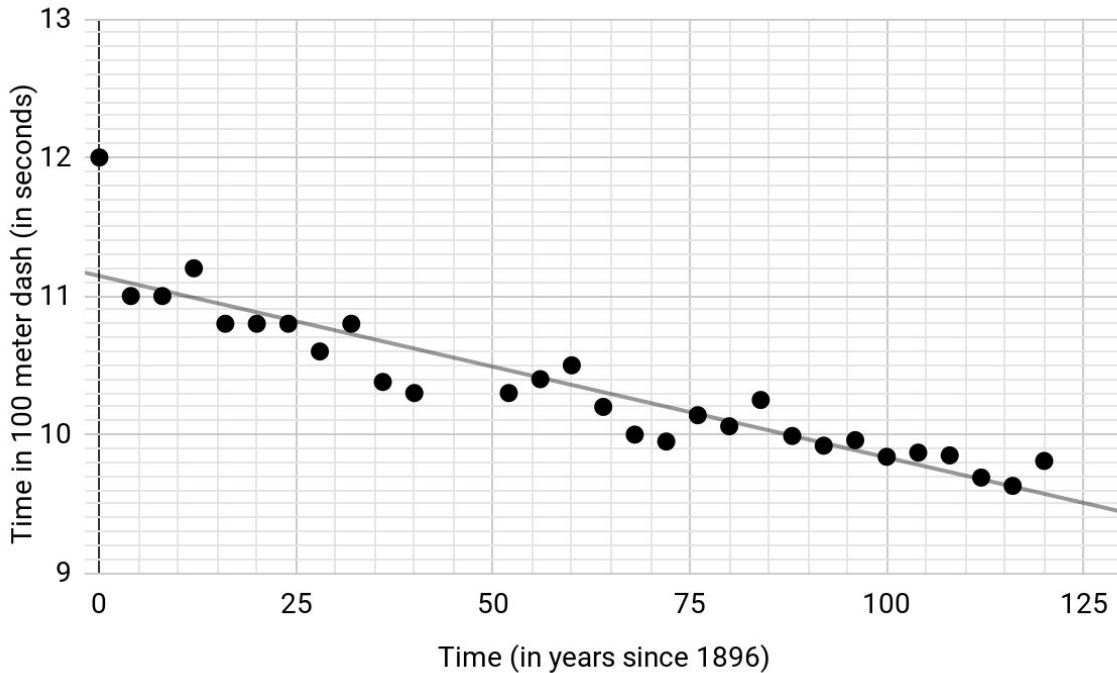
- The equation of the line of best fit is $y = -.013x + 11.14$.

- x represents the number of years after 1896.

- y represents the Olympic Gold medalists times, in seconds, for the 100-meter sprint.

What is the best interpretation of the slope of the line of best fit?

A. The time it takes to win the gold medal in the 100 meter sprint decreases an average of 0.013 seconds every year.

B. The time it takes to win the gold medal in the 100 meter sprint increases an average of 0.013 seconds every year.

C. The time it takes to win the gold medal in the 100 meter sprint decreases an average of 11.14 seconds every year.

D. The time it takes to win the gold medal in the 100 meter sprint increases an average of 11.14 seconds every year.

24) Which of the following tables shows a linear relationship?

A.

x	f(x)
0	-4
1	-2
2	4
3	14
4	28

B.

x	g(x)
-1	-3
0	-1
1	1
2	3
3	5

C.

x	h(x)
1	-1
2	1
3	5
4	13
5	29

D.

x	k(x)
0	8
1	9
2	12
3	17
4	24

25) Find the missing output for the function shown in the table.

x	2	4	6	8	16
y	11	21	31	41	

A. 39

B. 51

C. 81

D. 82

26) Consider the set of related x and y values. {(8, 67), (5, 28), (g, 19), (9, 84)}

Which value of g would make it *impossible* for the set to represent a function?

A. 2

B. 3

C. 4

D. 5

27) Shirley wants to see how far she can drive on a single tank of gas in her new hybrid car. She fills up her gas tank completely. The graph shows the number of gallons left in the fuel tank as a function of the number of miles driven for the duration of her trip.
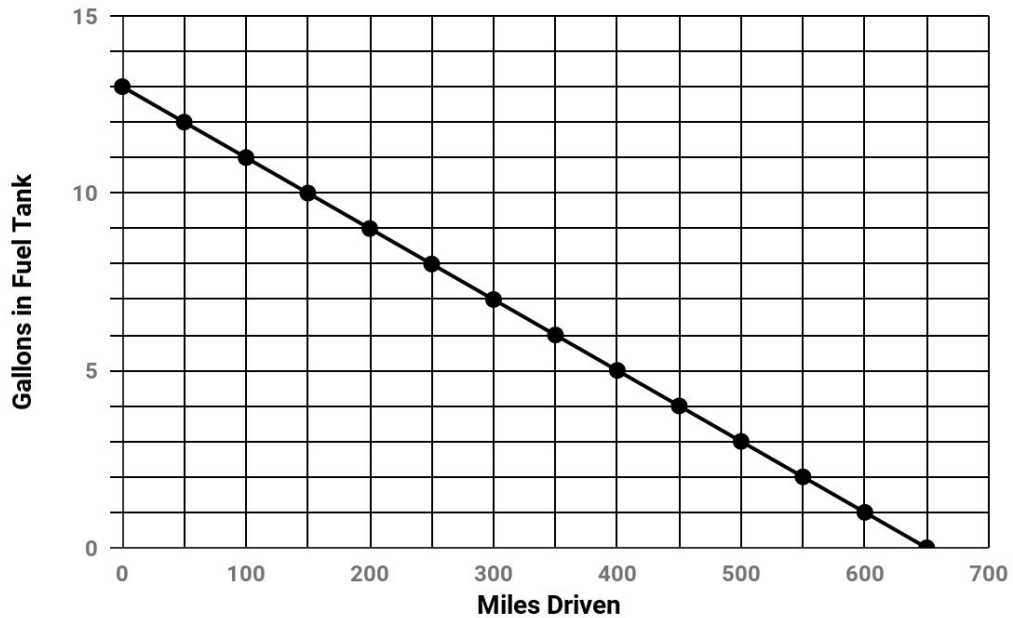
Which <u>two</u> statements can be justified using the graph?

A. Shirley was able to drive 700 miles on one full tank of gas.

B. The total capacity of the fuel tank is 13 gallons.

C. As the gallons of gas in the fuel tank decreases, the number of miles driven also decreases.

D. The car gets 50 miles per gallon.

28) The following function can be used to convert temperatures in Celsius to Fahrenheit:

$$F = 1.8C + 32$$

where C represents a temperature in Celsius and F represents the equivalent temperature in Fahrenheit.

Convert 15°C into Fahrenheit.

A. 32°F

B. 41°F

C. 49°F

D. 59°F

29) We can use the following function to model the exchange rate between the U.S. dollar and the Mexican Peso:

$$f(x) = mx$$

where $f(x)$ is the amount of Mexican Pesos and x is the amount of U.S. dollars. The table shows some values of x and $f(x)$.

Find m.

A. 10

B. 13

C. 15

D. 19

x	f(x)
Dollar (US)	Peso (Mexico)
150	2,850
300	5,700
600	11,400
900	17,100

30) The graph below represents the boiling point of water as a function of altitude above sea level. The line of best fit can be described with the function $y = 100 - .001x$, where x represents the altitude above sea level and y represents the boiling point of water.
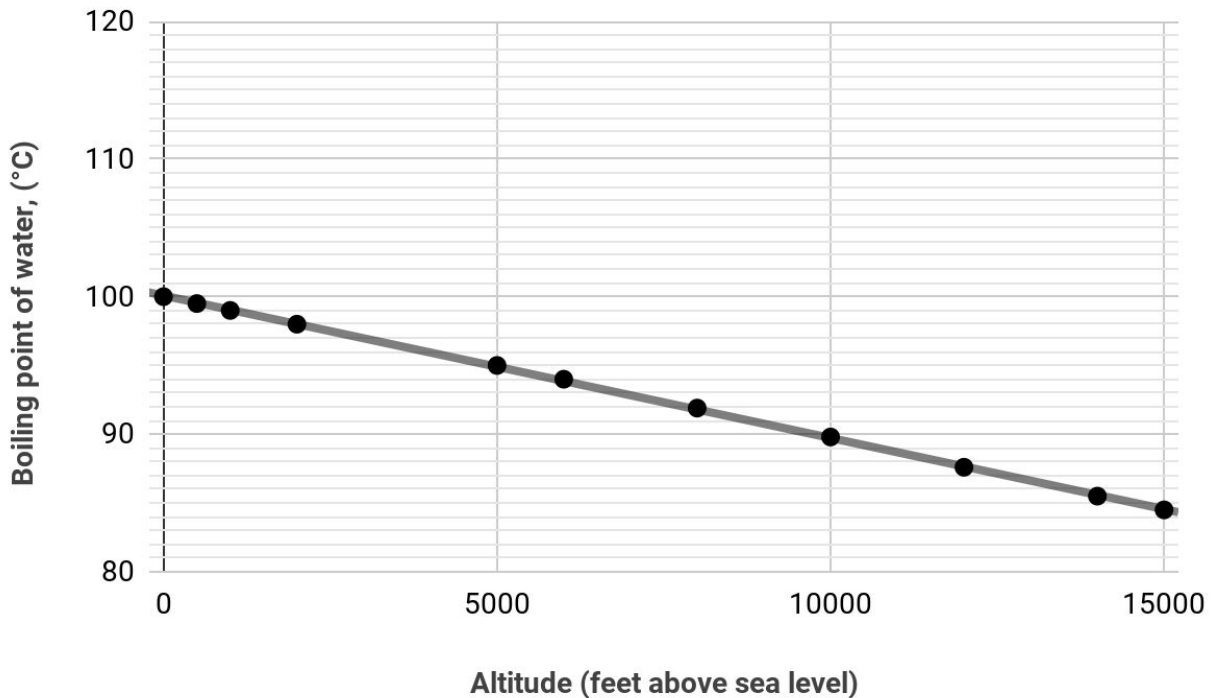
Which three statements are true, based on the line of best fit?

A. 100°C is the boiling point of water at sea level.

B. The boiling temperature of water decreases by 0.001 degrees Celsius for every 1 foot increase in altitude.

C. 100°C represents the rate of change for this function.

D. For every 5000 feet you go above sea level, the boiling point of water decreases by 5°C.

E. For every 5000 feet you go below sea level, the boiling point of water decreases by 5°C.

31) Which of the following graphs represents the equation $y = 3x - 2$?

A.

B.

C.

D.
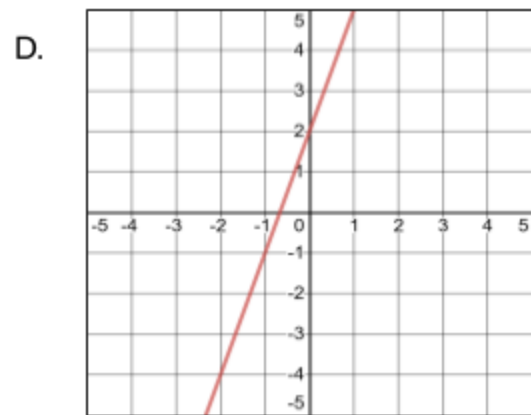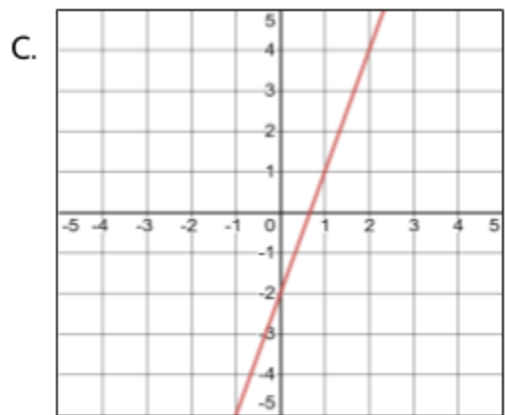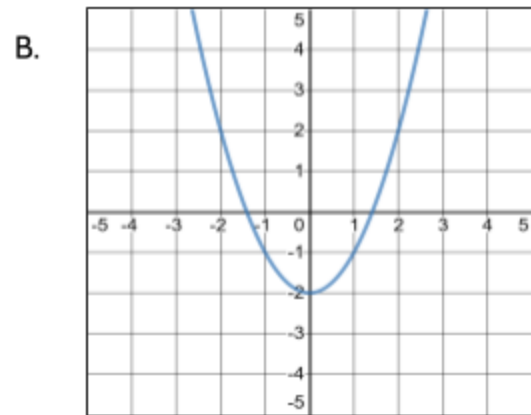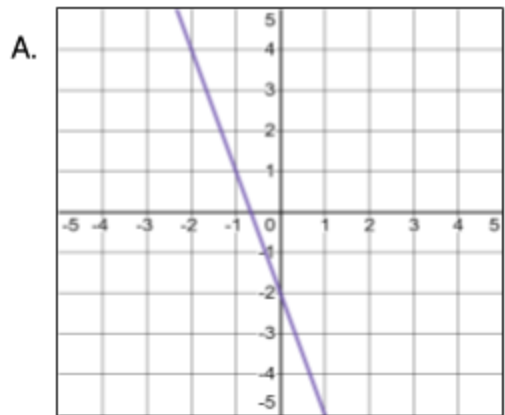
32) Consider the inputs and outputs in the table.

Which replacement for a makes this *not* a function?

A. 4

B. 5

C. 6

D. 7

x	f(x)
4	7
a	12
9	17
12	26

33) Consider the following graph.
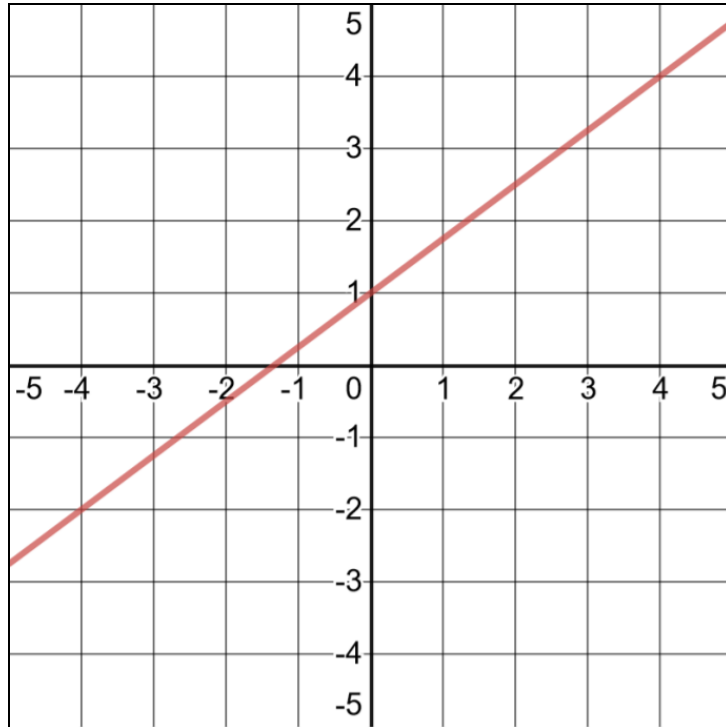
Which of the following function equations represents the line on the graph?

A. $y = \frac{4}{3} x + 1$

B. $y = -\frac{4}{3} x + 1$

C. $y = \frac{3}{4} x + 1$

D. $y = -\frac{3}{4} x + 1$

34) Which of the following functions could be graphed as a line parallel to $y = 3x + 8$?

A. $y = 3x - 2$

B. $y = 5x + 8$

C. $y = 8x + 3$

D. $y = 4x - 2$

35) Which of these graphs does *not* represent a function of *x*?

A.

Chance of Human Survival (y-axis) — Pollution (x-axis)

B.

Chance of Human Survival (y-axis) — Pollution (x-axis)

C.

Chance of Human Survival (y-axis) — Pollution (x-axis)

D.

Chance of Human Survival (y-axis) — Pollution (x-axis)
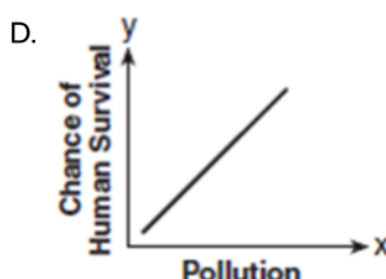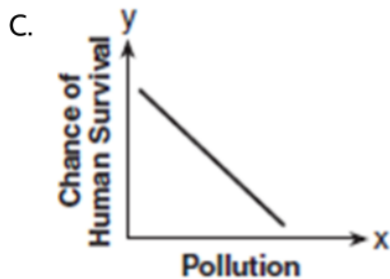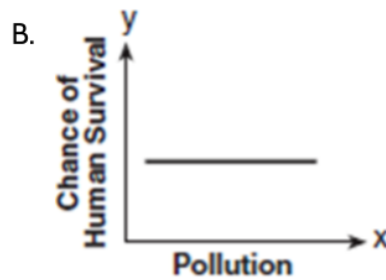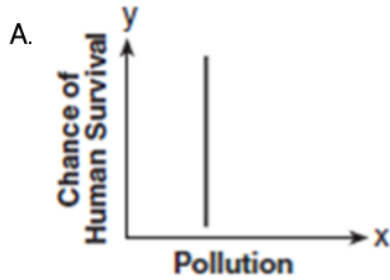
36) A plumber has a set fee for a house call and then charges by the hour for work done. The total cost of her services can be modeled by the function *c(t)* = 125*t* + 95. When working for friends, she reduces her hourly rate by $35. Which function can be used to calculate the total cost of her services for work done for friends?

A. $c(t) = 125t$

B. $c(t) = 90t + 95$

C. $c(t) = 125t + 60$

D. $c(t) = 90t + 60$

37) Alex had some money in a savings account and decided to save $20 a month for 12 months.
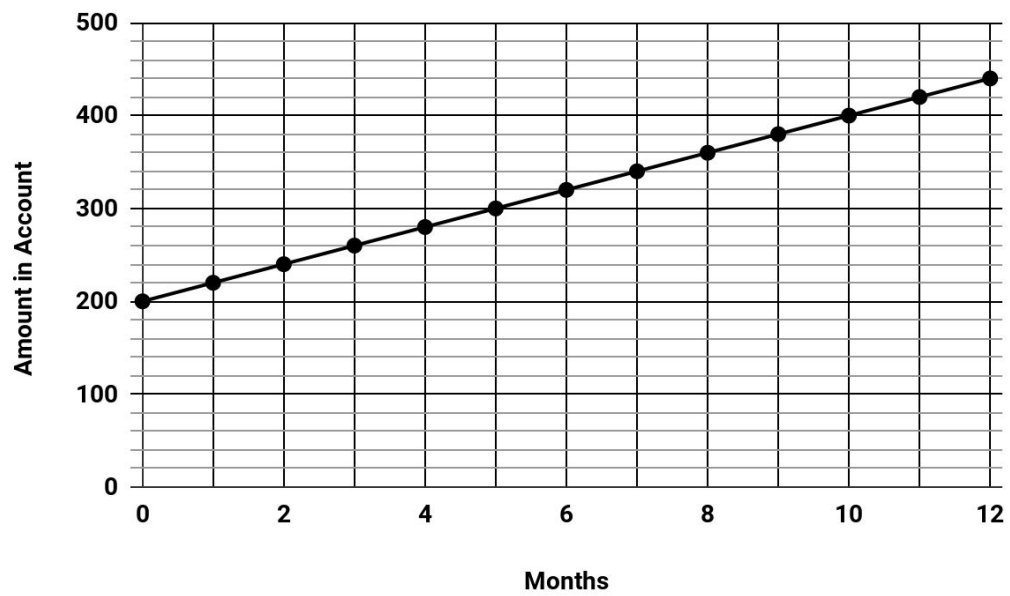
How much money did Alex have in the account at the start of the year?

A. $20

B. $200

C. $220

D. $460

38) Which table is the best match for the graph?
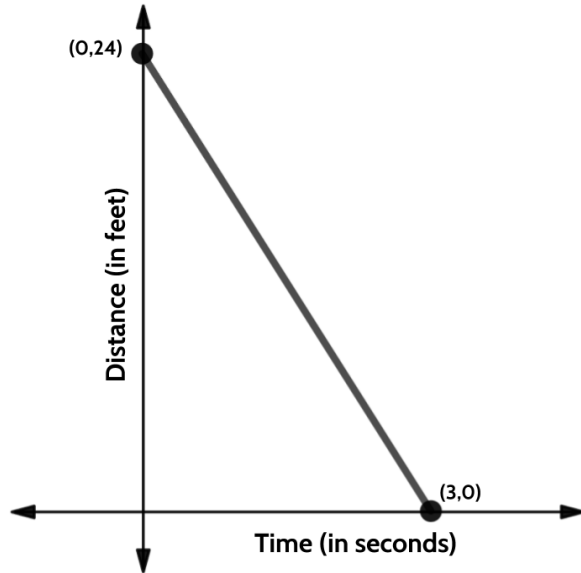
A.

Time (in seconds)	Distance (in feet)
0	24
1	16
2	8
3	0

B.

Time (in seconds)	Distance (in feet)
0	24
1	12
2	6
3	0

C.

Time (in seconds)	Distance (in feet)
0	24
1	18
2	12
3	0

D.

Time (in seconds)	Distance (in feet)
0	24
1	20
2	16
3	0

39) For the function $f(x) = 6x - 8$, what is $f(3)$?

    A. 10

    B. 26

    C. 30

    D. 42

40) An apple orchard charges an entrance fee of $15 for visitors to go apple picking. Any apples they pick can be purchased for $1.50 a pound.

    Part One: Choose the word that best completes the following statement.

    In this situation, the $15 can best be described as the _____ .

    A. rate of change

    B. slope

    C. input

    D. starting amount

    Part Two: If a visitor spent $24, how many pounds of apples did they buy?

    A. 3

    B. 6

    C. 8

    D. 16

41) Find the rate of change for this function.

    A. 8

    B. 24

    C. 32

    D. 104

x	f(x)
16	134
20	166
45	366
19	158
3	30

42) The graph below shows the distance in miles, m, hiked from a camp in h hours.
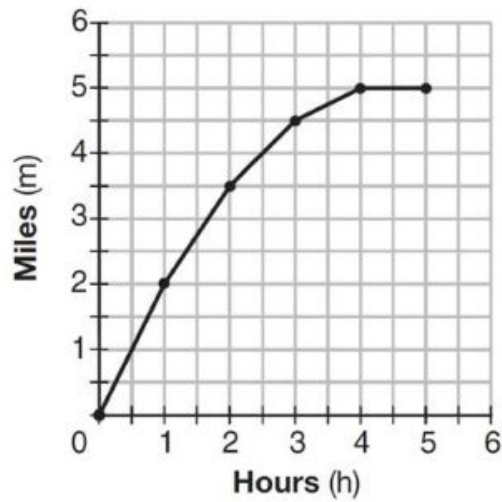
Which hourly interval had the greatest rate of change?

A. hour 0 to hour 1
B. hour 1 to hour 2
C. hour 2 to hour 3
D. hour 3 to hour 4

1. <u>Part One</u>: **Choice A.** This function is showing us the relationship between Samantha's profit and the number of cups of iced tea she sells. Her profit is a function of how many cups she sells. In the function $P(c) = .75c - 9.46$, the .75 represents the price of each cup of iced tea. Multiplying that number by c, the number of cups she sold, tells us the amount of money she made. Since profit is determined by taking the amount of money earned and subtracting the cost of what we produced, the 9.46 represents the $9.46 it must have cost to make the iced tea.

   <u>Part Two</u>: **Choice B**. In the function $P(c) = .75c - 9.46$, the .75 represents the price of each cup of iced tea. If Samantha raises the price of each cup by 50 cents, then the new price would be $1.25.

2. **Choice D.** In the function $C(m) = 50m + 75$, the rate of change is 50 and the starting amount is 75. We can think of the starting amount as a flat fee that we only pay once and we can think of the rate of change as a monthly cost. We can assume that the variable m represents the number of months. If you put a 2 (for two months) into the function, you get 50(2) + 75, which is $175. If you put a 3 in, you get 50(3) + 75, which is $225. That reflects the written description in Choice D of paying $75 to join and then $50 each month.

3. **Choice C.** The rate of change for this function is 16. One way to calculate that is to look at the two consecutive inputs, 6 and 7. The change in the outputs between those two consecutive inputs is 16. Evaluating all of the answer choices, choice C is the only one that does not equal 16.

4. **Choice D.** Choice B is a compelling answer choice because it has the correct rate of change (0.21) and the correct starting amount (0.50). A good strategy when you are given a written description to match an equation is to try a few numbers.

$y = .21x + .50$	
Ounces	Cost
x	y
1	0.71
2	0.92
3	1.13

| $y = .21(x-1) + .50$	
Ounces	Cost
x	y
1	0.50
2	0.71
3	0.92

The function table on the right is the one that matches the written description of the function. The USPS charges 0.50 for a letter weighing up to 1 ounce.

5. **Choice D.** Making a quick table can also be helpful when you are asked to evaluate statements like this. The table below allows us to see that statements A, B, and C are all true. Since Melvin is adding $15 a week the rate of change of the function describing his saving would be 15. The rate of change for Alexis' saving is 5, which is a smaller growth.

Week	Alexis	Melvin
1	$55	$25
2	$60	$40
3	$65	$55
4	$70	$70

6. **Choice C** has an input of 3 and an output of 42. The chart already has an output of 39 for an input of 3. A function can not have the same input resulting in different outputs, therefore including (3, 42) in the table would mean that the table doesn't represent a function

7. **Choice D.** A rate of 2 gallons per minute is equivalent to 32 cups of water per minute. If the rate is 32 cups per minute, it would be 32×15 or 480 cups of drinking water used in a 15 minute shower,

8. **Choice B** is the only choice that does not have two different outputs for the same input.

    A. {(1,9), **(2,15)**, (3,21), **(2,13)**, (4,27)}

    B. {(5,31), (2,13), (0,1), (4,25), (1,7)}

    C. {**(7,25)**, (1,7), (9,31), (3,13), **(7,21)**}
    D. {(0,2), (4,22), **(2,12)**, **(2,17)**, (5,27)}

9. **Choice A.** The function gives the starting height of the tree as 36 inches, which is equivalent to 3 feet. Choice B reverses the rate of change and the starting amount—you are likely to see this kind of misleading answer on the high school equivalency exam. Choices C and D are also common misleading answers. They are both *almost* true. Choice C is almost true – the tree does grow 51 inches in its first year, but the statement says it grows 51 inches <u>each</u> year, which is not true. Choice D is also almost true – in 5 years, the tree grows 111 inches, but the statement says <u>every</u> five years the tree grows 111 inches, which is not true.

10.
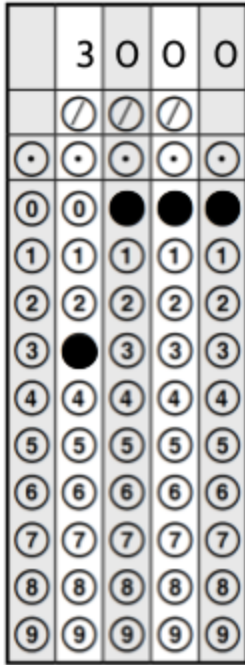
The function describing Eddie's savings is *f(x) = 20x + 4000*. The problem tells us that their accounts had the same amount of money after 10 months.

After ten months, Eddie has 20(10) + 4000, which equals $6000.

Brenda has been saving $300 a month. After ten months, she will have saved $3000. Since the problem tells us they had the same amount of money after the ten months, Brenda must have started with a starting amount of $3000.

11. Choices A, C, F.

12.
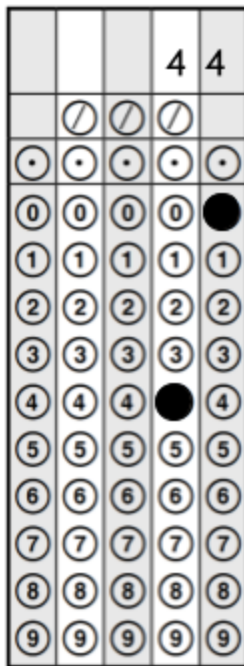
Term	Shaded Squares
1	8
2	12
3	16
4	20
5	24
6	28
7	32
8	36
9	40
10	44

One strategy is to make a table and continue the pattern.

Another strategy is to determine the function equation. The change in the number of shaded squares can be represented by the function *f(n) = 4n + 4* where *n* represents the term number. 4(10) + 4 is equal to 44.

If you count the number of shaded squares in the 3 terms given, you can see the rate of change is 4.

13. **Choice B.** It is not necessary, but using the equation to make a quick table can help when a question asks you to evaluate statements.

Years since 2004	Hourly Rate
0	$16
1	$18.50
2	$21
3	$23.50

14. The numbers in this sequence are increasing by 3. If the sequence continued until the 9th term, the sequence would be –12, –9, –6, –3, 0, 3, 6, 9, 12.

15. **Choice A.** The rate of change for the function represented by the graph is 0.5 or ½. The rate of change for the function in the table is 0.25. The rate of change on the table appears to be 0.5, making Choice C a compelling misleading choice. But the inputs in the table are not consecutive, they are going up by 2.

16. **Choice B.** A $80 flat fee charge plus a $40 flat fee insurance charge would result in a starting amount of $120.

17. The function equation that describes the relationship between the figure number and the number of squares in this sequence is $y = 4x + 1$, where x is the figure number and y is the number of squares. The problem gives us the number of squares, so we can set up the equation $89 = 4x + 1$.

    Another strategy is to think about thow the figure grows and ask what is changing and what is staying the same.
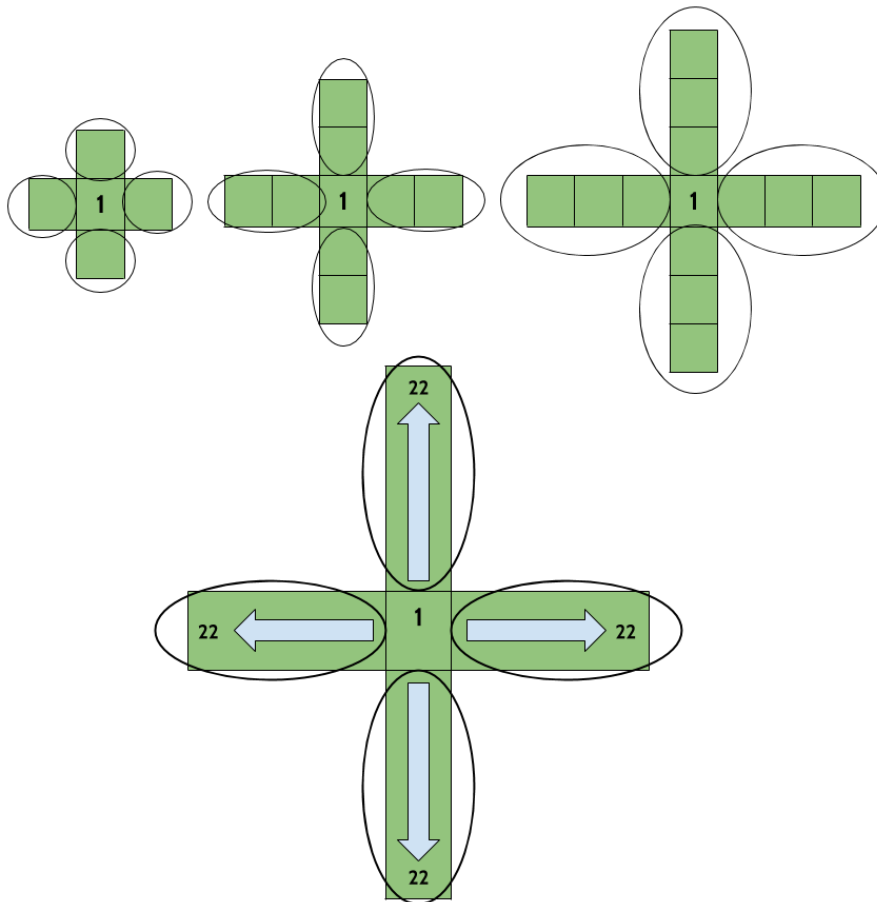
There is always 1 square in the middle. There are always 4 "arms" coming out of the center square. The number of squares in each arm is equal to the figure number. If we imagine 89 squares, we can put 1 in the middle, which leaves us with 88 squares, If we have 88 squares left and divide them into 4 equal groups, each group would have 22 squares. Figure 22 is the one that has 22 squares in each arm. $89 = 22 + 22 + 22 + 22 + 1$

18. **Choice B.** According to the table, when the input is zero, the output is 20. That means we are looking for an equation with a starting amount of 20.

19. **Choice C.** Based on the graph, the starting amount of this function is 120. That only matches one answer choice. Another strategy is to pick a point and put it into each answer choice. Point (4, 200) appears on the line of this function. 20(4)+120= 200.

20. **Choice D.** Making a quick table for each situation allows you to see that only one of them does not have a constant rate of change.

21. **Choice D.** Be careful of Choice A. The difference in the outputs shown in the table is 4, but the inputs are not consecutive.

22. **Choice B.** A key feature of linear functions is a <u>constant</u> rate of change, even if the outputs are decreasing in value..

23. **Choice A.** Usain Bolt won a gold medal in 2012 with a time of 9.63. Tom Burke won in 1896 with a time over 11 seconds.

24. **Choice B** is the only choice with a constant rate of change.

25. **Choice C.** The rate of change for this function is 5 and the starting amount is 1, so we can use the function $y = 5x + 1$. When the input is 16, the output is 81.

26. **Choice D.** Since one of the ordered pairs given has an input of 5 and an output of 28, the function cannot have another solution with an input of 5 and an output of 19.

27. **Choice B and Choice D.** A table based on the points on the graph allows us to identify the rate of change (-50) and the starting amount (13)

Number of Gallons in tank	Miles Driven
13	0
12	50
11	100
10	150

28. **Choice D.** 1.8(15) + 32 = 59

29. **Choice D.** The currency of the United States is the US dollar. When we talk about foreign currency, we are talking about the money that is used in other countries. We use exchange rates to determine the equivalent value between different currencies. There are different strategies you might use to answer a question like this. We are given inputs and outputs and possible rates of change. One method is to multiply each of the inputs by the given possible values of *m* and see which one results in the given outputs. Another method might be to use the information in the table to determine the rate of change, looking at the change in the inputs and the change in the outputs.

30. **Choice A, B, D.** High altitude cooking is cooking that is done at altitudes well above sea level. The higher you go, the less air pressure there is. A decrease in air pressure causes a decrease in the temperature required to make water boil. So basically, the higher your altitude, the lower the boiling point of water!

31. **Choice C.** When answering questions like these, eliminating choices can help. We can tell by the function equation given that the function is linear, so we can eliminate choice B. Choice A can also be eliminated because it has a negative slope. The main difference between choices C and D are the *y*-intercepts. The function equation has a *y*-intercept of -2, so we can eliminate choice D.

32. **Choice A.** Since the table already has an input of 4 with an output of 7, having another input of 4 and an output of 12 would make this not a function. This type of question is often used to test whether students understand that every input has exactly one output. If the same input results in two different outputs, then the relationship is not a function.

33. **Choice C.**

34. **Choice A**. Linear functions that have the same rate of change will have graphs that are parallel to each other.

35. **Choice A.** In this graph there are multiple *y* values for the same *x* value.

36. **Choice B.** The hourly rate is the rate of change in the function. If her standard rate, $125, is reduced by $35, then her "Friends and Family" hourly rate would be $90. The problem does not mention a discount in the house call fee.

37. **Choice B.** The *y*-intercept on the graph represents the starting amount of $200.

38. **Choice A** is the only table that represents a linear function with a constant rate of change between each of the outputs. If you were to graph the other tables, you would not get a straight line all the way from (0,24) to (3,0).

39. **Choice A.** With an input of 3, the function would produce an output of 10.

40. <u>Part One</u>: **Choice D.**

    <u>Part Two</u>: **Choice B.**

41. **Choice A.** You can use any of the input/output pairs in the table to calculate the rate of change. There are two consecutive inputs of 19 and 20. If you use those, and find the difference in their outputs, you can see the rate of change is 8.

42. **Choice A.** Between hour 0 and 1, the rate of change is 4. Between hour 1 and 2, the rate of change is 3. Between hours 2 and 3, the rate of change is 2. Between hours 3 and 4, the rate of change is 0.

The Language of Functions

Concept Circles

Explain the words in the circle and the connections you see between them.

_____

_____

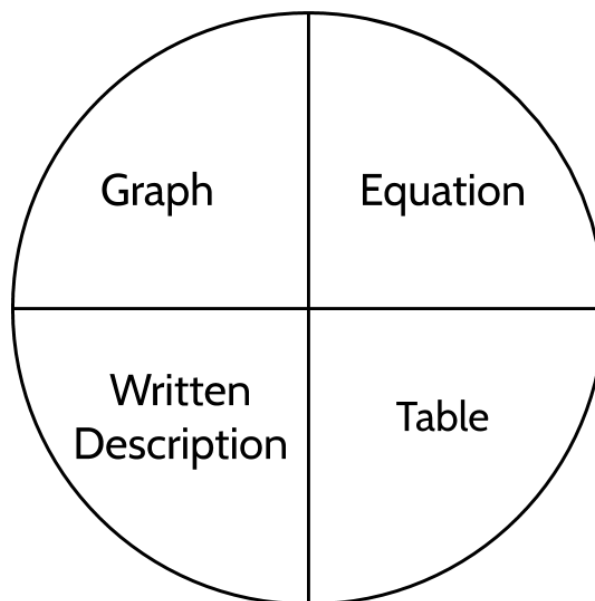_____

_____

_____

_____

_____

_____

_____

Graph	Equation
Written Description	Table

Explain the words in the circle and the connections you see between them.

Function Equation	Starting Amount
Rate of Change	Function Table

_____

_____

_____

_____

_____

_____

_____

_____

_____

Explain the words in the circle and the connections you see between them.

Graph | y-intercept

Slope | Origin

_____

_____

_____

_____

_____

_____

_____

_____

_____

Explain the words in the circle and the connections you see between them.

_____

_____

_____

_____

_____

_____

_____

_____

Relationship | Patterns

Variables | Functions
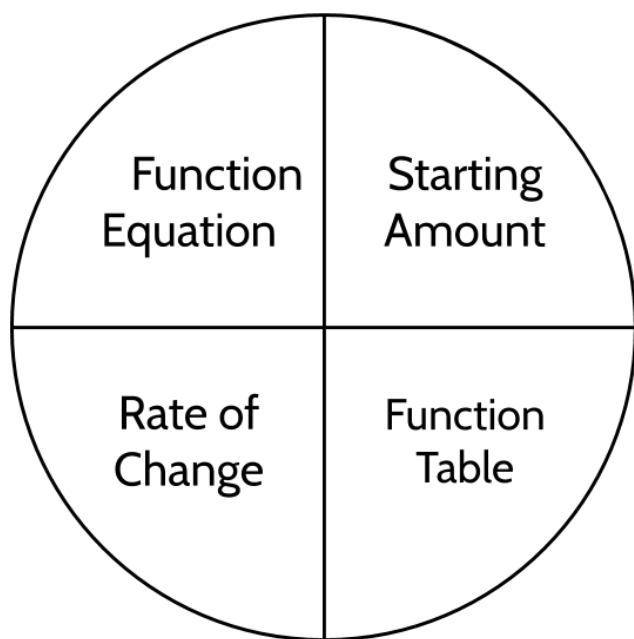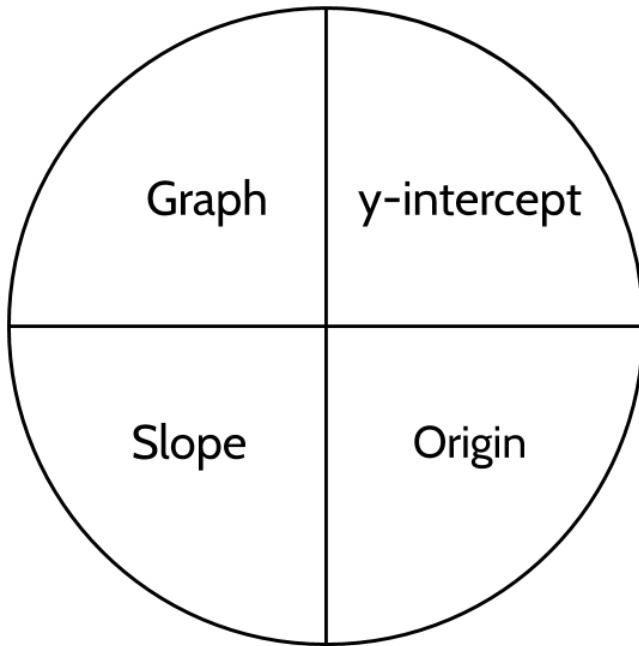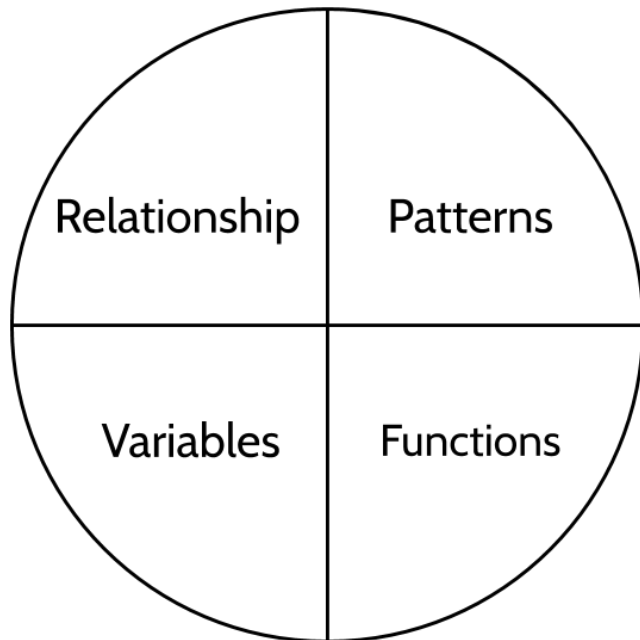
Questions

What questions do you still have about linear functions?

Vocabulary Review

You can use this section to look up words used in this math packet.

coefficient (noun): a number used to multiply a variable. In the linear equation y=7x+12, 7 is a coefficient. <see *RATE OF CHANGE, SLOPE*>

consecutive (adjective): Consecutive numbers are numbers that follow each other in order, without skipping, from smallest to largest. For example, 23, 24, 25, 26 are consecutive numbers.

coordinate plane (noun): a completely flat surface formed when two straight number lines go across each other at right angles. The point where the lines intersect is called the origin and represents the zero on each axis. An exact position on the grid can be described using coordinates. The coordinate plane is also called the coordinate grid. <see *ORIGIN, X-AXIS, Y-AXIS*>
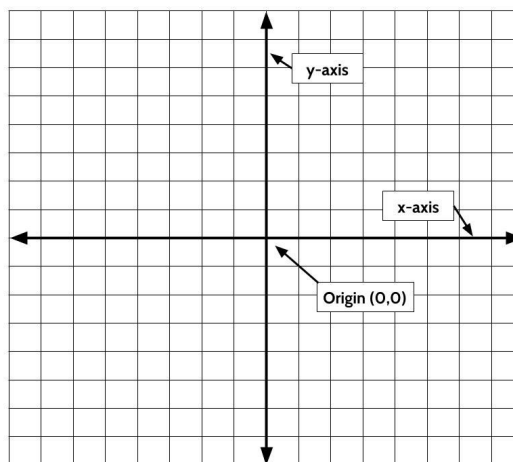
constant (noun): A value in an expression or equation that doesn't change. For example, in the function $y = 4x + 6$ the numbers 4 and 6 are both constants.

constant (adjective): Something that stays the same. A *constant* rate of growth means that the rate stays the same over time.

dependent variable (noun): The variable in a function that represents the output. <see IN*DEPENDENT VARIABLE*>

domain (noun): the set of possible values for the independent variable. <see *RANGE*>

equation (noun): A number sentence that shows two expressions are equal by using the equal sign.

evaluate (verb): to calculate the value of something.

function (noun): a relationship between two quantities. For every input into a function, there can only be one output. A function can be represented by an equation, a table, a graph, and a verbal or written description.

function notation (noun): a way of writing a function that emphasizes the relationship between the independent variable and the dependent variable used in the function. For example we can read f*(x)* = 2x + 5 as "the function of x is 2x +5."

graph (noun): a visual representation of a function relationship.

horizontal (adjective): going side to side. On the coordinate grid, the x-axis is horizontal.

independent variable (noun): The variable in a function that represents the input <see *DEPENDENT VARIABLE*>

input (noun): the number that goes into a function.

linear (adjective): linear means "a straight line". A Linear function is a function that grows in a constant way. The graph of a linear function is a straight line.

linear function (noun): A function that has a constant rate of change. The graph of a linear function is a straight line.

line of best fit / trend line (noun): a line on a graph that shows the general direction and trend that a set of points seem to follow. *<see SCATTER PLOT>*

model (verb): In math, to model a real-world situation means to create a mathematical representation of that situation.

origin (noun): the point where the x-axis and the y-axis intersect on the coordinate grid. The origin has the coordinates (0,0).

ordered pair (noun): Ordered pairs are used to show a position on a graph. An ordered pair is made up of an input and an output that make a function true. They are usually written in parentheses like this (14, 7), where the first number shows the x value (horizontal) and the second number shows the y value (vertical). <see *FUNCTION SOLUTIONS, COORDINATE GRID*>

output (noun): the unique value that comes out of a function when each input goes in. The output is dependent on the input.

pattern (noun): Patterns are things we can observe. We gather information about what has come before to make predictions about what will come next.

parallel (adjective): parallel lines are lines on a plane that never meet. The graphs of linear functions that have the same slope are parallel. For example: The line graph of *y = 4x+3* is parallel to the line graph of *y = 4x*

per (preposition): for each or for every

- The car was traveling 40 miles *per* hour.
- Most showers use 2.1 gallons of water *per* minute.
- The taxi charges $2.75 *per* mile.
- The air pressure in your car tires should be 32-35 pounds *per* square inch.

plot (noun): to draw a point on the coordinate grid or a graph that represents an ordered pair.

product (noun): the number you get from multiplying two or more numbers.

quantitative (adjective): something related to amounts or numbers.

quantity (noun): a measurement that is (or can be) specified by a number and a unit. For example 3 mangos, 16 feet, "the amount of money in a savings account", "the number of times a heart beats in a minute"

range (noun): the set of outputs for a function. the set of possible values for the dependent variable. <see *DOMAIN*>

rate (noun): A rate compares two different quantities.

- The sink was leaking at a rate of ½ an ounce of water per minute
- The train traveled at 80 miles per hour. For every hour, the train traveled 80 miles.
- The hybrid car gets a gas mileage rate of 50 miles per gallon. For every 1 gallon of gas, the car can travel 50 miles.

rate of change (noun): the rate of change in the outputs when the inputs are consecutive. <see *SLOPE*>

scatterplot (noun): A graph of plotted points that show the relationship between two sets of data. *<see LINE of BEST FIT>*

sequence (noun): a list of numbers, objects, or events in a special order.
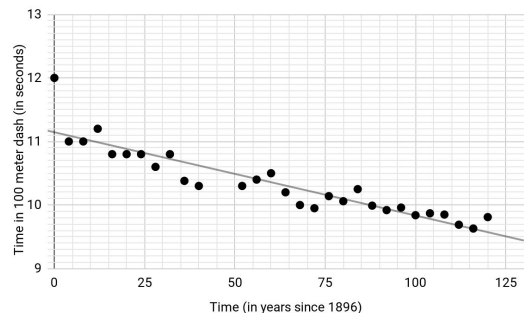
slope (noun): Slope refers to how steep a line is. The slope of a line is also known as the rate of change. The slope is how the rate of change appears in the graph of a linear function. <see *RATE of CHANGE*>

solution (noun): A solution to a function is any ordered pair (*x, y*) that can be used to make a function equation true. For example, (3, 14) and (4, 18) are both solutions to the function *y*=4*x*+2 because if you an input of 3 results in an output of 14 and an input of 4 results in an output of 18.

starting amount (noun): the value of the output when the input is zero. <see *Y-INTERCEPT*>

table (noun): information arranged in rows and columns. A function table can be used to list some of the input and output values for a function.

term (noun): Term is used to describe where a number is in a pattern. The first number in a pattern is Term 1, the second number is Term 2, and so on. Term 8 would be the eighth number in a pattern.

variable (noun): A letter or symbol that represents another value, either any number, a specific number, or a set of numbers.

vertical (adjective): Going up and down. On the coordinate grid, the y-axis is vertical.

x-axis (noun)**:** the line that goes from left to right in a graph. The horizontal axis on the coordinate grid.

x-intercept (noun): the point where a line crosses the x-axis of a graph.

y-axis (noun)**:** the line that goes from top to bottom of a graph. The vertical axis on the coordinate grid.

y-intercept (noun): the point where a line crosses the y-axis of a graph. <see *STARTING AMOUNT*>

Sources

Appleton, Eric C. 2016. The Paycheck Problem. www.mathmemos.org/the-paycheck-problem/

Desmos Graphing Calculator. 2019. www.desmos.com

Fulton, Brad S., Lombard Bill. 2001. The Pattern and Function Connection.

Hinds, S. 2007. Functions Rule (lesson set). The CUNY Adult Literacy Program.

Illustrated Mathematics Dictionary: https://www.mathsisfun.com/definitions/index.html

JMAP Regents Tests: http://www.jmap.org

Lappan, G., Fey, J., Fitzgerald, W.M., Friel, S.N., and Phillips, E.D. (2004).Variables and Patterns: *Introducing Algebra*. Connected Mathematics. New Jersey. (Sections:

Manly, M., & Ginsburg, L. (2010). Algebraic Thinking in Adult Education. National Institute for Literacy.

Nguyen, Fawn. www.visualpatterns.org

Open-Up 6-8 Math Resources. https://openupresources.org/math-curriculum/

Schmitt, M.J., Steinback, M., Curry, D., Donovan, T., Merson, M. (2005). Keeping Things in Proportion: Reasoning with Ratios. EMPower/TERC. McGraw Hill. Cambridge, Massachusetts.

Trushkowsky, M., Appleton, E., Holzer, T. (2015). CUNY HSE Math Curriculum Framework.

Van de Walle, J. A. (2003). Elementary And Middle School Mathematics. New York.

Van Dyke, Frances. (2002). A Visual Approach to Functions. Key Curriculum Press.