Area & Population Density

Fast Track GRASP Math Packet

1963 Civil Rights March on Washington, D.C. (US National Archives)

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http://www.collectedny.org/ftgmp

Table of Contents

Welcome!

Congratulations on deciding to continue your studies! We are happy to share this study packet on area and population density. We hope that these materials are helpful in your efforts to earn your high school equivalency diploma. This group of math study packets will cover mathematics topics that we often see on high school equivalency exams. If you study these topics carefully, while also practicing other basic math skills, you will increase your chances of passing the exam.

Please take your time as you go through the packet. You will find plenty of practice here, but it's useful to make extra notes for yourself to help you remember. You will probably want to have a separate notebook where you can recopy problems, write questions and include information that you want to remember. Writing is thinking and will help you learn the math.

After each section, you will find an answer key. Try to answer all the questions and then look at the answer key. It's not cheating to look at the answer key, but do your best on your own first. If you find that you got the right answer, congratulations! If you didn't, it's okay. This is how we learn. Look back and try to understand the reason for the answer. Please read the answer key even if you feel confident. We added some extra explanation and examples that may be helpful. If you see a word that you don't understand, try looking at the *Vocabulary Review* at the end of the packet.

We also hope you will share what you learn with your friends and family. If you find something interesting here, tell someone about it! If you find a section challenging, look for support. If you are in a class, talk to your teacher and your classmates. If you are studying on your own, talk to people you know or try searching for a phrase online. Your local library should have information about adult education classes or other support. You can also find classes listed here: **http://www.acces.nysed.gov/hse/hse-prep-programs-maps**

You are doing a wonderful thing by investing in your own education right now. You have our utmost respect for continuing to learn as an adult.

Please feel free to contact us with questions or suggestions.

Best of luck!

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Vocabulary

It is important to understand mathematical words when you are learning new topics. The following vocabulary will be used a lot in this study packet:

area · density · per · product · rate · rectangle · square · unit

In this first activity, you will think about each word and decide how familiar you are with it. For example, think about the word "area." Which of these statements is true for you?

- I know the word "area" and use it in conversation or writing.

- I know the word "area," but I don't use it.

- I have heard the word "area," but I'm not sure what it means.

- I have never heard the word "area" at all.

In the chart on the next page, read each word and then choose one of the four categories and mark your answer with a ✔ (checkmark). Then write your best guess at the meaning of the word in the right column. If it's easier, you can also just use the word in a sentence.

Here's an example of how the row for "area" might look when you're done:

Word	I know the word and use the word	I know the word but don't use it	I have heard the word, but I'm not sure what it means	I have never heard the word	My best guess at the meaning of the word (or use the word in a sentence)
area	✔				A place or location, like a neighborhood or town

Complete the table on the next page.

Complete the table based on your knowledge of the words in the left column. It's okay if you don't know the meaning of a word. Make your best guess or write where you have heard it.

Word	I know the word and use the word	I know the word but don't use it	I have heard the word, but I'm not sure what it means	I have never heard the word	My best guess at the meaning of the word (or use the word in a sentence)
area					
density					
per					
product					
rate					
rectangle					
square					
unit					

Introduction

Consider these two housing situations:

<div align="center">

2 people living in a 3-room apartment

3 people living in a 4-room house

</div>

Imagine that you have the choice of living as a roommate in one of the two homes above. Which would you choose? You probably want to live in the home that has more space. The apartment has fewer people, but it also has fewer rooms. The house has more rooms, but it also has more people.

Imagine you were also given some additional information:

<div align="center">

The apartment is 600 square feet in size.

The house is 1,000 square feet in size.

</div>

How would you decide which home would be less crowded?

You could use the idea of density to make a decision about which home to live in. Density helps us understand how crowded a place is.

Density is an idea that is used to measure many different things. For example, how many people live in a place, the number of animals that can survive in the wild, and the weight of gold are all related to density. This packet combines social studies, science and math. You will use the geometry concepts of area and volume to understand important topics on the high school equivalency math and science exams. You will also practice interpreting graphs, a crucial skill for high school equivalency success.

Population density is a way of describing how crowded a place is with people. Places where people are spread out, like the country, have low population density. Places where people live close together, like cities, have high population density. Population density is also used to describe how many animals or plants are in an area. For example, you might talk about the population density of pigeons in New York City.

In science, density is a measure of how heavy something is for its size. For example, steel is denser than foam rubber. If you hold a piece of foam rubber in one hand and a piece of steel the same size in the other hand, the steel would be heavier. Steel has a higher density which

makes it heavier for its size. (The second packet on density will give you practice in the **Density of Matter**.)

Answer: You can compare how crowded two homes are by measuring their density. You might calculate the number of rooms per person. The apartment has $1\frac{1}{2}$ rooms per person and the house has $1\frac{1}{3}$ rooms per person. Or you might calculate the number of square feet per person. The apartment has 300 square feet per person and the house has 333 square feet for each person. Can you think of any other ways of calculating how crowded the two homes are?

In this packet, you will practice comparing how crowded different places are, based on their size and population. The questions and answers you see in this packet will help you check your understanding of exercises and explanations as you read.

What is Area?

In order to understand population density, it is important to understand *area*, an idea from geometry. In everyday English, "area" means a place. A neighborhood could be an area, for example. A town could be an area. We might talk about the urban areas of New York State where a lot of people live or the rural areas of the state where fewer people live.

In mathematics, the definition of *area* is more specific. Area means the size of a surface. Examples of surfaces include a chalkboard, a table, a wall or a field. In mathematics, if you talk about the area of a city, you might ask, "What is the size of the surface the city covers?"

We measure surfaces by imagining how many squares it would take to cover the surface completely. For example, you might measure the area of a piece of paper in square inches. To do this, you can imagine lots of squares, each with a width and length of 1 inch, covering the paper.

Let's think about the area of the figure below. It's made of squares that are 1 inch on each side.

1 inch

1 inch
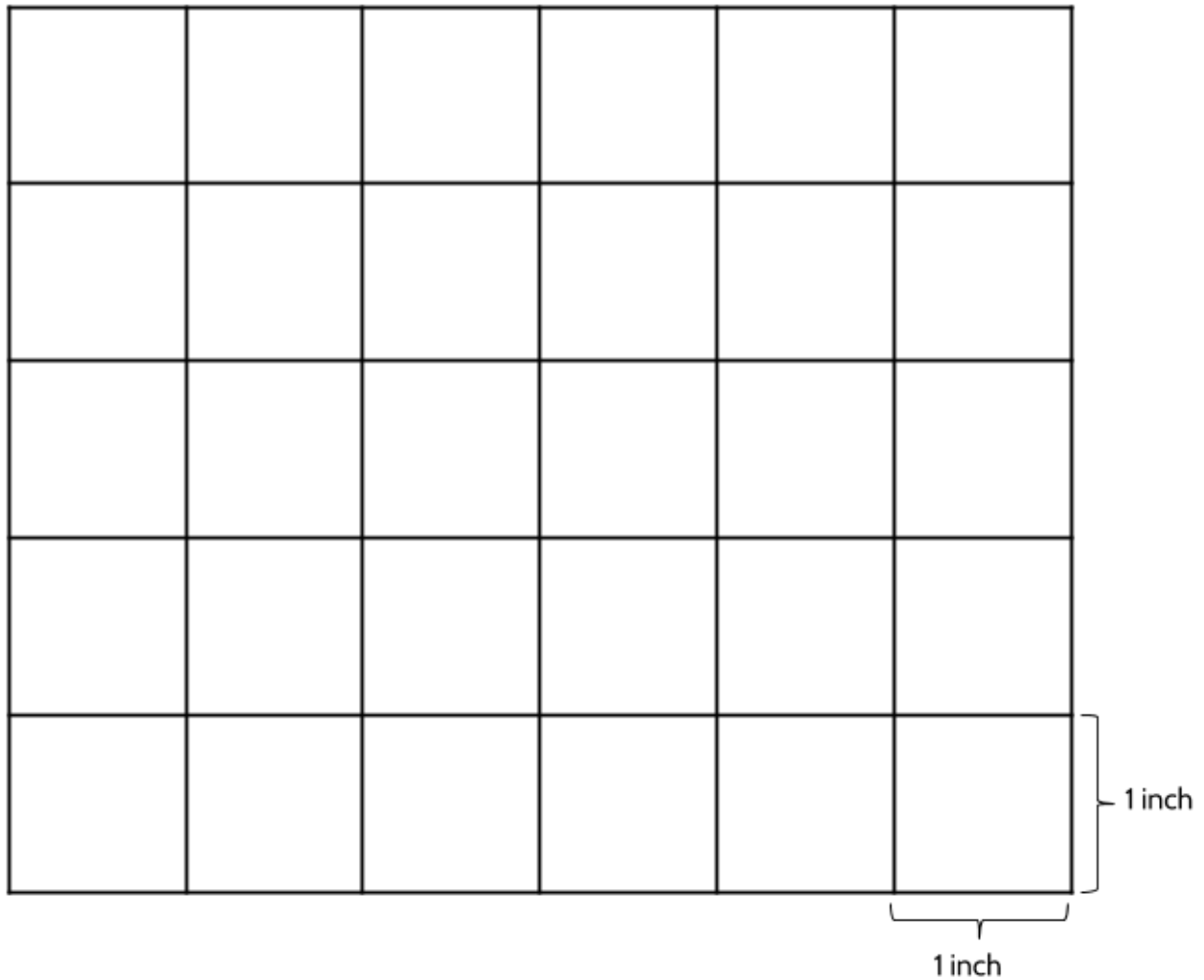
How many squares are there in total? _____

What is the area of the rectangular grid? _____ *square inches*

The answer to both of these questions is the same: *30*. How did you know there were 30 square inches in that figure?

Ways of finding the number of squares:

- Counting the number of squares is a perfectly good way to find out the area.

- Another way to find the number of squares in the grid is to count the number of rows and columns. There are 5 rows and each row has 6 squares. If you add 6's for each of the 5 rows, you end up with 30: 6 + 6 + 6 + 6 + 6 = 30

- Another way is to multiply the height by the length. The height is 5 squares and the length is 6 squares. The area of the grid = 5 inches × 6 inches = 30 square inches.

When we use the word *area*, we mean the number of squares it takes to cover a surface. The question, *What is the area?* means the same as *How many squares would you need to cover the entire surface?*

We need to know the size of the squares we are using to measure the surface. That is why it is important to include a measurement in your answer. When you write the number of squares in the area, always include a measurement: square inches, square centimeters, square feet, square meters, etc.

You can measure area with squares of many different sizes. For example, the area of an apartment might be measured with square feet or square meters, and the area of a city with square miles or square kilometers. We use the word *unit* to refer to the size of the squares. In the United States, our units for area are square inches, square feet and square miles. In the rest of the world, people use square centimeters, square meters and square kilometers as part of the International System (SI) of measurement, which is related to the metric system. American scientists, health care workers, and others also use SI measurements.

Here are some examples of possible area measurements:

The size of a...	is about...	and is also about...
postage stamp	$\frac{1}{2}$ square inch	3 square centimeters
playing card	9 square inches	56 square centimeters
football field	57,000 square feet	53,000 square meters
New York State	55,000 square miles	142,000 square kilometers

Check Your Understanding: If you look at real estate listings, you will see that the size of homes isn't usually measured in square inches. Why not? What unit is normally used?

To measure in different situations, you will use squares of different sizes. For example, if you measure the size of a table, you would probably use square inches or square centimeters. To measure the size of a house or apartment, you would probably use square feet or square meters. If you measure the size of a country, you would use square miles or square kilometers.

Square units can be written in complete words or they can be shortened:

square inches	*square feet*	*square miles*
sq. in.	*sq. ft.*	*sq. mi.*
in^2	ft^2	mi^2

Square inches, sq. in., and in^2 all mean the same thing.

Answer: The truth is, we *could* measure the size of homes in square inches though it probably wouldn't be practical. Usually, you will see that real estate ads describe the size of homes in square feet.

Let's say a 3-bedroom home has an area of 2,000 square feet. You could also say that the area is 288,000 square inches. This is correct since there are 144 square inches in a square foot, but 2,000 square feet is an easier measurement to remember. You could even say the area of the house is .00007 square miles!

The measurement in square inches and the measurement in square miles are both true, but I doubt you will ever hear someone measure a house that way. The number 2,000 in square feet is an easier number to talk about with other people. When we measure area, we usually choose the unit that gives us numbers that are easiest to communicate to other people.

Area & Population Density

Find the area of the following figures. Check your answers in <u>What is Area? – Answer Key</u>.

1) Area = ____42____ square inches

   How do you know the area is 42 in²?

   _____

   _____

   _____

   _____

   _____

   1 inch

   1 inch

2) Area = _____ square feet

   1 foot

   1 foot

Find the area of the following figures:

3) Area = _____ sq. mi.

2 miles

6 miles

4) Area =

4 inches

5 inches
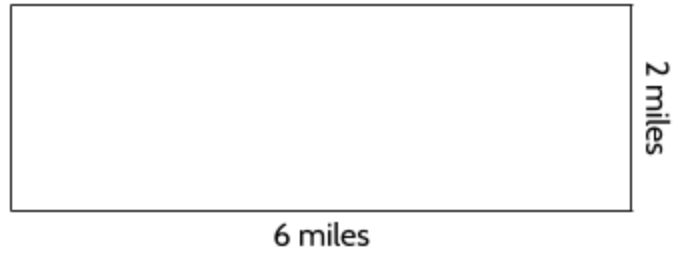
5) The area of the rectangle on the right is 15 square feet.

    What do you think the length and width of the rectangle could be?
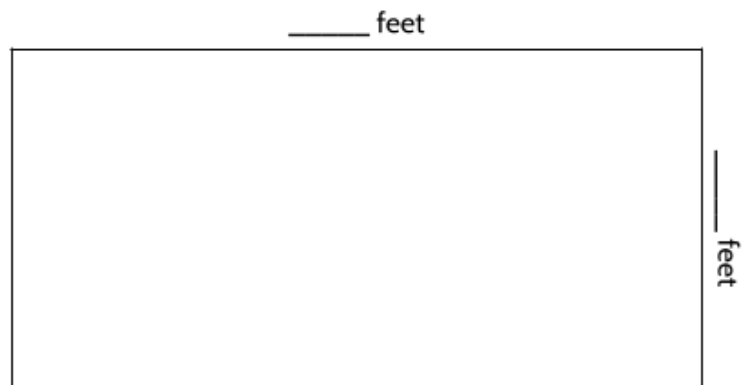
_____ feet

_____ feet

You can also find the area of shapes other than rectangles.

What are the areas of these figures?

6) Area = _____ sq. ft.

1 foot

7) Area =

1"
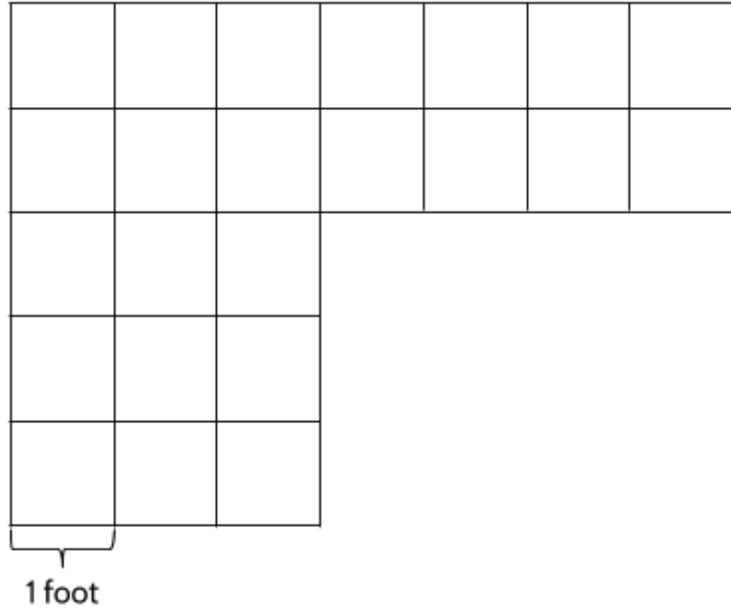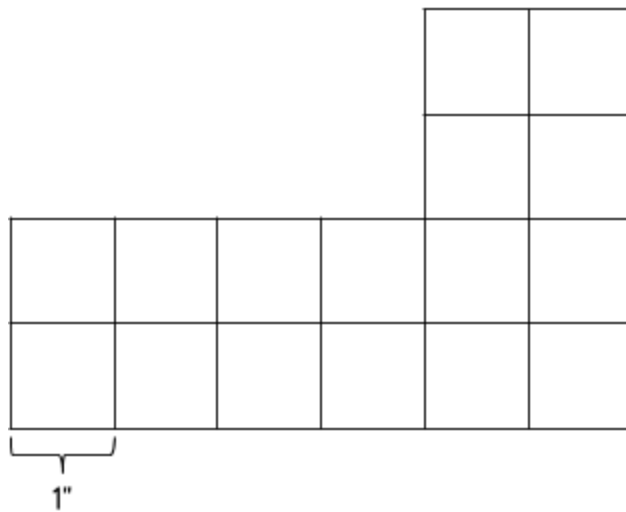
Note: The symbol ' means *feet* and " means *inches*.

8) Area =

8 in.

4 in.

8 in.

3 in.
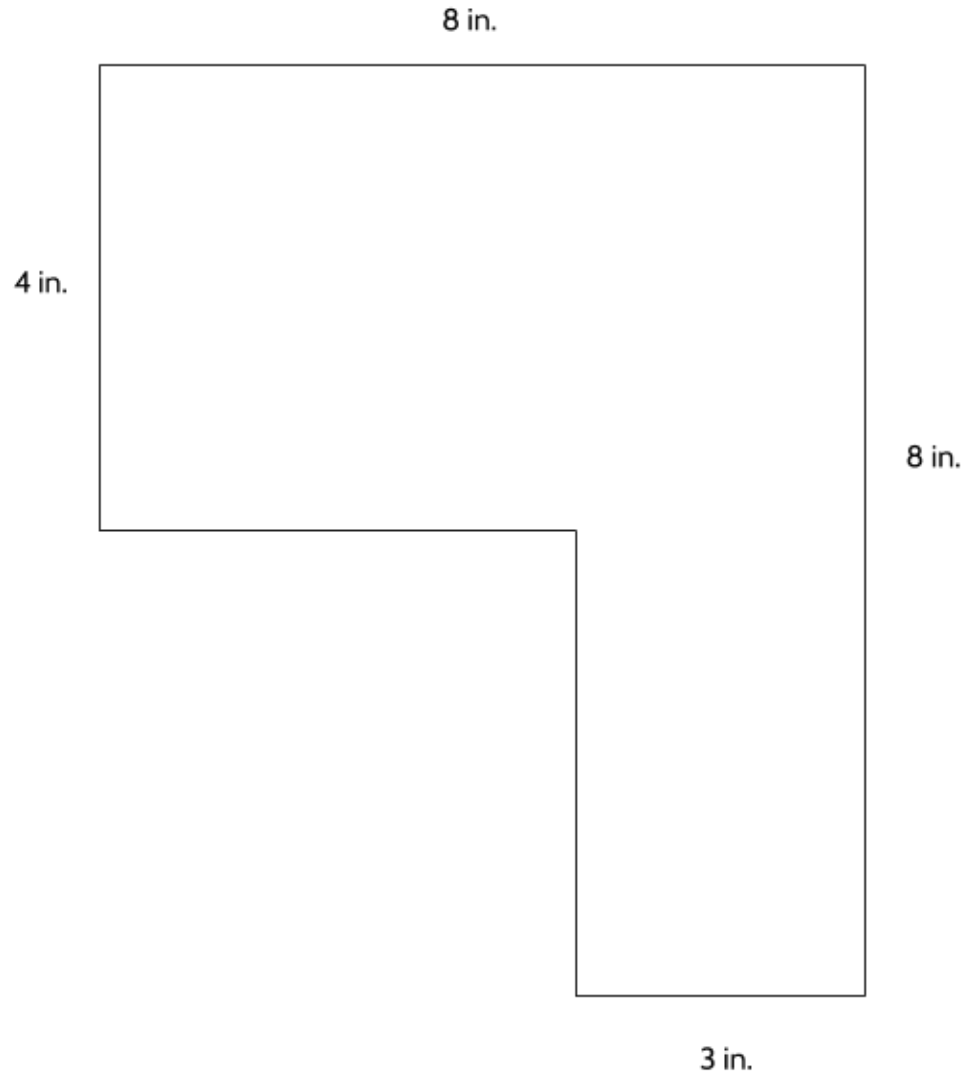
Question: What is the area of a standard piece of 8 ½" by 11" piece of paper?

What is Area? – Answer Key

1) 42 square inches. You could count 42 squares or multiply 6 by 7 to get 42.

2) 15 square feet

3) 12 square miles

4) 20 square inches

5) There are many possible correct answers. The dimensions of the rectangle could be 3 feet by 5 feet. The measurements could also be 2 feet by 7 ½ feet.

6) 23 square feet

7) 16 square feet

8) 44 square inches

What is Density?

Beans and Density

Now that we've explored area, let's consider density. When we calculate the density of a population, we need to divide the population over a certain area. For this activity, you will need about 100 beans (rice grains or other items about the same size are fine as well) and the grid paper on the next page. (You might also use graph paper if you have it.)

Check your answers in What is Density? – Answer Key.

1) On grid paper, color or fill in an area of 18 square inches.

2) Count out 54 beans and pour them on to your 18 square-inch area.

3) Spread the beans out equally so that there is the same number of beans in each of the 18 squares.

4) You should be able to put an equal number of beans on each square.
    Once you divide up all 54 beans equally, how many are there in each square? _____

5) This grid has a density of _____ beans per square inch. (In other words: *How many beans are in each square?*)

How we write measurements of density:

If you had 18 beans to spread out on the rectangular grid, the density is *1 bean per square inch*, since you can put 1 bean on each of the 18 squares in the grid. Try it on your grid.

If you had 36 beans, you could put 2 beans on each of the 18 squares. So, if the population is 36 beans and the area is 18 square inches, then the density is *2 beans per square inch*. This means there will be 2 beans for every square inch. Try it.

Density measurements can be written in different ways. Each of the following means the same thing and should be read as *"2 beans per square inch:"*

> *2 beans/square inch*   *2 beans/sq. in.*   *2 beans/in^2*

Answer: What is the area of a standard piece of 8 ½" by 11" piece of paper? In other words, how many squares would cover the paper?

The image below shows the paper covered with light grey 1-inch squares.

11 in.

8 ½ in.
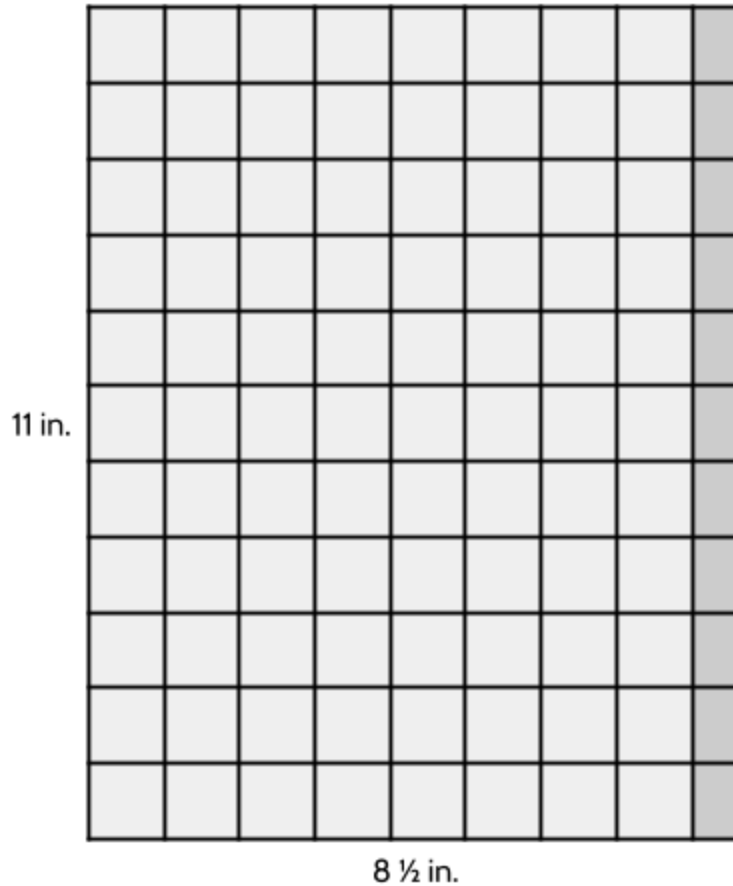
There are 88 light grey 1-inch squares. The dark grey pieces are actually each half of a square, since the paper is 8 ½ inches wide. There are 11 half-squares, which equals 5 ½ light grey square inches.

$88 + 5\frac{1}{2} = 93\frac{1}{2}$ square inches

Another way to figure out the area is to multiply the length and the width of the paper:

$8.5 \times 11 = 93\frac{1}{2}$ in² ← Try this with a calculator.

Density of Foxes in a State Park

In the last exercise, you filled in an area of 18 square inches. Imagine that this grid now represents a state park with an area of 18 square miles. There is a population of 54 foxes living in the park (represented by the beans). The population density of foxes is 3 foxes per square mile.

In the life sciences, the word **population** refers to all the organisms of a species living in a specific area. A **species** is a group of similar living things which interbreed among themselves.

In the spring, the fox population increased when 36 baby foxes were born.

6) Count out beans to represent the additional foxes and add them to the grid.

7) What is the total population of foxes now? _____

8) Can you put the same number of beans in each square? _____

    How many beans are there in each square? _____

9) The density after the population increase is _____ foxes per _____ .

The next winter was really cold and, unfortunately, 27 foxes died.

10) Remove 27 beans. What is the total population of foxes now? _____

11) You won't be able to divide an equal amount of whole beans into each square, but what if you could cut some beans in half?

    _____

12) The fox population density after the winter is _____ foxes/sq. _____ .

13) Take a look at Tayshawn's work on the fox population problem. Can you help him find the fox population density after the winter?

54 foxes
+ 36 baby foxes
———————————
90 foxes after the spring
− 27 foxes died
———————————
63 foxes after the winter

I found the fox density by dividing the fox population by the area.

$54 \div 18 \text{ mi}^2 = 3 \text{ foxes} / \text{mi}^2$

$90 \div 18 \text{ mi}^2 = 5 \text{ foxes} / \text{mi}^2$

$63 \div 18 \text{ mi}^2 = ?$

63 doesn't divide evenly by 18, so I don't know the fox density after the winter.

Finding Density from Area and Population

In order to find the density, you can *distribute* (spread out evenly) the population to each square in the area. Once you have distributed the population, the density is the number of things left in each square.

Try these practice exercises. Fill in the missing information.

14)   Population = 144 beans

Area = _24_ square _inches_

Density = _____ per sq. _in._

1 inch

1 inch
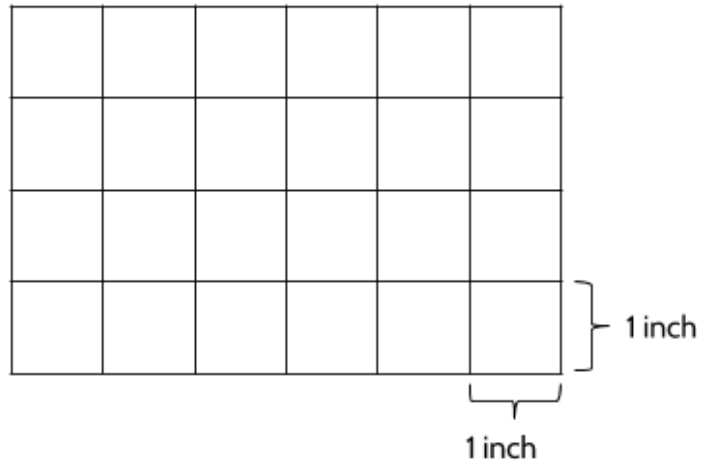
15)   Population = 180 foxes

Area = _____ sq. _____

Density = _____ / sq. _____

1 mile

1 mile
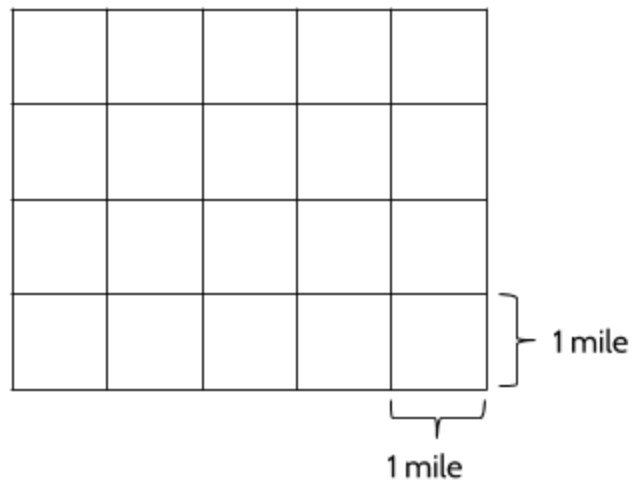
16)   Population = 132 squirrels

        Area = _____ square _____

        Density = _____ squirrels/sq. mi.
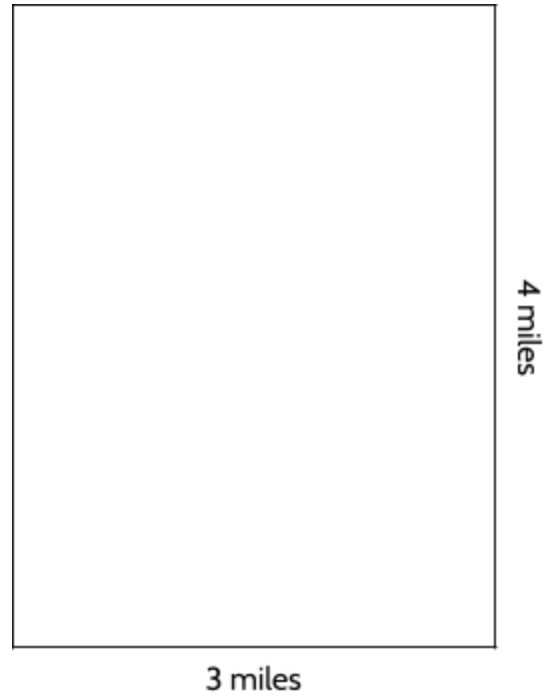
4 miles

3 miles

17)   Population = 27,000 pigeons

        Area = _____

        Density = _____
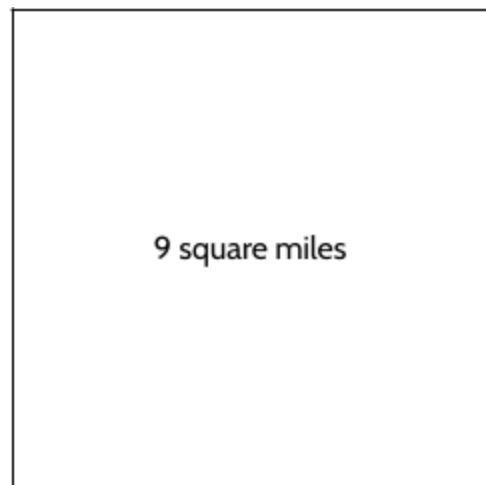
9 square miles

Fill in the missing blanks.

18)

Population = 350 people

Area = 14 square miles

Density = _____ people/_____

19)

Population = 100 rabbits

Area = 40 sq. mi.

Density = _____ _____ /_____

20)

Population = 1,035 pigeons

Area = 60 mi^2

Density = _____ pigeons/_____

21)

Population = 5,000 chickens

Area = 10,000 square _____

Density = _____ chickens/ft^2

22)

Population = 2000 people

Area = _____ mi^2

Density = 200 people/mi^2

23)

Population = _____ people

Area = 7 mi^2

Density = 5,000 people/mi^2

Answer the questions below.

24)   If a population of 200 cows lives on 5 square miles of land, what does it mean that there is a density of 40 cows/mile^2?

25)   Imagine a population of deer in a forest. In the winter, animals compete for fewer and fewer resources. Many deer don't survive and the population decreases while the area of the forest stays the same. What happens to the population density of deer?

26)   If parts of the forest are cut down to make room for houses, but the population stays the same, what happens to the population density of deer?

Challenge Question: There are about 2 million rats in New York City, which has a land area of about 300 square miles. What is the population density of rats in New York City?

25

What is Density? – Answer Key

1) The 18 square inch area could be different shapes on the grid paper. You might have a 2 by 9 rectangle or a 3 by 6 rectangle. You could even have an L shape as long as there are 18 squares outlined.

2) Count to make sure you have exactly 54 beans.

3) Each square should have the same number of beans.

4) 3

5) 3

6) You should count out 36 more beans.

7) 90

8) Yes, 5

9) 5, square mile

10) 63

11) There would be 3 and ½ beans on each square.

12) 3.5, mile

13) 63 ÷ 18 = 3.5. The fox population density after the winter is 3 ½ foxes per square mile. (You might have a different explanation.)

14) *24*, *inches*, 6, *in*

15) 20, mi., 9, mi.

16) 12, miles, 11

17) 9 square miles, 3,000 pigeons/square mile

18) 25, square mile

19) 2.5 rabbits/sq. mi.

20) 17.25, mi^2

21) feet, .5

22) 10

23) 35,000

24) Different answers are possible. It means that if 200 cows were distributed evenly across 5 square miles of land, every square mile would have 40 cows on it.

25) The population density goes down because there are fewer deer with the same amount of space.

26) The population density goes up because there are the same number of deer with less space.

Reviewing What We Have Learned about Population Density

Population density tells you how crowded a population is over a particular area. It shows the relationship between the population and the area. 500 people at the Department of Motor Vehicles would be really crowded, but a football stadium with 500 people would seem empty. This is because the area of a football stadium is much larger than the DMV's offices.

The *population density* of foxes in the state park is a measure of how crowded the park is with foxes. To find out the population density of foxes, we imagine the foxes spread around the park and distributed equally in each square mile.

We can divide the total number of foxes by the total area to find the population density.

54 foxes *divided by* ***18 square miles*** *equals* ***3 foxes per square mile****.*

Also written as:
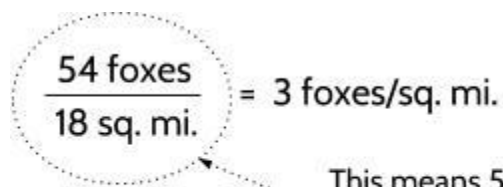
Population *divided by* ***area*** *equals* ***population density****.*

You can write this as a formula:

$$\frac{population}{area} = population \ density$$

or

$$\frac{p}{a} = d$$

If you insert numbers, you get:

$$\frac{54 \ foxes}{18 \ sq. \ mi.} = 3 \ foxes/sq. \ mi.$$

This means 54 divided by 18. The horizontal line means the top number is divided by the bottom number, so the whole equation means 54 ÷ 18 = 3.

This calculation confirms that 3 foxes would live in each square mile if they were distributed evenly around the park. (Of course, this is not how foxes actually live, but it's a way for us to understand how crowded the state park is with foxes.)

Sometimes the population can't be divided evenly by the area. In that case, your answer will have a decimal remainder.

$$\frac{63\ foxes}{18\ sq.\ mi.} = 3.5\ foxes/sq.\ mi.$$

> *You can find the population density by dividing the population*
>
> *(number of people, animals, plants or things) by the area.*

If you know the population and the population density, you can find out the area by dividing the population by the density: $\frac{p}{d} = a$

> *A population of 2,000 people with a population density of 200 people per square mile means that the area is _____ square miles.*

$$\frac{2000\ people}{?\ sq.\ mi.} = 200\ people/sq.\ mi.$$

There must be a total of 10 square miles, because 2,000 divided by 10 equals 200.

And if you know the area and the population density, you can find out the population by multiplying the area by the density: $a \times d = p$

> *An area of 7 square miles with a density of 5,000 people per square mile means that the total population is _____ people.*

$$\frac{?\ people}{7\ sq.\ mi.} = 5000\ people/sq.\ mi.$$

There must be a total of 35,000 people, because 35,000 divided by 7 equals 5,000.

Answer: There are about 2 million rats living in New York City. New York City has an area of about 300 square miles.

We can use these numbers to find the population density of rats in New York City, using the formula:

$$\frac{population}{area} = population\ density$$

Don't forget that *2 million* can be written as 2,000,000 before doing calculations.

$$\frac{2,000,000\ rats}{300\ sq.\,mi.} \approx 6,667\ rats/sq.\,mi.$$

We are using bigger numbers now, but our bean model still works. Imagine trying to fit 2 million beans into 300 squares. Each square would have about 6,667 beans on it.

Note: The symbol $\approx$ means "approximately equal to."

2,000,000 divided by 300 equals 6,666.66... (The 6 after the decimal point repeats forever, so we can round off the number to 6,667 per square mile.)

What Does "Per" Mean?

Here are some examples of density measurements:

3 beans *per* square inch

5 foxes *per* square mile

200 people *per* square mile.

Notice that "per" is used in all of these measurements. The word "per" means "for each," so the phrases above could also be written like this:

3 beans *for each* square inch

5 foxes *for each* square mile

200 people *for each* square mile

When you first spread out the 54 beans on the grid paper, there were 3 beans *for each* square inch. In the state park example, after the new foxes were born in the spring, there were 5 foxes *for each* square mile in the state park. *Per* is another way to say *for each*.

We use *per* a lot when we talk about driving.

Question: How fast was the car moving?
Answer: 65 miles *per* hour. This means the car went 65 miles *for each* hour of driving.

Question: How much did the gas cost?
Answer: $3.00 *per* gallon. This means you have to pay $3.00 *for each* gallon of gas you put in your car.

Question: What is your car's gas mileage?
Answer: 25 miles *per* gallon. This means your car travels 25 miles *for each* gallon in the tank.

Note: People often say "a" or "an" instead of "per." They might say "65 miles **an** hour," "3 dollars **a** gallon" or "25 miles **a** gallon." When used this way, the words "a" and "an" also mean *for each.*

Each of the answers above use a *rate* to compare two related quantities. The rate, "65 miles per hour," is the same thing as saying, "65 miles for each 1 hour." For each 1 hour that passes, the car travels 65 miles.

The rates for driving speed, cost and gas mileage above can be written in many ways:

speed	cost	gas mileage
65 miles per hour	*$3.00 per gallon*	*25 miles per gallon*
65 miles/hour	*3 dollars/gallon*	*25 miles/gallon*
65 mph	*$3.00/gallon*	*25 mpg*

The slash symbol / also means "per" or "for each." 65 *miles/hour* is another way to write "65 miles per hour."

1)   Complete the following tables. Check answers in <u>What Does "Per" Mean? – Answer Key</u>.

Rate: *65 Miles/Hour*

Hours Driving	Distance Traveled
1	65 miles/1 hour
2	130 miles/2 hours
3	
5	
10	

Rate: *$3.00/gallon*   *25 miles/gallon*

Number of Gallons	Cost	Gas Mileage
1	$3.00/1 gallon	25 miles/1 gallon
2	$6.00/2 gallons	
3		
5		
10		

A rate like 25 miles/gallon is called a *unit rate* because it connects 25 miles to 1 gallon. This answers the question, "How many miles can you drive on **1** gallon of gas?" The speed 65 miles/hour is also a unit rate since it answers the question, "How far did the car travel in 1 hour?" Unit rates answer the question, "How much (or how many) for 1?"

You can make a unit rate from other kinds of rates. For example, if you were told oranges cost $2 for 4 oranges, you could figure out that the unit rate is $.50 for 1 orange, which can be written as $.50/orange. This answers the question, "How much money is it for 1 orange?"

2)   Complete the following tables.

How many miles for 1 hour?   ____45____ *mph*

Hours Driving	Distance Traveled
1	
2	
3	135 miles/3 hours
5	
10	

How much for 1 gallon?   _____ */gallon*   _____ *mpg*

Number of Gallons	Cost	Gas Mileage
1		
2	$5.40/2 gallons	
3		
5		90 miles/5 gallons
10		

3)   Look up the words *per diem* and *percent*. What do these words mean?

Complete the following matching activity. Connect the quantity on the left with the unit rate on the right, then use the combined phrase to fill in the blanks in the sentences below.

190 heartbeats	person
19 students	pound
$15.00	hour
$2.00	mile
$3.00	class
270 eggs	gallon
1.6 kilometers	minute

per

Complete the following sentences, using phrases from the matching activity.

4) The doctor measured the patient's heart rate at 190 ___heartbeats per minute___ .

5) Kindergarten in Albany has an average of 19 _____ .

6) Starting in 2018, large employers in New York City paid a minimum wage of

 15 _____ .

7) The average cost of apples is about 2 _____ .

8) At many grocery stores, milk costs about 3 _____ .

9) Americans consume about 270 _____ each year.

10) To understand driving distances in other countries, you can use the conversion rate

 of 1.6 _____ .

What Does "Per" Mean? – Answer Key

1) *Distance Traveled*
65 miles/1 hour
130 miles/2 hours
195 miles/3 hours
325 miles/5 hours
650 miles/10 hours

Cost
$3.00/1 gallon
$6.00/2 gallons
$9.00/3 gallons
$15.00/5 gallons
$30.00/10 gallons

Gas Mileage
25 miles/1 gallon
50 miles/2 gallons
75 miles/3 gallons
125 miles/5 gallons
250 miles/10 gallons

2) 45 mph
Distance Traveled
45 miles/1 hour
90 miles/2 hours
135 miles/3 hours
225 miles/5 hours
450 miles/10 hours

$2.70 dollars/gallon

Cost
$2.70/1 gallon
$5.40/2 gallons
$8.10/3 gallons
$13.50/5 gallons
$27.00/10 gallons

18 mpg

Gas Mileage
18 miles/1 gallon
36 miles/2 gallons
54 miles/3 gallons
90 miles/5 gallons
180 miles/10 gallons

3) Some people get a *per diem* when they are traveling for work. A *per diem* is an amount of money that is given each day for expenses. "Diem" means *day* in Latin, so "per diem" literally means "for each day."

Percent means something out of 100, so 50 percent means 50 out of 100. 25% means 25 out of 100. "Percent" literally means "for every 100." By the way, you can find "cent" in many words that have to do with 100. A century is 100 years. A cent (penny) is $\frac{1}{100}$ of a dollar. A centimeter is $\frac{1}{100}$ of a meter. A centennial is the 100th anniversary of an important event. A centipede (supposedly) has 100 legs. A centenarian is a person who has lived to be 100 years old!

4) heartbeats per minute
5) students per class
6) dollars per hour
7) dollars per pound
8) dollars per gallon
9) eggs per person
10) kilometers per mile

Chickens and Eggs

Have you ever noticed a big difference in the prices of the eggs that are sold in grocery stores? Eggs can cost less than $2.00 per dozen and as much as $8.00 per dozen. There are different factors that affect the price of eggs, including population density. You might see words like "cage free," "free range," or "pasture-raised" on egg cartons. These phrases tell you how much space each chicken is given to move around.

Cage Free means the chickens are raised with at least one square foot per hen (1 chicken/ft^2). **Free Range** means that each bird has at least two square feet (0.5 chickens/ft^2) of space. To use the **Pasture-Raised** description, a farms must give each bird at least 108 square feet (0.01 chickens/ft^2). If the egg carton doesn't have one of these labels, each chicken gets less than half of a square foot and never goes outside.

graphic from Vital Farms

Check your answers in Chickens and Eggs – Answer Key.

1) As a consumer, which type of eggs would you prefer to buy? Why?

2) As a farmer, which eggs would you prefer to produce? Why?

Two Chicken Coops

10 feet

10 feet

100 chickens

5 feet

5 feet

50 chickens

Coop A

Coop B
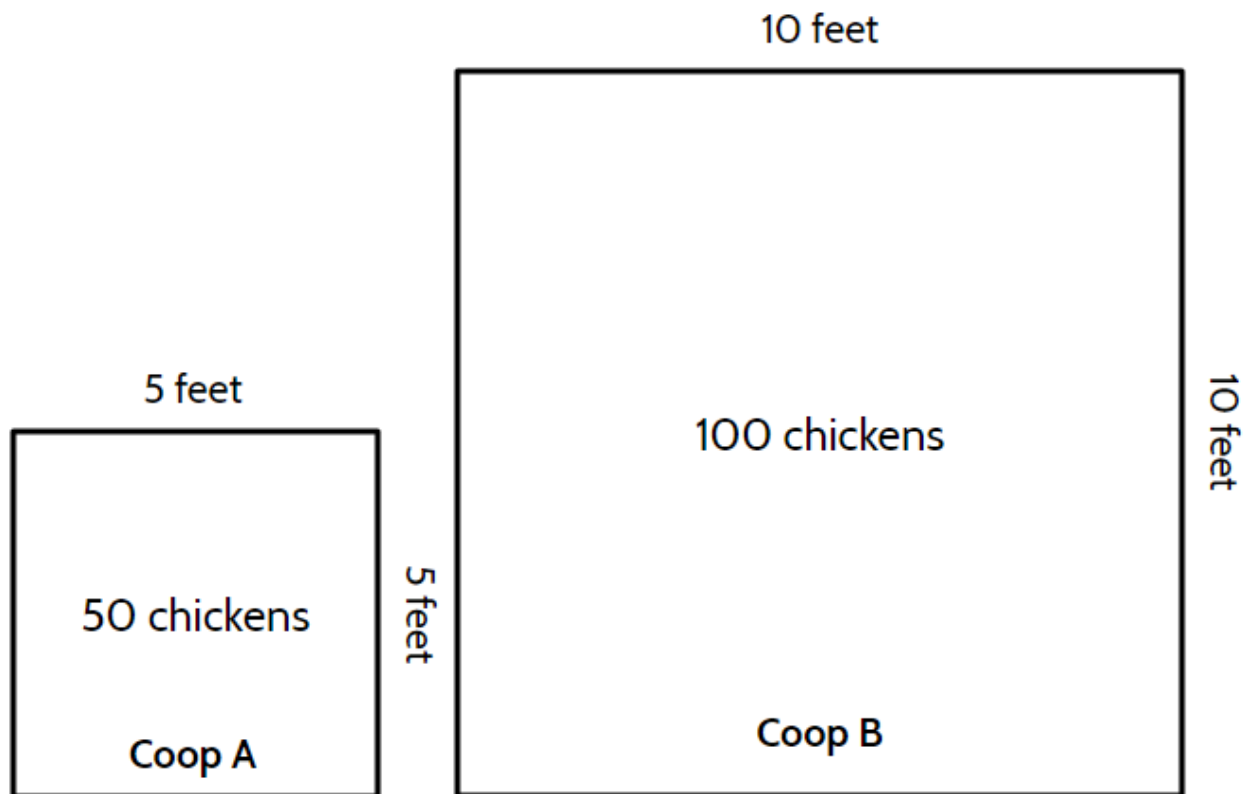
3)   What do you notice about the two chicken coops?

4) Record the data for the two chicken coops in a table:

Coop	Number of Chickens	Area (square feet)	Population Density (chickens per sq. ft.)
A			
B			

5) Is Coop A or Coop B more crowded? How do you know?

For the next few questions, create your own chicken coops. Decide on the size and how many chickens it will hold. Add the information to the table above.

6) Create a coop that is <u>less</u> crowded than both Coop A and Coop B.

7) Create a coop that is <u>more</u> crowded than both A and B.

Chicken Coop Practice Questions

8)   If you added 25 square feet to Coop A, what would the new density be?

9)   If you added 50 chickens to Coop B, what would the new density be?

10) What is the area of a chicken coop with a width of 4 feet and a length of 4.5 feet?

   A. 0.5 ft^2

   B. 8.5 ft^2

   C. 18 ft^2

   D. 180 ft^2

11) If you were a farmer, how many chickens could you raise in Coop B…

   …if you wanted to call the eggs *Cage-free*? _____

   …if you wanted to call the eggs *Free Range*? _____

   …if you wanted to call the eggs *Pasture-Raised*? _____

12) Chicken farmers sell eggs at a higher price when they have a lower population density

   because _____

   _____

13) Someone who buys eggs might be concerned about population density on poultry

   farms because _____

   _____

14) Challenge question: In many large chicken farms, each chicken has the space of an
     8 ½" by 11" sheet of paper. What is this area in square feet? _____ sq. ft.

   Note: To convert square inches to square feet, divide by 144.

Chickens and Eggs – Answer Key

1) This is your personal preference. Have you seen the prices of different kinds of eggs?

2) This probably depends on how much money you can make as a chicken farmer. Sometimes you can make money selling a high quantity at a low price for the consumer. You might also make money by selling few eggs at a higher price to the consumer.

3) There are lots of things you might notice:

   - Coop A is 25 ft^2 in area. Coop B is 100 ft^2.
   - Coop B is four times as big as Coop A. (At first, did you think it was twice as big?)
   - Coop B has twice as many chickens.
   - The density of Coop A is 2 chickens/ft^2. The density of Coop B is 1 chicken/ft^2. The density of Coop A is higher than Coop B.

4) The data from the table should match the numbers on page 37.

5) Coop A is more crowded. There are 2 chickens for every square foot. In Coop B, there is only 1 chicken for every square foot.

6) There are many possibilities. Just make sure that the density of this coop is **less** than 1 chicken/ft^2.

7) There are many possibilities. Just make sure that the density of this coop is **more** than 1 chicken/ft^2.

8) 1 chicken/ft^2

9) 1.5 chickens/ft^2

10) C

11) 100
    50

1*
*Actually, 100 ft^2 isn't quite enough for one chicken when it's pasture-raised. You would need 108 square feet to raise 1 chicken.

12) This answer should be based on your opinion, but think about how farmers can raise fewer chickens if they have a lower population density. Fewer chickens means fewer eggs.

13) It might depend on whether you can spend a bit more so that you know that the chickens that lay the eggs have better lives. However, if you are on a budget, you might just need to get the cheapest eggs.

14) About .65 or ⅔ ft^2

Looking at the Population of the United States
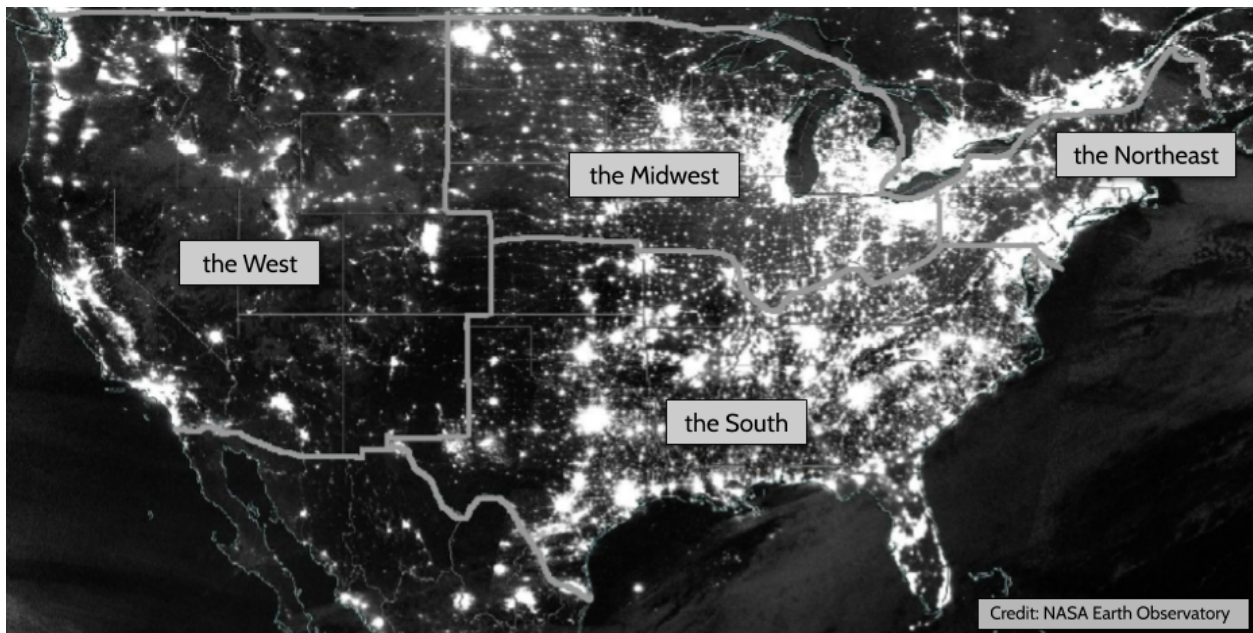
The United States can be separated into four regions: the West, the Midwest, the South, and the Northeast. This satellite image of the continental United States was taken in 2012. The photograph captures light at night.

Check your answers in [Looking at the Population of the United States – Answer Key](#).

1)   What do you notice?

Answer these questions using the photograph of the United States at night.

2)   Which region do you think has the biggest area?

3)   Which region do you think has the most people? How do you know?

4)   Which region do you think is the most crowded with people?

5)   What U.S. states are not included in this satellite photo?

Population of U.S. Regions (Chart)
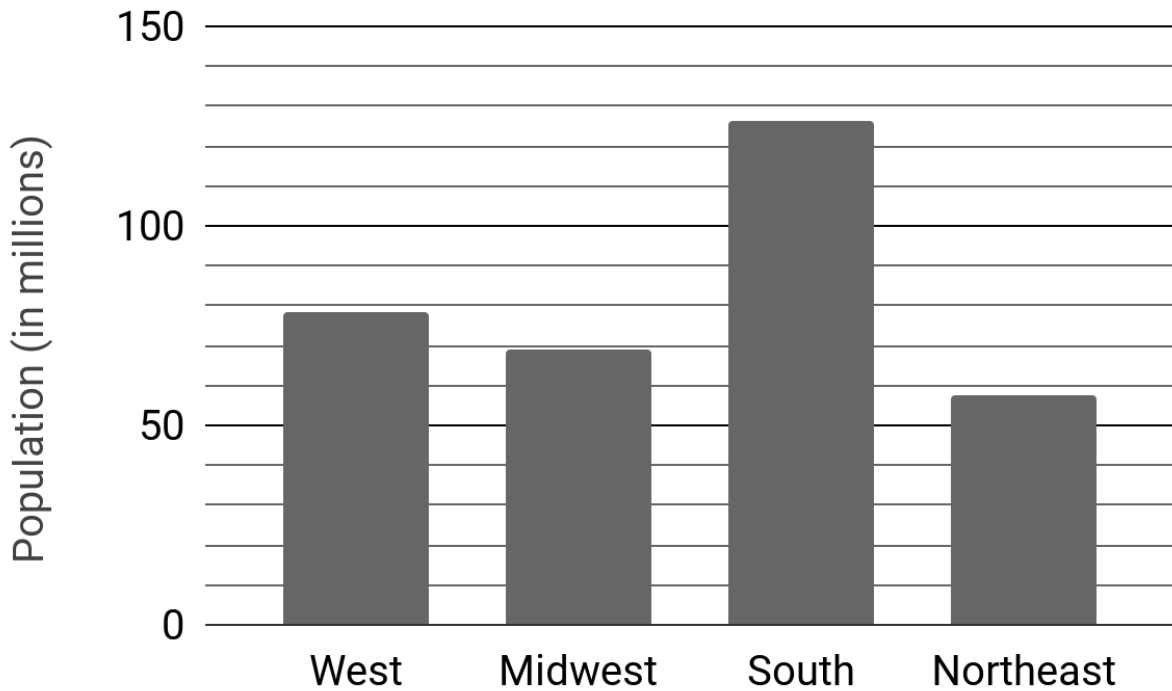
6) Based on the graph above, **estimate** the population of each region of the United States. Then calculate the population density of each region.

Region	Population	Approximate Area (sq. mi.)	Population Density (people per sq. mi.)
West	≈ 79,000,000	1,900,000	42
Midwest	≈	600,000	
South	≈	870,000	
Northeast	≈	160,000	

Answer the following questions using the population table above.

7)   The region of the US with the largest area is…

8)   The South has the largest population. Why doesn't it also have the largest population density?

9)   The Northeast has the smallest area. Does it also have the smallest population density? Why or why not?

10) Which region of the country has the most open space? Explain using your density calculations.

11) Which part of the country has the least open space? Explain using your density calculations.

12) The first European settlers who came to the United States came to the East coast across the Atlantic Ocean. Look back at the photo of the United States at night. What do you see in the photo that shows the history of European settlement?

Area and Population of U.S. States and Territories

Population

Area in Square Miles
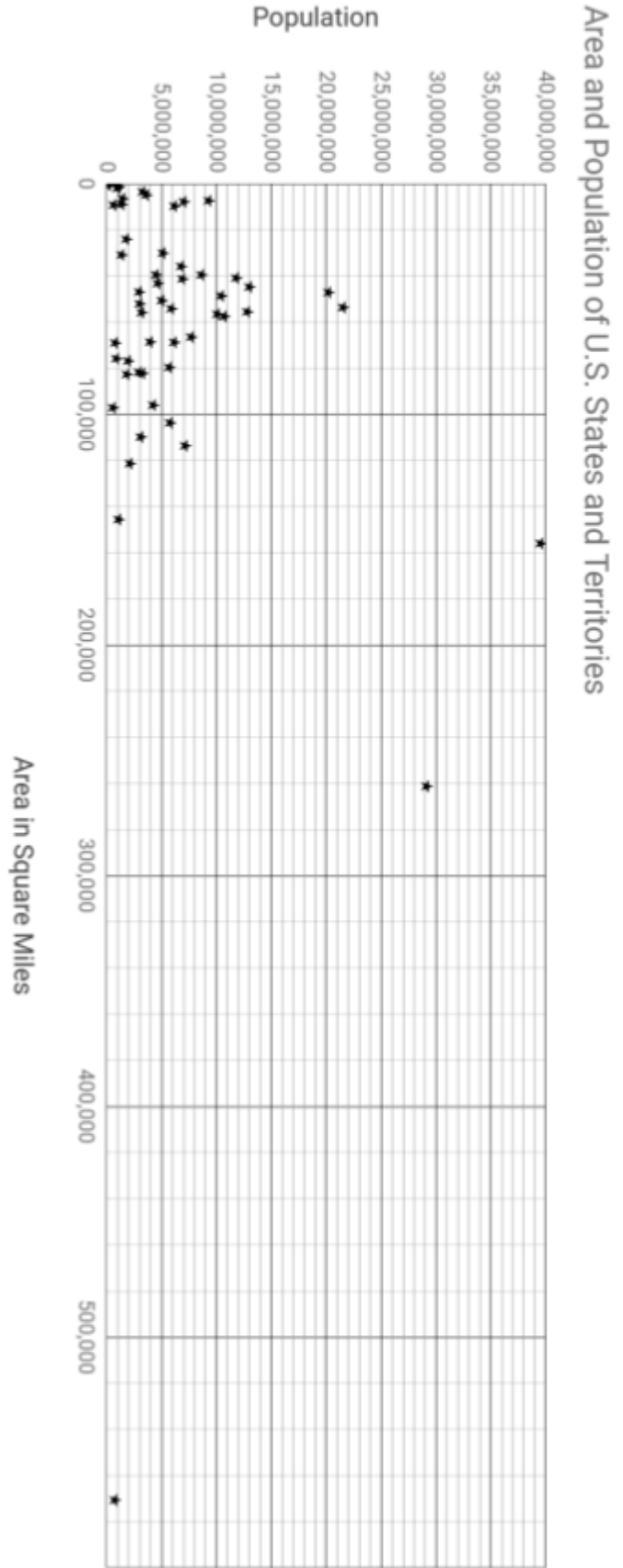
13)   Write three things you notice about this chart.

The chart on the previous page has a star for each of the 50 states and 5 territories of the United States. Did you try counting the stars? It might not seem possible, but there really are 55 stars on the chart. Some of the stars are very close or are overlapping, which makes it hard to count them all. A smaller version of the chart below shows a sample of places with high population density.

Five U.S. States and Territories with Highest Population Density
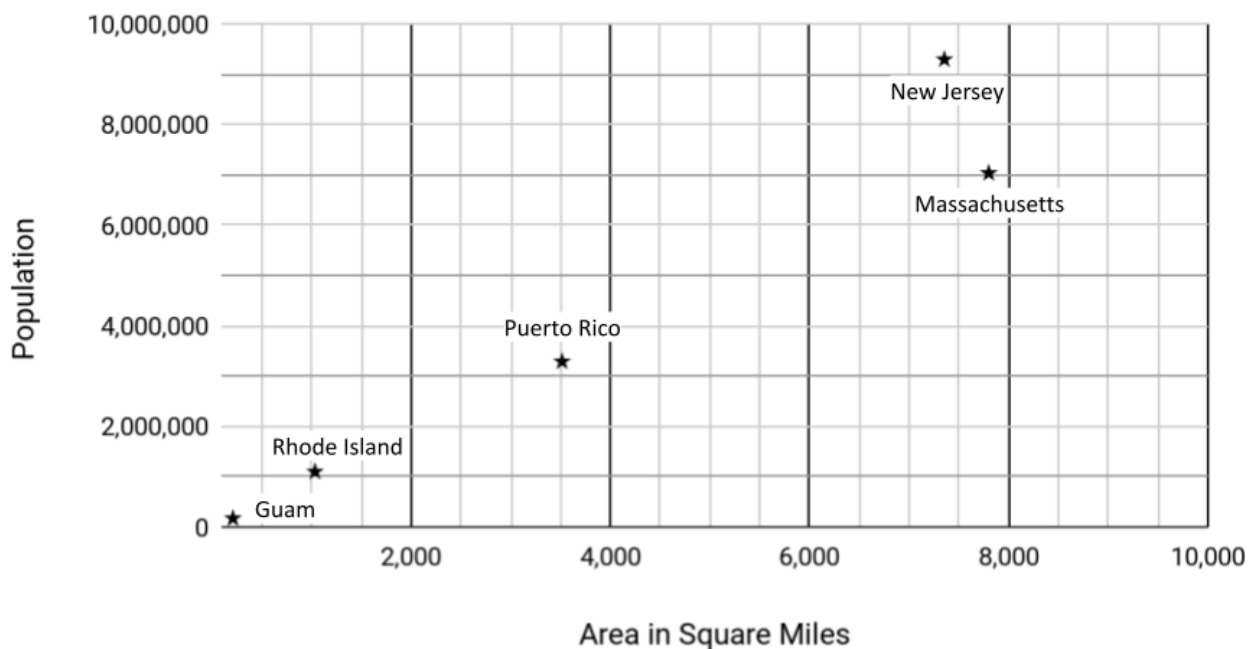
14) Use your best estimate of the population and area of each place to complete the table below. *Reminder*: The ~ symbol means that the numbers are approximate.

State or Territory	Guam	Massachusetts	New Jersey	Puerto Rico	Rhode Island
Population	~200,000	~7,000,000		~3,300,000	
Area (mi²)			~7,400		~1,000
Density (ppl/mi²)	~1,000			~950	

Area and Population of U.S. States and Territories (Table)

Use the table to check your estimates on the previous page. How close were you to the actual population, area and density?

State/Territory	Area (mi²)	Population	Density (ppl/mi²)
Alabama	50,645	5,024,279	99
Alaska	570,641	733,391	1
American Samoa	77	55,538	721
Arizona	113,594	7,151,502	63
Arkansas	52,035	3,011,524	58
California	155,959	39,538,223	254
Colorado	103,642	5,773,714	56
Connecticut	4,842	3,605,944	745
Delaware	1,949	989,948	508
Florida	53,625	21,538,187	402
Georgia	57,513	10,711,908	186
Guam	210	169,885	809
Hawaii	6,423	1,455,271	227
Idaho	82,643	1,839,106	22
Illinois	55,519	12,812,508	231
Indiana	35,826	6,785,528	189
Iowa	55,857	3,190,369	57
Kansas	81,759	2,937,880	36
Kentucky	39,486	4,505,836	114
Louisiana	43,204	4,657,757	108
Maine	30,843	1,362,359	44
Maryland	9,707	6,177,224	636
Massachusetts	7,800	7,029,917	901
Michigan	56,539	10,077,331	178
Minnesota	79,627	5,706,494	72
Mississippi	46,923	2,961,279	63
Missouri	68,742	6,154,913	90
Montana	145,546	1,084,225	7
Nebraska	76,824	1,961,504	26

State/Territory	Area (mi²)	Population	Density (ppl/mi²)
Nevada	109,781	3,104,614	28
New Hampshire	8,953	1,377,529	154
New Jersey	7,354	9,288,994	1,263
New Mexico	121,298	2,117,522	17
New York	47,126	20,201,249	429
North Carolina	48,618	10,439,388	215
North Dakota	69,001	779,094	11
Northern Mariana Islands	179	55,070	308
Ohio	40,861	11,799,448	289
Oklahoma	68,595	3,959,353	58
Oregon	95,988	4,237,256	44
Pennsylvania	44,743	13,002,700	291
Puerto Rico	3,515	3,285,874	935
Rhode Island	1,034	1,097,379	1,061
South Carolina	30,061	5,118,425	170
South Dakota	75,811	886,667	12
Tennessee	41,235	6,910,840	168
Texas	261,232	29,145,505	112
US Virgin Islands	134	106,906	799
Utah	82,170	3,271,616	40
Vermont	9,217	643,077	70
Virginia	39,490	8,631,393	219
Washington	66,456	7,705,281	116
West Virginia	24,038	1,793,716	75
Wisconsin	54,158	5,893,718	109
Wyoming	97,093	576,851	6

Questions about the Area and Population of U.S. States & Territories

15) Which two states or territories have the largest population?

16) Which two states or territories have the smallest population?

17) Which two states or territories have the largest land area?

18) Which two states or territories have the smallest land area?

19) Which two states or territories have the highest population density?

20) Which two states or territories have the lowest population density?

21) Missouri and Oklahoma are both about 69,000 square miles in area. Which state is more crowded? Explain your answer.

22) Utah and Puerto Rico both have about 3 million people. Which is more crowded? Explain your answer.

23) Look at the chart below and then answer the question below.

The History of Population Density in the United States
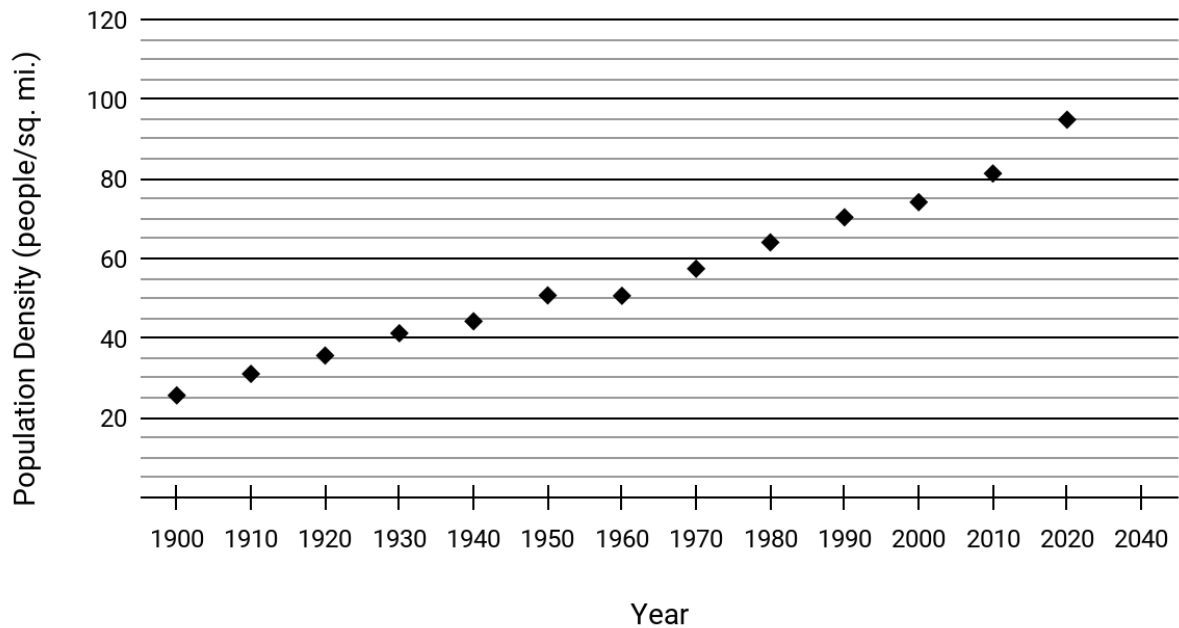
Which of these answers is the best prediction of what the population density of the United States will be in 2040, based on historical data?

    A. 90 people per square mile

    B. 95 people per square mile

    C. 100 people per square mile

    D. 110 people per square mile

Looking at the Population of the United States – Answer Key

1) There are lots of things you might notice:

   - The bright lights are heavily populated areas, like cities.
   - The West isn't as bright as the other regions.
   - The coast of the Northeast is really bright.
   - What else did you notice?

2-4) These are your predictions based on the photo. Were you right?

5) Alaska & Hawaii

6) These are estimates based on the graph. Your answers don't have to be exactly the same, but they should be close.

   Population:
   West 79,000,000
   Midwest 69,000,000
   South 126,000,000
   Northeast 58,000,000

   Density:
   West 42 ppl/mi^2
   Midwest 115 ppl/mi^2
   South 145 ppl/mi^2
   Northeast 363 ppl/mi^2

7) The West

8) The South also has a large area so the population is spread out. The Northeast has a higher population density because more people live in a smaller area.

9) The Northeast has a LOT of people living in a smaller area than the other regions, so its population density is high.

10) The West. There are only 42 people per square mile and each person has more space.

11) The Northeast has the least open space. There are 358 people per square mile, which means each person gets less space than the other regions.

12) The first Europeans sailed across the Atlantic and settled on the east coast. Their population grew as the settlements became cities. European Americans moved west and took lands as part of Western Expansion. The eastern part of the United States is still more populated than the West.

13) There are lots of things you might notice:

   - There are many stars on the chart.
   - The stars represents U.S. states and territories
   - The star on the far right side has a large area and small population.
   - The star at the top represents the state with the highest population, about 39,000,000 people.
   - Most states have less than 15 million people.
   - Most states are less than 100,000 square miles in area.

14) *Population*:
    NJ ~9,300,000
    RI ~1,100,000

Area:
Guam ~200 mi^2
MA ~7,800 mi^2
PR ~3,500 mi^2
RI ~1,000 mi^2

Density:
MA ~900 ppl/mi^2
NJ ~1,300 ppl/mi^2
RI ~1,100 ppl/mi^2

15) California & Texas

16) Northern Mariana Islands & American Samoa

17) Alaska & Texas

18) American Samoa & the U.S. Virgin Islands

19) New Jersey & Rhode Island

20) Alaska & Wyoming

21) Missouri is more crowded because it has a higher population in the same amount of space as Oklahoma.

22) Puerto Rico is more crowded because it has the same number of people as Utah, but with less space.

23) D

Population, Area, and Density Practice

Check your answers in Population, Area, and Density Practice – Answer Key.

1) What is the area of each of the towns below? Notice that they aren't in the shape of rectangles. Each square in the grid is 1 square mile.
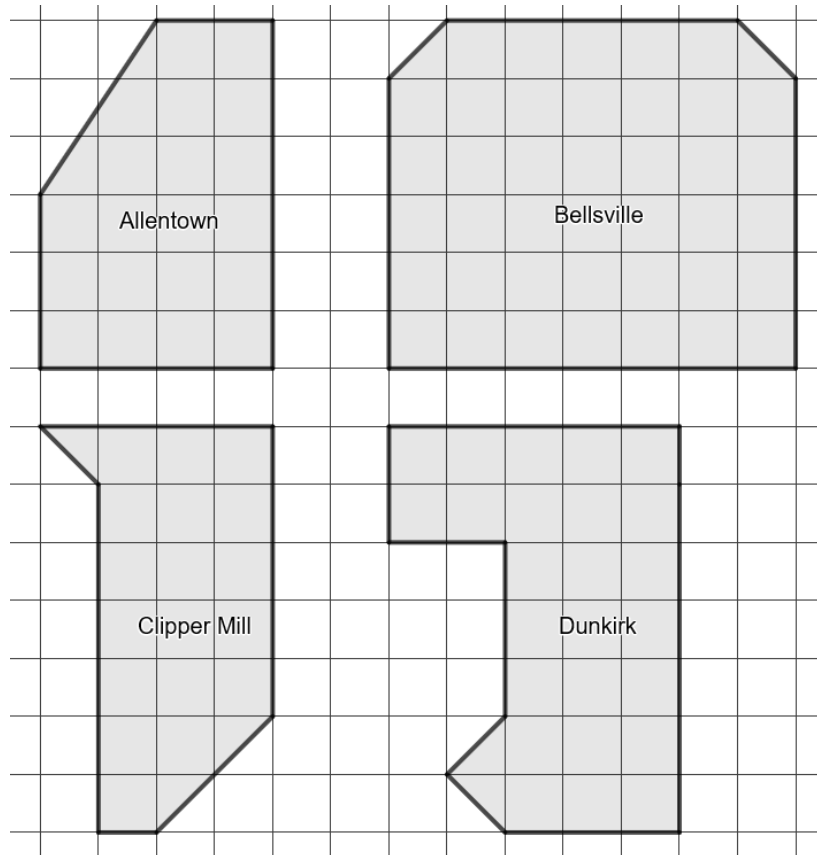
2) Complete the population, area and density table below. Round to the nearest person. For example, a density of 135.6 people/mi^2 would be rounded to 136 people/mi^2.

	Allentown	Bellsville	Clipper Mill	Dunkirk
Population	17,000	24,000	16,000	20,000
Area (mi^2)				
Density (ppl/mi^2)				

3) Fill in the missing information. Round to the nearest person.

Place	Population	Area (sq. mi.)	Population Density
Adams, NY	3,000	2	1,500
Alfred, NY	4,000	1	
Buffalo, NY	936,000	380	
Champaign, IL	145,000		3,085
Jamestown, ND	15,000		1,500
Los Angeles Metro, CA		1736	7,000
Miami, FL		1239	4,441
Montauk, NY	3,000		600
New York City Metro, NY	18,351,000		5,319
Oswego, NY	39,000	21	
Palmyra, NY	5,000	5	
Portsmouth, OH	36,000		1,800
Potsdam, NY	9,000		3,000
Red Hook, NY		7	1,000
Snowflake, AZ		5	1,200
Utica, NY	117,000		1,887
Walsenburg, CO	3,000		1,500

Provide the correct answer and explain your answer.

4) For each pair of locations, <u>underline</u> the place that has more people.

    A. Los Angeles Metropolitan Area, CA or <u>New York City Metropolitan Area, NY</u>

The NYC metro area has about 18,000,000 people and the LA metro area has about 12,000,000 people. I found out the LA population by multiplying the area (1736 mi^2) by the density (6,999 ppl/mi^2).

    B. Alfred, NY or Red Hook, NY

5) For each pair of locations, <u>underline</u> the place that is bigger in land area.

    A. Los Angeles Metropolitan Area, CA or New York City Metropolitan Area, NY

    B. Oswego, NY or Portsmouth, OH

6) For each pair of locations, <u>underline</u> the place that is more crowded.

    A. Champaign, IL or Buffalo, NY

    B. Montauk, NY or Palmyra, NY

7) The density of _____ is about 2500 people/mi^2.

8) The population of Concord, NY was 8,494 people in the 2010 Census. Its density was 121 people per square mile. How big is Concord?

   A. .014 sq. mi.

   B. 70 sq. mi.

   C. 8373 sq. mi.

   D. 1,0327,774 sq. mi.

9) What are some reasons places become densely populated with people? Choose all that apply.

   A. economic opportunity

   B. plentiful natural resources

   C. temperate climate

   D. extreme climate

10) What are some reasons places are less densely populated with people? Choose all that apply.

   A. economic opportunity

   B. plentiful natural resources

   C. temperate climate

   D. extreme climate

11) Which of these would cause a place to become more densely populated?

   A. low birth rate, low death rate

   B. low birth rate, high death rate

   C. high birth rate, low death rate

   D. high birth rate, high death rate

Your Own Population and Area Data

You can use this New York State web site with population and area data about NYS counties: *https://www.ny.gov/counties*

12)   What is the name of the county where you live?

13)   What year was your state founded?

14)   What is the population of your county?

15)   What is the area of your county?

16)   What is the population density of your county?

17)   Is your county more or less densely populated than Albany County? Explain your answer.

How Crowded is New York City?

18) Fill in the table below. Round to the nearest person.

Borough	Population (2020)	Area (sq. mi.)	Density (ppl/mi^2)
Bronx	1,472,654	42	
Brooklyn	2,736,074	71	
Manhattan	1,694,251	23	
Queens	2,405,464	109	
Staten Island	495,747	59	
Total			

19) _____ has the highest population.

20) _____ has the largest area.

21) With _____*73,663*___ people/sq. mi., _____ is the borough of NYC with the highest population density.

22) With _____ people/sq. mi., _____ is the borough of NYC with the lowest population density.

23) List the boroughs in order from least crowded to most crowded.

Public Libraries in NYC

24) Fill in the table below. Round to the nearest tenth. For example, a density of .22 libraries/mi² would be rounded to .2 libraries/mi².

Borough	Libraries	Area (mi²)	Density (libraries/mi²)
Bronx	35	42	
Brooklyn	59	71	
Manhattan	44	23	
Queens	65	109	
Staten Island	13	58	
Total			

25) There are about _____ libraries for every square mile in New York City.

26) _____ has the lowest density with _____ libraries/mi².

27) Based on libraries/mi², which borough is best for public libraries? _____.

Why? _____

28) Based on libraries per 100,000 residents, which is best for libraries? _____.

Why? _____

Borough	Libraries per 100,000 residents
Bronx	2.4
Brooklyn	2.2
Manhattan	2.6
Queens	2.8
Staten Island	2.7

29) Which do you think is a better way to measure the density of libraries, libraries/mi² or

libraries/100,000 residents? _____

_____

Population, Area, and Density Practice – Answer Key

1) Answers are in the next question.

2) *Area*:
   21 mi^2
   41 mi^2
   19.5 mi^2
   26 mi^2

   Density:
   810 ppl/mi^2
   585 ppl/mi^2
   821 ppl/mi^2
   769 ppl/mi^2

3) Adams – 1,500 ppl/mi^2
   Alfred – 4,000 ppl/mi^2
   Buffalo – 2,463 ppl/mi^2
   Champaign – 47 mi^2
   Jamestown – 10 mi^2
   LA – 12,152,000 ppl
   Miami – 5,502,399 ppl
   Montauk – 5 mi^2
   NYC – 3,450 mi^2
   Oswego – 1,857 ppl/mi^2
   Palmyra – 1,000 ppl/mi^2
   Portsmouth – 20 mi^2
   Potsdam – 3 mi^2
   Red Hook – 7,000 ppl
   Snowflake – 6,000 ppl
   Utica – 62 mi^2
   Walsenburg – 2 mi^2

4) A. NYC
   B. Red Hook

5) A. NYC
   B. Oswego

6) A. Champaign
   B. Palmyra

7) Buffalo

8) B

9) A, B, C

10) D

11) C

12-17) Answers will vary.

18) *Density:*
   Bronx 35,063
   Brooklyn 38,536
   Manhattan 73,663
   Queens 22,068
   Staten Island 8,402

   Total Population:
   8,804,190

   Total Area: 304

19) Brooklyn

20) Queens

21) 73,663, Manhattan

22) 8,402, Staten Island

23) Staten Island, Queens, Bronx, Brooklyn, Manhattan

24) *Density:*
Bronx 0.8
Brooklyn 0.8
Manhattan 1.9
Queens 0.6
Staten Island 0.2
Total Density 0.7
Total Libraries: 216
Total Area: 303

25) 0.7

26) Staten Island, 0.2

27) Manhattan. Based on the space, there are more libraries to spread around the borough.

28) Queens. Based on the population, there are more libraries for the people to share. By the way, the number of libraries for every 100,000 residents is an example of "per capita," which means "for each person." Queens has the most libraries per capita.

29) This answer should be based on your opinion, but someone could argue that libraries are created to serve people, so it's better to measure how many libraries there are in comparison to the population. Other people could argue that in order to use libraries, they have to be spread around the city, which requires measuring the density in comparison to the area of the city.

Test Practice Questions

Check your answers in <u>Test Practice Questions – Answer Key</u>.

1) There are about 20 million people in New York State, with a total land area of about 47,000 square miles.

   What is the population density of New York State?

   A. 0.0004 people per sq. mile

   B. 426 people per sq. mile

   C. 2,350 people per sq. mile

   D. 940,000 people per sq. mile

2) The following chart shows population and area of the five boroughs of New York City.

Borough	Population (2020)	Area (sq. miles)
Bronx	1,472,654	42
Brooklyn	2,736,074	71
Manhattan	1,694,251	23
Queens	2,405,464	109
Staten Island	495,747	59

   Based on the chart, which answer choice lists the boroughs from greatest to least population density?

   A. Manhattan, Brooklyn, Bronx, Queens, Staten Island

   B. Brooklyn, Queens, Manhattan, Bronx, Staten Island

   C. Queens, Brooklyn, Staten Island, Bronx, Manhattan

   D. Manhattan, Queens, Brooklyn, Bronx, Staten Island

3) In 2020, the total New York City population was about 8.8 million people. The area of New York City is about 300 square miles. By 2040, the population of New York City is projected to grow by 200,000 people from the 2020 population level. If this happens, what will the population density be for New York City in 2040?

   A. 34.9 ppl/mi^2

   B. 1,333 ppl/mi^2

   C. 28,667 ppl/mi^2

   D. 30,000 ppl/mi^2

Explain your answer:

4) Albany is about 22 square miles in area and has a population density of about 4,500 people/mi^2. What is the population of Albany?

   A. 0.005 people per square mile

   B. 205 people per square mile

   C. 99,000 people

   D. 205,000 people

Explain your answer:

5) Between 2000 and 2010 in New York State, the population density increased from 345 people per square mile to 352 people per square mile. The land area of New York State is about 55,000 square miles. What was the increase in population?

    A. 7 people

    B. 156 people

    C. 385,000 people

    D. 19,360,000 people

Explain your answer:

6) A farmer did some calculations while planning an addition to her farm. What does the number 1,200 represent in her notes below?

    A. How much land is needed to raise the chickens

    B. How many chickens will fit in each square foot of land

    C. How many chickens the farmer can raise

    D. The increase in the population of chickens

7) In New York City there are 7 libraries for every 10 square miles of area. New York City is about 300 square miles in size. About how many libraries are there in New York City?

    A. 30

    B. 43

    C. 70

    D. 210

Explain your answer:

8) The approximate 2020 U.S. state populations and population densities are shown in the table below.

State	Population Density (people/sq. mi.)	Population (2020)
Florida	401	21,500,000
Illinois	231	12,800,000
New York	429	20,200,000
Pennsylvania	291	13,000,000

Based on the table above, which list shows the states' areas in order from largest to smallest?

    A. New York, Florida, Pennsylvania, Illinois

    B. Illinois, Florida, New York, Pennsylvania

    C. Florida, New York, Pennsylvania, Illinois

    D. Pennsylvania, New York, Florida, Illinois

9) What is the area of this figure in square inches? Write your answer in the grid.

3 in.

9 in.

5 in.

9 in.
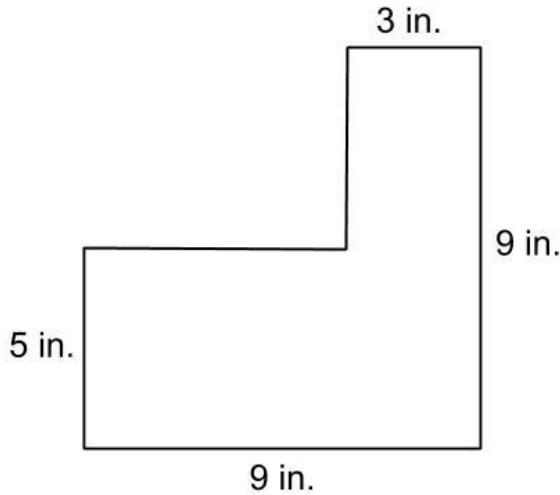
10) There are about 330 million people in the United States, with a total land area of about 3.8 million square miles.

What is the population density of the United States?

    A. 0.01 people per sq. mile

    B. 9 people per sq. mile

    C. 87 people per sq. mile

    D. 1,235 people per sq. mile

Explain your answer:

Look at the map below. You may want to also look at other maps online to identify countries in North America. Then answer the next three questions.

North America: Population Density, 2000

Persons per km²

☐	0
☐	1 - 4
☐	5 - 24
☐	25 - 249
☐	250 - 999
☐	1,000 +
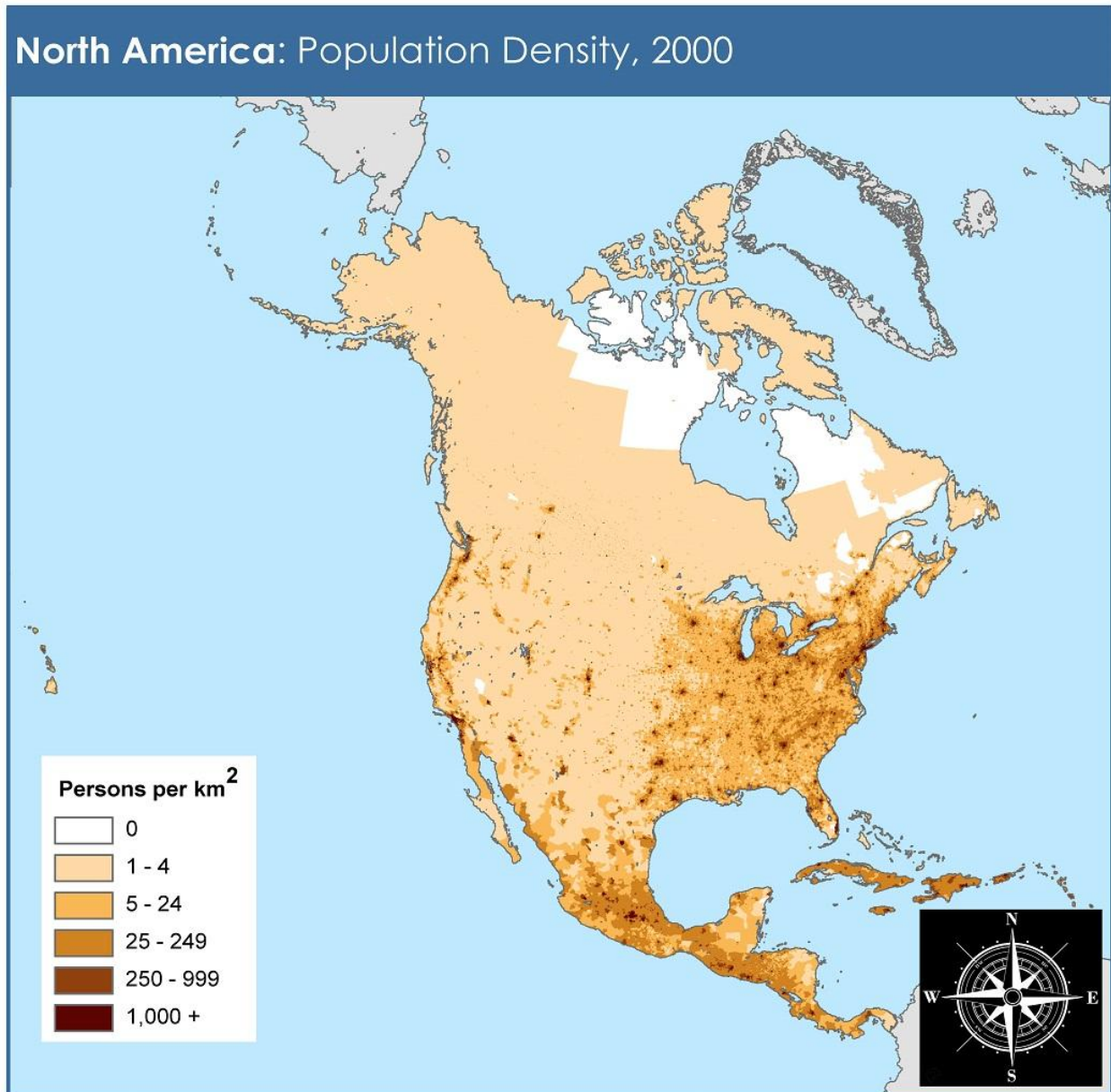
adapted from NASA Socioeconomic Data and Applications Center (http://sedac.ciesin.columbia.edu)

11) Which part of the United States is the most densely populated?

        A. West

        B. Midwest

        C. South

        D. Northeast

Explain your answer:

12) What is the main reason the far northern part of North America is sparsely populated?

        A. low economic opportunities

        B. low birth rates

        C. extreme weather

        D. war and conflict

Explain your answer:

13) One characteristic common to the geographical regions below is that they all...

> Siberian Plain
> Sahara Desert
> Amazon Basin
> Antarctica

    A. have a low population density

    B. are located between major river valleys

    C. are major religious centers

    D. have large areas of valuable farmland

14) If you know the density and the area, one way to find the population is to...

    A. Divide the area by the density

    B. Divide the density by the area

    C. Divide the population by the area

    D. Multiply the density and the area

Test Practice Questions – Answer Key

1) The correct answer is **B** (*426 people per sq. mile*). You can get this answer by imagining all the people in the state spread out evenly over each square mile. The population (20,000,000) divided by the area (47,000) is 425.53, which can be rounded to 426.

   A, C and D are *distractors*, which are answers that look like they might be correct but are not. The people who wrote the test write distracting answers to make you prove you really understand. Don't feel bad if you choose one of these answers. The distractors are based on common mistakes that many people make.

   Here is an explanation of the wrong answers:

   A (*0.0004 people per sq. mile*): A person might get this answer if they forget the zeros in 20 million. 20 divided by 47,000 equals 0.0004. We need to remember that 20 million means 20,000,000. That is the number that should be used in the calculation. We should also stop to think about what 0.0004 people per square mile would mean. That's much, much less than 1 person for every square mile in New York State. It would mean there were only 20 (twenty) people in the whole state. A population density of 0.0004 ppl/mi^2 would be similar to the population density of Antarctica, where only about 1,000 scientists live on 5.4 million square miles of land.

   C (*2,350 people per sq. mile*): You will get 2,350 people per sq. mile as an answer if you divide 47,000 by 20. However, it's 20 million, not 20. When you calculate the density, you should use 20,000,000. Also, in calculating population density we usually divide the population by the area, instead of the other way around.

   D (*940,000 people per sq. mile*): Sometimes, if we don't know what to do in a problem, we might grab a couple numbers from the problem and try something. 47,000 multiplied by 20 is 940,000 but this doesn't really make sense with the situation. Multiplying the population by the area doesn't give you the density. If there were 940,000 people per square mile all across New York State, the total population of the state would be 40 billion people. Since there are "only" 7.4 billion people on Earth, this isn't really possible.

2) The correct answer is **A** (*Manhattan, Brooklyn, Bronx, Queens, Staten Island*). To figure out the right answer, you should calculate the population density of each of the 5 boroughs and then see which borough has the most people per square mile. With 73,663 people per square mile, Manhattan has the highest population density of the 5 boroughs. You

can calculate the population density by dividing 1,694,251 (population) by 23 (area). If you live in New York City, you might already know that Manhattan is the most crowded borough from personal experience. It is the business center of the city and is a small island with lots of people crowded together. Brooklyn is the second most dense borough, with 38,536 people per square mile.

Here is an explanation of the wrong answers:

B (*Brooklyn, Queens, Manhattan, Bronx, Staten Island*): This lists the boroughs in order from greatest to least **population**, not population density. With 2.7 million people, Brooklyn has the highest population, but not the highest population density.

C (*Queens, Brooklyn, Staten Island, Bronx, Manhattan*): This lists the boroughs in order from greatest to least **area**, not population density. With 190 square miles in area, Queens is the biggest in land mass, but not in population density.

D (*Manhattan, Queens, Brooklyn, Bronx, Staten Island*): This list is almost correct, except Queens is out of order. The rest of the boroughs are in the right order. Someone might choose this answer if they figured out that Manhattan was the most densely populated, but didn't calculate the density of the other boroughs.

3) D

4) C

5) C

6) A

7) D

8) B

9) 57

10) C

11) D

12) C

13) A

14) D
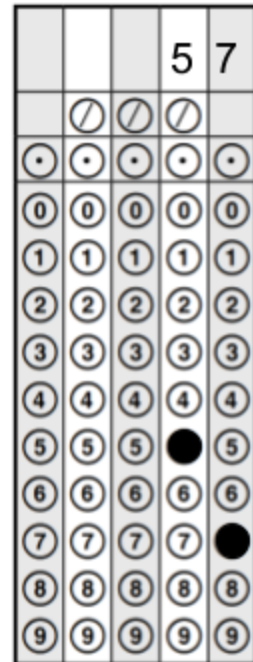
Key Ideas to Remember

- *Area* is the size of a surface measured by counting square units.

- *Density* is used to measure how crowded a space is.

- *Population density* is a measure of how crowded an area is with people or other living organisms. It is measured by the number per square unit. In the United States, this measure is often the number of people per square mile. The formula for *population density* is *population* divided by *area*:

$$\frac{p}{a} = d$$

When you see the words *population*, *area* and *density*, these questions should come to mind:

- Population*: How many are there?*

- Area*: How many square units are there?*

- Population Density*: How many are there in each square unit?*

Write notes for yourself so that you will remember the difference between population, area

and population density. _____

_____

_____

_____

_____

_____

_____

_____

The Language of Population Density

How Math is Written

In learning mathematics, knowing how to write your answer is important so that other people understand what you mean. Mathematics *notation* is the way in which mathematicians write to communicate with other mathematicians. Learning this kind of notation is like learning a new language, but it is helpful so that you understand other people and they will understand you.

In this notation for writing distance measurements, ' means *feet* and " means *inches*. Look at the example below:

$$4''$$

This means 4 inches, like the width of the grid below.

What's the area of this grid?
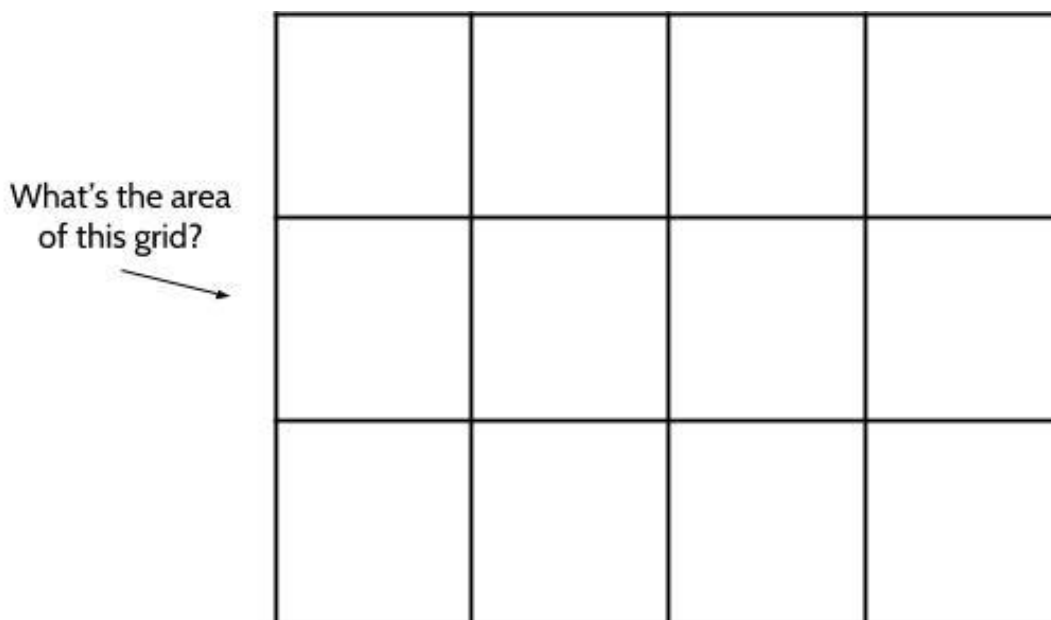
1) Make sure you know the difference between ' and ". For example, what's the difference between 10' and 10"? Can you think of two things in the real world with these measurements?

   10' _____   10" _____

As you know, area is the size of a flat surface, measured in square units. When you write an area measurement, you can use any of the following ways of saying the area of the grid on the previous page:

| 12 square inches | 12 sq. in. | 12 in^2 |

Note: ' and " are normally used for regular feet and inches, not square feet and inches.

When you see a measurement like **12 in²**, this is what it means:

12 in^2

in^2 means square inches are used to cover the surface you're measuring. "in" is short for "inches." Each square measures 1 inch on each side. In this case, the 2 **doesn't** mean "to the second power."

12 squares cover the surface without gaps or overlapping.

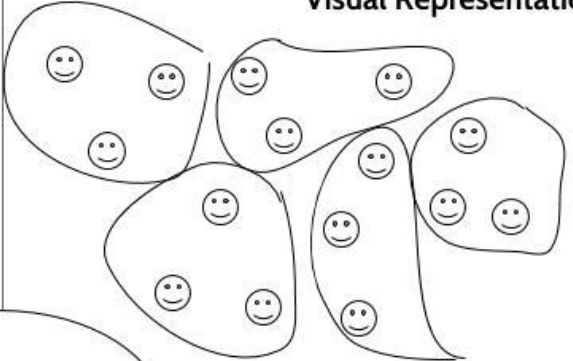2)   Fill in the missing boxes in this table.

|		
10 square inches →	10 sq. in.	10 in^2
5 square feet →		
→		9 in^2
→	7.5 sq. mi.	
1/2 square foot →		
25 square meters →		
→		12 cm^2

Using Graphic Organizers to Learn Vocabulary

In order to learn math vocabulary, we need to practice using it in different ways. In this activity, you will choose a few words from this packet that you want to practice, then you will complete a graphic organizer for each word. Look at the sample for the word *quotient* below.
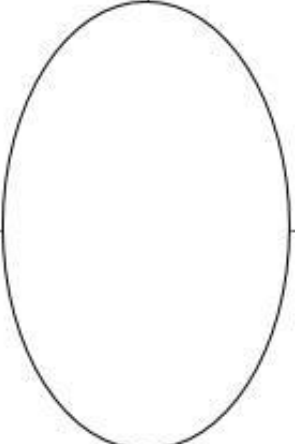
To complete the graphic organizer, you will choose a word and then answer four questions:

- What is the definition of the word? You can look at the vocabulary review on page 82 for help. Try to write the definition in your own words to really make the word yours.

- Make a visual representation. You can make a drawing or diagram that will help you remember what the word means.

- What are some examples of the word you're studying? Below you can see that there are examples of *quotients*, which are the answers to division problems.

- What are some non-examples of this word? These are things that are **not** the word you're studying. For example, 24 is **not** the quotient of 4 divided by 6.

What is it?	Visual Representation
A quotient is the result of dividing one number by another. It is the answer to a division question.	

quotient

What are some examples?	What are some non-examples?
15 divided by 3 equals **5** 66 ÷ 6 = **11** 63/18 = **3.5** 5, 11 and 3.5 are quotients in these calculations. population ÷ area = **density**	4 times 6 equals 24 18 + 5 = 23 17 – 2.5 = 14.5 3.5 × 18 = 63

What is it?

What are some examples?

Visual Representation
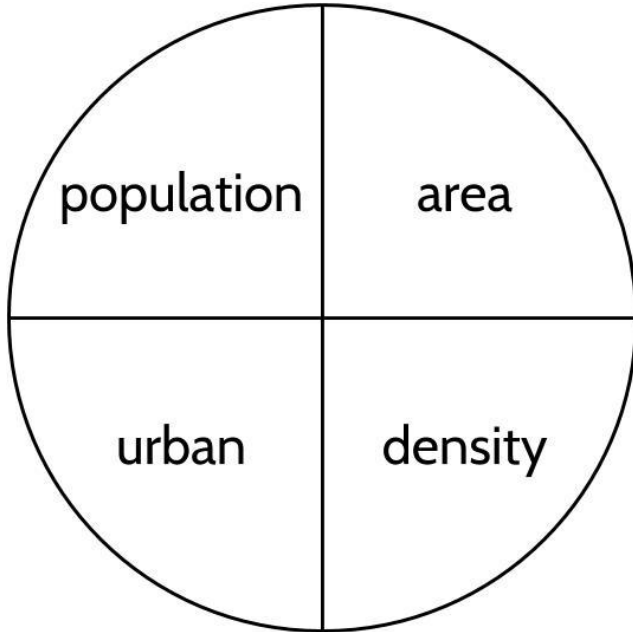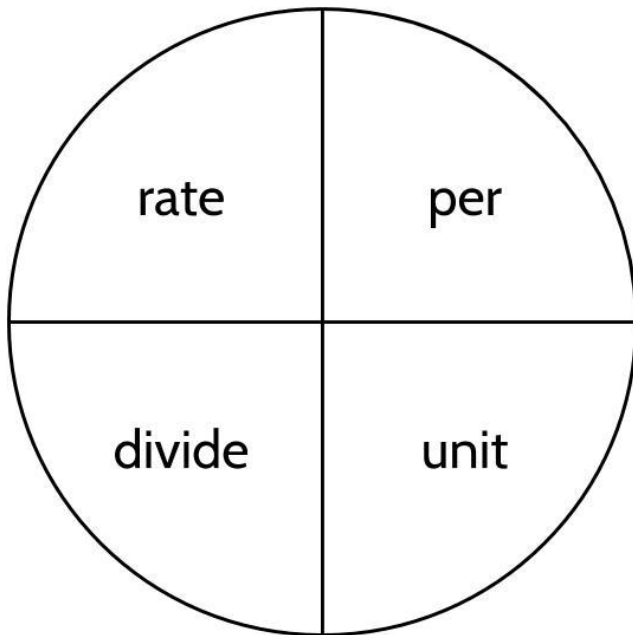
What are some non-examples?
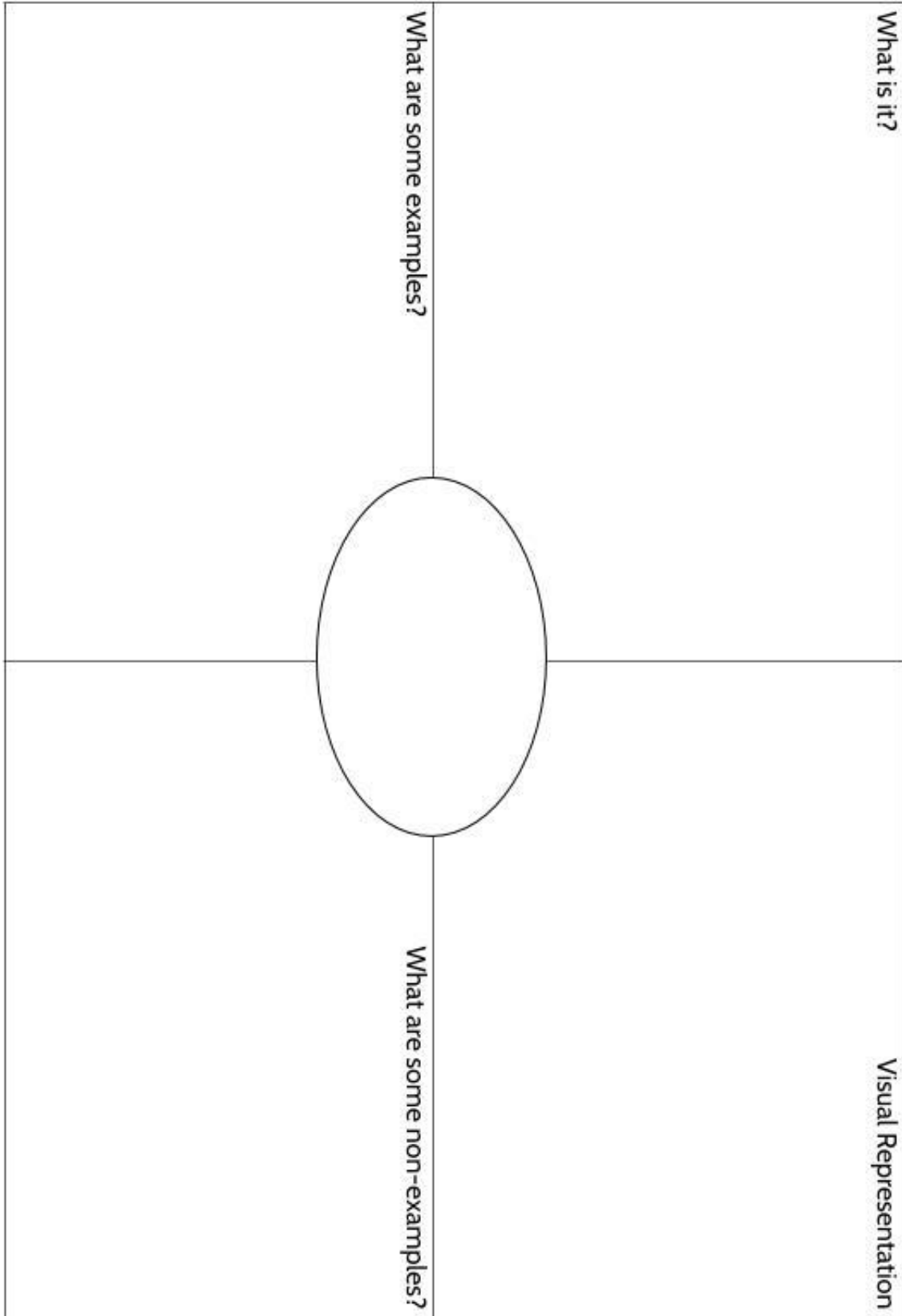
What is it?

What are some examples?

Visual Representation

What are some non-examples?

Concept Circle

3)   Explain these words and the connections you see between them.

population | area

urban | density

_____
_____
_____
_____
_____
_____
_____
_____

rate | per

divide | unit

_____
_____
_____
_____
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Fill in the Blanks

4)   Use the words and numbers below to fill in the blanks in the article.

few   population   high   41 ppl/sq. mi.   21 million people
square miles   water   deserts   climate
crowded   lowest   per   area
~~population density~~   highest   48,000 people/mi²   distributed
271   rural   world's population   urban

The ___population density___ of a country or a city or other place is a number that shows

how crowded that place is. It is calculated by dividing the _____ by the

_____ . For example, France has a population of 67,000,000 people and

an area of 247,368 square miles, so its population density is about _____

people per square mile.

A number of factors can affect population density. _____ is one of

those factors. Greenland has a very low population density because it is very cold there, so not

many people want to live there. Other places with harsh weather conditions such as

_____ or mountainous areas usually also have low population density.

Many cities were built near rivers, because people need _____ for

their daily needs, so places near rivers often have a _____ population

density. People are not _____ equally around the world. In satellite

photos of Earth at night, you can see lights surrounding oceans, lakes and rivers. This shows that

most of the _____ lives near water.

Many _____ places, such as cities, have high population densities and

can be really _____ . Mexico City is an example. With a population of

about _____ people and an area of about 3,000

_____, the city has a population density of about 7,000

people/mi^2. Other places with large _____ areas can have very low

population densities. For example, the population density of Sonora, a northern state in Mexico,

is only about _____ because much of the state is made up of mountains

and deserts.

The country with the _____ population density in the world is

Monaco, with _____ . The whole country is less than 1 square mile!

It's called a city-state because the country is the city. The country with the _____

population density is Greenland, which has only 0.07 people _____ square mile.

Greenland is really big, but very _____ people live there because it's so

cold.

Where You Live

5) Write a description of the place where you live. Use as many of the population density vocabulary words as you can. Look at page 82 for review.

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The Language of Population Density – Answer Key

1) Here are a couple possibilities:
   10' = 10 feet, the height of a basketball hoop
   10" = 10 inches, the length of a person's foot (maybe)

2) 10 square inches, 10 sq. in., 10 in^2
   5 square feet, 5 sq. ft., 5 ft^2
   9 square inches, 9 sq. in., 9 in^2
   7.5 square miles, 7.5 sq. mi., 7.5 mi^2
   1/2 square feet, 1/2 sq. ft., 1/2 ft^2
   25 square meters, 25 sq. m., 25 m^2
   12 square centimeters, 12 sq. cm., 12 cm^2

3) Each paragraph should use the four vocabulary words in the circle on the left. Be creative. There isn't one right way to do this activity!

4) population density
   population
   area
   271
   climate (You should capitalize "C.")
   deserts
   water
   high
   distributed
   world's population
   urban
   crowded
   21 million
   square miles
   rural
   41 ppl/mi^2
   highest
   48,000 $people/mi^2$
   lowest
   per
   few

5) Take your time with this activity. Look at the window and describe what you see. Think about whether the place where you live is crowded or not. You might even do some research so that you have some numbers to include in your writing. This is an opportunity to practice all the vocabulary and math skills you have learned.

Vocabulary Review

You can use this section to look up words used in this math packet.

approximate (adjective): close to the actual, but not completely accurate or exact

area (noun): The size of a flat surface, measured in square units

convert (verb): The change from one thing into another

- You can *convert* feet into inches by multiplying by 12.
- If you *convert* Euros into dollars today, you will get $1.15 for every Euro you exchange.
- To understand distances in different races, runners can convert miles to kilometers because they know that 1 mile equals 1.6 kilometers.

dense (adjective): crowded closely together or packed together

- The population of Buffalo, NY is less *dense* than New York City.
- Manhattan is the most *dense* borough of New York City.

density (noun): A measure of how crowded or "pushed together" things are in a space

The word dense comes from the Greek word *dasus*, which means "compact."

- The *den*sity of New York City is about 28,000 people per square mile.
- The *density* of libraries in Manhattan is about 2 per square mile.

population density: The average number of people or things per square unit of area

density in science: The amount of mass per unit of volume (This topic is covered in a separate packet.)

distribute (verb): to spread out or divide evenly

- The fox population is *distributed* evenly throughout the state park.

estimate (verb): to make a rough guess at a number, usually without making written calculations

per (preposition): for each or for every

- The car was traveling 40 miles *per* hour.
- Manhattan has about 73,000 people *per* square mile.

- The subway costs $2.75 *per* ride.
- The bicycle tire pressure should be 80 pounds *per* square inch.

per capita (adverb/adjective): for each person ("capita" means *head* in the language Latin)

- The *per capita* income, or average salary per person, in New York State is about $40,000 per year.
- The New York State egg industry produces a total of 1.5 billion eggs per year, which works out to about 75 eggs *per capita*.

population (noun): all the organisms of a particular species living in a given area at the same time.

product (noun): the result of multiplication

- The product of 2 and 3 is 6.

rate (noun): a ratio with two different quantities that are being compared

- The train traveled at a *rate* of 80 miles per hour.
- On October 10, 2018, the exchange *rate* for Mexican to U.S. money was 19.15 pesos per dollar.

    unit rate: How many (or much) of something for every **one** unit of another thing

- If the cost is $2.00 for 4 apples, there are two possible *unit rates*. One unit rate is 50 cents for 1 apple, which tells us how much money you would need to spend for **one** apple. Another unit rate would be $1.00 for 2 apples, which tells us how many apples you would get for **one** dollar.

ratio (noun): A relationship that shows the size of one value in comparison to one or more other values. For example, the ratio of teachers to students in a school might be 1:20, which means that there is one teacher for every 20 students.

rectangle (noun): A 4-sided flat shape with straight sides which has:

  - Four sides
  - Opposite sides of equal length
  - Four right angles (90°)

rural (adjective): related to the countryside, not a town or city

square (noun): a 4-sided, flat shape which has:

- ○ Four straight and equal sides
- ○ Four right angles (90°)

unit (noun): a quantity of one used to measure other quantities

- Inches, feet, centimeters and meters are all *units* for measuring distance.
- Square inches, square feet, square centimeters, and square meters are all *units* for measuring area.
- The cost of apples can be measured in *units* of apples or *units* of dollars.

urban (adjective): related to a city or town

Works Cited

NYS Regents Tests. www.nysedregents.org (Section: Practice Test Questions)

The Power of Per. 2018. Science Generation. Strategic Education Research Partnership. https://serpmedia.org/scigen/ (Section: What Does "Per" Mean?)

United States Census Bureau. 2021. Data.census.gov. (Sections: Looking at the Population of the United States and Propulation, Area, & Density Practice)

Versions

V.2.0, 9/14/2021

- Updated most national, state, and city population references with Census 2020 results. Will update town and small city data when full Census 2020 results are released.

V.2.3, 6/1/2022

- Corrected answer key error.