The Power of Exponents

Fast Track GRASP Math Packet

Part 2

 [1]

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Powers of Ten

In this section, we will use powers with a base of 10, which are called the *powers of ten*. We use the powers of ten every day as an important part of our number system. The decimal system is based on a Hindu-Arabic way of writing numbers that was developed about 800 A.D. One benefit of this number system is that it allows us to write any number we need using only 10 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. The reason people started using 10 digits to count is probably because humans have 10 fingers. The word *decimal* actually came from the Latin word for "10" and *digit* came from the Latin word for "finger."

Size

Powers of ten are often used to help us understand the size of things in the world. Look at the table below and then answer the questions on the next page.

Power of Ten	Size (meters)*	Size (feet)**	Approximate size (feet/inches)
10^4 meters	10,000 meters	33,000 feet	Mt. Everest, tallest mountain on Earth (29,000 feet)
10^3 meters	1,000 meters	3,300 feet	Burj Khalifa, tallest building in the world (2,717 feet)
10^2 meters	100 meters	330 feet	height of Statue of Liberty (305 feet)
10^1 meters	10 meters	33 feet	three-story building (30 feet)
10^0 meters	1 meter	3.3 feet	height of average 4-year-old child (3 ½ feet)
10^{-1} meters	$\frac{1}{10}$ meter	4 inches	length of a dollar bill (6 inches)
10^{-2} meters	$\frac{1}{100}$ meter	0.4 inches	width of average fingernail (0.4 inches)
10^{-3} meters	$\frac{1}{1,000}$ meter	0.04 inches	thickness of a credit card (0.03 inches)
10^{-4} meters	$\frac{1}{10,000}$ meter	0.004 inches	thickness of paper (0.006 inches)

*Scientists use meters for measuring distance. A meter is about 3.3 feet in length.

**A foot is 12 inches.

1) Burj Khalifa is about 10 times taller than…

   A. a three-story building

   B. the height of an average 4-year-old child

   C. Mt. Everest

   D. the height of the Statue of Liberty

2) A dollar bill is about 10 times longer than…

   A. the height of an average 4-year-old child

   B. the width of average fingernail

   C. the thickness of a credit card

   D. a three-story building

3) The height of the Statue of Liberty is about 100 times taller than…

   A. Mt. Everest

   B. the height of an average 4-year-old child

   C. a three-story building

   D. the thickness of a credit card

4) Burj Khalifa is about $\frac{1}{10}$ the size of…

   A. Mt. Everest

   B. the Statue of Liberty

   C. a three-story building

   D. an average 4-year-old child

Write your own comparison sentences:

5) ____________________ is about 10 times bigger than ____________________ .

6) ____________________ is about 100 times bigger than ____________________.

7) ____________________ is about $\frac{1}{10}$ the size of ____________________.

8) 10^3 meters is 10 times the size of ________ meters.

9) ____________________ is 10 times the size of 10^3 meters.

10) 10^1 meters is ____________________ the size of 10^3 meters.

11) As you go down in the Powers of Ten table, the sizes of objects ____________

____.

12) As you go up in the Powers of Ten table, the sizes of objects _______________

____.

Our number system is based on powers and exponents, but they are usually hidden. In this activity, you will learn to see the exponents. For example, before you turn the page, how do you think you might write 1 million using a base of 10 and an exponent?

13) Look at the chart on the next page and write what you notice.

Look at this table of the powers of 10.

Exponential Form	Factors	Numeral	How To Read the Numeral
10^{12}	$10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$	1,000,000,000,000	trillion
10^{9}	$10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$	1,000,000,000	billion
10^{6}	$10 \times 10 \times 10 \times 10 \times 10 \times 10$	1,000,000	million
10^{3}	$10 \times 10 \times 10$	1,000	thousand
10^{2}	10×10	100	hundred
10^{1}	10	10	ten
10^{0}	1	1	one
10^{-1}	$\frac{1}{10}$	0.1	tenth
10^{-2}	$\frac{1}{10 \times 10}$	0.01	hundredth
10^{-3}	$\frac{1}{10 \times 10 \times 10}$	0.001	thousandth
10^{-6}	$\frac{1}{10 \times 10 \times 10 \times 10 \times 10 \times 10}$	0.000001	millionth
10^{-9}	$\frac{1}{10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10}$	0.000000001	billionth
10^{-12}	$\frac{1}{10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10}$	0.000000000001	trillionth

What do you notice?

Answer these questions using the chart on the previous page.

14) One thousand is 10 times larger than...

A. One million
B. Ten thousand
C. One hundred
D. Ten

15) How many times is 10 used as a factor in the number 1000?

A. 100
B. 10
C. 3
D. 1

16) 10^{-2} multiplied by ______________ equals 1.

A. 100
B. 10
C. $\frac{1}{10}$
D. $\frac{1}{100}$

17) One million has the number 10 as a factor 6 times and has ______________ zeros.

18) ______________ has 9 zeros and has 10 as a factor 9 times.

As we saw above, the numbers we use, large and small, are powers of 10. Each power of 10 is made by multiplying 10s. Even numbers smaller than 1 can be made by dividing 1 by factors of 10.

Check your understanding: Sometimes, people use the words *million* and *billion* as if they are about the same number. For example, people talk about millionaires and billionaires as if they have about the same amount of money. To understand the difference, think about how much time passes in a million seconds and a billion seconds. How much time is each in weeks, months or years?

Place Value

Reading Numbers Larger Than One

The numbers 532 and 352 have the same three digits, yet we know which one is bigger. But how do we know?

Let's think about it in terms of money. 532 is equal to $532.00. We could count out this money with 5 hundred-dollar bills, 3 ten-dollar bills, and 2 one-dollar bills.

$532.00

5 hundreds + 3 tens + 2 ones

500 + 30 + 2

When we see the 5 in that position, we know that the number 532 includes 5 <u>hundreds</u>.

532.00
↑
hundreds

If the number were written 352, we would know that there are 5 <u>tens</u>, not hundreds. There is a big difference between $500 and $50!

352.00
↑
tens

The position of the 5 tells us its value. If 5 is in the hundreds position, it means 5 × 100. A 5 in the tens position, means 5 × 10.

So, when we look at all three digits in 532, we know the value is (5 × 100) + (3 × 10) + (2 × 1).

19) The value of 352 is…

(________ hundreds) + (________ tens) + (________ ones)

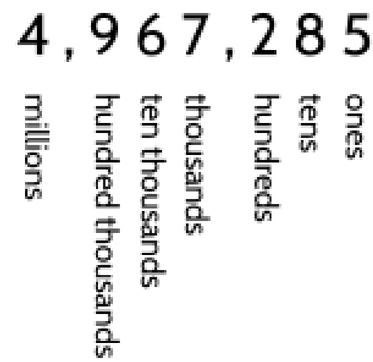
(________ × 100) + (5 × 10) + (________ × 1)

When we look at numbers like 532 and 352, we see the digits that are used (0, 1, 2, 3, 4, 5, 6, 7, 8, or 9) and the position of the digit. The combination of the digit and the position tells us the value of the number. In the decimal system, the position of the number is always a power of ten!

20) Fill in the blanks.

532	5 hundreds + 3 tens + 2 ones (5 × 100) + (3 × 10) + (2 × 1)
235	_______ hundreds + _______ tens + _______ ones (_______ × 100) + (_______ × 10) + (_______ × 1)
	8 hundreds + 4 tens + 7 ones (_______ × 100) + (_______ × 10) + (_______ × 1)
	_______ thousands + _______ hundreds + _______ tens + _______ ones (2 × 1,000) + (0 × 100) + (1 × 10) + (9 × 1)
12.5	_______ tens + _______ ones + _______ tenths (_______ × 10) + (_______ × 1) + (_______ × 0.1)

Place value is the value of each position in a number. For example, in the number 4,967,285, the place value of the 7 is 1,000 because 7 is in the thousands place. The 7 represents 7 thousands or 7 × 1,000. The digit multiplied by the place value tells us the quantity each digit represents. Because the number and the position together tell us the quantity, we only need 10 digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) to write any number you can think of.



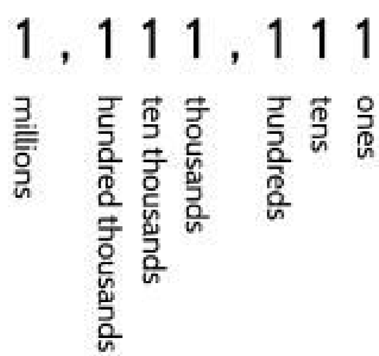
21) What is the quantity represented by the 7 in the number to the right?

    7 × ____________________ = ____________________

22) What is the quantity represented by the 9 in the number to the right?

    9 × ____________________ = ____________________

23) Look at the 1,111,111. What does it mean?



Add the missing digits.

(________ × 1,000,000) + (________ × 100,000) + (________ × 10,000) +
million *hundred thousand* *ten thousand*

(________ × 1,000) + (________ × 100) + (________ × 10) + (________ × 1)
thousand *hundred* *ten* *one*

The Hindu-Arabic or decimal number system is based on the number 10. Think about the place values of ones, tens, hundreds, thousands, and ten thousands. Each position is 10 times bigger than the one to its right. For example, one hundred is 10 times as big as ten.

ten thousands	thousands	hundreds	tens	ones	tenths	hundredths
10,000	1,000	100	10	1	0.1	0.01

Try multiplying by 10 with a calculator.

24) 5 × 10 =

25) 20 × 10 =

26) 0.5 × 10=

27) 0.03 × 10 =

28) Try multiplying some other numbers by 10. What do you notice?

Choose more than one correct answer.

29) Multiplying a number by 10

A. Moves the number's place value positions one place to the left.

B. Moves the number's place value positions one place to the right.

C. Moves the decimal point one position to the left.

D. Moves the decimal point one position to the right.

Answer to Check Your Understanding: A million seconds is about 11 ½ days. A billion seconds is about 11,500 days, which is almost 32 years! A billion is 1,000 times bigger than a million.

1 million seconds = 10^6 seconds
1,000,000 sec ÷ 60 sec/min = 16,667 min
16,667 min ÷ 60 min/hr = 278 hrs
278 hr ÷ 24 hr/day = 11.6 days

1 billion seconds = 10^9 seconds
1,000,000,000 sec ÷ 60 sec/min = 16,666,667 min
16,666,667 min ÷ 60 min/hr = 27,778 hrs
27,778 hr ÷ 24 hr/day = 11,574 days
11,574 days ÷ 365 days/yr = 31.7 years

Try multiplying by 100 with a calculator.

30) $7 \times 100 =$

31) $0.5 \times 100 =$

32) $0.02 \times 100=$

33) $0.003 \times 100 =$

34) Try multiplying some other numbers by 100. What do you notice?

Choose more than one correct answer.

35) Multiplying a number by 100

A. Moves the number's place value positions two places to the left.

B. Moves the number's place value positions two places to the right.

C. Moves the decimal point two positions to the left.

D. Moves the decimal point two positions to the right.

36) Try multiplying some numbers by 1,000. What do you notice?

37) What do you think will happen if you multiply a number by 1,000,000?

Reading Numbers Smaller than One

In the previous examples, we looked at numbers as small as 1, but you can write even smaller numbers with the powers of 10. How can we show numbers smaller than 1 as a power of 10? You might ask, if we multiply 10's to get different numbers, how can we show numbers that are smaller than 1?

Think about the following numbers:

1 0.10 0.01

We use these numbers every time when we deal with money.

$1 $0.10 $0.01

Let's use what we know about dollars, dimes, and pennies to make some observations. Moving from right to left, we know 10 pennies are equal to a dime and 10 dimes are equal to a dollar. A dime is worth 10 times as much a penny and a dollar is worth 10 times as much as a dime. (And a dollar is worth 100 times—10 × 10—as much as a penny).

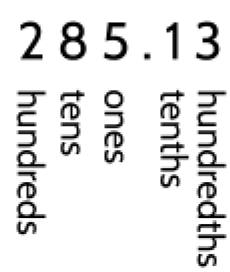
Moving in the other direction, we can see that a dime is worth $\frac{1}{10}$ as much as a dollar. A penny is worth $\frac{1}{10}$ as much as a dime. Another way to think about this is a dollar divided up into 10 equal pieces is a dime. A dollar divided up into 100 equal pieces in a penny.

On the right side of the decimal point, the value of each position is also a power of ten.

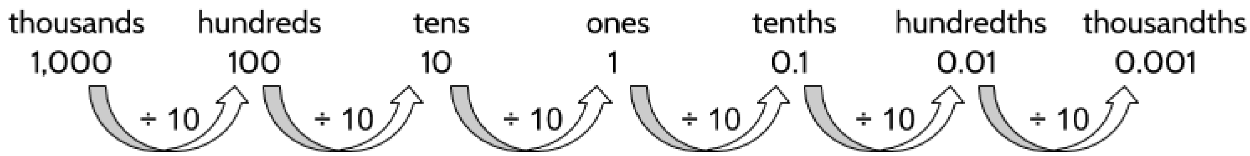
$$0.1 = 10^{-1} = \frac{1}{10} = 1 \text{ tenth} = \text{a dime}$$

$$0.01 = 10^{-2} = \frac{1}{100} = 1 \text{ hundredth} = \text{a penny}$$

In the number 285.13, the 3 represents the number of hundredths, also written as 0.03 or $\frac{3}{100}$ (and which could also represent 3 cents).



Look what happens when we move to the right in a number. Each position is $\frac{1}{10}$ the size of the position to its left. For example, 10 is $\frac{1}{10}$ of 1000. Put another way, 1,000 divided by 10 is 100.



Try these with a calculator.

38) $600 \div 10 =$

39) $40 \div 10 =$

40) $3 \div 10 =$

41) $0.7 \div 10 =$

42) Try dividing some other numbers by 10. What do you notice? What happens if the number ends in a zero? What happens if the number doesn't end in a zero? What happens if the number is already less than 1?

Choose more than one correct answer.

43) Dividing a number by 10

A. Moves the number's place value position to the left.

B. Moves the number's place value position to the right.

C. Moves the decimal point one position to the left.

D. Moves the decimal point one position to the right.

The number 0.5 is the decimal form of $\frac{5}{10}$.

44) What is the decimal form of $\frac{5}{100}$?

45) What is the decimal form of $\frac{7}{1,000}$?

Scientific Notation

The powers of 10 are very helpful in science, especially in exponential form. They can help us keep track of zeros and make numbers easier to write. Scientific notation is a way of writing large and small numbers so that you don't have to write out all the zeros. It uses the place value of the powers of 10.

Based on your understanding of exponents, fill in the missing blanks in these equations.

46) $10^4 =$ ____10,000____

47) ________________ $= 3 \times 10^3$

48) $8 \times 10^6 =$ ________________

49) $7 \times$ ____________ $= 70{,}000$

50) ____________ $\times 10^3 = 5{,}000$

51) $0.5 \times 10^2 =$ ________________

52) $1.5 \times 10^2 =$ ________________

53) $2.5 \times 10^3 =$ ________________

54) ____________ $\times 10^2 = 120$

55) $3.25 \times 10^8 =$ ________________

56) $7.5 \times 10^9 =$ ________________

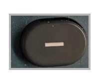
57) $8 \times 10^{-1} =$ ________________

58) ________________ $= 3 \times 10^{-4}$

59) $4 \times$ ____________ $= .04$

60) __________ $\times 10^{-5} = .00003$

61) $7 \times 10^{-8} =$ ________________

By the way, to enter negative exponents on the TI-30XS calculator, enter the negative sign with the negative button , not the subtraction button . To calculate 10^{-4}:

Press **10**, press **^**, press **(-)**, press **4**, press **enter**. Display should read **0.0001**. (TI-30XS)

Scientific notation is a way of writing numbers that are too big or too small to easily be written in the standard way (how we normally write them). Scientific notation makes long numbers into shorter numbers that can be easier to understand. It is often used by mathematicians, scientists, and engineers.

Here is the number 325,000,000 written in scientific notation: 3.25×10^8.
(This is the population of the United States, by the way.)

When written in scientific notation, numbers are in this form:

$$a \times 10^b$$

b will be a whole number exponent that indicates how many times 10 will be used as a factor. (10^b is a power of ten.)

a is a number greater or equal to 1 and less than 10.

Possible values for a			Values that aren't possible for a		
3.25	6.3	2.003	10.5	300	0.99
1	7.5	9.9	28.3	0.5	.02

62) In the number 5×10^3, written in scientific notation, what are the values of a and b?

a = ____________ b = ____________

63) If the value of a is 8 and the value of b is 2, what is the value of the number?

64) If the value of a is 2.5 and the value of b is 5, what is the value of the number?

65) If the value of a is 3 and the value of b is -4, what is the value of the number?

66) If you convert the number 90,000 into scientific notation, what would be the values of a and b?

a = ____________ b = ____________

Use the following information to complete the activity below.

Distance between Earth and the Sun: 93,000,000 miles
Distance between Neptune and the Sun: 2,800,000,000 miles
Speed of light: 671,000,000 miles per hour
Speed of sound: 767 miles per hour
Width of a red blood cell: 0.0003 inch
Width of an E. coli bacteria: 0.00002 inch
Diameter of an atom: 0.000000004 inch
Diameter of an electron: 0.000000000000004 inch

As you know, numbers in scientific notation are written in the form $a \times 10^b$.

67) Match a with the correct 10^b.

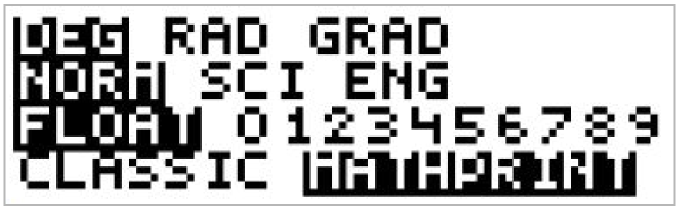
	a
Distance between Earth and the Sun	9.3
Distance between Neptune and the Sun	2.8
Speed of light	6.71
Speed of sound	7.67
Width of a red blood cell	3
Width of an E. coli bacteria	2
Diameter of an atom	4
Diameter of an electron	4

10^b
10^2
10^{-5}
10^7
10^{-9}
10^8
10^9
10^{-15}
10^{-4}

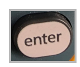
Using Scientific Notation on a Calculator

You can use the TI-30XS calculator to display numbers in scientific notation automatically.

Press the  button to show the following screen:



NORM, **SCI**, and **ENG** are modes for displaying numbers. **NORM** means *normal* mode or standard notation, **SCI** means *scientific notation*, and **ENG** means *engineering* mode.

Then use the arrow buttons  to move to SCI. Press  to choose scientific notation. Press  to go back to the main screen. All answers will now be shown in scientific notation.

Try entering a number into the TI-30XS to see it in scientific notation. Enter 95,000 and press enter. The display should show: $9.5*10^4$

To go back to normal mode, press the **mode** button, choose **NORM** and press **enter**. Then press **clear** to return to the regular screen.

Convert the following numbers into scientific notation, with a calculator or by hand.

68) Population of the world: 7,500,000,000 people

69) Annual spending of the U.S. federal government: $3,980,000,000,000

70) The age of the Earth: 4,600,000,000 years old

71) Size of a human cheek cell: 0.0024 inch

Powers of 10 - Answer Key

1) D

2) B

3) B

4) A

5-7) There are many correct ways to complete these sentences.

8) 10^2

9) 10^4

10) $\frac{1}{100}$

11) ...get smaller. Each row is $\frac{1}{10}$ the size of the row above. (You may have a different way of writing this.)

12) ...get larger. Each row is 10 times the size of the row below. (You may have a different way of writing this.)

13) There are many things to notice. Here are a few things you might notice:
    - Each positive power of ten (10^1, 10^2, 10^3, etc.) has one more factor of 10 than the previous power of ten.
    - Each negative power of ten (10^{-1}, 10^{-2}, 10^{-3}, etc.) has one more factor of 10 in the denominator (the bottom part) of a fraction, with 1 on top.
    - Positive powers of ten have the same number of zeros as factors of ten. For example, 10^4 is 10,000 and has four zeros.

14) C

15) C

16) A

17) 6

18) 1,000,000,000 (one billion).

19) (3 hundreds) + (5 tens) + (2 ones)
$(3 \times 100) + (5 \times 10) + (2 \times 1)$

20)

532	(5 hundreds) + (3 tens) + (2 ones) $(5 \times 100) + (3 \times 10) + (2 \times 1)$
235	(2 hundreds) + (3 tens) + (5 ones) $(2 \times 100) + (3 \times 10) + (5 \times 1)$
847	(8 hundreds) + (4 tens) + (7 ones) $(8 \times 100) + (4 \times 10) + (7 \times 1)$
2019	(2 hundreds) + (0 hundreds) + (1 tens) + (9 ones) $(2 \times 100) + (0 \times 100) + (1 \times 10) + (9 \times 1)$
12.5	(1 tens) + (2 ones) + (5 tenths) $(1 \times 10) + (2 \times 1) + (5 \times 0.1)$

21) $7 \times 1{,}000 = 7{,}000$

22) $9 \times 100{,}000 = 900{,}000$

23) $1{,}111{,}111 = (1 \times 1{,}000{,}000) + (1 \times 100{,}000) + (1 \times 10{,}000) + (1 \times 1{,}000) + (1 \times 100) + (1 \times 10) + (1 \times 1)$

24) 50

25) 200

26) 5

27) 0.3

28) What did you notice when you multiply whole numbers by 10? What if you multiply a number with a decimal (2.5, 1.25, 31.3, etc.) by 10?

29) A, D

30) 700

31) 50

32) 2

33) 0.3

34) What did you notice when you multiply whole numbers by 100? What if you multiply a number with a decimal (2.5, 1.25, 31.3, etc.) by 100?

35) A, D

36) It moves the number's place value positions three places to the left. The decimal point moves three positions to the right.

37) Try multiplying by different powers of ten to see what happens. Can you predict the answer before doing a calculation with a calculator?

38) 60

39) 4

40) 0.3

41) 0.07

42) There are a lot of things you might have noticed.

43) B, C

44) 0.05

45) 0.007

46) 10,000

47) 3,000

48) 8,000,000

49) 10^4

50) 5

51) 50

52) 150

53) 2500

54) 1.2

55) 325,000,000

56) 7,500,000,000

57) 0.8

58) 0.0003

59) 10^{-2}

60) 3

61) 0.00000007

62) a = 5, b = 3

63) 800

64) 250,000

65) 0.0003

66) a = 9, b = 4

67)

Measurement	$a \times 10^b$
Distance between Earth and the Sun	9.3×10^7 miles
Distance between Neptune and the Sun	2.8×10^9 miles
Speed of light	6.71×10^8 mph
Speed of sound	7.67×10^2 mph
Width of a red blood cell	3×10^{-4} inches
Width of an E. coli bacteria	2×10^{-5} inches
Diameter of an atom	4×10^{-9} inches
Diameter of an electron	4×10^{-15} inches

68) 7.5×10^9 people

69) 3.98×10^{12} dollars

70) 4.6×10^9 years old

71) 2.4×10^{-3} inches

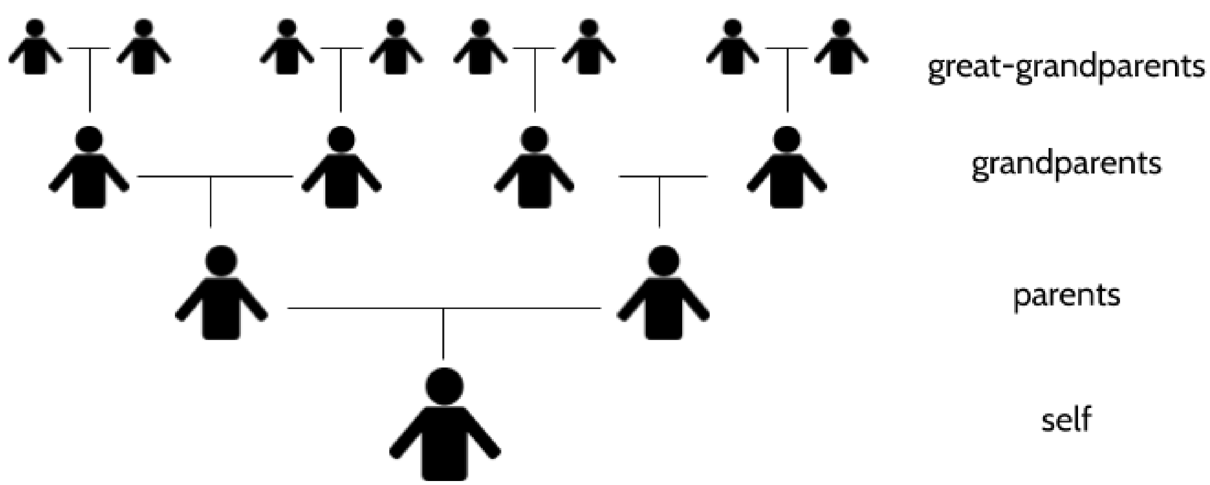
Exponential Growth and Decay

Powers can be used to show how quantities get larger or smaller over time. For example, calculations using exponents can help us understand how a population grows or how an investment can increase in value. If a quantity gets larger over time by multiplying the same factor over and over, it is an example of exponential growth. If a quantity gets smaller over time by multiplying the same factor over and over, it is an example of exponential decay.

In this section, we will take a second look at some of the problems from the introduction. Using exponents as a tool can make it much easier to solve the problems. It can also help us understand examples of exponential growth or decay in the real world. For example, when we started counting our ancestors at the beginning of the packet, that was an example of exponential growth. The number of ancestors grows exponentially as we go back in time: 2 times 2 times 2 times 2 times... (and so on).

Powers of Two: Counting Ancestors

In the introduction, you were shown this diagram:



Then we asked you how many 30th great-grandparents you have (that is "grandparents" with 30 "greats"). Finding the answer involves multiplying 1 (self) × 2 (parents) × 2 (grandparents) and then multiplying by 2 again for each of the "greats." As you know, we could do this by multiplying by 2 over and over again on paper or with a calculator, but using exponents is so much faster!

Numbers that are in the form 2^1, 2^2, 2^3, etc. are called powers of 2. We can use powers of 2 to find out how many ancestors you have.

1) Complete the following table.

Generation	Number of People	Prime Factorization	Exponential Form
self	1	1	2^0
parents	2	2	2^1
grandparents	4	2×2	2^2
1st-great-grandparents	8	$2 \times 2 \times 2$	
2nd-great-grandparents		$2 \times 2 \times 2 \times 2$	
3rd-great-grandparents		$2 \times 2 \times 2 \times 2 \times 2$	
4th-great-grandparents		$2 \times 2 \times 2 \times 2 \times 2 \times 2$	
5th-great-grandparents		$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$	
6th-great-grandparents		$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$	
7th-great-grandparents		$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$	

When you finish filling out the table, look at the number of "greats" in each generation and the number of 2's that are being multiplied in the exponential form.

2) What do you notice?

You may have noticed that the exponent is always 2 more than the number of "greats." For example, the number 4th great-grandparents is 2^6 and the number of 7th great-grandparents is 2^9. To calculate the number of people in each generation, you can count the number of "greats," add 2 and use that number as an exponent with 2 as the base (since you want the numbers to double).

3) What is the exponential form for the number of 10th-great-grandparents you have?

4) How many 10th-great-grandparents do you have? (Use the exponent form to calculate an answer.)

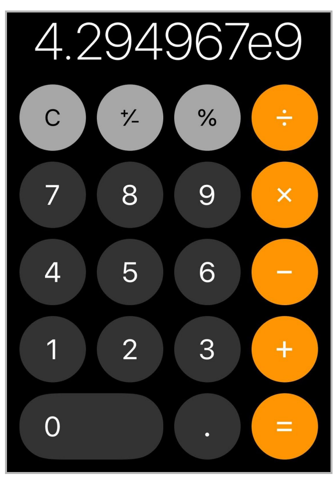
5) What is the exponential form for the number of 30th-great-grandparents you have?

To calculate the number of 30th-great-grandparents you have, take out a calculator!

Press **2**, press **^**, press **32**, press **enter**. (TI-30XS)

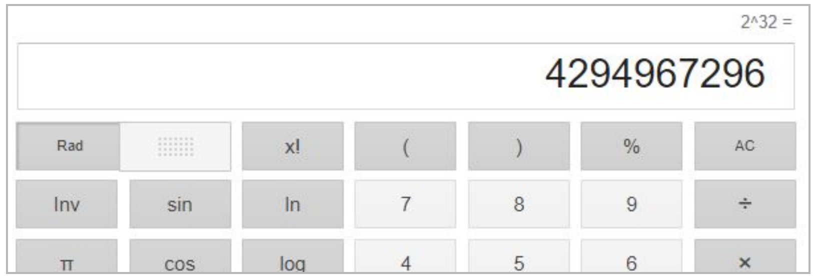
or

Turn phone, press **2**, press **x^y**, press **32**, press **=**. (smartphone)

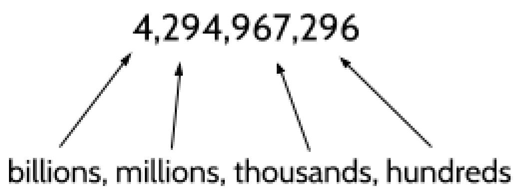


By the way, did you know that an Internet search will calculate 2^{32}?

Type 2^32 into a search window and the answer will appear.



After calculating the number of 30th-great-grandparents, you will get:



You can read this number as "four billion, two hundred and ninety-four million, nine hundred and sixty-seven thousand, two hundred and ninety-six." That's how many 30-great grandparents you had. Surprised?

Depending on how you calculated it, the number may look different:

On a TI-30XS calculator or a computer :

2^{32} 4294967296 or 2^32 4294967296

On a smartphone:

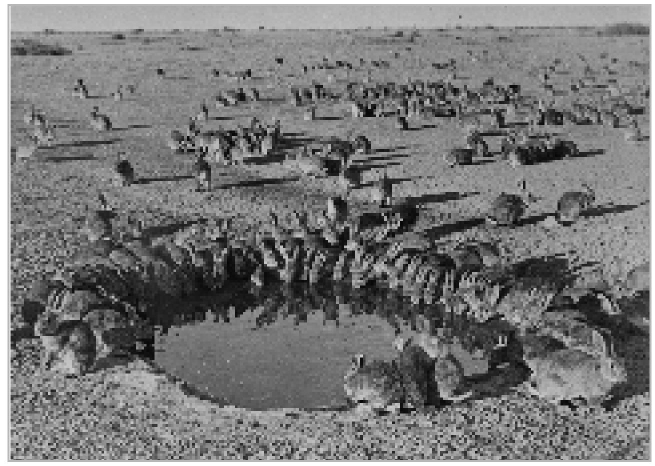
4.294967e9

6) What do you think the "e" and the "9" mean on the smartphone display?

Challenge: Find out the total number of ancestors you have had in the last 800 years (this is an estimate of how long ago your 30th-great-grandparents lived). You could add the number of 30th-great-grandparents to the number of 29th-great-grandparents to the number of 28th-great-grandparents to the number of...

Exponential Growth

Rabbits in Australia



Rabbits are native to Morocco, Algeria, Spain, Portugal and France. They were brought to other parts of the world by humans. Rabbits were first introduced to Australia in 1788, but they were kept in pens and didn't live in the wild. In 1858, a group of 24 rabbits was released for people to hunt. There are no natural predators of rabbits in Australia, meaning there was nothing to stop the population from growing. Even though there weren't very many rabbits at the beginning, they quickly multiplied, reaching a population of about 10 billion by 1920!

How is it possible that 24 rabbits became 10 billion rabbits so quickly? We'll need to look at exponential growth to understand.

7) First of all, how many years passed between 1858 and 1920?

Let's look at the possible population growth of rabbits in the first few years after they were released. Year 0 refers to 1858, the year when the first rabbits were released into the wild.

Year	Population
0	24
1	36
2	54
3	81
4	122
5	182

8) What do you notice about how the population is growing in the first few years?

You may have noticed that the population increased by different amounts each year. In the first year, it grew from 24 rabbits to 36 rabbits, an increase of 12 rabbits. In the second year, the population went up by 18 rabbits. In the third year, the rabbit population had increased 27 rabbits. Each year, the increase in rabbits was greater than the year before. The population growth started off relatively slow, but grew faster and faster.

Populations go up or down over time. A *growth rate* will tell you how much the population grows each year. A growth rate of 50% means that a beginning population of 100 will have 50 new individuals in the second year. In the second year, the new population would be 150, after the new individuals are added. This is an increase of 50%, because we added 50% of the population to the original population number. This would also be considered a percent change of 50%.

The increases in the rabbit population table above are based on a 50% growth rate. In Year 0, the beginning population was 24 and 12 rabbits were added by the end of the year. The new population total was 36 (24 + 12).

9) Complete the table.

Year	Beginning Population (100%)	Growth (50%)	Population + Growth New Population (150%)
0	24	12	36
1	36	18	
2	54		81
3	81		121.5

You could also find the new population in the first year by multiplying the beginning population of 24 by 1.5. Try it now to see if you get the same total. Did you get 36?

Multiplying a number by 1.5 is the same as taking 150% of the number. This is because multiplying by .5 gives you 50% a number and multiplying by 1 gives you 100% of a number. 1.5 is considered a *growth factor*, since it is multiplied by the original population to find the new population. Adding 50% each year as a growth rate is equal to multiplying by a growth factor of 1.5.

10) To calculate the population in the second year, you can multiply the beginning population of 24 by 1.5 and then again by 1.5. What answer do you get?

11) Complete the table.

Year	Multiplying by Growth Factor	Population
1	24×1.5	36
2	$24 \times 1.5 \times 1.5$	
3	$24 \times 1.5 \times 1.5 \times 1.5$	
4	$24 \times 1.5 \times 1.5 \times 1.5 \times 1.5$	

You should have gotten the same population numbers you saw in the original rabbit population table.

As you can see, the population is growing by multiplying the original population by the growth factor 1.5 over and over. In Year 1, we multiply 24 by 1.5 once. In Year 2, we multiply 24 by 1.5 twice. In Year 3, we multiply 24 by 1.5 three times. And so on.

12) What would the rabbit population be in Year 6?

To calculate the rabbit population of any year, multiply the original population of 24 by 1.5 once for every year. So, the population in Year 6 would be $24 \times 1.5 \times 1.5 \times 1.5 \times 1.5 \times 1.5 \times 1.5$. Using exponents, you can write this as 24×1.5^6. This is equal to 273.375, which we would call 273 rabbits. (We want to count whole rabbits.)

13) How could you write the calculation for the rabbit population in Year 10?

A. $24 \times 1.5 \times 10$

B. 24×1.5^{10}

C. 1.5×10^{24}

D. $24 \times 10^{1.5}$

Scientists use a formula for predicting population growth. In the case of the rabbit population in Australia, they could use a function similar to this one:

$$P(t) = 24 \cdot 1.5^t$$

Let's look at the parts of the function:

P(t) is read as "P of t." ***P*** stands for *Population* and ***t*** stands for *time*. This means that the population size is a function of the number of years that have passed. Important: $P(t)$ does <u>not</u> mean P × t.

24 is the starting population, which is the number of rabbits in the beginning (year 0). This could be a different number for other populations.

1.5 is the growth factor. This is the number that the beginning population is multiplied by each year. Each year, the population of rabbits is growing by a factor of 1.5. This means that 50% more rabbits are added to the population each year.

t stands for *time* and refers to the number of years that the population has been growing. In order to calculate the population growth, we can substitute different numbers of years for *t* to find the population after that number of years.

For example, after 10 years, the population of rabbits would be about 1,384:

$$\begin{aligned} P(10) &= 24 \times 1.5^{10} \\ &= 24 \times 57.665\ldots \\ &= \mathbf{1383.96} \text{ rabbits} \end{aligned}$$

In the formula above, P(10) would be read as, "P of 10" and refers to the population after 10 years. If you want to know the rabbit population after 20 years, you might set up the formula like this...

$$P(20) = 24 \times 1.5^{20}$$

...and then calculate the answer.

14) What is $P(20)$?

15) What is the rabbit population after 30 years?

16) The rabbit population in Australia did start with 24 rabbits, but the actual growth factor was about 1.38 (not 1.5). Is it possible that there were 10 billion rabbits after 62 years? Explain.

Population growth also shows up as a topic on the science portion of the high school equivalency exam. Try this question. By the way, metabolic rate refers to how quickly individual organisms use the energy that they consume through food.

Sample science question:

17) Rabbits introduced into Australia over one hundred years ago have become a serious pest. Rabbit populations have increased so much that they have displaced many native species of herbivores. Which statement best explains the reason for their increased numbers?

(1) Rabbits have a high metabolic rate.

(2) There are few native predators of rabbits.

(3) Additional rabbit species have been introduced.

(4) There is an increase in rabbit competitors.

Growth Rate and Growth Factor

In the rabbit problem above, the population went up with a growth rate of 50%. Adding 50% each year is equivalent to multiplying by a growth factor of 1.5 each year. The growth rate of 50% is added to 100% of the original population for a total of 150%. If all the numbers are kept in percent, the growth rate plus the original amount equals the growth factor.

18) Complete the following table.

Original Amount		Growth Rate		Growth Factor	
Percentage	Decimal	Percentage	Decimal	Percentage	Decimal
100%	1	50%	0.5	150%	1.5
100%	1	40%	0.4		
100%	1	38%		138%	
100%	1	20%			
100%	1	10%			1.1
100%	1	8%	0.08		
100%	1	2%			

19) If a population's growth rate is 25% each year, what growth factor can be used to calculate the increase?

20) If a growth factor of 1.05 is used to calculate a population's increase, what is the growth rate of the population?

Calculating the Growth of Investments (and Debt)

[2]

If you are interested in business, you might want to learn how to invest money. There are also times in our lives when we need to borrow money, either with a credit card or when buying a car or a house. You might want to predict the amount of money you will make in an investment or how much you will owe in a loan.

Investing Money

Let's say you invest money in a bank savings account that pays 2% yearly interest. This additional 2% each year is the growth rate. Your investment is growing by 2% each year. Percent means "for every one hundred," so 2% means for every $100 you have in the account, the bank will pay you $2 in interest at the end of the year.

To make more money over time, investors try to let investments sit and continue to earn interest for many years. If you are investing for your retirement, for example, hopefully you are able to wait many years before you take your money out of the account.

Here is an investment of $1,000 at 2% interest. To calculate the yearly interest, you can multiply the initial investment of $1,000 by the growth factor of 1.02. Each year, you have an extra factor of 1.02.

21) Complete the table.

Year	Multiplying by Growth Factor	Investment
1	1000×1.02	$1020.00
2	$1000 \times 1.02 \times 1.02$	
3	$1000 \times 1.02 \times 1.02 \times 1.02$	
4	$1000 \times 1.02 \times 1.02 \times 1.02 \times 1.02$	

[2] Image courtesy of Karl Thomas Moore (wikimedia.org)

22) Which of these expressions shows the amount of money you would have after 10 years?

    A. $1000 \times 1.02 \times 10$

    B. 1.02×1000^{10}

    C. 1000×1.02^{10}

    D. $1000 \times 10^{1.02}$

23) Calculate the amount of money you would have after 10 years.

24) Did you make more or less money than you thought you would?

You can use this formula to calculate growth of investments.

$$A = P \cdot g^t$$

Let's look at the parts of the function:

A is the amount of money you have left at the end.

P is the *principal*, which is the amount of money you invest in the beginning

g is the growth factor. This is the number that principal is multiplied by each year.

t stands for *time* and refers to the number of years that the investment has been growing.

In order to calculate the growth of the investment, we can substitute different amounts of money for the beginning investment, a growth factor for the interest rate, and the number of years that the money was invested.

For example, with an investment of $2,000 invested for 5 years at 3%, the calculation would look like this:

$$\begin{aligned} A &= 2000 \times 1.03^5 \\ &= 2000 \times 57.665... \\ &= \mathbf{2318.55} \end{aligned}$$

25) Use the investment formula above to complete this table.

Beginning Investment	Interest Rate	Growth Factor	Years Invested	Final Amount
\$1,000	3%	1.03	10	
\$2,000	3%		10	
\$500		1.04	20	
\$5,000	5%		20	
\$5,000	7%		30	

Borrowing Money

Unfortunately, if you borrow money through a credit card, you will probably pay a much higher interest rate (as much as 21%!) than you can usually earn yourself from investing money on your own. Imagine if you had to borrow \$1,000 at 20% interest (Annual Percentage Rate or APR). If you aren't able to pay off the credit card before the next month, the company will charge interest on the money you still owe. The math works the same as an investment. In this case, a bank is investing in you and you are paying their interest. It's not near as much fun as the other way around.

Here is a debt of \$1,000 at 20% interest. To calculate the yearly interest in the first year, you can multiply \$1,000 by the growth factor of 1.2. Each year, you have an extra factor of 1.2.

26) Complete the table.

Year	Multiplying by Growth Factor	Debt
1	1000×1.2	\$1200.00
2	$1000 \times 1.2 \times 1.2$	
3	$1000 \times 1.2 \times 1.2 \times 1.2$	
4	$1000 \times 1.2 \times 1.2 \times 1.2 \times 1.2$	

27) How much money do you think we would owe after 10 years, assuming we borrowed $1,000, paid 20% interest and were never able to pay any money back?

Incredibly, after 10 years, we could owe $6,191.74 (if we didn't pay any money back during that time). This is why credit cards are so dangerous. The high interest rates and compounding interest can make the debt go up quickly. How is it possible that $1,000 grew to be $6,000? Exponential growth. The debt grows faster and faster each year, unless it's paid off.

By the way, if you need to use credit cards (and most of us do), try to find a credit card that has a low APR (Annual Percentage Rate). The low APR will mean that they will charge you less interest if you aren't able to pay off your balance at the end of the month. You should also do your best to pay off your credit card before the end of your grace period each month. If you do this, the credit card company can't charge you interest on your purchases.

Is It Growing or Decaying?

On the high school equivalency exam, you may be asked if an exponential function represents growth or *decay*. Growth means that the values get larger as the exponent gets larger. Normally, when we hear the word *decay*, we think of something rotting or breaking down into parts. In mathematics, *decay* means that something is getting smaller. *Exponential decay* means that the value goes down as the exponent goes up.

Growth

Look at the example of 2^n:

$2^1 =$ **2** $\quad 2^2 =$ **4** $\quad 2^3 =$ **8** $\quad 2^4 =$ **16**

The numbers are getting bigger: **2**, **4**, **8**, **16**, etc. 2^4 is a bigger number than 2^1. The numbers will keep growing and growing as you calculate larger exponents for base 2. This is exponential growth.

We have looked at many examples of exponential growth. Populations going up over time is an example. Investment and debt can be understood with exponential growth as well.

Decay

Look at the example of $(.5)^n$.

$(0.5)^1 =$ **0.5** $\quad (0.5)^2 =$ **0.25** $\quad (0.5)^3 =$ **0.125** $\quad (0.5)^4 =$ **0.06125**

These numbers are getting smaller as the exponent goes up. It might be easier to see when the numbers are rounded to the hundredths place:

0.50 **0.25** **0.13** **0.06**

Think about these amounts as money: 50 cents, 25 cents, 13 cents, and 6 cents. The values of each power is going down as the exponent gets larger. The power $(0.5)^4$ is a smaller number than $(0.5)^2$. This is exponential decay.

Exponential decay is helpful for understanding science. For example, exponential decay explains the amount of radiation given off over time by a radioactive material like uranium. The amount of radioactive material decreases over time. In archeology, scientists have used this information to determine the age of ancient human civilizations. They have also determined when dinosaurs lived and the age of Earth by using exponential decay calculations for Carbon-14, a radioactive substance found in the fossils and living things.

So, how can you tell the difference between exponential growth and exponential decay? Try different powers and see whether the values get larger or smaller.

28) Complete the following table. Feel free to use a calculator. You may want to round decimals to the hundredths place.

Base	Base to the 2nd power	Base to the 3rd power	Base to the 4th power	Growth or Decay?
2	4	8	16	Growth
0.5	0.25	0.13	.06	Decay
1.5		3.38		
0.9			0.66	
$\frac{1}{2}$	$\frac{1}{4}$			
1.1		1.3		
1				

29) It's possible to look at the base of a power and know whether it will lead to exponential growth or decay. How?

30) The function $M(t) = 500 \cdot 1.2^t$ represents an investment of \$500 at 20% interest. Does it represent exponential growth or decay? Why?

31) Every day, Ramon takes 20 milligrams of a medication to help with his allergies. The amount of medication in his body is modeled by the function $f(t) = 20 \cdot (0.85)^t$, where *t* stands for *time* in hours. How much medication is in Ramon's body after 2 hours?

32) How much medication is left after 4 hours?

33) Does the function for allergy medication represent exponential growth or decay? Why?[3]

34) The function $P(t) = 1.4^t$ models a population over time. $P(t)$ is the size of the population and *t* is the time in years. Does this function represent exponential growth or decay?

35) What is the percent change of the population above?

[3] Thanks to Illustrative Mathematics (https://www.illustrativemathematics.org).

Exponential Growth and Decay - Answer Key

1)

Generation	Number of People	Prime Factorization	Exponential Form
self	1	1	2^0
parents	2	2	2^1
grandparents	4	2×2	2^2
1st-great-grandparents	8	$2 \times 2 \times 2$	2^3
2nd-great-grandparents	16	$2 \times 2 \times 2 \times 2$	2^4
3rd-great-grandparents	32	$2 \times 2 \times 2 \times 2 \times 2$	2^5
4th-great-grandparents	64	$2 \times 2 \times 2 \times 2 \times 2 \times 2$	2^6
5th-great-grandparents	128	$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$	2^7
6th-great-grandparents	256	$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$	2^8
7th-great-grandparents	512	$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$	2^9

2) One thing to notice is that the exponent is always 2 more than the number of greats.

3) 2^{12}

4) 2^{12} is 4,096 10th-great-grandparents

5) 2^{32}

6) The "e" stands for exponent. It's a short way of writing scientific notation. In the display, "e" means the same thing as "$\times$ 10," so if you see 5e3, that means 5×10^3. As another example, 2.6e5 means 2.6×10^5.

7) 62

8) One thing you might have noticed is that the population is going up by larger amounts each year.

9)

Year	Beginning Population (100%)	Growth (50%)	New Population (150%)
0	24	12	36
1	36	18	54
2	54	27	81
3	81	40.5	121.5

10) 54

11)

Year	Multiplying by Growth Factor	Population
1	24×1.5	36
2	$24 \times 1.5 \times 1.5$	54
3	$24 \times 1.5 \times 1.5 \times 1.5$	81
4	$24 \times 1.5 \times 1.5 \times 1.5 \times 1.5$	122 (rounded down)

12) 273 (rounded)

13) B

14) 79,806

15) 4,602,025

16) Yes. A population of 24 rabbits, growing at a rate of 1.38 a year, for 62 years would be 24×1.38^{62}, which is 1.13×10^{10} or 11,300,000,000 (11.3 billion rabbits!)

17) (2)

18)

Original Amount		Growth Rate		Growth Factor	
Percentage	Decimal	Percentage	Decimal	Percentage	Decimal
100%	1	50%	0.5	150%	1.5
100%	1	40%	0.4	140%	1.4
100%	1	38%	0.38	138%	1.38
100%	1	20%	0.2	120%	1.2
100%	1	10%	0.1	110%	1.1
100%	1	8%	0.08	108%	1.08
100%	1	2%	0.02	102%	1.02

19) 125% or 1.25

20) 5% or 0.05

21)

Year	Multiplying by Growth Factor	Investment
1	1000×1.02	\$1020.00
2	$1000 \times 1.02 \times 1.02$	\$1040.40
3	$1000 \times 1.02 \times 1.02 \times 1.02$	\$1061.21
4	$1000 \times 1.02 \times 1.02 \times 1.02 \times 1.02$	\$1082.43

22) C

23) $1,218.99

24) You made $218.99, which is probably less than what you thought you would make. To increase the amount of money you make on investments, you can invest more money (maybe add a small amount each money so your initial investment continues to grow) or find investments that pay a higher interest rate. A savings account in a bank will pay as much as 2%. Other kinds of investments can pay a higher interest rate.

25)

Beginning Investment	Interest Rate	Growth Factor	Years Invested	Final Amount
$1,000	3%	1.03	10	$1,343.92
$2,000	3%	1.03	10	$2,687.83
$500	4%	1.04	20	$1,095.56
$5,000	5%	1.05	20	$13,266.49
$5,000	7%	1.07	30	$38,061.28

26)

Year	Multiplying by Growth Factor	Debt
1	1000×1.2	$1200.00
2	$1000 \times 1.2 \times 1.2$	$1440.00
3	$1000 \times 1.2 \times 1.2 \times 1.2$	$1728.00
4	$1000 \times 1.2 \times 1.2 \times 1.2 \times 1.2$	$2073.60

27) $6,191.74

28)

Base	2nd power	3rd power	4th power	Growth or Decay?
2	4	8	16	Growth
0.5	0.25	0.13	0.06	Decay
1.5	2.25	3.38	5.06	Growth
0.9	0.81	0.73	0.66	Decay
$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	Decay
1.1	1.21	1.33	1.46	Growth
1	1	1	1	Neither

29) Yes. If the base is greater than 1, it will lead to exponential growth. If the base is less than 1, it will lead to exponential decay.

30) Exponential growth, because 1.2 is greater than 1. You can also calculate what the value of the investment over time: $500, $600, $720, $864, etc. The investment is growing.

31) 14.45 milligrams

32) 10.44 milligrams

33) Exponential decay, because 0.85 is less than 1. You can also calculate how much medicine is left in Ramon's body over time: 17 milligrams, 14.45 milligrams, 12.28 milligrams, etc. The amount of medication is going down over time.

34) Exponential growth

35) 40%

Operations with Exponents

On the high school equivalency exam (and college placement exams), you may be given problems that look like these:

Evaluate the following expression for $x = 3$. $2^x + 10$	Which of the following are equivalent to $10^2 \cdot 10^5$?	What is the value of x? $n^5n^3 = n^x$
How else could you write $(x^4)(x^{-2})$?	What is another way to write $\frac{x^7}{x^2}$?	Evaluate $(x^3)^5$ for $x = 2$.

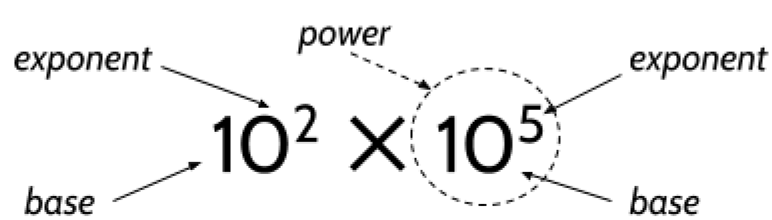
Note: The word *evaluate* means to calculate the value of something. For example, to evaluate $2^x + 10$ for $x = 3$, you would replace x with 3 to get 2^3 and then add 10. The answer is 18. The word *equivalent* means having the same value. For example, 4^3 and 8^2 are equivalent. Do you know why?

By the time you finish the packet, you will learn how to answer each of the problems above.

Multiplying Numbers with Exponents

Knowing how to multiply powers with the same base will help you solve problems like $10^2 \times 10^5$ or $(5^4)(5^6)$. You will also be able to solve equations like $n^5n^3 = n^x$ and find expressions equivalent to $(x^4)(x^{-2})$, both which use variables as the base of the power. In this section, we will only multiply powers that have the same base.

Just a reminder that *power* refers to the base and the exponent together. The *base* is the number being multiplied. The *exponent* is the number of times the base is used as a factor.



1) What is the value of $2^5 \cdot 2^3$?

One way to find the value of $2^5 \cdot 2^3$ is to convert 2^5 and 2^3 into standard numbers, then multiply them.

$$2^5 \cdot 2^3 = \quad ?$$

$$32 \cdot 8 = 256$$

That works well for this example. However, if you were asked to find the value of $8^6 \cdot 8^{10}$, this strategy could be a little difficult to use. 8^6 is 262,144 and 8^{10} is 1,073,741,824! Multiplying those two numbers might be exhausting. Maybe there's an easier way to multiply 8^6 and 8^{10}?

There is also another reason to look for a different strategy. Sometimes you will be asked a question in this form:

What is the value of x? $\qquad 2^5 \cdot 2^3 = 2^x$

In this situation, you have to give your answer as a power of 2. Another way to ask this question is: How many times is 2 used as a factor in $2^5 \cdot 2^3$?

Let's expand 2^5 and 2^3 to show all their factors of 2.

$$2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$$

$$2^3 = 2 \cdot 2 \cdot 2$$

Now we need to multiply the two sets of factors by each other.

$$(2 \cdot 2 \cdot 2 \cdot 2 \cdot 2)\ (2 \cdot 2 \cdot 2)$$

$$\underbrace{(2 \cdot 2 \cdot 2 \cdot 2 \cdot 2) \times (2 \cdot 2 \cdot 2)} = 2^?$$

How many times are we using 2 as a factor in this calculation?

If we take away the parentheses, you can count eight 2's as factors in the answer:

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$$

This means that $2^5 \cdot 2^3$ is equal to 2^8. The value of x is 8.

Expanding powers to see all the factors is a good strategy to use when multiplying powers.

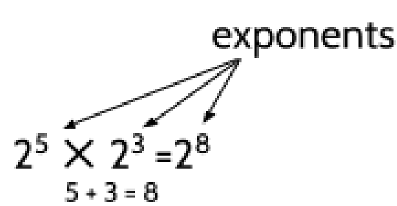
2) Complete the table.

Multiplication	Expanded Form	Product as a Power
$2^5 \cdot 2^3$	$(2 \cdot 2 \cdot 2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2)$	2^8
$3^2 \cdot 3^4$	$(3 \cdot 3)(3 \cdot 3 \cdot 3 \cdot 3)$	
$4^4 \times 4^3$		
$7^2 \cdot 7^6$		7^8
	$(10 \cdot 10 \cdot 10 \cdot 10)(10 \cdot 10)$	
$(5^5)(5^3)$	$(5 \cdot 5 \cdot 5 \cdot 5 \cdot 5)(5 \cdot 5 \cdot 5)$	
$8^3 \times 8^3$		

3) What do you notice when you look at the completed table?

You might notice the product on the right always has the same base as the powers being multiplied on the left. For example, the product of $2^5 \cdot 2^3$ has 2 as a base and the product of $7^2 \cdot 7^5$ has 7 as a base.

You might also notice that if you add the exponents in the two powers being multiplied you will get the exponent of the product. Here's an example:



Here are a few to try on your own:

4) Does $(2^4)(2^2) = 2^6$? Explain why you think so.

5) Does $3^2 \times 3^3 = 3^5$? Explain why you think so..

6) Does $10^4 \cdot 10^{-2} = 10^2$? Explain why you think so.

7) $(2^4)(2^4) =$

8) $3^6 \cdot 3^b = 3^8$ What is *b*?

9) $7^2 \cdot 7^8 = 7^a$ What is *a*?

10) $10^{12} \times 10^{-9} =$

11) $(3^2)(3^3)(3^4) =$

12) Bonus: $9^{\frac{1}{2}} \cdot 9^{\frac{1}{2}} =$

In the section above, you practiced multiplying powers with the same base. When multiplying powers with the same base, we can write the product as the base raised to the sum of the exponents of the powers that are being multiplied.

Since this will always work, we can use variables to talk about what is going on here.

$$x^5 \cdot x^4$$

$x^5 \cdot x^4$ means some number raised to the 5th power times that same number raised to the 4th power. The variable *x* could be replaced with any number. The following equations are true.

$$2^5 \cdot 2^4 = 2^9$$

$$3^5 \cdot 3^4 = 3^9$$

$$4^5 \cdot 4^4 = 4^9$$

In fact, we can say that any number raised to the 5th power times the same number to the 4th power is equal to that number raised to the 9th power.

Or $$x^5 \cdot x^4 = x^9$$

Let's practice multiplying powers with variables as bases:

13) $(b^4)(b^4) =$

14) $x^4 \cdot x^b = x^7$ What is b?

15) $n^2 \cdot n^5 = n^a$ What is a?

16) $x^5 \times x^{-3} =$

17) $(y^2)(y^5)(y^3) =$

18) What is the value of x?

$n^2 n^3 = n^x$

19) How else could you write $(x^4)(x^{-2})$?

Some expressions may include regular numbers as factors along with the variables that are multiplied. In this situation, you can turn the powers back into factors in order to see what is going on.

$$3x^2 \cdot 2x^5$$
$$(3 \cdot x \cdot x)(2 \cdot x \cdot x \cdot x \cdot x \cdot x)$$

The total factors being multiplied are 2, 3 and seven x's.

$$2 \cdot 3 \cdot x^7$$

$$6x^7$$

20) Complete the table.

Multiplication	Expanded Form	Product as a Power
$3x^2 \cdot 2x^5$	$(3 \cdot x \cdot x)(2 \cdot x \cdot x \cdot x \cdot x \cdot x)$	$6x^7$
$4x^3 \cdot 3x^2$	$(4 \cdot x \cdot x \cdot x)(3 \cdot x \cdot x)$	
$3b^4 \cdot 8b^3$		
	$(5 \cdot y)(2 \cdot y \cdot y \cdot y \cdot y)$	
$1.5x^3 \cdot 3x$		

It can be useful to do calculations with numbers written in scientific notation. For example, when using very large or very small numbers, it can be easier to multiply the numbers in scientific notation rather than using the original numbers. Below is an example of a situation where you might multiply numbers in scientific notation rather than using the original numbers.

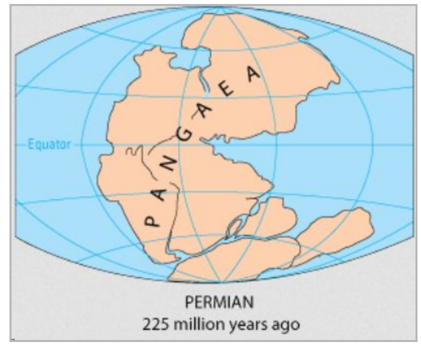
The map of Earth has not always looked the way it does now. Earth's continents are part of tectonic plates that move very, very slowly. The arrangement of the continents has changed over time.

> If continents move at an approximate speed of 0.000000006 miles per hour, how far could a continent move in 225,000,000 years?

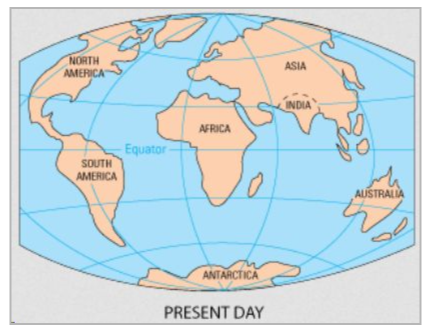
To find the distance a continent could move in this time period, we can multiply the speed by the time that passed. (225,000,000 years is equal to about 2,000,000,000,000 hours!) How would you feel about calculating an answer to the following multiplication problem?

$$0.000000006 \times 2{,}000{,}000{,}000{,}000$$

Scientific notation makes this multiplication much easier to deal with. This is because it is much easier to multiply 6 by 2 than to multiply the original numbers.



PERMIAN
225 million years ago



PRESENT DAY

$$\begin{aligned} 0.000000006 \times 2{,}000{,}000{,}000{,}000 &= (6 \times 10^{-9})(2 \times 10^{12}) \\ &= 12 \times 10^{3} \\ &= 12{,}000 \text{ miles} \end{aligned}$$

$$\begin{array}{r} 6 \times 10^{-9} \\ 2 \times 10^{12} \\ \hline 12 \times 10^{3} \end{array}$$

The answer is 12,000 miles! This explains how the map looked different in the ancient past.

Here are the steps followed above:

A. Convert numbers to scientific notation: 6×10^{-9} and 2×10^{12}
B. Multiply the powers of ten: $10^{-9} \times 10^{12} = 10^{3}$
C. Multiply the remaining numbers: $6 \times 2 = 12$
D. Convert the answer to a standard number: $12 \times 10^{3} \rightarrow 12{,}000$

Dividing Numbers with Exponents

When you know how to divide powers with the same base, you will be able to solve problems like $10^4 \div 10^2$ or $\frac{5^6}{5^4}$. You will also be able to solve equations like $n^5/n^3 = n^x$ and find expressions equivalent to $\frac{x^6}{x^2}$, both which use variables as the base of the power. In this section, we will only divide exponents that have the same base.

21) What is $4^5 \div 4^3$?

One way to find the value of $4^5 \div 4^3$ is to convert 4^5 and 4^3 into standard numbers, then divide them.

$$4^5 \div 4^3 = \quad ?$$

$$1{,}024 \div 64 = \quad 16$$

So, the answer is 16. That worked pretty well. However, you might also be asked a question in this form:

22) What is the value of *x* in the equation $4^5 / 4^3 = 4^x$?

We know that 4^x is 16, because $1{,}024 \div 64$ is 16. To find the value of *x*, we need to know how many times 4 is used as a factor in 16. Well, 16 is the same as $4 \cdot 4$, so *x* should be 2. The completed equation should be $4^5 / 4^3 = 4^2$.

To answer this question above, we converted the powers into standard numbers, divided 1,024 by 64, then converted 16 into 4^2. That worked well for this problem, but what if you needed to calculate the value of $5^{10} \div 5^8$? The power 5^{10} is 9,765,625 and 5^8 is 390,625. I don't know about you, but I would rather not do that division problem. There must be an easier way to divide 5^{10} by 5^8. Let's see if we can find it.

First, we need to back up for a second and look at something that happens with division.

What happens when you divide a number by itself? Each of these mean division.

$\frac{5}{5}$ = 30 ÷ 30 = 1,000 ÷ 1,000 = 1.5 / 1.5 = $\frac{0.25}{0.25}$ =

Try the calculations above with a calculator. Then you might try it with some other numbers. You should find that any number divided by itself is 1.

Yes, each of these division problems are equal to 1. Any number or expression divided by itself equals 1.

It doesn't matter how big the numbers are: $\frac{50{,}000}{50{,}000} = 1$

It doesn't matter how small the numbers are: $\frac{0.00002}{0.00002} = 1$

It's also true for expressions: $\frac{2 \cdot 3 \cdot 5}{2 \cdot 3 \cdot 5} = 1$

And even for variables: $\frac{x}{x} = 1$ $\frac{x \cdot x \cdot x \cdot x}{x \cdot x \cdot x \cdot x} = 1$ and $\frac{x^3}{x^3} = 1$ and $\frac{5x}{5x} = 1$

We can say that x divided by x equals 1, even though x could be any number.

This rule even works for numbers in exponential or factored form: $\frac{x^3}{x^3} = 1$ and $\frac{2 \cdot 3 \cdot 5}{2 \cdot 3 \cdot 5} = 1$

Now, let's look at how this fact can help us divide powers.

$$4^5 \div 4^3 = \ ?$$

Instead of converting the powers into normal numbers, we will expand them to show their factors. Expanding powers to see all the factors is a good strategy to use when dividing powers.

$$4^5 = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$$

$$4^3 = 4 \cdot 4 \cdot 4$$

Then we'll set up division with a fraction bar.

$$\frac{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4}{4 \cdot 4 \cdot 4}$$

When we look at this division, we can see an opportunity to divide a number by itself and make the number 1.

$$\frac{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4}{4 \cdot 4 \cdot 4}$$

The expression (4 · 4 · 4) divided (4 · 4 · 4) equals 1. This means we can replace $\frac{4 \cdot 4 \cdot 4}{4 \cdot 4 \cdot 4}$ with 1.

Our new calculation is: $1 \cdot 4 \cdot 4$

So the answer is 4 · 4, which is 4^2.

Here's another way to see what we just did:

$$\frac{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4}{4 \cdot 4 \cdot 4} =$$

Find an equivalent expression on the top and bottom.

$$\frac{4 \cdot 4 \cdot 4}{4 \cdot 4 \cdot 4} \cdot 4 \cdot 4 =$$

Separate it and turn it into 1.

$$1 \cdot 4 \cdot 4 =$$

$$4 \cdot 4 = 4^2$$

Try a few on your own:

23) $\frac{5 \cdot 5 \cdot 5}{5 \cdot 5} =$

25) $\frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2} =$

24) $\frac{7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7}{7 \cdot 7 \cdot 7} =$

26) $\frac{4^8}{4^3} =$

27) $\frac{10^7}{10^4}$ =

28) Look back at this equation: $4^5 \div 4^3 = 4^2$. Do you notice anything interesting?

You might notice that if you subtract the exponents in the two powers being divided you get the exponent of the *quotient* (a number that is the result of division).

$$4^5 \div 4^3 = 4^2$$
$$5 - 3 = 2$$

Does this always work? Look back at the other problems you did above to see it's true.

Here are a few more to try:

29) Does $\frac{2^4}{2^2} = 2^2$? Explain.

30) $10^{13}/10^8$ =

31) Bonus: $\frac{10^6}{10^7} =$

32) Does $\frac{3^3}{3^2} = 3$? Explain.

33) $2^5 \div 2^2 = 2^a$ What is a?

34) Bonus: What is the value of $\frac{10^4}{10^{-2}}$?

In the section above you practiced dividing powers that have the same number as a base. Now, we try dividing powers that have a variable like *x* or *y* as a base.

The following equations are true. You might want to check the calculations of the standard numbers on the right.

Exponential Form		Standard Notation
$2^5 \div 2^2 = 2^3$	⟶	$32 \div 4 = 8$
$3^5 \div 3^2 = 3^3$	⟶	$243 \div 9 = 27$
$4^5 \div 4^2 = 4^3$	⟶	$1{,}024 \div 16 = 64$
$10^5 \div 10^2 = 10^3$	⟶	$100{,}000 \div 100 = 1{,}000$

Each of the equations in exponential form on the left is equivalent to the equations in standard notation on the right. In fact, we can say that any number raised to the 5th power divided by the same number to the 2nd power is equal to that number raised to the 3rd power. This is a bit complicated to say, so we shorten it to following equation that means the same thing:

$$x^5 \div x^2 = x^3$$

Let's practice dividing powers with variables as bases:

35) $\frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x}$ =

36) $\frac{x^6}{x^2}$ =

37) n^5/n^3 =

38) $(b^4)/(b^4)$ =

39) $x^5 \div x^2 = x^b$ What is *b*?

40) $n^5 \div n^a = n^4$ What is *a*?

41) $(y^2)/(y^3)$ =

42) What is the value of *x*?

$n^5/n^3 = n^x$

43) How else could you write $\frac{x^6}{x^2}$?

Numbers written in scientific notation can also be divided. When using very large or very small numbers, scientific notation makes a calculation like division easier.

The U.S. government borrows money to pay its bills. As of 2019, the national debt of the United States is about \$20,000,000,000,000. This is how much money the U.S. has borrowed to pay its bills (and how much it still owes). Here's a common question that is asked about the national debt:

> With a population of about 300,000,000 people in the United States, how much would it cost each person in the United States if we paid off the national debt?

To find each person's share of the national debt, we can divide the national debt by the United States population. How would you feel about calculating an answer to the following division problem?

$$20{,}000{,}000{,}000{,}000 \div 300{,}000{,}000$$

It's awful, right? Scientific notation makes this division much easier to deal with. This is because it is much easier to divide 2 by 3 than to divide the original huge numbers.

This is another way to write division. The top number is divided by the bottom number, just like in a fraction.

$$\frac{20{,}000{,}000{,}000{,}000}{300{,}000{,}000} = \frac{2 \times 10^{13}}{3 \times 10^{8}}$$

$$= \frac{2}{3} \times 10^{5}$$

$2 \div 3 = 0.67$

$$= 0.67 \times 10^{5}$$

$$= \$67{,}000.00$$

$10^{13} \div 10^{8} = 10^{5}$

That's a lot of money. My guess is that we aren't going to pay off the national debt any time soon.

Here are the steps followed above:

A. Convert numbers to scientific notation: 2×10^{13} and 3×10^{8}
B. Divide the powers of ten by subtracting exponents: $10^{13} \div 10^{8} = 10^{5}$
C. Divide the remaining numbers: $2 \div 3 = 0.67$
D. Convert the answer to a standard number: $0.67 \times 10^{5} \rightarrow 67{,}000$

44) Michael is looking at the speed of sound and the speed of light. It seems to him that sound is faster than light, since 7.67 is a bigger number than 6.71.

Speed of light: 6.71×10^8 miles per hour
Speed of sound: 7.67×10^2 miles per hour

Can you explain the use of scientific notation so that Michael understands why light is faster than sound?

45) The mass of Earth and the Moon in kilograms is shown below.

The mass of Earth: 6×10^{24} kilograms
The mass of the Moon: 7×10^{22} kilograms

True or False: Earth weighs about 100 times more than the Moon. (Explain your answer.)

46) The mass of the Sun is shown below in scientific notation.

The mass of the Sun: 2×10^{30} kilograms

The Sun weighs about __________ times as much as Earth.

A. $\frac{1}{3}$

B. 3

C. 333,333

D. 3,000,000

47) Directions: Find 3 whole numbers that add up to 10. Place each number into one of the blanks to find the largest possible result.

$$(\boxed{})\cdot(\boxed{})^{\boxed{}}$$

Attempt	Expression	Numeral
1st try (example)	$(5)\cdot(3)^2$	45

Thanks to Open Middle!

48) Which is bigger? $2 \cdot 3^5$ or $2 \cdot 5^3$

Raising a Power to a Power

Look at the following expression. What do you think it means?

$$(4^3)^2$$

Remember that an exponent tells you how many times to use the number of expression below as a factor. In this case, 4^3 is being used as a factor twice. So, we can split $(4^3)^2$ into 4^3 multiplied by 4^3.

$$(4^3)^2 = 4^3 \times 4^3$$

It's helpful to continue to expand the factors. 4^3 is equal to $4 \cdot 4 \cdot 4$, so we can substitute the factors of 4.

$$(4^3)^2 = (4 \cdot 4 \cdot 4) \times (4 \cdot 4 \cdot 4)$$

Then we can just count all the factors of 4 and turn the answer back into a power.

$$(4^3)^2 = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$$

$$= 4^6 \text{ or } 4{,}096$$

49) Complete the table.

Power to a Power	Expanded Form	Exponential Form	Standard Notation
$(4^3)^2$	$(4 \cdot 4 \cdot 4)(4 \cdot 4 \cdot 4)$	4^6	4,096
$(5^2)^3$	$(5 \cdot 5)(5 \cdot 5)(5 \cdot 5)$		
$(3^2)^4$			6,561
	$(2 \cdot 2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2 \cdot 2)$		
	$(1.5 \cdot 1.5 \cdot 1.5)(1.5 \cdot 1.5 \cdot 1.5)$		11.39
Bonus: $(10^4)^{1/2}$	$(10 \cdot 10 \cdot \cancel{10 \cdot 10})$		

Try solving these problems using what you learned on the previous page.

50) Evaluate $(x^3)^5$ for $x = 2$.

If x is 2, then the expression can be rewritten as $(2^3)^5$. The expanded version of this power is $(2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2)$, which can be written as 2^{15}. If you are asked to evaluate the expression for $x = 2$, it means that you should give a final answer in standard notation: 32,768.

51) Evaluate $(x^3)^5$ for $x = 3$.

52) Evaluate $(x^2)^3$ for $x = 4$.

53) Evaluate $(y^4)^2$ for $y = 5$.

54) What is the value of $(3^3)^2$?

Check your understanding. The following statements are all INCORRECT. They all show common mistakes.

For each:

- Identify the mistake.
- Correct the statement.
- Explain your thinking.

55) $2^5 = 10$

56) $9^2 \cdot 9^4 = 9^8$

57) $100^{1/2} = 50$

58) $5^{-2} = -25$

59) $8^0 = 0$

60) $\frac{7^5}{7^9} = 7^4$

61) $(5^3)^5 = 5^8$

62) $4^5 \div 4^3 = 4^{5/3}$

Operations with Exponents - Answer Key

1) 256

2)

Multiplication	Expanded Form	Product as a Power
$2^5 \cdot 2^3$	$(2 \cdot 2 \cdot 2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2)$	2^8
$3^2 \cdot 3^4$	$(3 \cdot 3)(3 \cdot 3 \cdot 3 \cdot 3)$	3^6
$4^4 \times 4^3$	$(4 \cdot 4 \cdot 4 \cdot 4)(4 \cdot 4 \cdot 4)$	4^7
$7^2 \cdot 7^6$	$(7 \cdot 7)(7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7)$	7^8
$10^4 \cdot 10^2$	$(10 \cdot 10 \cdot 10 \cdot 10)(10 \cdot 10)$	10^6
$(5^5)(5^3)$	$(5 \cdot 5 \cdot 5 \cdot 5 \cdot 5)(5 \cdot 5 \cdot 5)$	5^8
$8^3 \times 8^3$	$(8 \cdot 8 \cdot 8)(8 \cdot 8 \cdot 8)$	8^6

3) There are a lot of things you could notice. Continue reading to see what we noticed.

4) It does. How do you know?

5) This is also true. Can you explain it?

6) This equation is true. Try converting 10^4 and 10^{-2} into standard numbers and use a calculator. You should get 100. Do you know why?

7) 2^8 or 256

8) 2

9) 10

10) 10^3 or 1,000

11) 3^9 or 19,683

12) 9 or 9^1

13) b^8

14) 3

15) 7

16) x^2

17) y^{10}

18) 5

19) x^2 is another way to write $(x^4)(x^{-2})$. You could also write it as $\frac{x^4}{x^2}$. Do you know why?

20)

Multiplication	Expanded Form	Product as a Power
$3x^2 \cdot 2x^5$	$(3 \cdot x \cdot x)(2 \cdot x \cdot x \cdot x \cdot x)$	$6x^7$
$4x^3 \cdot 3x^2$	$(4 \cdot x \cdot x \cdot x)(3 \cdot x \cdot x)$	$12x^5$
$3b^4 \cdot 8b^3$	$(3 \cdot b \cdot b \cdot b \cdot b)(8 \cdot b \cdot b \cdot b)$	$24b^7$
$5y \cdot 2y^4$	$(5 \cdot y)(2 \cdot y \cdot y \cdot y \cdot y)$	$10y^5$
$1.5x^3 \cdot 3x$	$(1.5 \cdot x \cdot x \cdot x)(3 \cdot x)$	$4.5x^4$

21) 16 or 4^2

22) 2. (Look for the explanation in the paragraph below the question.)

23) 5^1 or 5

24) 7^3

25) 2^2

26) 4^5

27) 10^3

28) 5 - 3 = 2

29) Yes. 16 ÷ 4 = 4.

30) 10^5

31) 10^{-1}

32) Yes. 27 ÷ 9 = 3.

33) 3

34) 10^6. This is because $4 - (^-2) = 6$.

35) x^3

36) x^4

37) n^2

38) 1

39) 3

40) 1

41) y^{-1}

42) 2

43) x^4

44) 10^2 is 100. 10^8 is 100,000,000. 10^8 is a million times bigger than 10^2. The speed of sound is about 700 miles per hour. The speed of light is about 700 million miles per hour.

45) True. 6,000,000,000,000,000,000,000,000 ÷ 70,000,000,000,000,000,000,000 = 86, which is pretty close to 100.

46) C

47) There are many possible answers here. Make sure that the 3 numbers you use add up to 10. In our example, 5 + 3 + 2 = 10. I'm sure that you can find a combination that beats our high of 45.

48) $2 \cdot 3^5 = 486$; $2 \cdot 5^3 = 250$

49)

Power to a Power	Expanded Form	Exponential Form	Standard Notation
$(4^3)^2$	$(4 \cdot 4 \cdot 4)(4 \cdot 4 \cdot 4)$	4^6	4,096
$(5^2)^3$	$(5 \cdot 5)(5 \cdot 5)(5 \cdot 5)$	5^6	15,625
$(3^2)^4$	$(3 \cdot 3)(3 \cdot 3)(3 \cdot 3)(3 \cdot 3)$	3^8	6,561
$(2^4)^2$	$(2 \cdot 2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2 \cdot 2)$	2^8	256
$(1.5^3)^2$	$(1.5 \cdot 1.5 \cdot 1.5)(1.5 \cdot 1.5 \cdot 1.5)$	1.5^6	11.39
Bonus: $(10^4)^{1/2}$	$(10 \cdot 10 \cdot \cancel{10 \cdot 10})$	10^2	100

50) $(2^3)^5$ is 32,768

51) 14,348,907 (3^{15})

52) 4,096 (4^6)

53) 390,625 (5^8)

54) 729

55) Maybe they thought this means 2×5? It really means $2 \times 2 \times 2 \times 2 \times 2$.

56) Maybe they thought you were supposed to multiply the exponents? Maybe expanding 9^2 and 9^4 to show all the factors would help.

57) When ½ is used as an exponent, it means square root, not half. What's the square root of 100?

58) Negative exponents aren't the same as negative numbers. 5^{-2} means 1 divided by 5^2.
$5^{-2} = \frac{1}{5^2}$ or $\frac{1}{25}$

59) Any number to the zero power is 1.

60) $7^9 \div 7^5$ does equal 7^4, but $7^5 \div 7^9$ equals ______________.

61) $(5^3)^5 = (5 \cdot 5 \cdot 5)(5 \cdot 5 \cdot 5)(5 \cdot 5 \cdot 5)(5 \cdot 5 \cdot 5)(5 \cdot 5 \cdot 5) = 5^{15}$

62) $4^5 \div 4^3 = 4^2$. Can you explain why?

Test Practice Questions

Answer the following questions. You can check your answers in Test Practice Questions - Answer Key.

1) What is the product of 4^3 and 4^5?

A. 4^{-2}

B. 4^2

C. 4^8

D. 4^{15}

2) What is the product of x^2 and x^3?

A. x^6

B. x^5

C. x

D. x^{-1}

3) What is 5^6 divided by 5^2?

A. 5^{-4}

B. 5^3

C. 5^4

D. 5^{12}

4) What is x^{10} divided by x^6?

A. x^{60}

B. x^4

C. $x^{5/3}$

D. x^{-4}

5) Evaluate the expression below.

$$(5^4 \times 5^7) \div 5^2$$

A. 5^7

B. 5^9

C. 5^{14}

D. 5^{20}

6) Which of the following is equal to the $\frac{(x^5)(x^4)}{x^3}$?

A. $\frac{1}{3}$

B. x^3

C. $x^{20/3}$

D. x^6

7) Simplify the expression below.

$3x + 9x$

A. 12

B. $12x$

C. $12x^2$

D. $12(x + x)$

8) Evaluate $(5^3)^4$.

A. 5^7

B. 5^{12}

C. $5^{3/4}$

D. $5^{4/3}$

9) What is the value of $\sqrt{5^4}$?

A. 25

B. $\frac{1}{25}$

C. $\frac{1}{625}$

D. 625

10) Which of the following numerical expressions is equivalent to $3^2 \cdot 3^{-5}$?

A. 27

B. $\frac{1}{27}$

C. 59,049

D. $\frac{1}{59,049}$

11) Which two equations represent exponential decay? Select two correct answers.

A. $y = (1.02)^x$

B. $y = (\frac{1}{2})^x$

C. $y = (0.98)^x$

D. $y = (\frac{3}{2})^x$

12) Alicia has invented a new app for smartphones that two companies are interested in purchasing for a 2-year contract.

Company A is offering her \$10,000 for the first month and will increase the amount each month by \$5000.

Company B is offering \$500 for the first month and will double their payment each month from the previous month.

Monthly payments are made at the end of each month. What is the first month in which Company B's payment will be higher than company A's payment?

A. 9

B. 7

C. 6

D. 8

13) The number of rabbits in a population over time can be calculated with the function $R(t) = (100)(1.4)^t$, where $R(t)$ is the population and t is the amount of time in years.

I. What is the percent change in the population?

A. 0.4%

B. 4%

C. 40%

D. 140%

II. What kind of function is this?

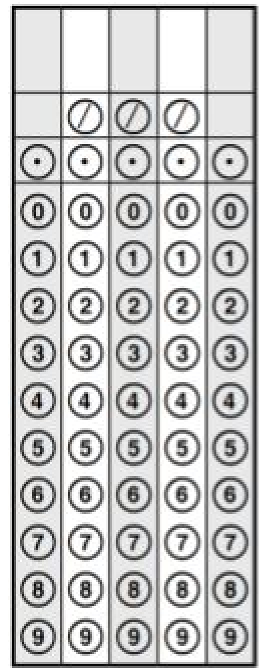
A. linear growth

B. linear decay

C. exponential growth

D. exponential decay

III. What was the rabbit population after 2 years? (Enter your answer in the gridded response area to the right.)



IV. What is the approximate rabbit population after 5 years?

A. 140

B. 538

C. 700

D. 952

14) What is the value of 3^{-2} ?

A. $\frac{1}{9}$

B. -6

C. -9

D. 1.7

15) Evaluate $64^{1/3}$.

A. $\frac{3}{64}$

B. $\frac{1}{192}$

C. 8

D. 4

16) Which of these is equivalent to $\frac{(x^{-2})(x^{3})}{2}$?

A. $\frac{x^{-6}}{2}$

B. $\frac{x}{2}$

C. x^{-3}

D. $\sqrt{x^{-6}}$

17) The population growth of a colony of E. coli bacteria is shown in the table below.

Hours	Population
0	1
1	8
2	64
3	512
4	4,096
5	32,768

Which of the functions below determines the population of bacteria, P, after t hours?

A. $P = 8^t$

B. $P = 2^t$

C. $P = 8t$

D. $P = t^8$

Why did you choose this answer?

18) The population of mosquitoes in a swamp is estimated over the course of twenty weeks, as shown in the table.

Time (weeks)	Population
0	100
1	200
2	400
3	800
4	1600

Which of the following best describes the estimated population of mosquitos during the four weeks?

A. Increasing linear

B. Decreasing linear

C. Exponential growth

D. Exponential decay

19) Choose all expressions below that are equivalent to $64^{1/2}$.

A. $\frac{1}{64}$

B. $\frac{1}{128}$

C. 8

D. 4

E. $64 \cdot 64$

F. $\sqrt{64}$

G. $\frac{1}{\sqrt{64}}$

H. 2^3

20) Andrea is comparing two options for investing an initial amount of \$5,000 in an investment account .

> Investment option 1: An additional \$52 is added to the investment account every month.
>
> Investment option 2: An amount equal to 1% of the current value of the account is added to the account every month.

As the amount of money in the account grows over time, which investment option will make the <u>most</u> money for Andrea? How many months will pass before the better investment option exceeds the other option?

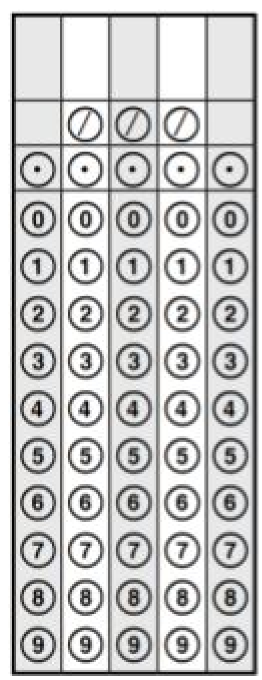
A. Investment option 1 is better; it will always exceed option 2.

B. Investment option 1 is better; it will exceed option 2 in less than a year.

C. Investment option 2 is better; it will always exceed option 1.

D. Investment option 2 is better; it will exceed option 1 in less than a year.

21) What is the value of $(3^4)^{\frac{1}{2}}$? (Enter your answer in the gridded response area to the right.)



Test Practice Questions - Answer Key

1) C
2) B
3) C
4) B
5) B
6) D
7) B
8) B
9) A
10) B
11) B, C
12) D
13) I. 40%
    II. exponential growth
    III. 196
    IV. B
14) A
15) D
16) B
17) A
18) C
19) C, F, H
20) D
21) 9

Reviewing What We Have Learned About Exponents

Write a checkmark ✓ next to each statement you agree with:

I understand...

- ❏ how to read square roots and cube roots.
- ❏ how to find the square roots of small perfect squares: 9, 16, 25, 36, etc.
- ❏ that the square root of x^2 is x.
- ❏ that the cube root of x^3 is x.
- ❏ the relationship between x^2 and $\sqrt[2]{x}$.
- ❏ the relationship between x^3 and $\sqrt[3]{x}$.
- ❏ how to multiply a base number raised to powers: for example, $3^3 \times 3^4$.
- ❏ how to divide a base number raised to powers: for example, $3^5/3^2$.

The Language of Exponents and Roots

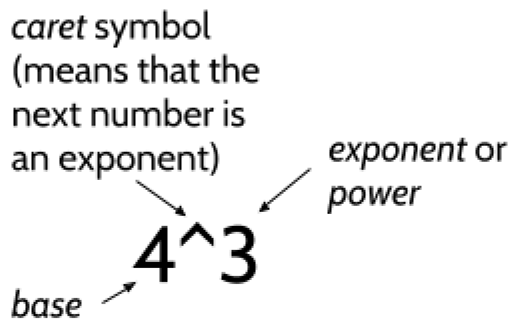
Review of How Exponents are Written

You can use this table to review the meaning of different ways of writing exponents and roots.

Notation	What it Means	Example
2^x	What is 2 multiplied by itself *x* times? How many times am I multiplying the base? (I'm multiplying the base *x* times).	2^3. I'm multiplying 2 three times. $2 \times 2 \times 2 = 8$.
n^4	What is *n* multiplied by itself 4 times? I am multiplying some number by itself four times.	$n^4 = n \times n \times n \times n$
3^4	3 times 3 times 3 times 3	$3^4 = 3 \times 3 \times 3 \times 3 = 81$
$\sqrt{x}$ $x^{½}$	What is the square root of *x*? What number times itself is x?	$4^{1/2} = 2$ $25^{1/2} = 5$
$\sqrt[3]{x}$ $x^{⅓}$	What is the cube root of x? What number times itself and then times itself again is *x*?	$27^{1/3} = 3$, $\sqrt{27} = 3$ $64^{1/3} = 4$, $\sqrt[3]{64} = 4$
2^{-x}	What is 1 divided by 2^x?	$2^{-3} = \frac{1}{8}$ $4^{-1} = \frac{1}{4}$

How to Read and Write Exponents on a Computer

On calculators and computers, exponents are often displayed in the following manner. Instead of showing the exponent as a small number above the base, the *caret* symbol ^ is placed between the base and the exponent. Both 4^3 and 4^3 mean 4 × 4 × 4.



To write a square root on a computer, you can also use the *caret* symbol ^. The square root of 16 is equal to 16 to the $\frac{1}{2}$ power:

$\sqrt{16}$ = 16^(1/2)   <--- This way of writing the square root will also work on the TI-30XS.

The cube root of 27 is equal to 27 to the $\frac{1}{3}$ power:

$\sqrt[3]{27}$ = 27^(1/3)   <--- This way of writing the cube root will also work on the TI-30XS.

1) What do you think 9^(.5) means?

2) How would your write $\sqrt{64}$ on a computer?

3) How would your write $\sqrt[3]{125}$ on a computer?

4) Evaluate 144^(1/2).

5) Evaluate 125^(1/3).

Factors, Multiples, Primes, and Composites

6) Fill in the blanks with words, numbers, and explanations.

5 and 8 are factors of 40 because ________________________________ . The numbers _______ and _______ are also factors of 40 because ______________________________ ____. Factors are numbers we can multiply with other numbers to get another number. A number's factors are the numbers that can be multiplied to get that number. These are some factors of 24: _______, _______, _______, and _______.

3, 6, 9, 12, and 15 are multiples of 3. Some other multiples of 3 are _______, _______, and _______. A multiple is a number that can be divided by another number evenly, with no remainder. These are some multiples of 5: _______, _______, and _______.

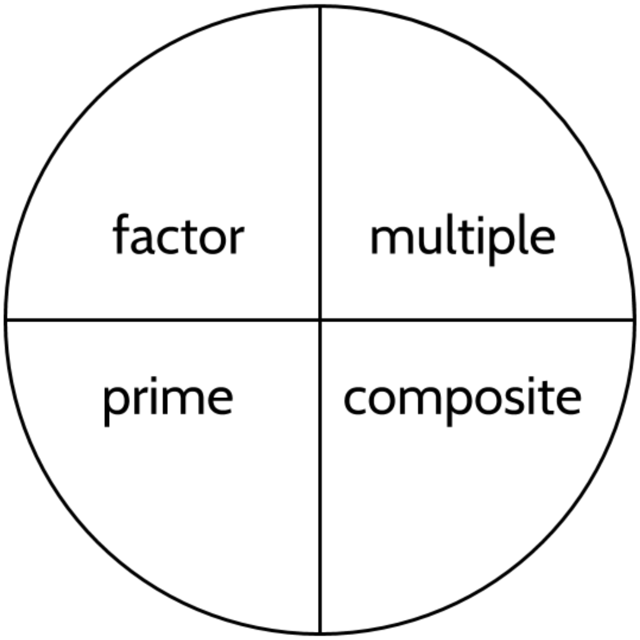
7, 19, and 23 are some examples of prime numbers. A prime is a number that has only two factors, 1 and itself. Some other examples of prime numbers are _______, _______, and _______.

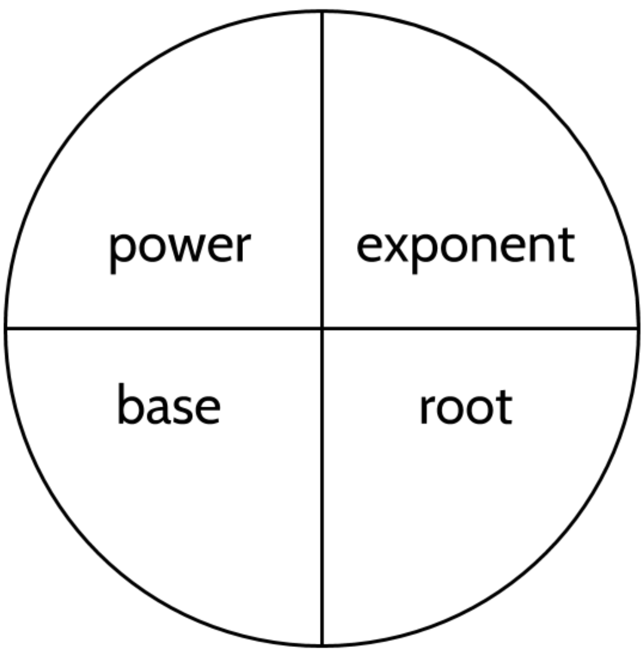
8, 26, and 35 are some examples of composite numbers. A composite is a number that has more than two factors. For example, the factors of 8 are 1, 2, 4, and 8. So, 8 has 4 factors. Some other examples of composite numbers are _______, _______, and _______.

1 is not a prime number or a composite number. It only has one factor: 1.

Concept Circle

7) Explain these words and the connections you see between them.





Exponents and Roots in Your Life

8) Look around you. Where do you see exponents and roots? Describe the world you see using as many of the exponents and roots vocabulary words on page 82 as you can.

Language Review - Answer Key

1) The square root of 9.

2) 64^(1/2)

3) 125^(1/3)

4) $\sqrt{144}$ is 12

5) $\sqrt[3]{125}$ is 5

6) (There are different possible answer for the blanks. The following answers aren't the only correct ones.)

   because they both divide into 40 with no remainder
   2 and 20
   both numbers can be multiplied by other whole numbers to get 40
   1, 2, 3 and 4
   18, 21, and 24
   10, 25, and 45
   29, 37, and 41
   100, 16, and 85

7) Each paragraph should use the 4 vocabulary words in the circle on the left. Be creative. There is no one way to do this activity!

8) Take your time with this activity. Look around you and look for evidence of exponents and roots. You might even do some research so that you have some numbers to include in your writing. This is an opportunity to practice all the vocabulary and math skills you have learned.

Vocabulary Review

You can use this section to look up words used in this math packet.

area (noun): The size of a flat surface, measured in square units

array (noun): An arrangement of objects in columns and rows

> *rectangular array*: An *array* in the shape of a rectangle

base (noun): In a quantity represented as a power, the *base* is the factor being multiplied. For example, in the power 2^3, the *base* is 2.

composite number: A number that has more than two factors

cube (noun): A box-shaped solid object that has six identical square faces

cube number (noun): A number which is the product of three numbers which are the same

> *perfect cube*: Numbers like 1, 8, 27, 64, and 125 which can be formed into a *cube* of this number of blocks. All *perfect cubes* are a whole number to the third power. For example, $27 = 3^3$ and $125 = 5^3$.

digit (noun): The numbers 0-9 and the numerals that represent them: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. There are 5 digits in this number: 14,692. *Digit* can also mean "finger."

divisible (adjective): A number is *divisible* by another number if it can be divided into the second number with no remainder. 21 is divisible by 3. 25 is <u>not</u> divisible by 3.

equation (noun): An expression that shows two mathematical expressions are equal (using = sign). $2^3 = 8$ is an *equation*. $5x + 3$ is an expression, <u>not</u> an *equation*.

equivalent (adjective): Having the same value. 4^3 and 64 are *equivalent*

estimate (verb): To make a rough guess at a number, usually without making written calculations

evaluate (verb): To calculate the value of something. If asked to *evaluate* 4^3, your answer should be 64.

even (adjective): Numbers that are divisible by 2.

expanded form (noun): A way of showing all the factors that are multiplied in a power. The *expanded form* of 3^4 is $3 \times 3 \times 3 \times 3$.

expression (noun): Numbers and symbols that show the value of something. 100, $5x + 3$, and 2^3 are all *expressions*. $5x + 3 = 23$ is an equation, not an *expression*. *Expressions* don't use equal signs.

exponent (noun): In a quantity represented as a power, the *exponent* shows how many times the base is multiplied. The exponent is shown as a smaller number up and to the right of the base. For example, in the power 2^3, the *exponent* is 3.

exponential form: A way of writing values as a power using exponents. 3^4 is an *exponential form* of 81.

exponential growth: A growth pattern in which the unknown quantity or variable is an exponent and the base (factor) is larger than 1. The power 2^x produces *exponential growth* based on doubling or multiplying by 2. The power 3^x produces *exponential growth* based on tripling or multiplying by 3.

exponential decay: A change in numbers where the variable is an exponent and the base (factor) being multiplied is less than 1. As the exponent gets larger, the values get smaller. The power $(.5)^x$ represents *exponential decay*.

factor (noun): Whole numbers that are multiplied together to get another number. A number that can be divided <u>into</u> another number evenly, with no remainder.

prime factor: A factor of another number that itself has only 2 factors. 2, 3, and 5 are the prime factors of 30. The number 15 is a factor of 30, but it is <u>not</u> a *prime factor* of 30.

prime factorization: A way of representing numbers with all of their prime factors. $2 \times 3 \times 5$ is the *prime factorization* of 30. $2 \times 3 \times 3$ is the *prime factorization* of 18.

factor (verb): To split a number into its factors (see above definition of factors).

multiple (noun): A number that can be divided <u>by</u> another number evenly, with no remainder. 25 is a *multiple* of 5.

numeral (noun): A symbol or name for a number. 12 and twelve are both *numerals*.

place value (noun)**:** The value of each position in a number. In the number 4,967,285, the *place value* of the 7 is 1,000.

power (noun): A way of showing repeated multiplication. The base of a *power* shows what value is being multiplied. The exponent shows how many times it is multiplied.

powers of ten: 10^{-2}, 10^{-1}, 10^{0}, 10^{1}, 10^{2}, etc.

powers of two: 2^{-2}, 2^{-1}, 2^{0}, 2^{1}, 2^{2}, etc.

prime number (noun): A number that has exactly two factors (1 and itself)

product (noun): The result of multiplication. 4 times 5 gives a *product* of 20.

quotient (noun): The result of division. 20 divided by 5 gives a *quotient* of 4.

radical (noun): A symbol that means "root." *Radicals* are used for square roots, cube roots, and other roots.

square root symbol: $\sqrt{}$

cube root symbol: $\sqrt[3]{}$

reciprocal (noun): Equal to 1 divided by a number or value. The *reciprocal* of 10 is $\frac{1}{10}$. The *reciprocal* of x^3 is $\frac{1}{x^3}$.

remainder (noun): A number left over after division. 20 divided by 8 equals 2 with a *remainder* of 4.

root (noun): The solution to an equation, usually similar to $a^2 = 25$ or $a^3 = 8$

square root: A square root of a number is a value that, when multiplied by itself, gives the number. The *square root* of 25 is 5.

cube root: A cube root of a number is a value that, when multiplied by itself and then multiplied by itself again, gives the number. The *cube root* of 8 is 2.

square (noun): a 4-sided, flat shape which has four straight and equal sides, and four right (90°) angles

square number (noun): A number which is the product of two numbers which are the same *perfect square*: Numbers like 1, 4, 9, 16, 25, and 36, which can be formed into a square array of rows and columns. All *perfect squares* are a whole number to the second power. For example, $9 = 3^2$ and $25 = 5^2$.

variable (noun): A letter that represents any number or a specific number. In the expression x^3, *x is a variable* that could mean any number.

volume (noun): A measurement of the 3-dimensional space something takes up, measured in cubes