# Essential Algebra Practice
# for Students of TASC-math

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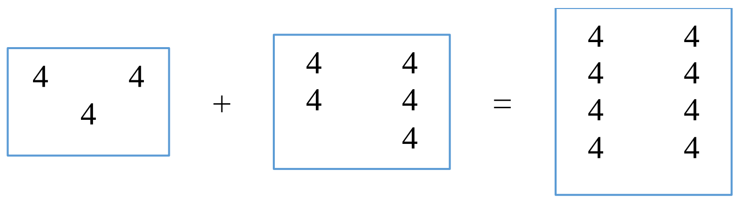
# Essential Algebra Practice for Students of TASC-math

Approximately 26% of the questions on the TASC math subtest focus on algebra. That is at least 1 in 4 questions. It's clear that you need to be strong in this area of math to pass this test.

But, what is algebra, and what do you need to know about algebra to pass the TASC math subtest?

***Algebra: the use of symbols in place of numbers to perform generalized arithmetic.***

"*Generalized arithmetic*"??? – Let's look at a simple example. If we have 3 turkeys and we add 5 more turkeys, we have 8 turkeys. That's obvious. It's also true that if we have 3 fours and we add 5 more fours, we have 8 fours.




12 + 20 = 32

It turns out, that no matter what we have, whether it be turkeys, rocks, apples, or fours, for example, if we add 3 of some quantity to 5 of the same quantity we have 8 of that quantity. That's pretty wordy though. We can use a symbol, let's say $x$, to represent *some quantity*. That allows to write:

$$3x + 5x = 8x$$

This equation says that no matter what $x$ stands for, 3(of a quantity) + 5(of the same quantity) = 8(of that quantity). Writing $3x + 5x = 8x$ is just so much simpler. See, *generalized arithmetic*, or algebra, isn't so bad after all.

Of course, there's more to it than that. Some of the most common types of algebra questions you will find on the TASC math subtest will require that you know how to:

1. Solve linear equations and inequalities in one variable;
2. Read graphs of equations and inequalities in two variables;
3. Write equations and inequalities in one variable;
4. Write equations in two variables;
5. Write a system of equations;
6. Interpret expressions;
7. Identify equivalent expressions;
8. Solve quadratic functions graphically;
9. Solve quadratic functions and equations algebraically.

<u>Essential Algebra Questions</u>

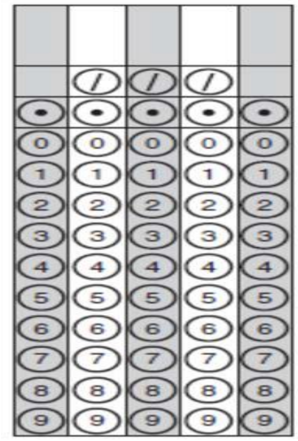
Below you will find practice algebra questions like those you will see on the TASC. There are many more types of questions you will see on the exam, but these represent some of the most common topics on the test.

## I. Solve Linear Equations and Inequalities in One Variable

Example 1

The equation $0.25x - 60 = 40$ can be used to calculate the profit of a lemonade stand given the number of lemonades sold, $x$.

What is the value of $x$?



**Solution to Example 1**

According to the problem, the equation $0.25x - 60 = 40$ can be used to calculate the profit of a lemonade stand. However, since the equation is equal to 40, we already know the profit we are looking for, $40. But, what input value for $x$ will give us an output of 40? How many lemonades would that be? Let's try a guess of 100 lemonades.

First, replace $x$ with 100 in the equation: $0.25(100) - 60 = 40$

Next, evaluate using the order of operations: $25 - 60 = 40$

$-35 = 40$

-35 = 40? Of course not. Selling 100 lemonades does not give us the output of 40 we wanted. We could continue this guess-and-check process and probably find the correct answer eventually. Or, we could use another strategy.

Let's agree on some terms first: **coefficient** **variable** **constant**

$0.25x - 60 = 40$

Remember, an equation is like a balance. Both sides will remain balanced, or equal, as long as we do the same thing to both sides. We can get $x$ by itself by getting rid of the coefficient and the constant. Here it's easiest to deal with the constant, – 60, first. To do so, we can add 60 to both sides. We'll show that both vertically and horizontally.

<u>Vertical Method</u>

$$\begin{array}{rr} 0.25x - 60 = & 40 \\ \underline{+\ 60} & \underline{+60} \\ 0.25x + 0 = & 100 \end{array}$$

$0.25x = 100$

<u>Horizontal Method</u>

$0.25x - 60 + 60 = 40 + 60$

$0.25x + 0 = 100$

$0.25x = 100$

Both methods bring us to $0.25x = 100$. This means that some number of lemonades, $x$, times 0.25 must give us 100. To work backwards from multiplication, we divide. Remember, we must do the same thing to both sides.

$$\frac{0.25x}{0.25} = \frac{100}{0.25}$$

$x = 400$ **The Correct Answer is 400**

Testing Tip

Check Your Algebra! – You can usually check to see that your solution to an algebra problem is correct. In Example 1, we determined that the answer is 400 lemonades. We can make sure this is true by **substituting our answer, $x = 400$, back into the original equation $0.25x - 60 = 40$.**

$0.25x - 60 = 40$
$0.25(400) - 60 = 40$
$100 - 60 = 40$
$40 = 40$ We get a true statement, therefore, **400 is the correct answer.**

**You Try I – Solve Linear Equations and Inequalities in One Variable**

1. The equation \$2.25*m* + \$3.50 = \$21.50 can be used to calculate the cost of riding in a taxi-cab, given the number of miles traveled, *m*, in miles. What is the value of *m*?

   A. 8 miles

   B. 11 miles

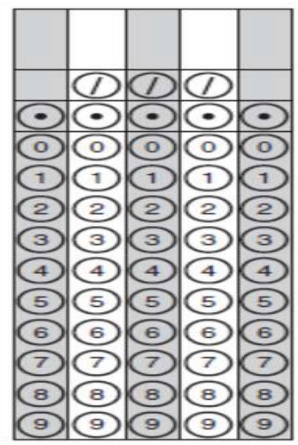
   C. 21.5 miles

   D. 25 miles

2. Consider this equation

$$0.3x - 4.2 = 0.75$$

   What is the value of $x$?



3. Lewis took the following steps to solve the equation $4(x - 1) = 40$.

   Equation: $4(x - 1) = 40$

   Step 1: $4x - 4 = 40$

   Step 2: $4x = 44$

   Solution: $x = 11$

   Which statement explains why the solution following Step 1 is a valid step?

   A. If you add 4 to both sides of an equation, the sides remain equal.

   B. If you subtract 4 from both sides of an equation, the sides remain equal.

   C. If you multiply both sides of an equation by 4, the sides remain equal.

   D. If you divide both sides of an equation by 4, the sides remain equal.

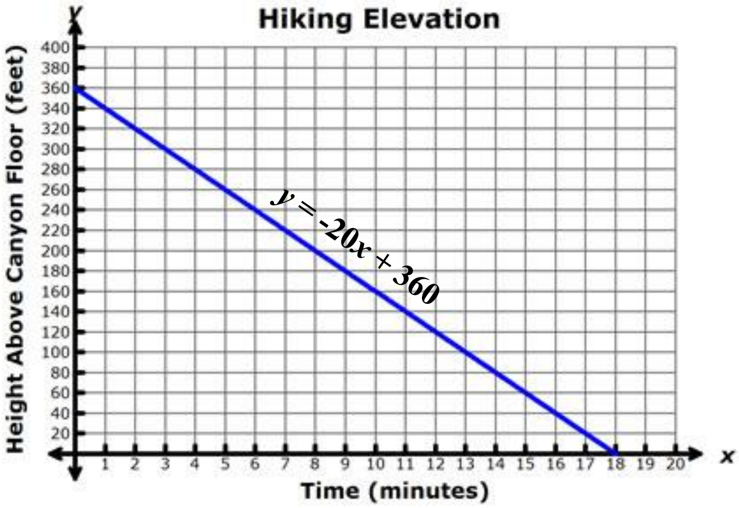
4. Which number is in the solution set of the inequality $5x + 3 > 38$?

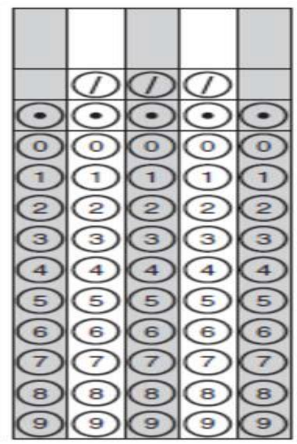
   A. 5
   B. 6
   C. 7
   D. 8

## II. Read Graphs of Equations and Inequalities in Two Variables

Example 2

The progress of Paul's hike at the Grand Canyon is shown below.

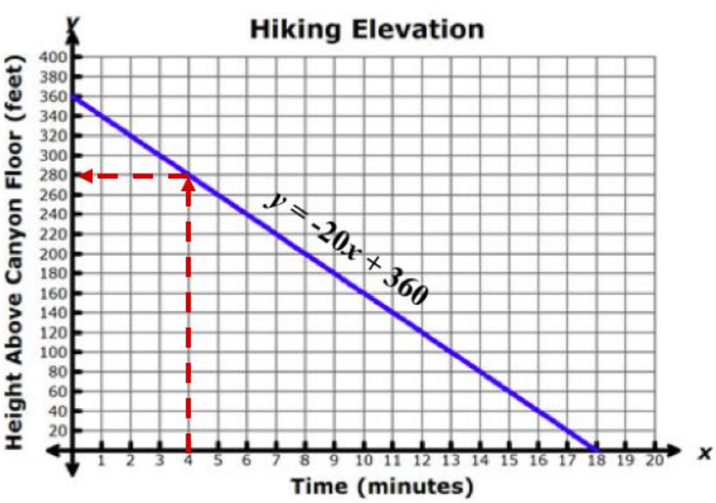


How high above the canyon floor is Paul after 4 minutes of hiking?

**Solving Example 2**

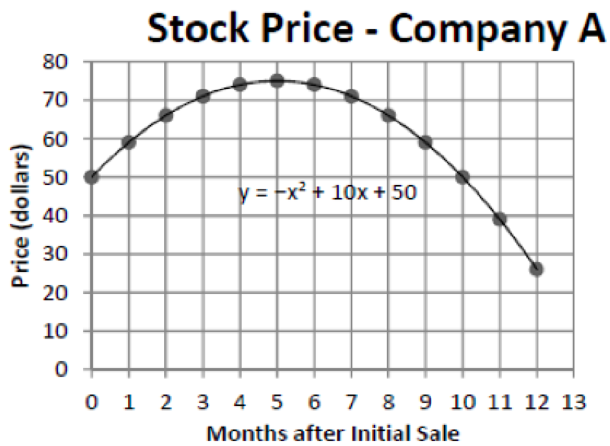
Looking at the graph, we see that the horizontal axis, or $x$-axis, measures the time since Paul started hiking. The time goes from 0 minutes to 18 minutes, so it must take Paul 18 minutes to hike to the canyon floor. The, vertical axis, or $y$-axis, measures Paul's height above the canyon floor. The height goes from a height of 360 feet down to a height of 0 feet, so Paul must hike 360 down to the bottom of the canyon. The question asks, "how high above the canyon floor is Paul after 4 minutes of hiking?" To find out this information, we look along the $x$-axis to 4, and then we follow the vertical line that represents 4-minutes up until we hit the line $y = -20x + 360$. Where we hit the line, we then look left to the $y$-axis to see the measurement we have for *Height Above Canyon Floor (feet)*.




**Here, we find the correct answer, 280 feet.**

**You Try II – Read Graphs of Equations and Inequalities in Two Variables**

5. The stock price of Company A after its initial sale is shown below.




What is Company A's stock price 5 months after its initial sale?

A. \$10

B. \$50

C. \$75

D. \$125

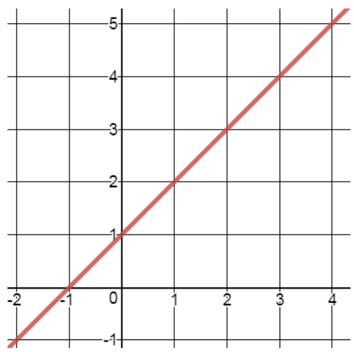
6. Which of the following is NOT a solution to the equation $y = x + 1$ shown by the graph below?

A. (1, 1)

B. (1, 2)

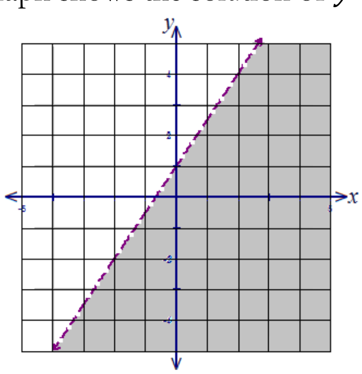
C. (2, 3)

D. (4, 5)

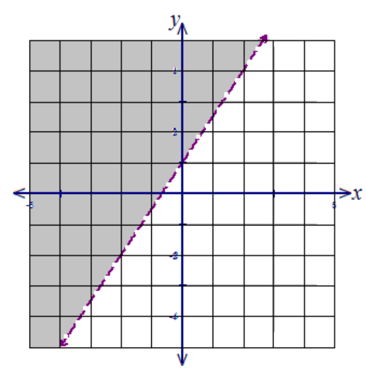


7. Which graph shows the solution of $y \geq \frac{3}{2}x + 1$?

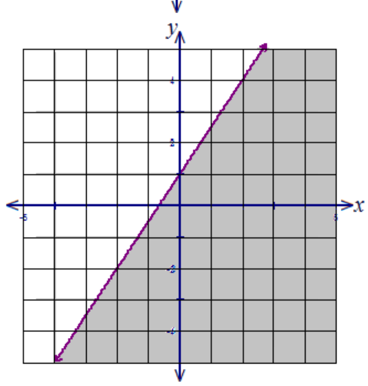
A.



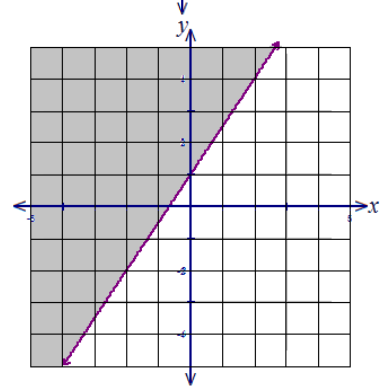
B.



C.



D.



## III. Write Equations and Inequalities in One Variable

Example 3

A car wash pays their four employees a total of $2000 for one day of work. Each of the employees are paid the same amount of money for the day. Which equation can be used to find the pay each employee receives, $p$, for the day?

A. $4p = 2000$

B. $2000p = 4$

C. $p^4 = 2000$

D. $4p = 500$

**Example 3 – Solution**

In the example, we know that there are 4 employees that make a total of $2000 for one day of work. Knowing that all of the employees make $2000 in total, we can write the following simple equation:

*employee 1's pay* + *employee 2's pay* + *employee 3's pay* + *employee 4's pay* = $2000.

However, since they each make the same amount, we can use a variable, $p$, to represent the pay each employee makes. We can replace each pay amount with the variable $p$. Our equation now becomes:

$$p + p + p + p = 2000$$

We have the same amount, $p$, four times, so we can write the equation as:

$4p = 2000$ **The correct answer is Choice A.**

Testing Tip

Many questions do not ask us for a numerical answer. Example 3, for example, asks us for an equation that could be used to model the situation using algebra. However, we can use numbers to check to see that our algebra and the numbers support each other.

In Example 2, we have four employees that earn a total of $2000. Since we know that each employee makes the same amount, that means that each employee makes $500 since $500 + $500 + $500 + $500 = $2000. Even though the question doesn't ask us this, we can still use the $500 in place of the variable to see which answer will give us a true statement. Let's look at each answer choice, but replace the variable, $p$, with $500.

A. $4p = 2000 \longrightarrow 4(500) = 2000 \longrightarrow$ TRUE!

B. $2000p = 4 \longrightarrow 2000(500) = 4 \longrightarrow$ FALSE!

C. $p^4 = 2000 \longrightarrow (500)^4 = 2000 \longrightarrow$ FALSE!

D. $4p = 500 \longrightarrow 4(500) = 500 \longrightarrow$ FALSE!

Here, we can see that Choice A. provides us with the only algebraic equation that holds true using the $500 we can determine that each employee must make.

**You Try III – Write Equations and Inequalities in One Variable**

8. Carlisa has saved $1400 to buy a car. Altogether, she needs $5000. Her plan is to save money each month for one year to save the rest of the money she needs. Which of the following equations could be used to determine how much money per month, $m$, Carlisa needs to save?

   A. $12m + 1400 = 6400$

   B. $1400m + 12 = 5000$

   C. $12m + 1400 = 5000$

   D. $12m - 1400 = 5000$

9. During a sale, a couch that costs $1200 is being sold at a discount for $900. Which equation can be used to find the rate of discount, $d$, during the sale?

   A. $1200 - 1200d = 900$

   B. $1200 - \mathrm{d} = 900$

   C. $1200 - d = 300$

   D. $300 - 300d = 900$

10. Krystal bought $x$ boxes of cookies to bring to a party. Each box contains 12 cookies. She decides to keep two boxes for herself. She brings 60 cookies to the party. Which equation can be used to find the number of boxes, $x$, Krystal bought?

    A. $2x - 12 = 60$

    B. $12x - 2 = 60$

    C. $12x - 24 = 60$

    D. $24 - 12x = 60$

11. Your brother has $3,000 saved for a 10-day vacation. His airplane ticket costs $820. Which of the following inequalities could be used to find the average amount of money he can spend each day on his vacation, $x$?

    A. $10x + 820 \geq 3000$

    B. $10x + 820 \leq 3000$

    C. $10x \leq 3820$

    D. $10x \geq 820$

## IV. Write Equations in Two Variables

Example 4

The graph below shows Josh's drive from college back to home.

Which equation can be used to calculate the distance in miles, $x$, Josh is from home in relation to the number of hours, $y$, that he has been driving?

A. $y = 40x$

B. $y = -40x$

C. $y = -40x + 160$

D. $y = 40x - 160$


**Solution to Example 4**

Looking at the graph showing Josh's Car trip, we see that the data forms a straight line. Whenever we have a straight line on a graph, we know that there is a *linear* equation that can be used to model the straight line. All linear equations can be written in the form:

*Output value = (rate of change)(input value) + starting amount* OR $y = mx + b$

To help us write the linear equation that can model Josh's Car trip, we can make a table of values. To do that, we need to recognize that the graph measures two things: *Time (in hours)*, or $x$, which represents our *input values*; and *Number of Miles from Josh's Home*, or $y$, which represents our *output values*. We can use these variables for the columns of our table. To fill in the table, we need to locate points on the graph. Our table looks like this:

| Time (in hours) | Number of Miles from Josh's Home |
|---|---|
| $x$ | $y$ |
| 0 | 160 |
| 1 | 120 |
| 2 | 80 |
| 3 | 40 |
| 4 | 0 |

Looking down our table, we can see that every time the hours increase by 1 in the $x$-column, the number of miles decreases by 40 in the $y$-column. Therefore, our **rate of change, or $m$, is -40**. Then, to find our *starting amount*, or $b$, we need to look where the input is 0 on the table. When the time is 0 hours, the start, Josh is 160 miles from home. So, our **starting amount**, or ***b***, is 160. We have defined:

Output value = $y$
Rate of change ($m$) = -40
Input value = $x$
Starting amount ($b$) = 160

We can then write our equation: $y = -40x + 160$ **The correct answer is Choice C.**

### You Try IV – Write Equations in Two Variables

12. The results of an experiment testing the effectiveness of a medication in raising the number of antibodies in a sample of blood are shown in the graph at right.

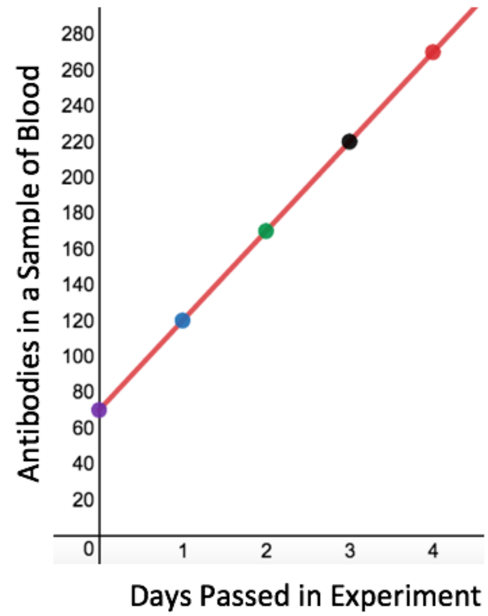
    Which of the following equations correctly models the relationship between *d*, the days that have passed in the experiment, and *a*, the number of antibodies in the sample of blood?

    A. $d = 70a + 50$

    B. $d = 50a + 70$

    C. $a = 70d + 50$

    D. $a = 50d + 70$




13. The graph shows Jerry's hike down Bryce Canyon.

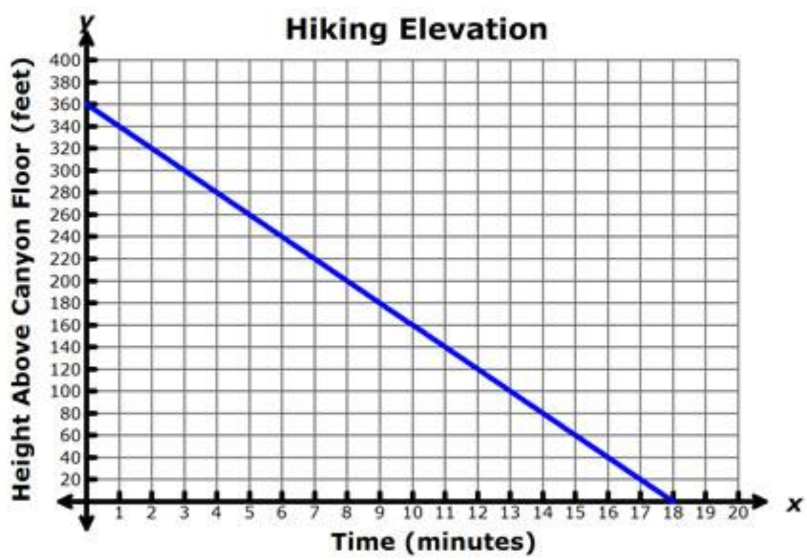
    Which equation can be used to calculate Jerry's height above the canyon floor, *h*, in relation to time in minutes, *t*, since he started hiking?

    A. $h = 18t + 360$

    B. $h = 360 - t$

    C. $h = 20t - 360$

    D. $h = 360 - 20t$




14. The table below shows the cost of riding in a taxi where *m* represents the number of miles driven in the taxi, and *c* represents the total cost in dollars. Which equation below could be used to calculate the total cost of riding in this taxi?

    A. $c = m + 2$

    B. $c = 2m + 3$

    C. $c = 3m + 2$

    D. $c = m + 3$

| Miles Driven | Total Cost in $ |
|---|---|
| *m* | *c* |
| 0 | 2 |
| 1 | 5 |
| 2 | 8 |
| 3 | 11 |

V. **Write a System of Equations**

Example 5

A group of adults and children took a ride on a Ferris Wheel. The ticket collector collected all of their tickets.

- The total number of tickets collected is 28.
- Child tickets cost \$3 each.
- Adult tickets cost \$5 each.
- The total amount of money collected is \$118.

Which system of equations can be used to find the number of child tickets, $x$, and the number of adult tickets, $y$, that the ticket collector collected?

A. $x + y = 78$ and $3x + 5y = 118$

B. $x + y = 8$ and $3x + 5y = 28$

C. $x + y = 28$ and $8x + 5y = 118$

D. $x + y = 28$ and $3x + 5y = 118$

**Solving Example 5**

Here, we have two distinct quantities being measured for the same group of people. There is the total amount of tickets collected, or 28 tickets, and the total amount of money collected, or \$118. However, there is only one group of adults and children riding the ride. That means that the correct number of child and adult tickets must do two things at once – add up to 28 and cost a total of \$118. In cases like these, we can use what's called a *system of linear equations* to find one solution to two problems.

We are given $x$ to stand for the total number of child tickets, and $y$ to stand for the total number of adult tickets. Together, we need a total of 28 tickets. As an equation we can write:

(child tickets) + (adult tickets) = 28 tickets  OR  $x + y = 28$

We also know that \$3 is collected for each child ticket and \$5 is collected for each adult ticket. So, when thinking about the money collected, we must multiply \$3 for each child ticket, $x$, and \$5 for each adult ticket, $y$. Knowing this, we can use $3x$ to represent the total money collected for child tickets and $5y$ to represent the total money collected for adult tickets. Altogether, all of those tickets account for \$118 in ticket sales. Therefore, we can write:

(\$3 for each child ticket) + (\$5 for each adult ticket) = \$118  OR  $3x + 5y = 118$

To solve the problem, we would first write a system of equations that includes both of the equations we have described. Only Choice D has both equations. **Therefore, Choice D is the correct answer.**

Testing Tip

In problems like Example 5, we are not asked to solve the problem, only to write a system of equations that could be used to solve the problem. However, if time allows, you could solve the problem and then look to make sure the algebra you chose matches the situation. One way to do this is to use tables. We could start by using 14 children and 14 adults to make 28 tickets, and then see how much money that would give us to begin.

| Child Tickets $x$ | Adult Tickets $y$ | Total Tickets $= x + y$ |
|---|---|---|
| 14 | 14 | = 14 + 14 = 28 |

| Cost of Child Tickets $3x$ | Cost Adult Tickets $5y$ | Total Money $= 3x + 5y$ |
|---|---|---|
| 3(14) = 42 | 5(14) = 70 | = 42 + 70 = 112 |

With 14 child and 14 adult tickets we have 28 tickets, but only \$112. We want \$118. Let's try adding adults. Remember, we need to keep 28 total tickets. If we increase adult tickets, we need to decrease child tickets.

| Child Tickets $x$ | Adult Tickets $y$ | Total Tickets $= x + y$ |
|---|---|---|
| 13 | 15 | $= 13 + 15 = 28$ |

| Cost of Child Tickets $3x$ | Cost Adult Tickets $5y$ | Total Money $= 3x + 5y$ |
|---|---|---|
| $3(13) = 39$ | $5(15) = 75$ | $= 39 + 75 = 114$ |

We're going in the right direction. Let's keep going till we get \$118.

| Child Tickets $x$ | Adult Tickets $y$ | Total Tickets $= x + y$ |
|---|---|---|
| 12 | 16 | $= 12 + 16 = 28$ |
| 11 | 17 | $= 11 + 17 = 28$ |

| Cost of Child Tickets $3x$ | Cost Adult Tickets $5y$ | Total Money $= 3x + 5y$ |
|---|---|---|
| $3(12) = 36$ | $5(16) = 80$ | $= 36 + 80 = 116$ |
| $3(11) = 33$ | $5(17) = 85$ | $= 33 + 85 = 118$ |

Our tables tell us that 11 child tickets and 17 adult tickets will give us both the 28 total tickets and \$118 we need. We can substitute 11 for $x$ and 17 for $y$ into Choice D to make sure that our algebra does what we think it should.

$x + y = 28$ and $3x + 5y = 118$

$11 + 17 = 28$ √ and $3(11) + 5(17) = 118$ √ Both equations hold true. Choice D must be correct.

**You Try V – Write a System of Equations**

15. Justin, $J$, worked 5 hours longer this week than Andy, $A$. They worked a total of 59 hours. Which system of equations can be used to find how long each worked?

    A. $A = J + 5$ and $J + A = 59$

    B. $J = A + 5$ and $J + A = 59$

    C. $A = J + 5$ and $J - A = 59$

    D. $J - A = 59$ and $J = A + 5$

16. A ribbon 56 centimeters long is cut into two pieces. One of the pieces is three times longer than the other. Which system of equations could be used to find the length of the shortest piece of ribbon in centimeters?

    A. $x + y = 56$ and $3x = y$

    B. $x + 3 = y$ and $3x = 56$

    C. $x + y = 56$ and $y + 3 = x$

    D. $xy = 56$ and $y = 3x$

17. Joey, J, ate 13 more hot dogs than Kobayashi, K. Together they ate 125 hotdogs. Which system of equations could be used to find out how many hot dogs they each ate?

    A. $J + K = 125$ and $J = K - 13$

    B. $K = J + 13$ and $J + K = 125$

    C. $J + K = 125$ and $J = 13$

    D. $J = K + 13$ and $J + K = 125$

## VI. **Interpret Expressions**

Example 6

The equation $D = 95 - 65t$ can be used to represent the distance in miles, $D$, a car must travel given the number of hours travelled, $t$, on a drive from Philadelphia to New York.

Which interpretation of $95 - 65t$ is correct?

A. The car is travelling at a speed of 65 mph and it is 95 miles from Philadelphia to New York.

B. The car is travelling at a speed of 95 mph and it is 65 miles from Philadelphia to New York.

C. It is 30 miles from Philadelphia to New York.

D. It is 160 miles from Philadelphia to New York.

**Solving Example 6**

To get a better sense of the car's trip from Philadelphia to New York we can make a table of values. We know that the equation is set up to find $D$, the distance a car must travel in miles, so this will be our output value. We also know that we need to know $t$, or the hours travelled, to find the distance left to travel, so $t$, will be our input value.

| $t$<br>Hours travelled | $D$<br>Distance left to travel |
|---|---|
| 0 | $95 - 65(0) = 95 - 0 = 95$ miles |
| 1 | $95 - 65(1) = 95 - 65 = 30$ miles |

In the first row of the table, when the car has travelled for 0 hours, the distance left to travel is 95 miles. This means that at the start, before any driving, the distance from Philadelphia to New York is 95 miles. Then, after 1 hour of driving, there is 30 miles left to go. Therefore, the car travelled $95 - 30 = 65$ miles in one hour. That means that the car has travelled at a speed of 65 mph (miles per hour). **The correct answer is Choice A.**

**You Try VI – Interpret Expressions**

18. The formula $C = \pi D$ is used to find the circumference of a circle, where $D$ represents the diameter of a circle. Which interpretation of $\pi D$ is correct?

    A. The circumference of a circle is double pi.

    B. To find the circumference of a circle add pi to the diameter of the circle.

    C. The circumference of a circle is equal to pi times the diameter of the circle.

    D. The circumference of a circle is half the radius of the circle

19. The expression $y + 5(2x + 5) - 25$ can be used to model the steps of a card trick where two playing cards are chosen. Here, $x$ represents the first card chosen, and $y$ represents the second card chosen. Which interpretation of the expression correctly identifies the first steps of the card trick after the first card is chosen?

    A. Double the value of the first card chosen and then add five to that amount.

    B. Multiply the value of the first card chosen by seven and then multiply by five.

    C. Multiply the value of the first card chosen by five and then add five.

    D. Subtract 25 from the total value, and then multiply that amount by five.

20. At the gym, Latrell can burn 6 calories/minute on the treadmill and 5 calories/minute using an exercise bike. If $t$ represents the number of minutes Latrell spends on the treadmill and $b$ represents the number of minutes Latrell spends on the exercise bike, which expression represents the greatest number of calories Latrell can burn at the gym if he only has 45 minutes to spend at the gym?

    A. $6t$

    B. $5b$

    C. $11tb$

    D. $45tb$

## VII. **Identify Equivalent Expressions**

Example 7

An accountant is using the expressions $(4a + 3b - 6c)$ and $(5a + c)$ to calculate the total compensation for employees in two different departments of a business.

What is the sum of the two expressions?

A. $9a - 3b - 7c$

B. $9a + 3b - 5c$

C. $9a + 3b + 5c$

D. $9a - 3b + c$

**Solving Example 7**

A *sum* is the result of addition. For example, 7 is the sum of 3 + 4. In the problem above, we need to find the sum of the two expressions $(4a + 3b - 6c)$ and $(5a + c)$. In other words, we have to add these two expressions together.

| | |
|---|---|
| Step 1 – Write out the problem: | $(4a + 3b - 6c) + (5a + c)$ |
| Step 2 – Remove the parentheses. | $4a + 3b - 6c + 5a + c$ |
| Step 3 – Rearrange according to *like terms**. | $4a + 5a + 3b - 6c + c$ |
| Step 4 – Combine like terms by adding the coefficients and keeping the variable(s). | $9a \quad + 3b \quad - 5c$ |

Written out, our sum is $9a + 3b - 5c$, so **the correct answer is Choice B.**

* *Like terms* are terms with the same variable and the same exponent. For example, $3x$ and $5x$, or $6y^2$ and $10y^2$.

Testing Tip

In Example 7, we are looking for an expression that is equivalent to $(4a + 3b - 6c) + (5a + c)$. Another way to check for equivalence is to use values for our variables. We can use almost any value we like for our variables. Let's say that $a = 2$, $b = 3$, and $c = 4$. You could use other values, but let's try plugging in these.

$$(4a + 3b - 6c) + (5a + c) = (4(2) + 3(3) - 6(4)) + (5(2) + 4)) = (8 + 9 - 24) + (10 + 4) = -7 + 14 = 7$$

Next, we can evaluate each answer choice and look for the expression that gives us the same output of 7 when we use the same values of $a = 2$, $b = 3$, and $c = 4$.

A. $9a - 3b - 7c = 9(2) - 3(3) - 7(4) = 18 - 9 - 21 = -12$

B. $9a + 3b - 5c = 9(2) + 3(3) - 5(4) = 18 + 9 - 20 = 7$

C. $9a + 3b + 5c = 9(2) + 3(3) + 5(4) = 18 + 9 + 20 = 47$

D. $9a - 3b + c = 9(2) - 3(3) + 4 = 18 - 9 + 4 = 13$

Only Choice B. gives us 7, the output we are looking for. The correct answer is Choice B.

**You Try VII – Identify Equivalent Expressions**

21. A store owner is using the expressions $150x + 1000$ and $125x + 500$ to calculate the total cost of daily operations per hour at two different stores where $x$ stands for the hours of operation.

    What is the sum of the two expressions?

    A. $1775x$
    B. $25x + 500$
    C. $275x + 1500$
    D. $275x^2 + 1500$

22. Which expression is equivalent to $(5a + 2b - \mathrm{c}) + (3a - 2b + 6c)$?

    A. $8a + 4b + 7c$
    B. 18abc
    C. $8a + 6c$
    D. $8a + 5c$

23. Simplify $(12x^2 + 8x) \div 4x$

    A. $5x$
    B. $3x + 2$
    C. $8x + 4x$
    D. $3x^2 + 2x$

## VIII. Solve Quadratic Functions Graphically

Example 8

A quadratic function is shown on the graph at right.

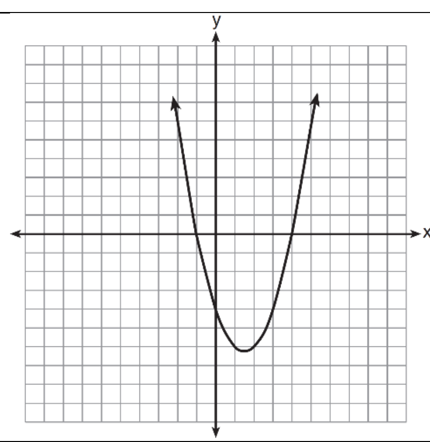
What are the zeros of the function shown on the graph?

A. -4 only

B. -4 and -1

C. -1 and 4

D. -4, -1, and 4



**Solving Example 8**

A "quadratic function" is one in which the *independent variable*, often $x$, has 2 as an exponent, as in $x^2$, which is also called "*x-squared*". For example, $y = x^2 - 3x - 4$ is a quadratic function. When we are asked to find the *zeros* of a function such as $y = x^2 - 3x - 4$, we are actually being asked to find the input values, or $x$-values, that give us an output value of 0. That means we are actually solving the equation $0 = x^2 - 3x - 4$. *Zeros* are also sometimes referred to as *roots* or *solutions*. There are technical differences, but those differences don't really matter here too much.

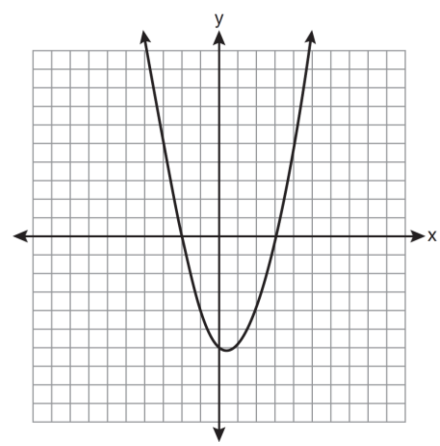
You will notice, though, that our function is only shown as a U-shaped graph called a *parabola*. We are not given a function like $y = x^2 - 3x - 4$. We do know that the *zeros* that we are looking for have an output of 0. That means that we are looking for a value or values of $x$ that will fit the form $(x, 0)$. On a graph, we always find points like these on the $x$-axis. So, to find the *zeros* of the function above, we look to find where the *parabola* meets the $x$-axis. These points are also known as $x$-intercepts. In Example 8, the $x$-intercepts are found at the two points (-1, 0) and (4, 0). **The correct answer is Choice C,** -1 and 4.

**You Try VIII – Solve Quadratic Functions Graphically**

24. A quadratic function is shown on the graph at right.

    What are the zeros of the function shown on the graph?

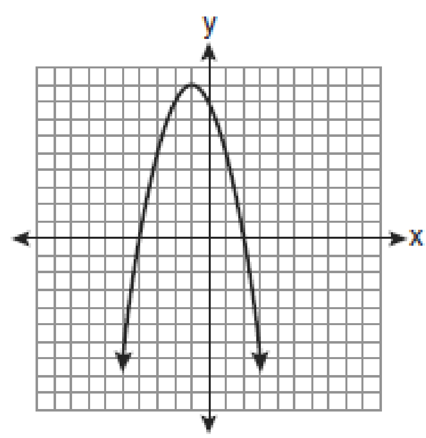
    A. -6 and 3
    B. -6 and 0
    C. -3 and 2
    D. -2 and 3




25. The quadratic equation $y = -x^2 - 2x + 8$ is shown on the graph at right.

    What are the zeros of the function shown on the graph?

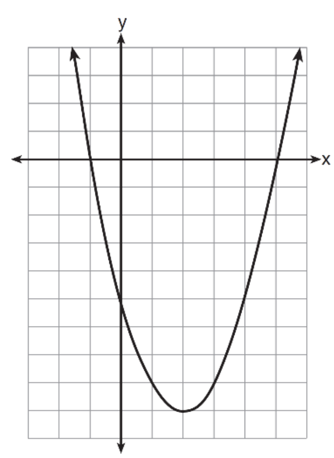
    A. 2 and -4
    B. 8 and 0
    C. 9 and -1
    D. -2 and 4




26. The graph of a quadratic function is shown at right.

    What are the zeros of the function shown on the graph?

    A. -5 and -1
    B. 1 and -5
    C. 5 and -1
    D. 2 and -9



## IX. Solve Quadratic Functions and Equations Algebraically

Example 9

A small rocket is launched from a height of 80 feet above the ground. The height of the rocket in feet, $h$, is represented by the equation $h(t) = -16t^2 + 64t + 80$ where $t$ = time, in seconds. Determine the number of seconds that it will take for the rocket to hit the ground.

A. -1 second

B. 5 seconds

C. 8 seconds

D. 10 seconds

**Solving Example 9**

It bears noting, that the equation $h(t) = -16t^2 + 64t + 80$ is different than most equations we have looked at so far. Here, we have *function notation* with $h(t)$. This means that we will be finding height, $h$, as a function of time, $t$.

We can't "solve" $h(t) = -16t^2 + 64t + 80$ until we have some amount that we want the output, or $h(t)$, to be. The problem asks us to determine how many seconds it will take for the rocket to hit the ground. This means that we want to set $h(t)$ to 0, because the height of the rocket will be 0 feet when it hits the ground. So the first thing we will do is rewrite our quadratic function as an equation like this $0 = -16t^2 + 64t + 80$.

We now need to solve the equation $0 = -16t^2 + 64t + 80$. *Two* of the more common methods for solving a problem like this are factoring or using the quadratic formula. We will show both solution methods below.

**Factoring Method**

$$-16t^2 + 64t + 80 = 0$$

This function has three terms and each term has -16 as a factor. That means we can start by factoring -16 from each term by dividing each term by -16.

$$-16(t^2 - 4t - 5) = 0$$

We then simplify by dividing both sides by -16.

$$\frac{-16(t^2-4t-5)}{-16} = \frac{0}{-16}$$

$$t^2 - 4t - 5 = 0$$

We can then factor our trinomial.

$$(t - 5)(t + 1) = 0$$

Then, find the two solutions that will give us 0.

$$(t - 5) = 0 \quad \text{OR} \quad (t + 1) = 0$$

$$t = 5 \quad \text{OR} \quad t = -1$$

**Quadratic Formula Method**

First, identify the $a$, $b$ and $c$ values for the formula

$$t = \frac{-b \pm \sqrt{b^2-4ac}}{2a}.$$

In $-16t^2 + 64t + 80 = 0$, $a = -16$, $b = 64$, and $c = 80$.

Next, substitute the values for $a$, $b$, and $c$, and evaluate.

$$t = \frac{-64 \pm \sqrt{64^2-4(-16)(80)}}{2(-16)}$$

$$t = \frac{-64 \pm \sqrt{4096+5120}}{-32}$$

$$t = \frac{-64 \pm \sqrt{9216}}{-32}$$

$$t = \frac{-64+96}{-32} \quad \text{OR} \quad t = \frac{-64-96}{-32}$$

$$t = \frac{32}{-32} \quad \text{OR} \quad t = \frac{-160}{-32}$$

$$t = 5 \quad \text{OR} \quad t = -1$$

You will notice, we obtain the same solutions using both methods $t = \{-1, 5\}$. This means that the solution to the quadratic equation is -1 or 5. Indeed, both inputs of -1 or 5 will give us an output of 0. However, when we consider the context of the question we see that $t$ cannot equal -1 since $t$ stands for time in seconds and it is not possible to have -1 seconds here. Therefore, **the correct answer is Choice B. 5 seconds.**

Testing Tip – Use the multiple choice answers to work backwards!

If solving a problem like Example 9 using the above methods is not working for you at test time, you can try plugging in the answers to see which one gives you an output of 0. Remember, though, you can eliminate -1 seconds since there is no such thing as -1 seconds in this context. Let's try and separately plug in the answer choices, 5 seconds, 8 seconds, and 10 seconds in for $t$ in the equation $h(t) = -16t^2 + 64t + 80$.

| 5 seconds | 8 seconds | 10 seconds |
|---|---|---|
| $h(5) = -16(5^2) + 64(5) + 80$ | $h(8) = -16(8^2) + 64(8) + 80$ | $h(10) = -16(10^2) + 64(10) + 80$ |
| $h(5) = -16(25) + 320 + 80$ | $h(8) = -16(64) + 64(8) + 80$ | $h(10) = -16(100) + 640 + 80$ |
| $h(5) = -400 + 320 + 80$ | $h(8) = -1024 + 512 + 80$ | $h(10) = -1600 + 640 + 80$ |
| $h(5) = 0$ | $h(8) = -432$ | $h(10) = -880$ |

Using 5 as our input for $t$ gives us an output of 0. Therefore, **the correct answer is Choice B. 5 seconds.**

**You Try IX – Solve Quadratic Functions and Equations Algebraically**

27. Which equation could be used to find the zeros of $y = x^2 - 3x - 4$?

    A. $(x - 3)(x + 1)$
    B. $(x + 4)(x - 1)$
    C. $(x - 4)(x + 1)$
    D. $(x - 3)(x - 1)$

28. The formula $h(s) = 42s - 12s^2$ can be used to calculate the height of a ball thrown straight up into the air at an initial velocity of 42 feet per second. How many seconds after the ball is thrown, $s$, will it take for the ball to hit the ground?

    A. 3.5
    B. 12
    C. 21
    D. 30

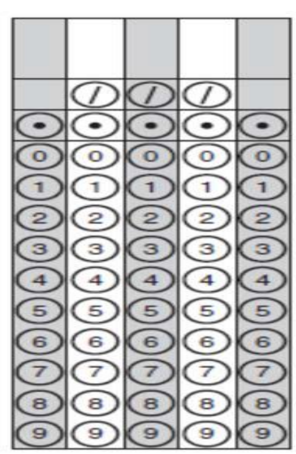
29. Which quadratic equation will have solutions 4 and -6?

    A. $x^2 - 2x - 24 = 0$
    B. $x^2 + 10x + 24 = 0$
    C. $x^2 - 10x + 24 = 0$
    D. $x^2 + 2x - 24 = 0$

30. A student was given the equation $x^2 + 6x - 13 = 0$ to solve by completing the square. The first step that was written is shown below.

    $$x^2 + 6x = 13$$

    The next step in the student's process was $x^2 + 6x + c = 13 + c$. State the value of $c$ that creates a perfect square trinomial.



# Essential Algebra Practice for Students of TASC-math

## Answer Key

**Solutions to: You Try I – Solve Linear Equations and Inequalities in One Variable**

| # | √ | Explanation |
|---|---|---|
| 1 | A | $\$2.25m + \$3.50 = \$21.50$<br>$\$2.25m + \$3.50 – \$3.50 = \$21.50 – \$3.50$<br>$\$2.25m = \$18.00$<br>$\frac{\$2.25m}{\$2.25} = \frac{\$18.00}{\$2.25}$<br>$m = 8$ |
| 2 | 16.5 | $0.3x – 4.2 = 0.75$<br>$0.3x – 4.2 + 4.2 = 0.75 + 4.2$<br>$0.3x = 4.95$<br>$\frac{0.3x}{0.3} = \frac{4.95}{0.3}$<br>$x = 16.5$ |
| 3 | A | On Step 1 we have $4x – 4 = 40$. Following this, to get to the next step, we will add 4 to both sides of the equation like so $4x – 4 + 4 = 40 + 4$, to then get us to $4x = 44$. |
| 4 | D | To solve the inequality $5x + 3 > 38$, we work to isolate the variable $x$ as we would an equation.<br>$5x + 3 – 3 > 38 – 3$<br>$5x > 35$<br>$\frac{5x}{5} > \frac{35}{5}$<br>$x > 7$, therefore our solution set is all numbers greater than 7. The only choice greater than 7 is 8. |

**Solutions to: You Try II – Read Graphs of Equations and Inequalities in Two Variables**

| | | |
|---|---|---|
| 5 | C | Look along the $x$-axis to find 5 "Months after Initial Sale". Then, go up to find the point (5, 75). |
| 6 | A | The point (1, 1) is the only choice <u>not</u> found along the line that is graphed for the equation $y = x + 1$. Therefore (1, 1) is not a solution to this equation. |
| 7 | D | To graph inequalities with two variables, such as $y \geq \frac{3}{2}x + 1$, we show the solution set by graphing a boundary line and shade an area of the coordinate plane according to the following:<br><u>Symbol</u>   <u>Boundary Line</u>   <u>Half-plane (shaded area)</u><br>$y >$   dashed   shade up<br>$y \geq$   solid   shade up<br>$y <$   dashed   shade down<br>$y \leq$   solid   shade down<br>The inequality $y \geq \frac{3}{2}x + 1$ has the ≥ ("greater than or equal to") symbol, so we need a solid boundary line and a shaded area above the line. Only Choice D has this. |

**Solutions to: You Try III – Write Equations and Inequalities in One Variable**

| | | |
|---|---|---|
| 8 | C | Carlisa needs to save $m$ dollars per month for 1 year. This can be modeled by $12m$ since there are 12 months in a year. She can add this amount to the \$1400 she already has saved to get a total of \$5000. We can write this $12m + 1400 = 5000$. |
| 9 | A | full price – (full price)(discount rate) = discount price<br>$1200 – 1200d = 900$ |
| 10 | C | (12 cookies per box)(the number of boxes of cookies) – (two boxes of cookies) = 60 cookies<br>$12x – 24 = 60$ |
| 11 | B | (10 days)(average spending per day) + (airplane cost) must be less than, or equal to, \$3000<br>$10x + 820 \leq 3000$ |

**Solutions to: You Try IV – Write Equations and Inequalities in Two Variables**

<table>
<tr><td>12</td><td>D</td><td>A table of the values shown by the graph could look like this.
<table>
<tr><td>Days Passed (d)</td><td>0</td><td>1</td><td>2</td><td>3</td></tr>
<tr><td>Antibodies (a)</td><td>70</td><td>120</td><td>170</td><td>220</td></tr>
</table>
Rate of change = 50, Starting amount = 70. $a = 50d + 70$</td></tr>
</table>

<table>
<tr><td>13</td><td>D</td><td>A table of the values shown by the graph could look like this.<br><table><tr><td>Time ($t$)</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>Height ($h$)</td><td>360</td><td>340</td><td>320</td><td>300</td></tr></table>Rate of change = -20, Starting amount = 360. $h = -20t + 360$. This is equivalent to $h = 360 - 20t$.</td></tr>
<tr><td>14</td><td>C</td><td>Rate of change = 3, Starting amount = 2. $c = 3m + 2$.</td></tr>
</table>

**Solutions to: You Try V – Write a System of Equations**

| | | |
|---|---|---|
| 15 | B | Andy's work hours + 5 = Justin's work hours and Justin's work hours + Andy's work hours = 59.<br>$A + 5 = J$ and $J + A = 59$ |
| 16 | A | Let $x$ = the shorter piece of ribbon, and let $y$ = the longer piece of ribbon.<br>The shorter piece of ribbon + the longer piece of ribbon = 56 centimeters, or $x + y = 56$.<br>The longer piece is three times longer than the shorter piece, or $3x = y$.<br>$x + y = 56$ and $3x = y$. |
| 17 | D | Joey, J, ate 13 more hot dogs than Kobayashi, K. Together they ate 125 hotdogs.<br>$J = K + 13$ and $J + K = 125$. |

**Solutions to: You Try VI – Interpret Expressions**

| | | |
|---|---|---|
| 18 | C | The circumference of a circle is equal to pi times the diameter of the circle. $C = \pi D$ |
| 19 | A | In the expression $y + 5(2x + 5) - 25$, we would first multiply $x$, the first card chosen, by 2 (or "double"), and then add 5 to that amount. That appears inside the parentheses $(2x + 5)$. |
| 20 | A | Latrell would maximize his workout by only using the treadmill. He burns 6 calories/minute for every minute he uses the treadmill, $t$. The expression $6t$ would represent the greatest number of calories he could burn. |

**Solutions to: You Try VII – Identify Equivalent Expressions**

| | | |
|---|---|---|
| 21 | C | To find the sum of the two expressions, we add $(150x + 1000) + (125x + 500)$. We can rewrite this, $150x + 1000 + 125x + 500$, and then, $150x + 125x + 1000 + 500$. Combining like terms, we get $275x + 1500$. |
| 22 | D | We rewrite, $(5a + 2b - c) + (3a - 2b + 6c)$ as $5a + 2b - c + 3a - 2b + 6c$, and then<br>$5a + 3a + 2b - 2b - c + 6c$. Combining like terms, we get $8a + 5c$. |
| 23 | B | We can rewrite $(12x^2 + 8x) \div 4x$ as $\frac{12x^2}{4x} + \frac{8x}{4x}$. And then, we divide coefficients by coefficients and variables by variables to get $3x + 2$. |

**Solutions to: You Try VIII – Solve Quadratic Functions Graphically**

| | | |
|---|---|---|
| 24 | D | The parabola intercepts the $x$-axis at (-2, 0) and (3, 0). So, the zeros are -2 and 3. |
| 25 | A | The parabola intercepts the $x$-axis at (-4, 0) and (2, 0). So, the zeros are -4 and 2 or 2 and -4. |
| 26 | C | The parabola intercepts the $x$-axis at (-1, 0) and (5, 0). So, the zeros are -1 and 5 or 5 and -1. |

**Solutions to: You Try IX – Solve Quadratic Functions and Equations Algebraically**

| | | |
|---|---|---|
| 27 | C | Factoring the expression $x^2 - 3x - 4$, we get $(x - 4)(x + 1)$. |
| 28 | A | When the ball hits the ground it will be at a height of 0. We set the equation equal to 0, by writing $0 = 42s - 12s^2$. We can then:<br>Factor out $6s$ to get $0 = 6s(7 - 2s)$;<br>Divide both sides by $6s$ to get $0 = 7 - 2s$;<br>Add $2s$ to both sides to get $2s = 7$;<br>Divide both sides by 2 to get $s = 3.5$ seconds. |
| 29 | D | Separately substituting 4 and -6 into each equation we can see that only $x^2 + 2x - 24 = 0$ has both 4 and -6 as solutions.<br>If $x = 4$, then $(4)^2 + 2(4) - 24 = 0$, then $16 + 8 - 24 = 0$ , $0 = 0$ √<br>If $x = -6$, then $(-6)^2 + 2(-6) - 24 = 0$, then $36 - 12 - 24 = 0$, $0 = 0$ √ |
| 30 | 9 | To solve by completing the square, we find $c$ in the equation $x^2 + 6x + c = 13 + c$ by using $c = \left(\frac{6}{2}\right)^2 = 3^2 = 9$. The left side of the equation becomes a perfect square $x^2 + 6x + 9$. |