## TABE 11/12: Concepts in Math

This document is an overview of the content assessed on the TABE forms $11 / 12$ for Level $E, M$, and D . It is based on an analysis of the questions on the actual exams, as well as the TABE 11/12 Blueprints and the TABE 11/12 Transition Toolkit for Mathematics.

Here is a list of the domains covered at each level of the TABE $11 / 12$.

| Content Area Reporting Domains | Covered Domains |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | L | E | M | D | A |
| MATHEMATICS |  |  |  |  |  |
| Measurement and Data | $\checkmark$ | $\checkmark$ | $\checkmark$ |  |  |
| Number and Operations in Base Ten | $\checkmark$ | $\checkmark$ | $\checkmark$ |  |  |
| Number and Operations - Fractions |  | $\checkmark$ | $\checkmark$ |  |  |
| Operations and Algebraic Thinking | $\checkmark$ | $\checkmark$ | $\checkmark$ |  |  |
| Geometry | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| Expressions and Equations |  |  | $\checkmark$ | $\checkmark$ |  |
| Ratios and Proportional Relationships |  |  | $\checkmark$ | $\checkmark$ |  |
| The Number System |  |  | $\checkmark$ | $\checkmark$ |  |
| Statistics and Probability |  |  | $\checkmark$ | $\checkmark$ |  |
| Functions |  |  | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| Algebra |  |  |  | $\checkmark$ | $\checkmark$ |
| Number and Quantity |  |  |  |  | $\checkmark$ |

A few things to notice:

- the range of topics covered on the Level M form of TABE $11 / 12$
- the overlap between Levels $E$ and $M$, and between Levels $M$ and $D$

One other big takeaway is that all of the content strands are assessed on every level of the TABE 11/12. For example, we can see there is algebra on the Level $E$ and $M$ assessed as Operations and Algebraic Thinking.

To begin to help teachers understand the TABE 11/12, we examined each question on the Level $E, M, D$ forms of the TABE $11 / 12$ with two simple questions: (1) What content strand is this?, and (2) What do students need to need to know and be able to do to work on this

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problem? Our answers to the second question were informed by the College and Career Readiness Standards for Adult Education.

We hope this document (and later drafts) will be useful to teachers in at least one of the following ways:

- To see connections between the math taught at all levels and the math on the HSE exam
- To better understand what the strands look like for each level of the TABE $11 / 12$ (i.e. What does geometry look like on Level E? Level M? Level D?)
- To have a clear sense of what they want students to be able to do at the end of a lesson. There is a difference between, "Today, I'm teaching fractions" and "By the end of today's lesson, students will be able to identify equivalent fractions using visual representations".
- To build coherence between lessons by connecting prior understandings, to what students are doing in our class, to what they will need to understand in the future.

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## E LEVEL MATH

## Number \& Operations: 40\%

## Fractions

- Understand fractions represent a whole broken up into some number of equal pieces and we are talking about a certain portion of those pieces. The number of total pieces the whole is broken into is represented by the denominator. The number of equal pieces we are talking about is represented by the numerator.
- Match a shaded visual representation to a fraction (a circle or rectangle divided up into equal pieces).
- Compare fractions with the same denominators (greater than/less than notation).
- Recognize fractions equivalent to one-half $(1 / 2)$.
- Recognize fractions equivalent to one whole (ex: 6/6).
- Add unit fractions with the same denominator $(1 / 4,1 / 6,1 / 8$, are all examples of unit fractions).
- Identify equivalent fractions using visual representations (for example, given a fraction or a picture of a fraction, choose the answer with the visual representation that matches).
- Recognize numerical and written fractions for "thirds", "halves" and "quarters."
- Identify fraction increments on a number line, between O and 1 .


## Place Value

- Model with base ten blocks to understand $10 \mathrm{~s}, 100 \mathrm{~s}$.
- Understand that the three digits of a three-digit number represent the amount of $100 \mathrm{~s}, 10 \mathrm{~s}$ and 1 s .
- Use place value understanding to round three-digit numbers to the nearest ten and to the nearest hundred.
- Use place value understanding to compare three-digit numbers (greater than/less than notation).
- Use place value understanding to identify numbers in written form as when you write a check for "three hundred forty four dollars."


## Addition \& Subtraction

- Use models or drawings like the bar model to represent situations and solve word problems.
- Solve one and two-step word problems with two and/or three-digit numbers.
- Understand the commutative and associative properties of addition.
- Represent addition and subtraction with arrows on the number line.
- Attend to precision in units (not in conversion, but in choosing the correct unit).
- Add and subtract using strategies based on place value (e.g. making groups of ten).


## Multiplication \& Division

- Interpret products of whole numbers, e.g., interpret $5 \times 7$ as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as $5 \times 7$.
- Interpret whole number quotients of whole numbers. e.g. interpret $\frac{56}{8}$ as the number of objects in each share when 56 objects are divided up into 8 shares or as the number of shares when 56 objects are divided into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as $\frac{56}{8}$.
- Understand division as an unknown factor problem - for example $\frac{32}{8}$ can be understood as asking what number we multiply by 8 to get 32 .
- Use visual representations of groups to represent multiplication and division.
- Recognize relationship between multiplication and division.
- Illustrate and explain multiplication calculations using area models (e.g. Match area model pictures to calculations).
- Use "model" and "represent" when discussing methods.
- Understand the commutative, associative and distributive properties of multiplication.
- Skip count by $5 \mathrm{~s}, 10 \mathrm{~s}$, and 100 s .


## Geometry \& Measurement: 10+\%

## Area and Perimeter

- Area and perimeter of squares and rectangles
- Know the difference between area and perimeter in a question that might involve both.

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- Choose a picture of a rectangle with the same area but different perimeter (or the same perimeter but different area) of a given rectangle.
- Given the area of a rectangle, choose the picture that has the matching dimensions that produce that area.
- Understand that area is covering a surface with square units. (For example, given a shaded area within a larger area and being asked to count the number of square units.)
- Composite Areas
- Calculate a composite area made up of squares and rectangles, using the given dimensions and the attributes of squares and rectangles to find any missing and needed dimensions.
- Show ways to break composite areas into smaller rectangles and squares.


## Measurement with Tools

- Measure objects with a ruler with increments of whole numbers, halves, and quarters.
- Measure objects with non-traditional units (i.e. hands, paperclips, pencil lengths, etc.)
- Estimate and have general sense of the length of a centimeter, inch, foot, and meter.
- Estimate length of everyday objects.


## Attributes of Shapes

- Understand how attribute charts are used to identify similarities and differences.
- Know basic definition and features of shapes (number of sides, number of angles, etc.) for squares, rectangles, triangles, quadrilaterals, rhombus.
- Know basic features of a cube (number of faces, edges, etc.)
- Understand associated vocabulary: "polygon," as well as the names of the shapes and attributes mentioned above.


## Algebra/Functions: 22\%

## Operations and Algebraic Thinking

- Identify patterns from a sequence of numbers and (1) name the recursive pattern ("add 6"), or (2) predict the next number.
- Evaluate multiple strategies for solving an operation and choose.


## Expressions and Equations

- Use parentheses in numerical expression to group operations (as with number talks or area models).
- Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example: $36 \square=9$. Which number goes in the box to make the equation true?
- Write, read, match and evaluate expressions with one variable to a given situation.


## Data, Statistics, \& Probability: 28\%

## Graphs

- Match tables and bar graphs that have the same data.
- Construct bar graphs from tables.
- Interpret bar graphs and choose the relevant information to answer questions.
- Interpret line plots.


## Tables and Charts

- Use information in tables and charts to answer word problems. Decide which data in the table is relevant to the problem.


## M LEVEL MATH

## Number \& Operations: 43\%

## Number

- Determine whether a whole number in the range 1-100 is prime or composite.
- Find the greatest common factor of two numbers.


## Fractions

- Recognize equivalent fractions.
- Identify correct bar model for representation of sum of two fractions with the same denominator.
- Interpret word problem and subtract fractions with same denominator.
- Subtract benchmark fractions (halves, quarters, eighths, etc.) from 1.
- Subtract benchmark fractions from each other (e.g. subtraction with eighths and fourths).
- Read a number line made up of fractions with different denominators (like a ruler) but equal divisions (eighths, fourths, and halves).
- Solve word problems by multiplying a whole number by a fraction.
- Divide a unit fraction by a whole number to get another unit fraction.
- Divide fractions.
- Find the area of a rectangle with fractional side lengths. Find the side length of a rectangle given a fractional side length and a fractional area.


## Place Value

- Use understanding of place value to decide which inequality is correct: . 37 < . 38 . Question is given with shaded hundred grids for reference.
- Understand visual models using hundred grids to represent decimals.
- Use understanding of place value to decide if an inequality is correct (e.g. . 391 > .389).
- Compare two decimals to hundredths by reasoning about their size: Which numbers are between 5.43 and 5.45 ?
- Use place value understanding to round multi-digit whole numbers to any place (from hundredths to the ten thousandths place).
- Write a decimal fraction in numbers (e.g. twenty five thousandths $\rightarrow .025$ ).
- Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right: e.g. 700 is 10 times bigger than 70,

7,000 is 100 times bigger than 70, etc. How many times bigger is $300,000,000$ than 3,000 ?

## Addition \& Subtraction

- Represent operations on a number line (addition and subtraction).
- Add two multi-digit numbers.
- Add two numbers with with hundredths in the context of dollars and cents (e.g. \$24.35 + \$35.79).


## Multiplication \& Division

- Interpret a multiplication equation as a comparison, e.g., interpret $35=5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5 .
- Understand that multiplying a number larger than 1 makes the original number bigger.
- Understand that multiplying a number smaller than 1 makes the original number smaller.
- Multiply multi-digit numbers (4 digits by 2 digits).
- Multiply two numbers expressed as prime factors to a power: $2^{3} \times 3^{2}$.
- Find whole-number quotient for four-digit dividend and two-digit divisor: 72,096 $\div$ 24 (presented using long division form).
- Find whole-number quotient for four-digit dividend and one-digit divisor: 72,096 $\div 4$ (presented using long division form).


## Proportional Reasoning

- Understand the concept of unit rate and convert a proportion into a unit rate: 4 pounds for $\$ 5.00$ is equal to $\$ 1.25 /$ pound.


## Algebra/Functions: 27\%

## Operations and Algebraic Thinking

- Determine the rule from a number pattern/sequence: $45,42,39,36, \ldots$ is subtract 3 .
- Choose correct number pattern/sequence from a rule: add 6 , subtract 2.


## Expressions and Equations

- Evaluate expressions with parentheses or brackets: $5+(3 \times 4)$.
- Write expressions to express a relationship: "x multiplied by 4 " is equivalent to " $4 x$."
- Use variables to write an equation representing a real-world situation.
- Choose an expression that matches a given situation with 1 variable.
- Find and create equivalent expressions with two variables using the distributive property.
- Find and create equivalent expressions with two variables by combining like terms
- Evaluate expressions in which letters stand for numbers.
- Looking at an equation with one variable, evaluating which value would make the statement true: e.g. $7 \mathrm{~b}+21=63$, or $5 \times \square=5 / 7$, or $r \div 4=32$.
- Write equation using variables to represent two quantities in a real-world problem: e.g. Mark ( $m$ ) drinks 3 times as much coffee as Eric (e). The equation $m=3 e$ can be used to express the relationship between the two quantities. If Eric drinks 2 cups of coffee, Mark drinks 6 cups..
- Understand the words "expression" and "equation."


## Functions

- Use variables to represent two quantities in an equation that change in relationship to one another (dependent variable and independent variable).
- Understand function machines, including finding an output for a given input and finding an input for a given output.


## Data, Statistics, \& Probability: 5+\%

## Graphs \& Charts

- Interpret a line plot.
- Interpret numerical data in a histogram.
- Create histograms.
- Match data sets and graphs (e.g. Which graph can be used to represent the data?)
- Given a set of data based, identify the correct graph (dot plot or histogram).
- Understand and use the words "interval", "frequency", and "center" in the context of histograms.


## Statistics

- Recognize a statistical question as one that looks for variability in the data related to the question: e.g. "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because you can expect variability in students' ages in the collected data.
- Write statistical questions and use them to collect data.


## Geometry \& Measurement: 10+\%

## Area and Perimeter

- Area
- Calculate the area of a rectangle with fractional side lengths.
- Calculate the area of a rectangle with side lengths in decimal fractions.
- Volume
- Use visual representations of rectangular prisms made up of unit cubes.
- Measure volume in a rectangular prism by counting unit cubes or multiplying length, width and height.
- Understand that the volume of a rectangular prism is its Area multiplied by its height.
- Given the volume of a rectangular prism and the dimensions of a section of that prism, calculate the volume of the remaining section.
- Angles
- Understanding that angles are the opening between lines and that opening is measured with degrees.
- Understand that a one-degree angle is 1 out of $360^{\circ}$ (similar to a unit fraction).
- Measure angles in whole-number degrees using a protractor (protractor necessary).
- Estimate and have a sense of benchmark angles: $90^{\circ}, 30^{\circ}, 60^{\circ}, 180^{\circ}$.
- Find measure of a missing angle if a neighboring angle is known and the angle measure of the whole is known. Understand how to read angle notation: $\angle A B C$.
- Coordinate grid
- Understand how to place and move ordered pairs as points on a coordinate grid.
- Understand the words "origin", "x coordinate", "y coordinate", and "ordered pair."


## Measurement with Tools

- Add a measurement in inches to a measurement in feet and give an answer in feet and inches: e.g. 28 inches +1 foot $=3$ feet 4 inches.
- Attend to precision in units.
- Find quotients to real world problems in which the remainder must be interpreted as fraction of a foot.
- Convert between meters to centimeters, inches and feet.


## Attributes of Shapes

- Look at diagrams of shapes and lines and determine which statement is true.
- Identify rays, parallel and perpendicular line segments in a figure (rhombus). Know difference between a line and a line segment.
- Understand the attributes of two-dimensional figures such as rectangles and rhombuses.
- Understand the definitions of "parallel lines", "perpendicular lines", "rays", "line segments."
- Identify a point at the intersection of two lines.
- Identify a three-dimensional figure by picture of its net (2-dimensional "unfolded" object).
- Recognizing the three-dimensional visual representations of cones, rectangular prisms, triangular prisms and pyramids.


## D LEVEL MATH

## Number \& Operations: 31\%

## Number

- Solve multi-step problem involving multiplication and subtraction of decimal amounts (dollars and cents).
- Recognize the decimal equivalents for a half, quarters and eighths.
- Find quotient to a division problem, involving signed numbers, in which the remainder is represented as a fraction.
- Calculate and recognize percent increase and decrease using benchmarks: $1 / 2-50 \%, 1 / 4$ $-25 \%, 3 / 4-75 \%, 1 / 5-20 \%, \frac{1}{10}-10 \%$.


## Exponents

- Evaluate square roots of small perfect squares: 9, 16, 25, etc.
- Recognize square and cube root notation.
- Understand the general case that the square root of $x^{2}$ is $x$.
- Understand relationship between $\mathrm{x}^{2}$ and $\sqrt[2]{x}$ and $\mathrm{x}^{3}$ and $\sqrt[3]{x}$.
- Use exponent rules to multiply and divide a base number raised to different powers: e.g. $\left(3^{3} \times 3^{4}\right) /\left(3^{2} \times 3^{2}\right)$.
- Estimate approximate values of non-perfect squares (e.g. The $\sqrt{32}$ is between what two whole numbers?). Place approximations of irrational numbers as square roots on a number line: $\sqrt{7}, \sqrt{10}, \sqrt{15}$, etc.
- Starting with a number in scientific notation, multiply or divide by a whole number to find a quotient also expressed in scientific notation.
- Also see Triangles in the Geometry section below.


## Signed Numbers

- Create horizontal and vertical number lines with positive and negative numbers.
- Use a number line to show positive numbers are larger than negative numbers.
- Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line (e.g. 16 is 16 units larger than zero, -16 is 16 units less than 0 ).
- Represent operations with signed numbers on a number line (addition and subtraction).


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- Understand that positive and negative numbers are used together to describe quantities having opposite directions or values: e.g., temperature above/below zero, elevation above/below sea level, credits/debits.
- Represent the absolute value of a signed number on a number line.
- Multiply and divide signed numbers.
- Calculate the final result after a sequence of adding positive and negative numbers.


## Fractions

- Find quotient to a division problem in which the remainder is represented as a fraction.
- Compute unit rates associated with ratios of fractions (e.g. if a recipe calls for $51 / 2$ cups of an ingredient to make $123 / 8$ of something, how much could you make with 1 cup of the ingredient?


## Proportional Reasoning

- Given a ratio, build a proportion table of equivalent ratios.
- Determine whether a relationship between quantities shown in a table are proportional. Recognize and calculate proportional relationships between quantities.
- Understand the concept of unit rate and convert a proportion into a unit rate: 4 pounds for $\$ 5.00$ is equal to $\$ 1.25 /$ pound.
- Compute unit rates associated with ratios of fractions: e.g. if a recipe calls for $51 / 2$ cups of an ingredient to make $123 / 8$ of something, how much could you make with 1 cup of the ingredient?
- Create scale drawings of a geometric figure. Solve problems using the scale including computing actual lengths from a scale drawing.
- Use proportional relationship to convert from one unit rate to another: 2 feet per second is what mph?


## Algebra/Functions: 29\%

## Expressions and Equations

- Use variables to write an equation representing a real-world situation.
- Looking at an equation with a variable, evaluating which value would make the statement true.
- Find and create equivalent expressions with two variables using the distributive property.
- Find and create equivalent expressions with two variables by combining like terms.
- Find a solution to a system of linear equations by looking at the intersection of lines on a graph.
- Identify the correct inequality to represent a real-world situation.


## Functions

- Create any of these representations of a function (situation, rule, equation, table, or graph) if given one (or more) of the representations.
- Create a function equation to model a linear relationship between two quantities in a real-world context.
- Determine the rate of change and starting amount of a function from a table of $(x, y)$ values.
- Determine the rate of change and initial value of the function from a description of a relationship.
- Understand words "rate of change", "starting amount", "slope", "y-intercept."
- Identify the equation of a linear function among equations of nonlinear functions.
- Recognize whether a function is linear or nonlinear based on a table of values.
- Describe the functional relationship between two quantities by analyzing a graph (for example, where the function is increasing or decreasing, linear or nonlinear).
- Interpret the functional relationship represented in an ordered pair within the graph of a line: What is the significance of the point $(3,5)$ on the graph?
- Choose graphs that represent a function. More than one value of y for a given x value means that the graph is not a function.
- By observing the graph, identify features of a function - is it increasing or decreasing?, is it linear or nonlinear?
- Identify the graph of a linear function among graphs of nonlinear functions.
- Identify the correct graph of a linear function among graphs of other linear functions.
- Recognize slope as rate of change in the graph of a function.
- Comfort with functions in both real-world and abstract contexts.
- Identify the correct inequality to represent a real-world situation.


## Data, Statistics, \& Probability: 22\%

## Graphs \& Charts

- Understand how to read a scatter plot.
- Determine a line of best fit for a scatter plot of data.
- Based on a scatterplot of scientific data, use an equation for line of best fit to solve problems, interpreting the slope and intercepts ( $x$ and $y$ ) in a real-world context.
- Interpret scatter plots to determine association and see patterns such as positive/negative, linear/nonlinear, association/correlation, clustering and outliers.
- Interpret box-and-whiskers plots.


## Statistics

- Choose which measure of central tendency (mean, median, mode, range) is most appropriate for a given set of sample data.
- Interpret tables to evaluate which of several possible conclusions can be drawn.
- Interpret 2-way frequency tables.
- Analyze data using interquartile range.


## Probability

- Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood.
- Find probabilities of an event using organized lists, tables, or tree diagrams. Represent all possible outcomes in the sample space of the experiment.
- Determine probability for an event based on previous data of previous results. Use data from a survey to draw inferences. Based on past experience, what is the best prediction of an outcome?
- Understand percentage as an expression of probability (favorable outcomes divided by total outcomes). Calculate best guess of outcome using percentage. OR Which model (dice, spinner) represents the probability of the situation?
- Find probabilities of compound events. Compound probability is equal to the probability of the first event multiplied by the probability of the second event.


## Geometry \& Measurement: 18\%

## Area and Perimeter

- Area
- Determine the area of a rectangular space while removing the area of smaller rectangles within the space.
- Perimeter

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- Write an equation to find an unknown side length given the perimeter of a polygon.
- Write equation to find the perimeter of a polygon after changing the dimensions: i.e. If we triple the perimeter, write an equation to find the perimeter of the new polygon.
- Triangles
- Apply the Pythagorean Theorem to determine unknown side lengths in right triangles.
- Apply the Pythagorean Theorem to determine unknown side length in right triangles, then find perimeter.
- Looking at a triangle on a coordinate grid, estimate the perimeter using the approximate length of hypotenuse using a non-perfect square. For example, if the hypotenuse of a right triangle has a length of $\sqrt{52}$ units, we can estimate the perimeter by recognizing that length is between 7 and 8 .
- On a coordinate grid, find the length between two points by applying the Pythagorean Theorem. Recognize the length can be represented as a range or left in square root form. If the length of the line is $\sqrt{145}$, it can either be left in that form or expressed as being between 12 and 13 .
- Also see Exponents in the Number section above.
- Angles
- Find complementary and supplementary angles, given then complements or supplements.
- Use facts about supplementary and complementary angles in a multi-step problem to write equations for an unknown angle.
- Circles
- Know the formulas for the area and circumference of a circle and use them to solve problems.
- Coordinate grid
- Plot points on a coordinate grid in order to create a quadrilateral.
- Find distance between points on a coordinate grid, following straight vertical and horizontal path (not distance formula, which is listed in the Triangle section).
- Reflect a point across the $y$-axis (no coordinate grid given).
- Identify transformation that can be used to turn one figure into another on a coordinate grid (translation, rotation, reflection, and dilation).
- Identify two transformations that can be used to turn one figure into another on a coordinate grid (translation, rotation, reflection, and dilation).


## NEXT STEPS

The CUNY Framework Posts section of CollectEdNY.org has hundreds of lesson plans, problems, videos, and activities for use in the adult numeracy and high school equivalency math classroom. In the coming months, additional materials will be added which focus especially on developing some of the foundational concepts that student will need in algebra, functions, geometry, number and statistics and probability to develop a deeper understanding of the math they need for the HSE exam as well as to find success on the TABE 11/12.

This document and later versions will be available in the CUNY Framework Posts section of CollectEdNY.org.

