Rigid Transformations: Shapes on a Plane

Fast Track Math GRASP Packet

Part 1

Detail of tile work from the Alhambra Palace in Granada, Spain

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Welcome!

Congratulations on deciding to continue your studies! We are happy to share this study packet on the topic of rigid transformations. We hope that these materials are helpful in your efforts to earn your high school equivalency diploma. This group of math study packets will cover mathematics topics that we often see on high school equivalency exams. If you study these topics carefully, while also practicing other basic math skills, you will increase your chances of passing the exam.

Please take your time as you go through the packet. You will find plenty of practice here, but it's useful to make extra notes for yourself to help you remember. You will probably want to have a separate notebook where you can recopy problems, write questions and include information that you want to remember. Writing is thinking and will help you learn the math.

After each section, you will find an answer key. Try to answer all the questions and then look at the answer key. It's not cheating to look at the answer key, but do your best on your own first. If you find that you got the right answer, congratulations! If you didn't, it's okay. This is how we learn. Look back and try to understand the reason for the answer. Please read the answer key even if you feel confident. We added some extra explanation and examples that may be helpful. If you see a word that you don't understand, try looking at the *Vocabulary Review* at the end of the packet.

We also hope you will share what you learn with your friends and family. If you find something interesting here, tell someone about it! If you find a section challenging, look for support. If you are in a class, talk to your teacher and your classmates. If you are studying on your own, talk to people you know or try searching for a phrase online. Your local library should have information about adult education classes or other support. You can also find classes listed here: **http://www.acces.nysed.gov/hse/hse-prep-programs-maps**

You are doing a wonderful thing by investing in your own education right now. You have our utmost respect for continuing to learn as an adult.

Please feel free to contact us with questions or suggestions.

Best of luck!

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Vocabulary

It is important to understand mathematical words when you are learning new topics. The following vocabulary will be used a lot in this study packet:

slide ● turn ● flip ● congruent ● reflect ● rotate ● translate ● transformation

In this first activity, you will think about each word and decide how familiar you are with it. To start, think about the word “slide .” Which of these statements is true for you?

- I know the word “slide” and use it in conversation or writing.
- I know the word “slide ,” but I don’t use it.
- I have heard the word “slide ,” but I’m not sure what it means.
- I have never heard the word “slide ” at all.

In the chart on the next page, read each word and then choose one of the four categories and mark your answer with a ✔ (checkmark). Then write your best guess at the meaning of the word in the right column. If it’s easier, you can also just use the word in a sentence.

Here’s an example of how the row for “slide” might look when you’re done:

Word	I know the word and use the word	I know the word but don’t use it	I have heard the word, but I’m not sure what it means	I have never heard the word	My best guess at the meaning of the word (or use the word in a sentence)
slide	✓				To move slowly over a surface while continuing to touch it

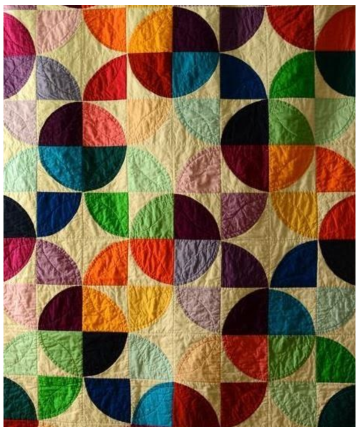
Complete the table on the next page.

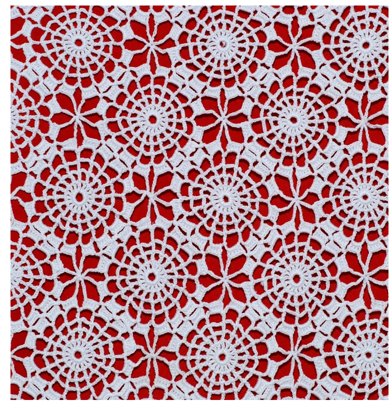
Word	I know the word and use the word	I know the word but don't use it	I have heard the word, but I'm not sure what it means	I have never heard the word	My best guess at what the word means (or use the word in a sentence)
slide					
turn					
flip					
congruent					
reflect					
rotate					
translate					
transformation					

Introduction

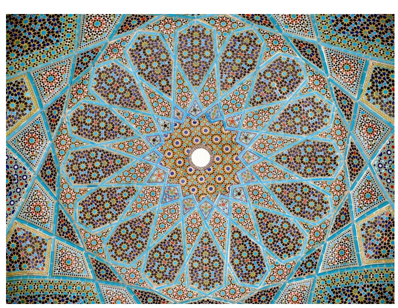
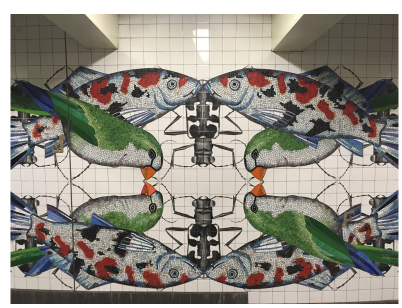
In this packet we are going to be working on a topic in geometry called rigid transformations.

We see rigid transformations everywhere. Nature, art, design, architecture, textiles, clothing, furniture, computer animation, advertisements, logos. and video games are just some of the places where we see rigid transformations everyday.







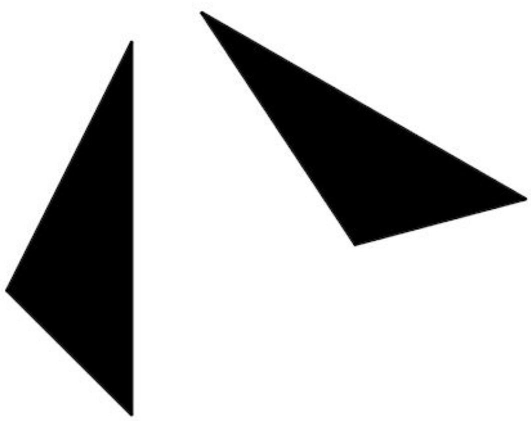
In everyday language, “rigid” and “transformation” might seem like a strange pair of words to put together. “Rigid” means “stiff, not bending or moving, very difficult to change.” On the other hand, “transformation” means to change into something else—for example, a caterpillar transforms into a butterfly.

In math, a transformation still means a change. But with rigid transformations, only some things are changing. With rigid transformations, we change the position of shapes, but we keep their size and shape the same.

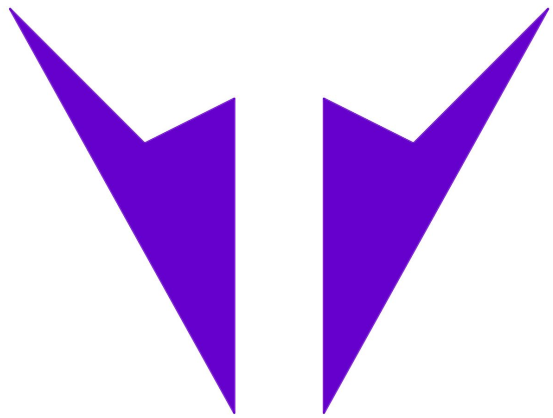
In this packet you will learn about the three types of rigid transformation and how they work. You’ll also learn about the coordinate grid and how it can be used to describe rigid transformations. You will have the opportunity to practice the kinds of questions you might see on the TASC dealing with rigid transformations. You will also have a chance to use rigid transformation to identify and even create designs and patterns.

Congruence and Rigid Transformations

The mathematics of rigid transformations begins with two or more shapes being congruent to each other. But what does it mean for two shapes to be congruent to each other? Here are three examples of shapes that are congruent to each other.



These are congruent shapes.

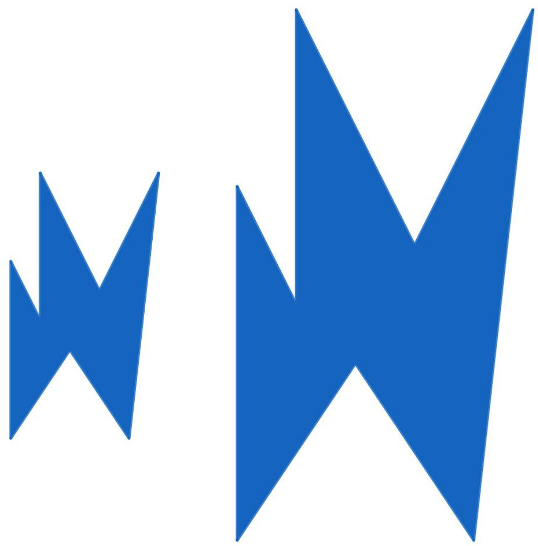


These are congruent shapes.

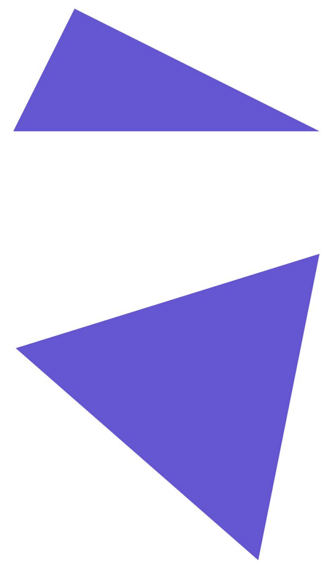
These are congruent shapes.

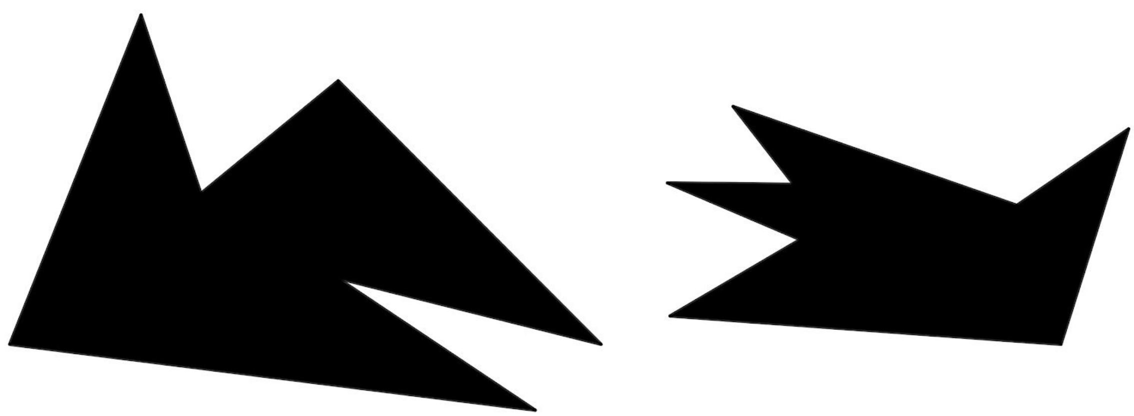
Here are three examples of shapes that are not congruent to each other:

These are not congruent shapes.



These are not congruent shapes.





These are not congruent shapes.

I think congruent shapes are shapes that ____________________________________

____

To help you think more about the definition of congruence, use the congruent and not congruent shapes to fill in this chart.

1) Based on the examples provided, circle whether the description is sometimes true, always true, or never true.

   For example:

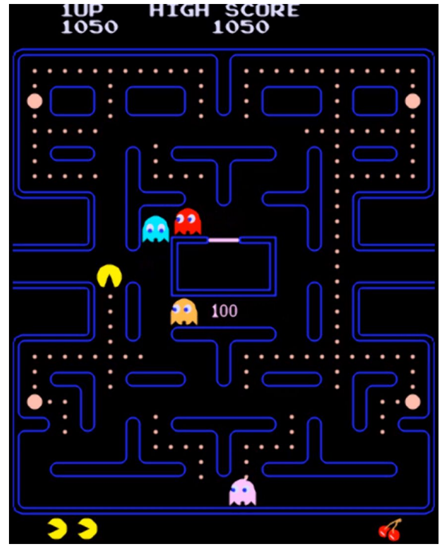
   Shapes that are congruent Sometimes / Always / Never have the same number of sides.

Two shapes that are congruent to each other...	**Two shapes that are not congruent to each other...**
...sometimes / always / never have the same number of sides.	**...sometimes / always / never** have the same number of sides.
...sometimes / always / never have the same shape.	**...sometimes / always / never** have the same shape.
...sometimes / always / never are the same size.	**...sometimes / always / never** are the same size.
...can **sometimes / always / never** fit exactly over each other.	...can **sometimes / always / never** fit exactly over each other.

We can see that if two shapes are congruent to each other, they always have the same number of sides, but that is not enough for a definition. Two shapes that are not congruent to each other can still sometimes have the same number of sides.

If we put the different characteristics together we can make a definition that is true for all of congruent shapes and not true for the shapes that were not congruent.

Key idea: Congruent shapes are the same size and have the same shape. Two shapes are congruent if you can move one so that it fits exactly over the other one.



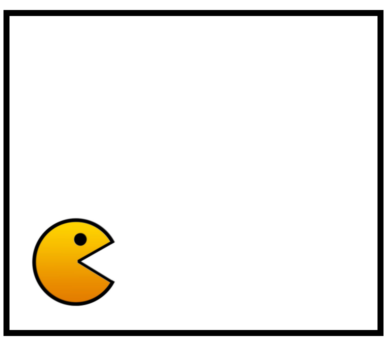
Rigid transformations are rules for <u>moving shapes into different positions *without changing their shape or their size.*</u> We call them *rigid* because they do not change their size or shape but we describe them as *transforming* because they do change their position.

One of the most influential video games of all time was released in 1980. It was called Pac-Man. In 1981, an improved version was developed called Ms. Pac-Man. In both versions of the game, players move a character around a board as he/she ate power pellets and tried to avoid ghosts.

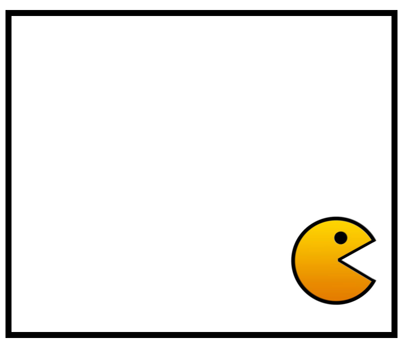
The computer programmers who wrote Pac-Man and Ms. Pac-Man used rigid transformations to move the characters on the video screen.

There are three types of rigid transformations. In other words, there are 3 ways we can change the position of a shape without changing its size or shape.

Let's look at the three ways that Pac-Man was programmed to move.

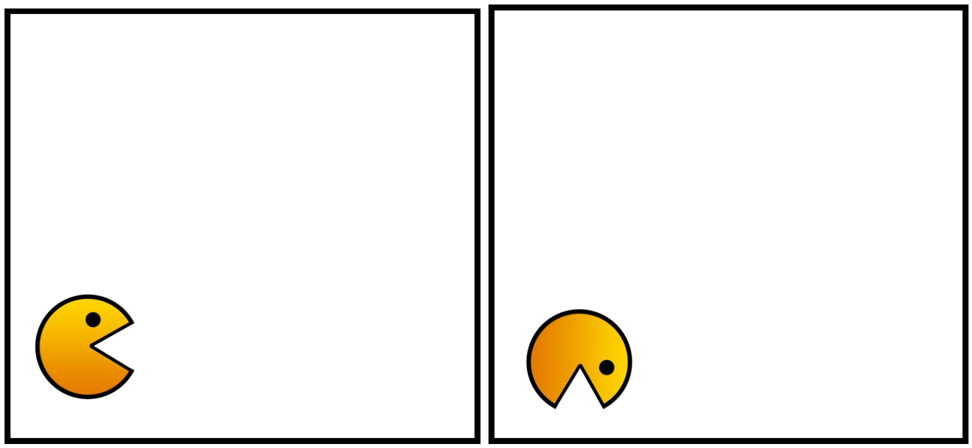


Frame 1



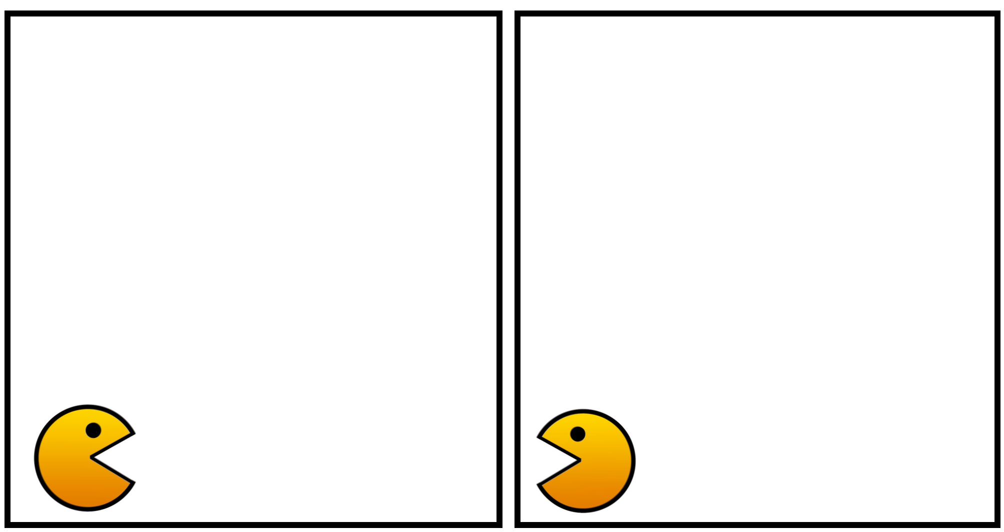
Frame 2

2) What language would you use to describe how Pac Man moved from frame 1 to frame 2?
    a) Slide
    b) Turn
    c) Flip



Frame 1 **Frame 2**

3) What language would you use to describe how Pac Man moved from frame 1 to frame 2?
    a) Slide
    b) Turn
    c) Flip

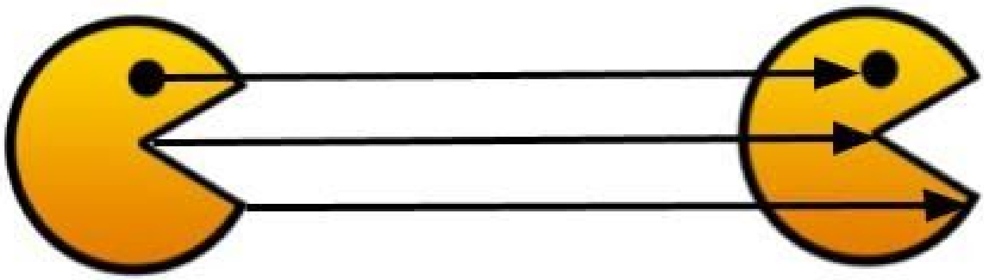


Frame 1 **Frame 2**

4) What language would you use to describe how Pac Man moved from frame 1 to frame 2?
    a) Slide
    b) Turn
    c) Flip

Translation

The first type of rigid movement is a slide.



Original Figure **Image or Translated Image**

You can see this translation by tracing the Original Figure of Pac-Man and then sliding it along a straight line without turning it. When we slide a figure all points in that figure slide the same distance in the same direction. That is what produces the congruent image. If you cut out the translated image, and slide it over, you would see that the image fits exactly over the original Pac-Man

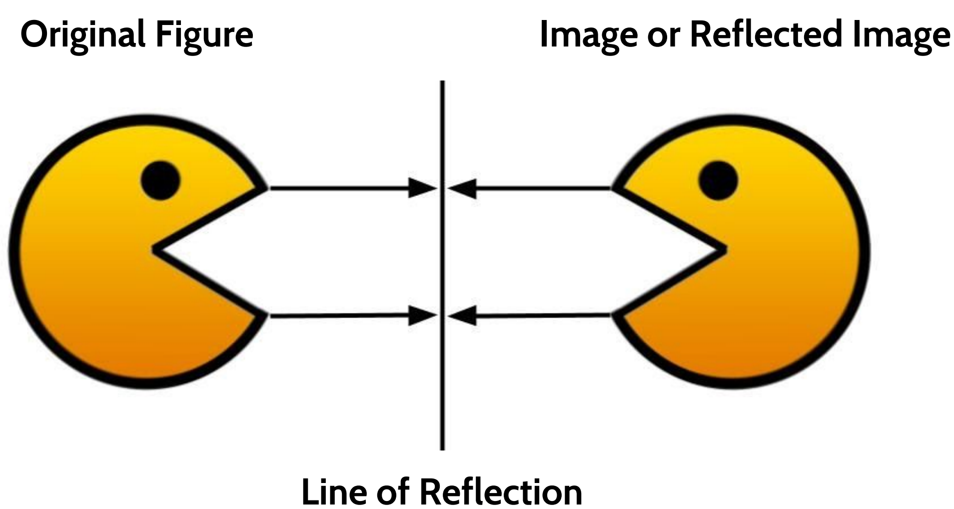
Key Idea: The rigid transformation of sliding a figure is called **translation**.

Reflection

The second type of rigid transformation is the flip. This is the rigid transformation you probably see every morning when you are brushing your teeth. This type of rigid transformation produces a “mirror image”.

Key Idea: The rigid transformation of flipping a figure is called **reflection.**

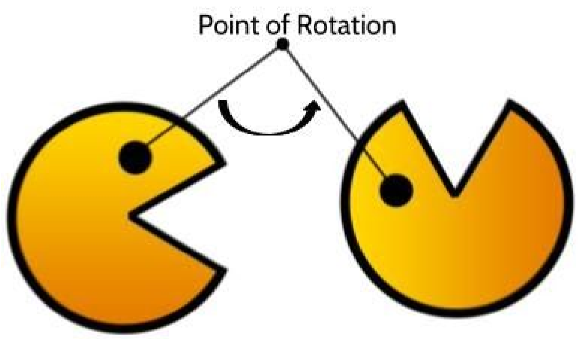
In geometry, when we reflect figures we use a **line of reflection**, which is the halfway point between the original figure and its reflected image. If you fold this page at the line of reflection, you will see that the reflected image fits exactly over the original Pac-Man.



Rotation

The third type of rigid transformation is a turn. Here we are turning Pac-Man around the fixed point, which we call the point of rotation. When we turn a figure, we turn, or rotate all the points in that figure the same number of degrees around the point of rotation. If you drive, when you make a turn, the fixed point is the spot outside the car that your body moves towards as you turn the car.

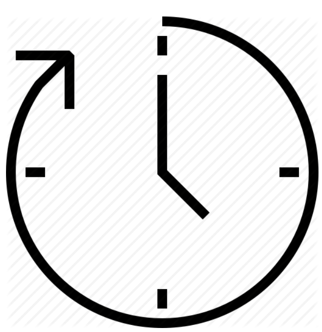
Original Figure **Image or Rotated Image**



Key Idea: The rigid transformation of turning a figure is called **rotation.**

A rotation requires three things: (1) the number of degrees the figure is turned, (2) the direction of the turn (is it a clockwise or counterclockwise turn), and (3) a point of rotation. The Pac-Man above was rotated 90° counterclockwise around the point of rotation.

Note: When we talk about the point of rotation, we can say "a rotation *around* the point of rotation." We can also say the figure was rotated *about* the point of rotation. You will encounter both ways of saying it on your high school equivalency exam and you will have a chance to practice both as you work through this packet.



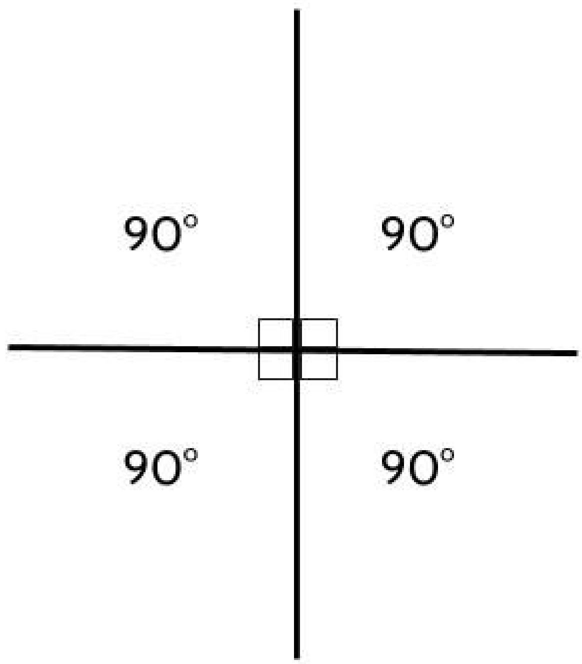
Clockwise turns move in the same direction that the hands move around a clock. We open most doorknobs by turning them clockwise. We screw in most light bulbs by turning them clockwise.

Counterclockwise turns move in the opposite direction from clockwise. When we unscrew a light bulb, we turn it counterclockwise.

In this packet, we will focus on turns of 90 degrees, 180 degrees and 270 degrees.

The missing number in all of this is 360°. If you watch competitive skiing or skateboarding you have probably heard the announcers refer to an athlete doing a 360°. That means the turned their bodies in a full circle.

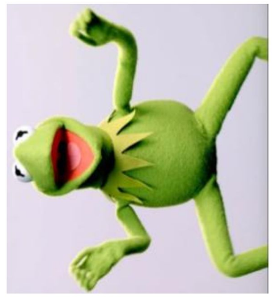
If turning in a full circle is 360°, we can imagine dividing that full-circle turn into four 90° turns. Two of those 90° turns is 180°. Three of those 90° is 270°. Four of those turns is 360°



Stretch your left hand out straight in front of you. Stretch your right arm straight out from your right side. Line your head up with the arm in front of you and turn until your head is lined up with the arm out at your side. You just turned your head 90 degrees (also written as 90°).

Now stretch both arms straight out from your sides. Line your head up with one arm and turn it until it is lined up with the other arm. You just turned your head 180°. Notice that your head is facing in the opposite direction from where you started.

Three rigid transformations are shown below. Label the **reflection**, the **translation**, and the **rotation**. Explain how you know.

5)  

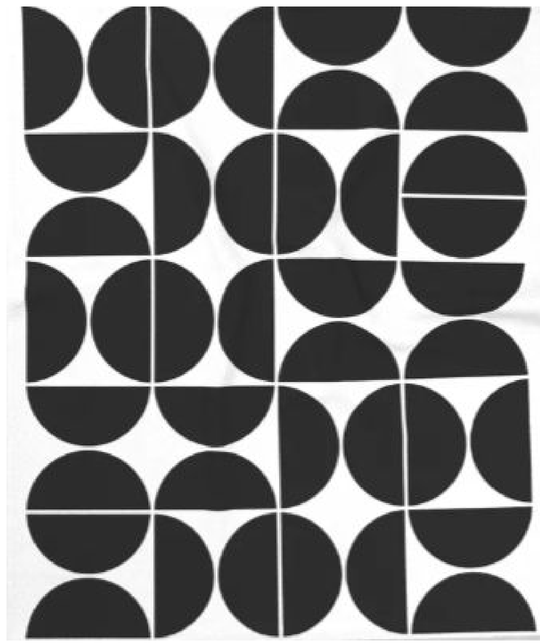
6)  

7)  

Textile Design with Rigid Transformations

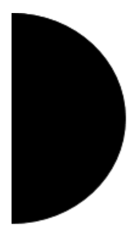
Rigid transformations are often used in the design and production of textiles like blankets and rugs.

Here is a blanket designed using rigid transformations of geometric shapes.



Designs often begin with a *core element*. Rigid transformations are then used to create congruent copies of that core element.

The core element in the blanket above is a half circle.

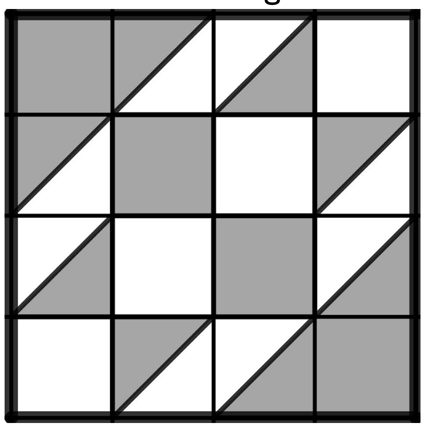


Rigid transformations and congruent images can take a single shape and turn it into something beautiful.

8) Find at least five examples of rigid transformations of that core element in the blanket and label them on the picture above.

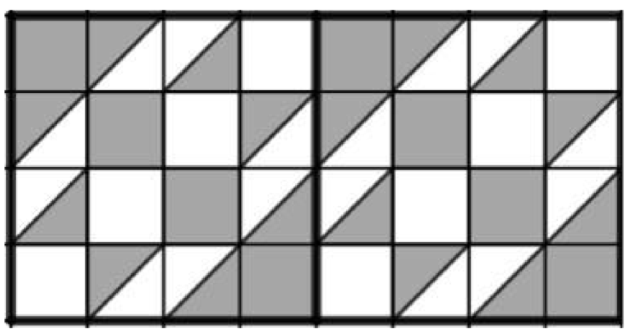
How does the process work?

We start with a design of a core element..

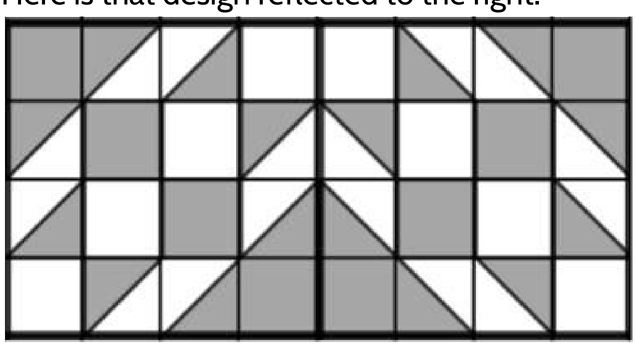


We can use rigid transformations to build off that design.

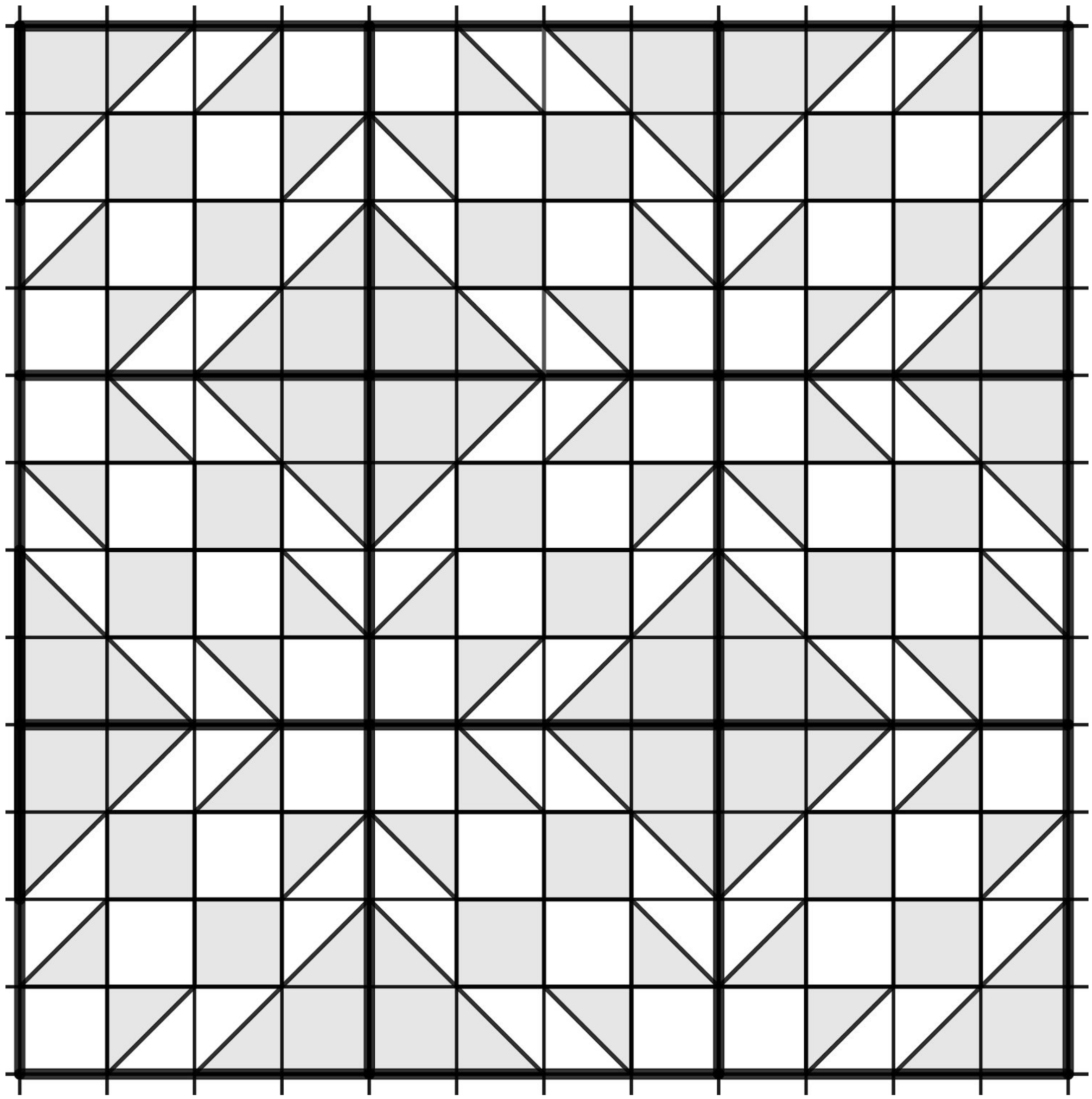
Here is that design translated to the right



Here is that design reflected to the right.



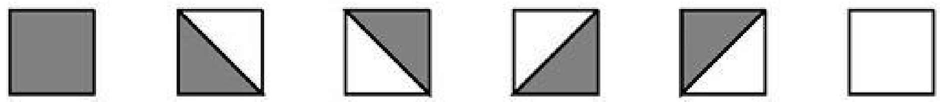
Here is an example of that first design being used to create congruent sections with rigid transformations.

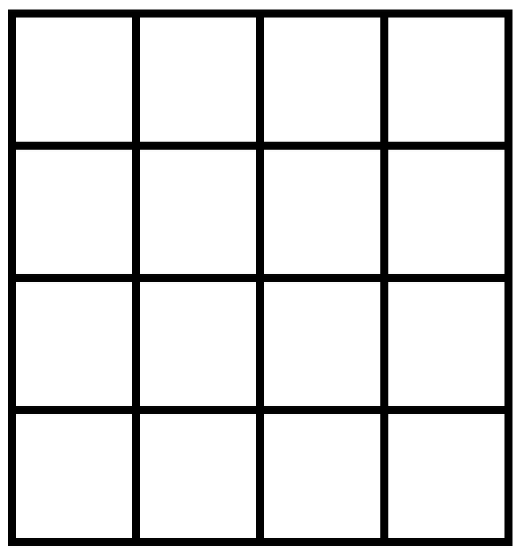


Designing a Blanket with Rigid Transformations

For this activity, you are going to practice rigid transformations by designing a blanket. Your design is going to start in the blank 4 by 4 section below.

Fill in each of the 16 squares above using any of these possible ways of shading in these squares.





9) The blanket below is divided up into 9 sections. Add your design to one of the sections. Use rigid transformations to cover the entire blanket with your core element design. For this activity, you can either slide the original design with a translation, or flip it with a reflection.

HINT: Tracing your design on a piece of paper can help (especially with the reflections).

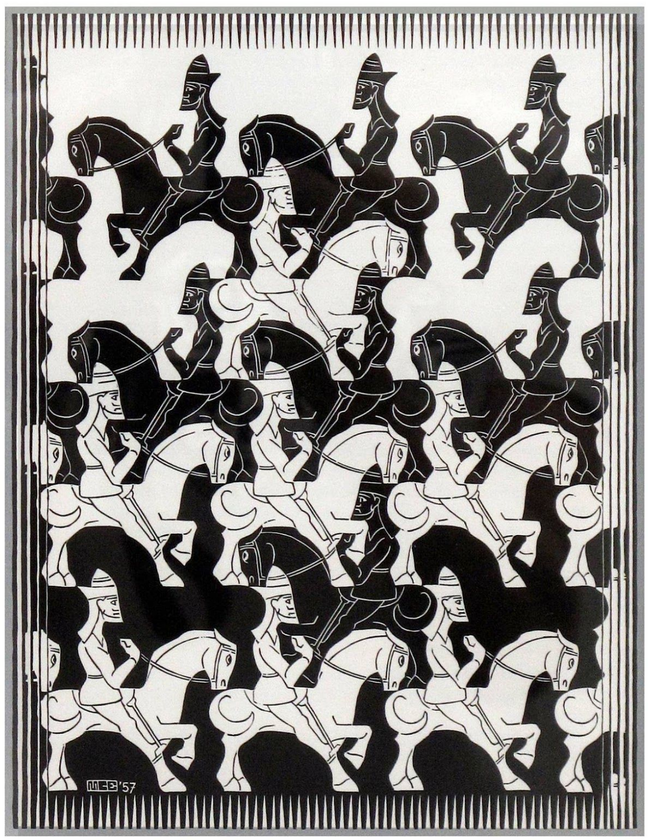
Tessellations

A tessellation is an image that includes repeated congruent figures that cover a surface without gaps or overlapping. The designs of Dutch artist M.C. Escher often used tessellations.

Below is a print by M.C. Escher called *Horsemen*. It is an example of a tessellation.

10) Let's practice using some of our new vocabulary. Describe what you see and try to use some of the following words:

Slide	**Turn**	**Flip**	**Congruent**	**Reflection**	**Translation**	**Rotation**



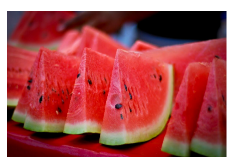
The Coordinate Plane

The computer programs that are used to design the images for the computer animation in movies, TV shows, and video games all use something called *coordinate graphing systems*. Coordinate graphing systems can be used to communicate two or more pieces of information at once. In addition to it being useful, it is definitely something you will see on the high school equivalency exam, so it is good to have a strong foundation. In this section you will learn how to use coordinates to prepare for your work on rigid transformations.

The Fruit Graph[1]

Think about your favorite fruit. There are many attributes of fruit we could think about. For now, we are going to focus on how tasty a fruit is and how easy it is to eat.

Here are ten fruits:

Pineapples	Red Apples	Green Apples	Bananas	Grapes
				
Cherries	Strawberries	Watermelons	Oranges	Lemons
				

Which of these fruits do you like the best? ________________________________

Explain why you like that fruit best.

[1] This activity was adapted from the Desmos.com activity, *Pomegraphit*.

I ranked some fruits by how tasty they are to me (with 1 being the tastiest and 10 being the least tasty):

1. Pineapples
2. Strawberries
3. Red Apples
4. Watermelon
5. Bananas
6. Cherries
7. Oranges
8. Grapes
9. Lemons
10. Green Apples

Here are the same fruits, ranked in order of how difficult I think they are to eat (with 1 being the most difficult to eat and 10 being the easiest to eat):

1.	Pineapples
2.	Watermelons
3.	Cherries
4.	Oranges
5.	Lemons
6.	Bananas
7 and 8 (tie).	Green Apples/Red Apples
9.	Grapes
10.	Strawberries

1) Write four things you notice about my fruit preferences.

    a)

    b)

    c)

    d)

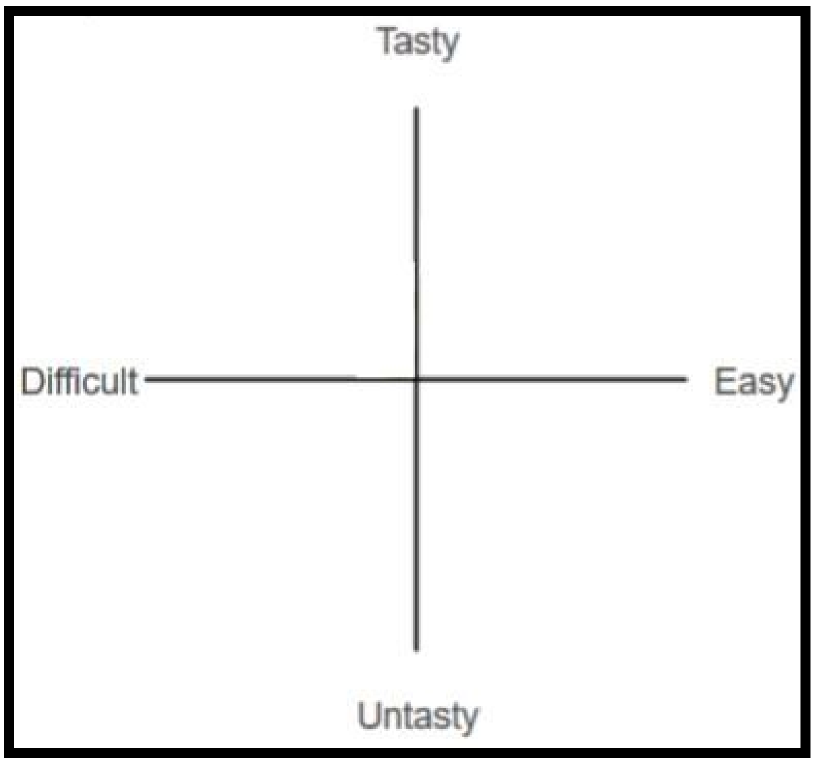
With the lists above, you can make plenty of statements about my fruit preferences, but there is still a lot you can't see. You can see that I like cherries more than green apples, but you don't know how much more. You can tell I think pineapples are more difficult to eat than cherries, but you don't know how much more difficult.

Here is where a coordinate graph can help us look at both pieces of information - tasty/untasty and difficult/easy - at the same time.

Here's an example of a coordinate graph.

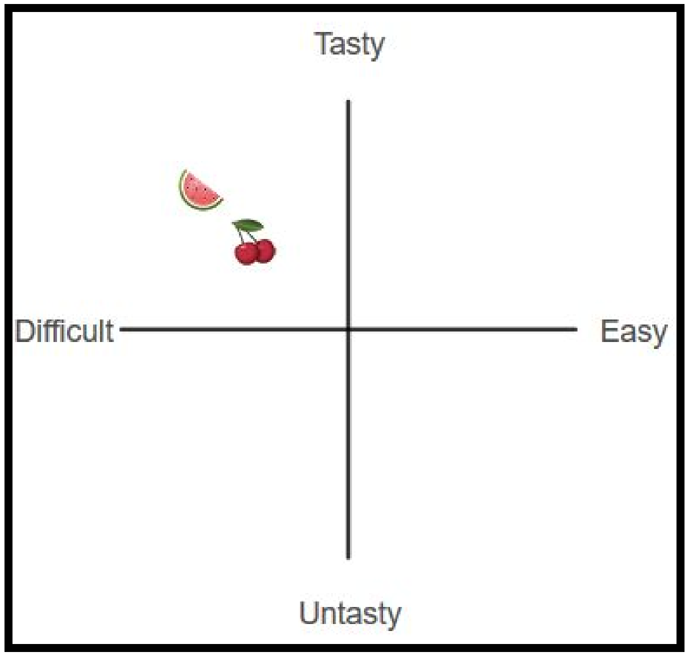
We draw two lines - one horizontal (across) and one vertical (up and down). For this graph, the horizontal line shows how difficult/easy the fruit is to eat. The vertical line shows how tasty/untasty the fruit is.

It is important to pay attention to is the place where the two lines cross.



Let's put some fruits on this coordinate graph and learn more about how it works.

Let's start with cherries and watermelons. Here are what those two fruits might look like if we made a coordinate graph of my fruit preferences.



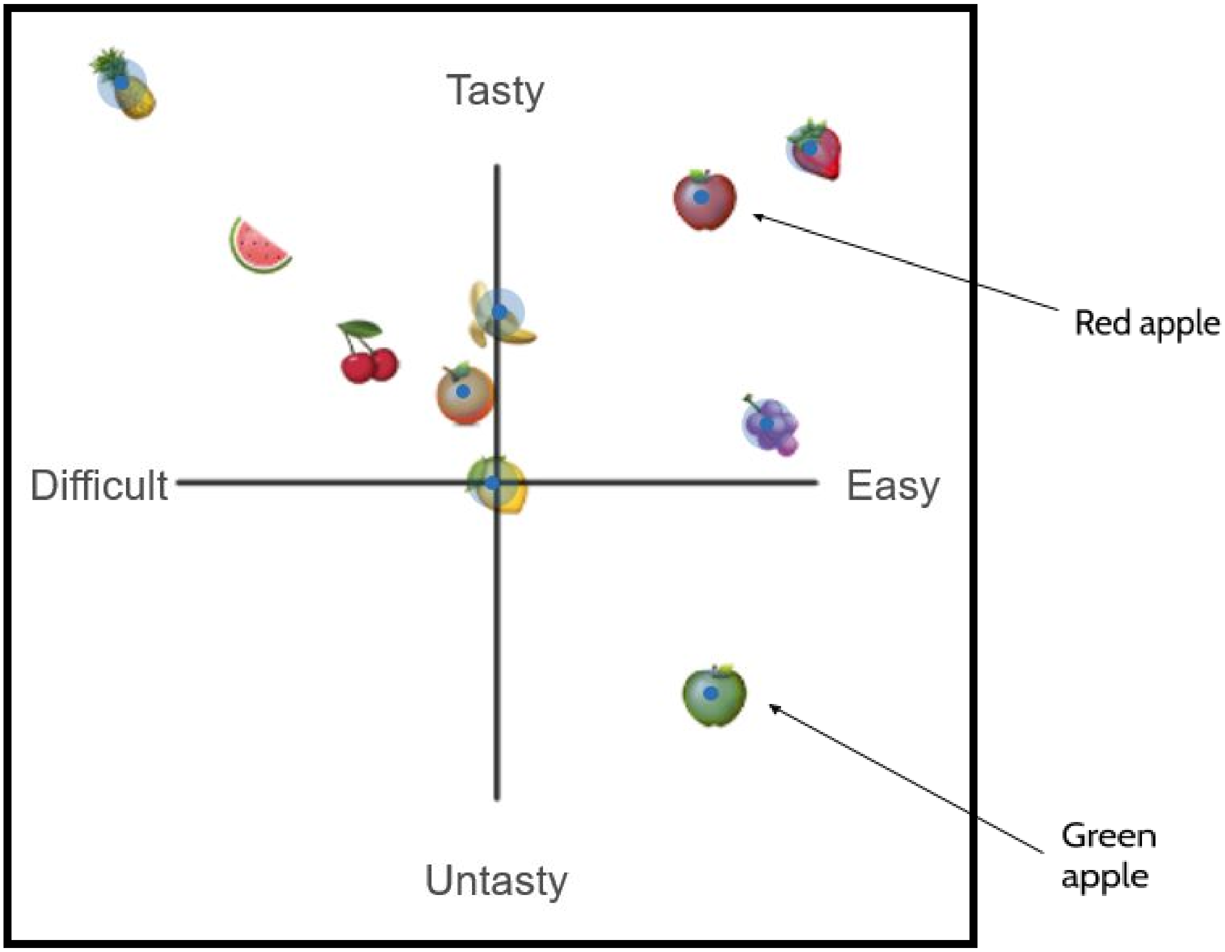
2) Which of these two fruits do I think is tastier? How do you know?

3) Which of these two fruits do I think is more difficult? How do you know?

4) Why are both of these fruits on the left side of the Tasty/Untasty line?

5) Why are both of these fruits above the Difficult/Easy line?

Let's see what it looks like with all of the fruits on the coordinate graph.



6) Which fruits do I think are tasty and difficult to eat?

7) Write everything you can about my opinion on strawberries.

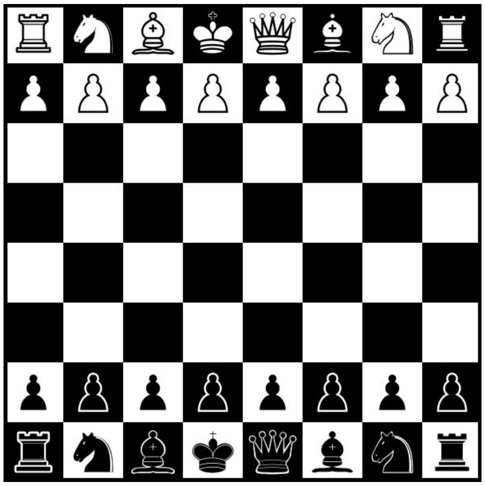
8) Write everything you can about my opinion on apples (green and red).

9) Write everything you can about my opinion about bananas.

10) The lemon is at the place where the two lines cross. What does that mean about my opinion about lemons?

The Chessboard

In the fruit activity, we looked at how a coordinate graph can be set up by drawing a horizontal line and a vertical line that cross. Now we are going to look at the way grids can help us make precise statements about location on the coordinate grid.



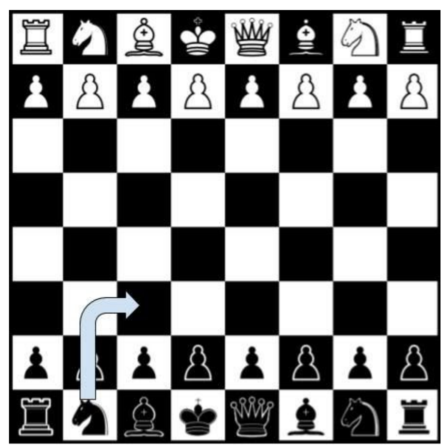
Chess is said to have its roots in Eastern India in the 6th century in a game called chaturanga. It spread to Persia and came to be known as shatranj. The game reached Europe around the 9th century, and by the 15th century the game was essentially played the same way it is today.

Chess has a long history and has spent more than a thousand years becoming the game we know today.

Fans of chess wanted a way to record each move so they could study strategies. They also wanted to save dramatic games.

It is possible to recreate chess games that took place hundreds of years ago. Gioachino Greco was an Italian chess player in the 17th century. He is responsible for some of the earliest recorded chess games, including one he played in the year 1619 with an amateur player.

In those days they would write out a full sentence for each move. The opening move on the board to the right might have been recorded as: *"The black king commands his own knight into the third house before his own bishop."*



A chess game can involve many moves. In the average game, each player will make about 40 moves each. Writing a full sentence for each of those moves would take up a lot of time (and paper!). Over the years, the notation for chess has changed. The goal was to make it easier and more efficient to keep track of moves.

Today, the move on the board would be written as: ♘c3.

How does that work?

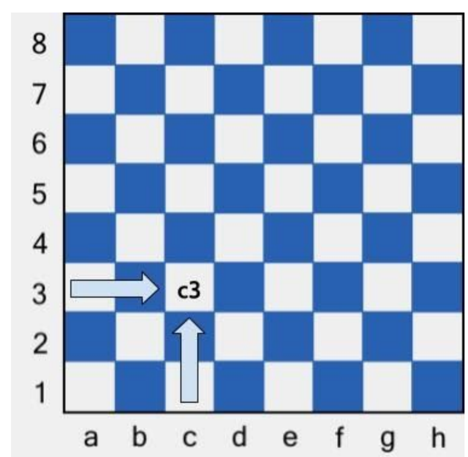
We use letters and numbers together to identify each square on the chessboard.

The letters name the columns (up and down) and the numbers name the rows (across). If you have used computer spreadsheets, the columns and rows are also labeled with letters and numbers.

♘c3 means the piece moves into the square in the **c** column and the **3** row.

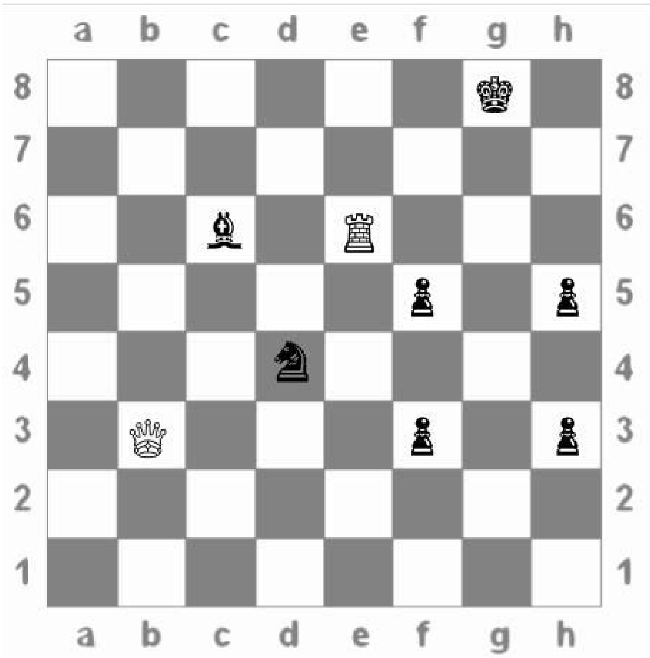
To name each square, we say the letter first and then the number. We start in the bottom left corner. First we go across and then we go up.

This way of naming the exact location on the board is an example of how we use **coordinates** on a coordinate graphing system. Coordinates are a pair of numbers or letters showing the exact position of a point on a line, a map, a graph, or in this case, a chessboard.



In the example on the left, we can say the knight is being moved into the square at the coordinates, **<u>c3.</u>**

Let's practice. There are nine chess pieces on the board below. Write the coordinates for the location of each chess piece. For example, the ♔ is on g8.

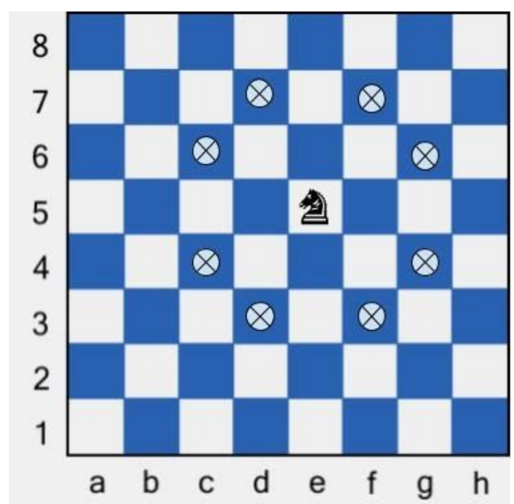


11) ♕	
12) ♖	
13) ♗	
14) ♞	
15) ♟	
16) ♟	
17) ♟	
18) ♟	

19) In chess, the Knight (or Horse) piece moves like a translation.
To figure out where your knight can move:

- slide two spaces horizontally and one space vertically, or
- slide one space horizontally and two spaces vertically.

The Knight on the board to the right is on **e5**. From that position the knight can move to eight possible squares, shown by the ⊗.



On the 8 lines below, write the coordinates for each square where the Knight can move.

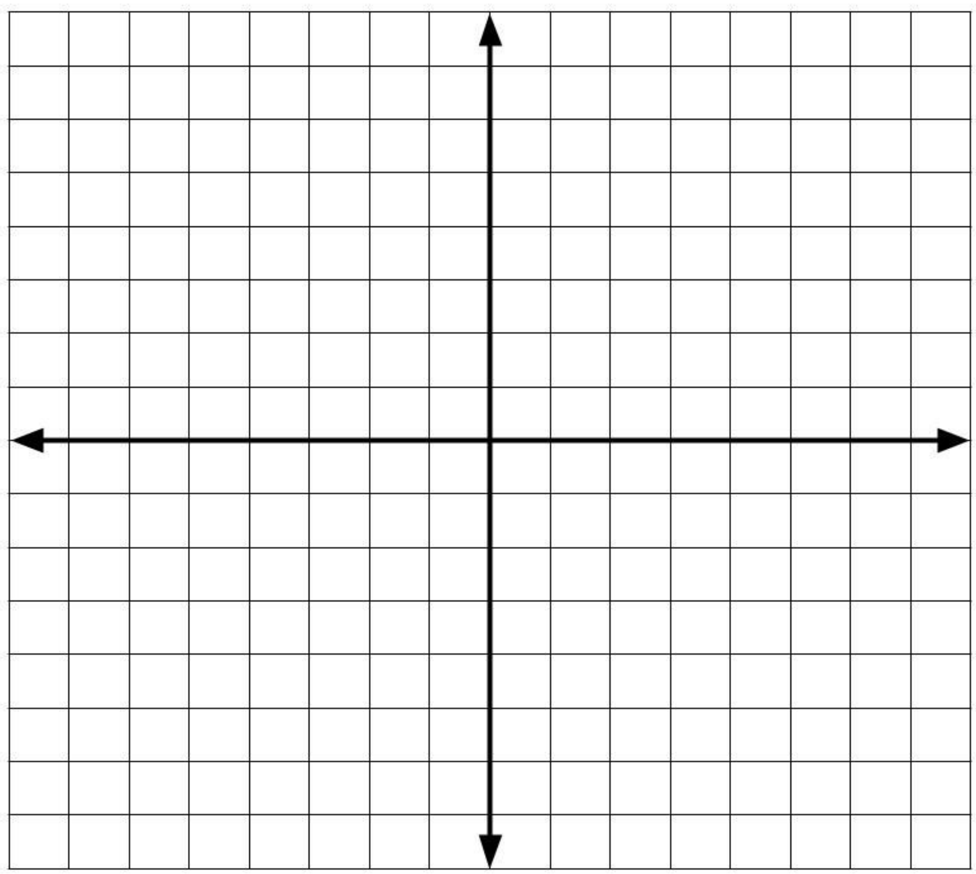
______ ______ ______ ______ ______ ______ ______ ______

The Coordinate Plane

We are now going to review some of the important concepts you've been working on.

This is a coordinate plane.

There are some differences between this and the fruit graph and the chessboard, but let's focus on how they are similar. They all share some important features that are worth noticing.



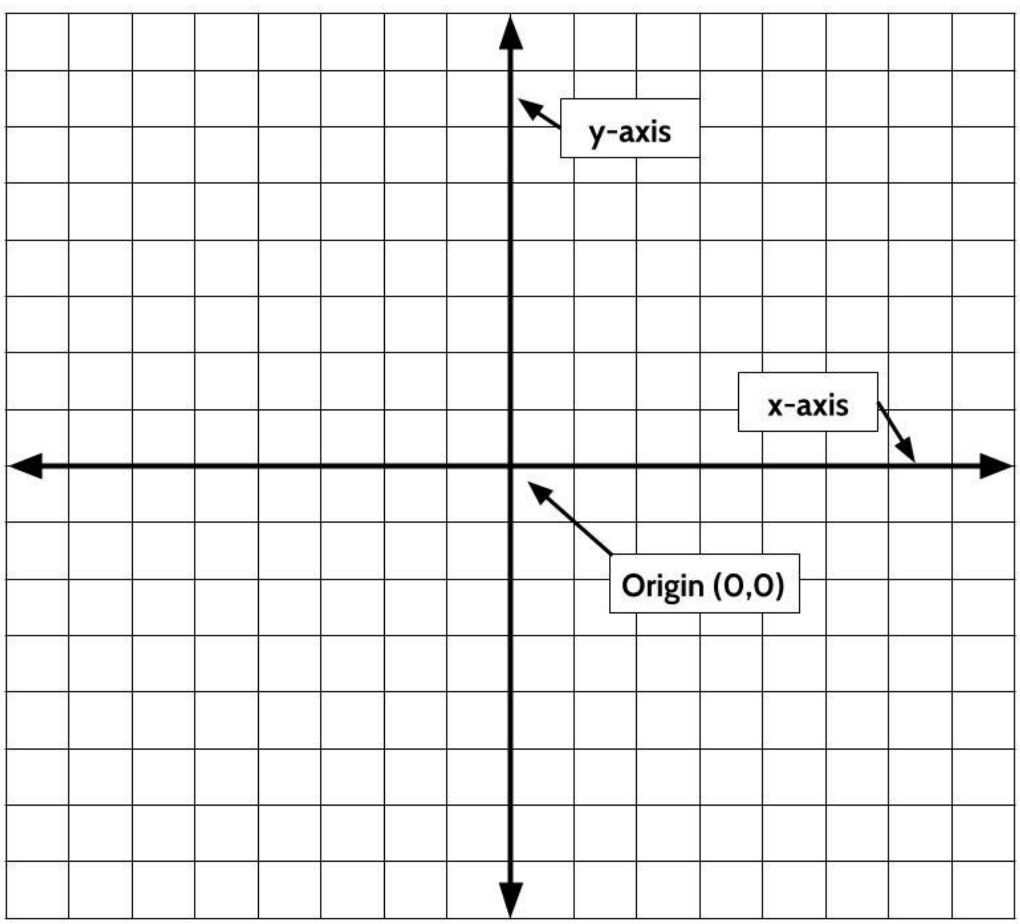
How is this grid is similar to the fruit graph or the chessboard?

One similarity is that all three are formed by a horizontal line and a vertical line that go across each other at right angles.

- In the fruit graph, those lines were the Difficult/Easy line and the Tasty/Untasty line.
- On the chessboard, those lines were made by the letters going across and the numbers going up.

When we talk about coordinate planes in general, we can use the names **x-axis** and **y-axis** to talk about those two lines that cross. The x-axis is always the horizontal line. The y-axis is always the vertical line.

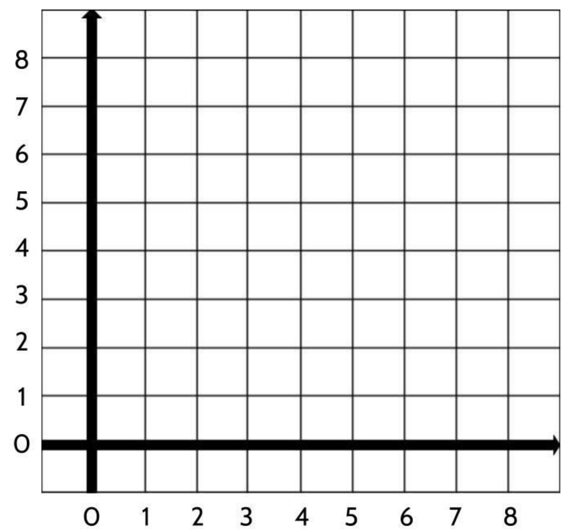
Remember how the lemon was in the center of the fruit graph? On the coordinate plane, the point where the x-axis and the y-axis cross is called the **origin**. The origin is the place where we are at 0 on the y-axis and 0 on the x-axis. We write the combination of those two coordinates as (0,0).



Plotting Points on the Coordinate Plane

In chess and on the coordinate plane we use coordinates to describe an exact location.

For now, let's focus on the top right part of the coordinate plane. We call this section Quadrant 1.



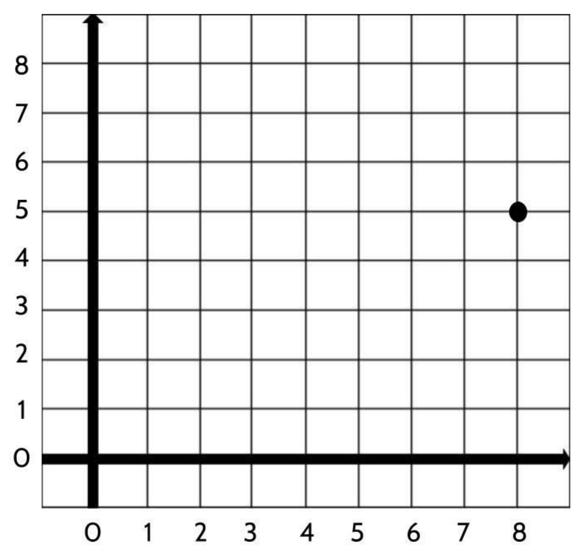
This now resembles the 8 by 8 chessboard. On the chessboard, we used letters across the bottom to tell us how many squares to move to the right. Then we used the numbers up the side to tell us how many squares to move vertically.

The coordinate plane has two important differences.

The first difference is we use numbers for both directions. The numbers along the x-axis tell us how to move horizontally and the numbers along the y-axis tell us how to move vertically.

The second difference is that on the coordinate plane, we count where the grid lines meet (instead of counting squares they way we did on the chessboard).

On the coordinate plane, coordinates work like giving someone directions. You always start in the origin. That point is 0 on the x-axis and 0 on the y-axis. To describe an exact point on the coordinate grid, start at the origin, give the horizontal directions and then give vertical directions.



For example, if I wanted to tell you to place a point on the graph, I might tell you to *plot* it at

(8,5)

The 8 tells you to move 8 lines to the right from the origin. Then 5 tells you to move 5 lines up from the origin. (8, 5) is the point where the vertical line at 8 intersects with the horizontal line at 5.

The coordinates we use on the coordinate plane are sometimes called *ordered pairs* to help us remember that the order is always the same. The first number always tells us how far to move along the x-axis and the second number tells us how far to move along the y-axis. (8,5) is an example of an ordered pair.

1) Let's practice writing coordinates out as ordered pairs.

   Write the ordered pairs for each point on this coordinate plane.

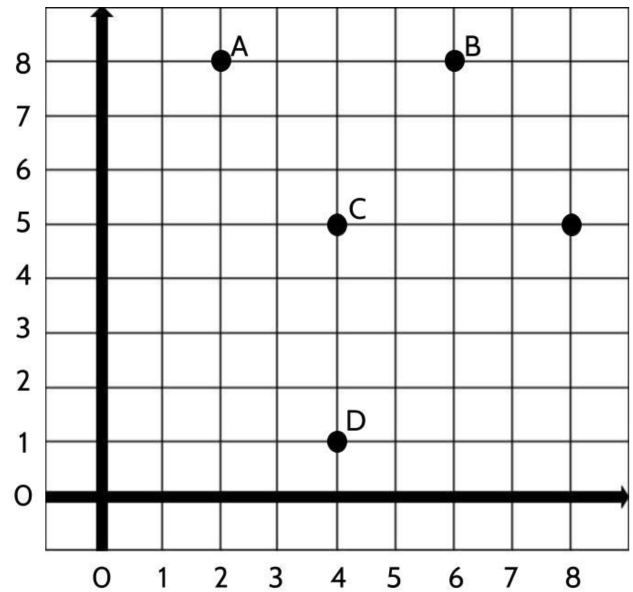
   Point (8, 5) is included as an example.

   Point A:

   Point B:

   Point C:

   Point D:



2) Let's practice using ordered pairs to draw points.

   Use these coordinates to draw the following ordered pairs on the coordinate plane. Make sure to label each point with the letter given.

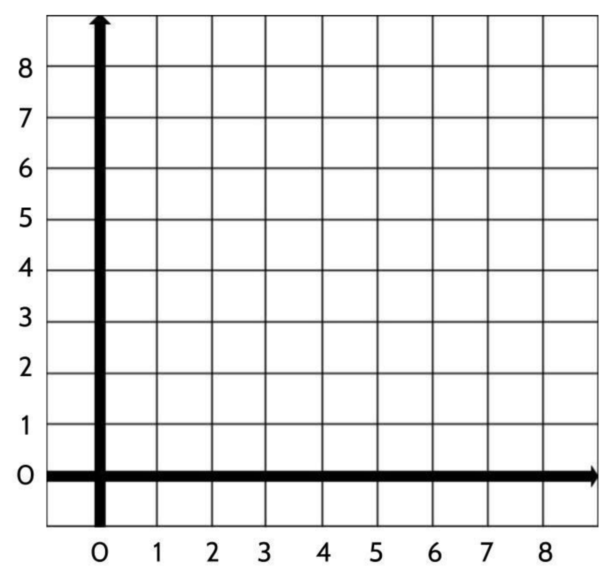
   H (2,1)

   O (2,6)

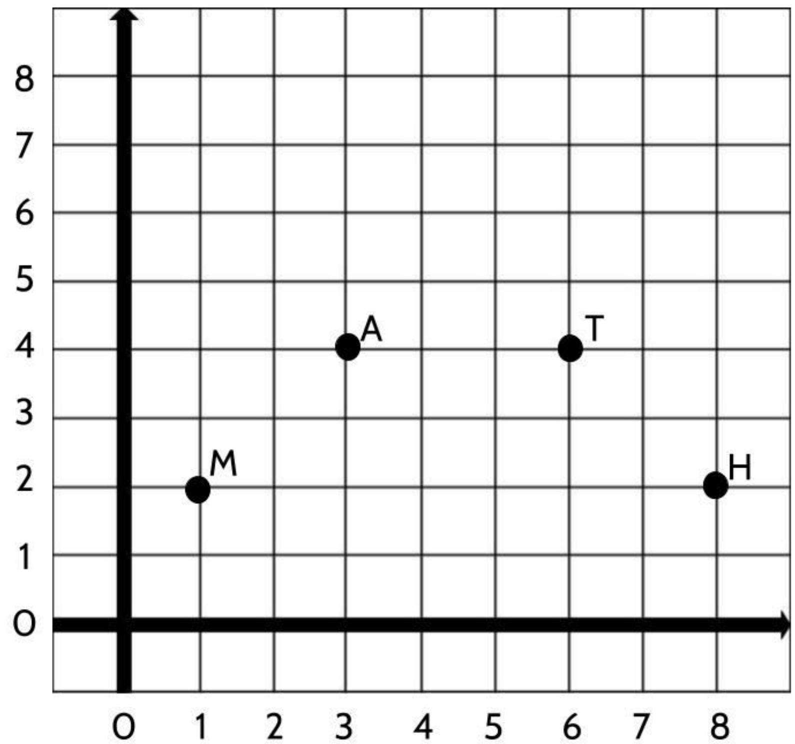
   S (6,6)

   E (6,1)

   U (4,8)



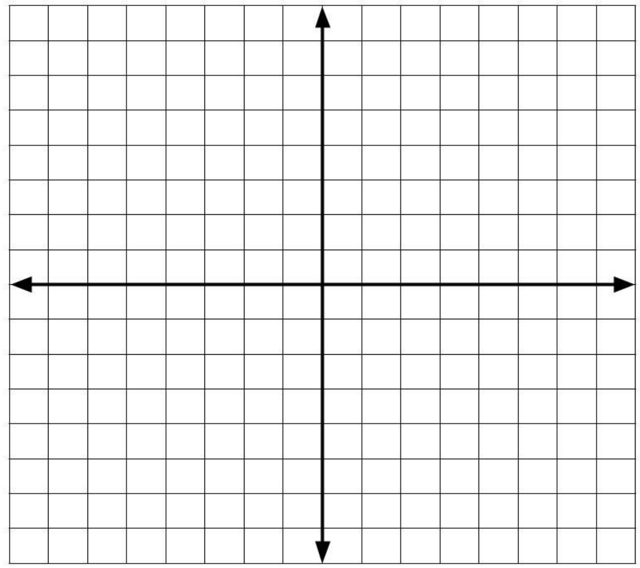
3) Look at the points on this coordinate grid.



Which of the following statements are true? Underline the <u>two</u> true statements.

A. Point T is 6 units to the right and 4 units up from the origin.
B. The distance between Point M and Point H is 8 units.
C. Point A and Point T have the same x coordinate.
D. The ordered pair for Point M is (2, 1)
E. To move from Point M to Point T, move 5 units to the right and two units up.

So far, we've been working in the top right section of the coordinate plane. Let's see what happens when we look at all four sections.



We know the coordinates of the origin are (0,0) because the origin represents the point of zero on the x-axis and zero on the y-axis.

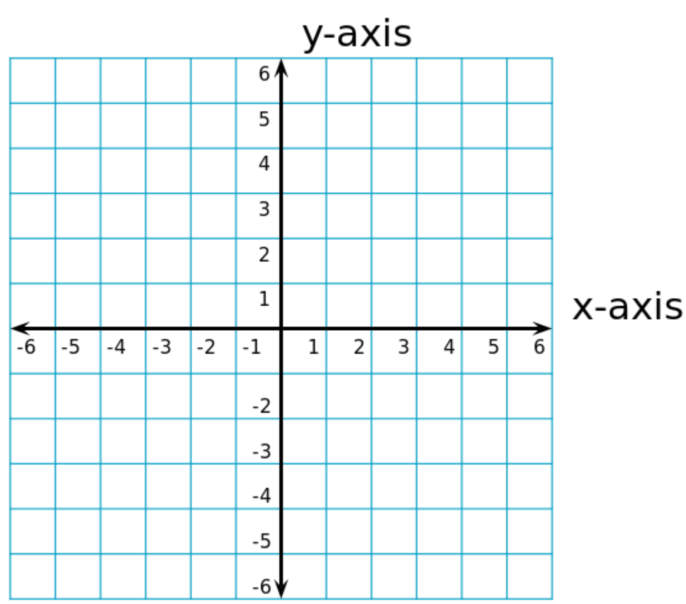
As you move to the right along the x-axis from the origin, the numbers are getting bigger. As you move up along the y-axis from the origin, the numbers are also getting bigger.

But what happens to the numbers as you move to the left along the x-axis from the origin?

And what happens to the numbers as you move down along the y-axis from the origin?

You can think of the x-axis and the y-axis as number lines. The same way number lines can continue below zero, so can the x-axis and y-axis, with negative numbers!

This is similar to what we saw on the fruit graph.

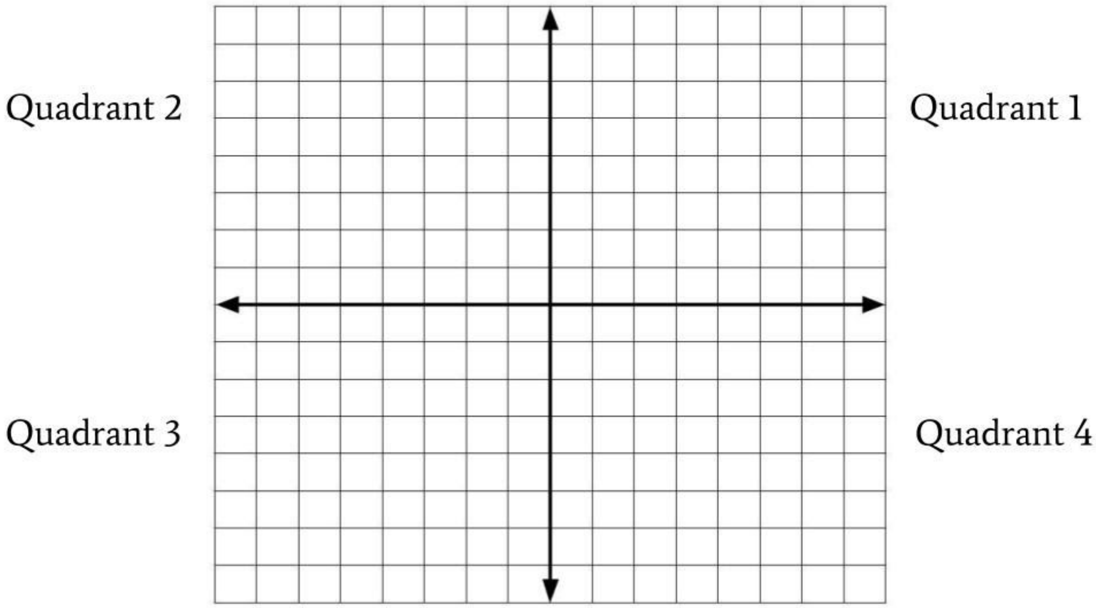


- The Tasty/Untasty axis was the dividing line between fruits that were difficult to eat and fruits that were easy to eat.

  In the same way, the y-axis is the dividing line between positive numbers on the right and negative numbers on the left.

- The Difficult/Easy axis was the dividing line between tasty fruits above and untasty fruits below. In the same way, the x-axis is the dividing line between positive numbers above and negative numbers below.

As we've seen, the top right section of the coordinate plane is called Quadrant 1. The other Quadrants are numbered 2, 3, and 4, going in order, counterclockwise.

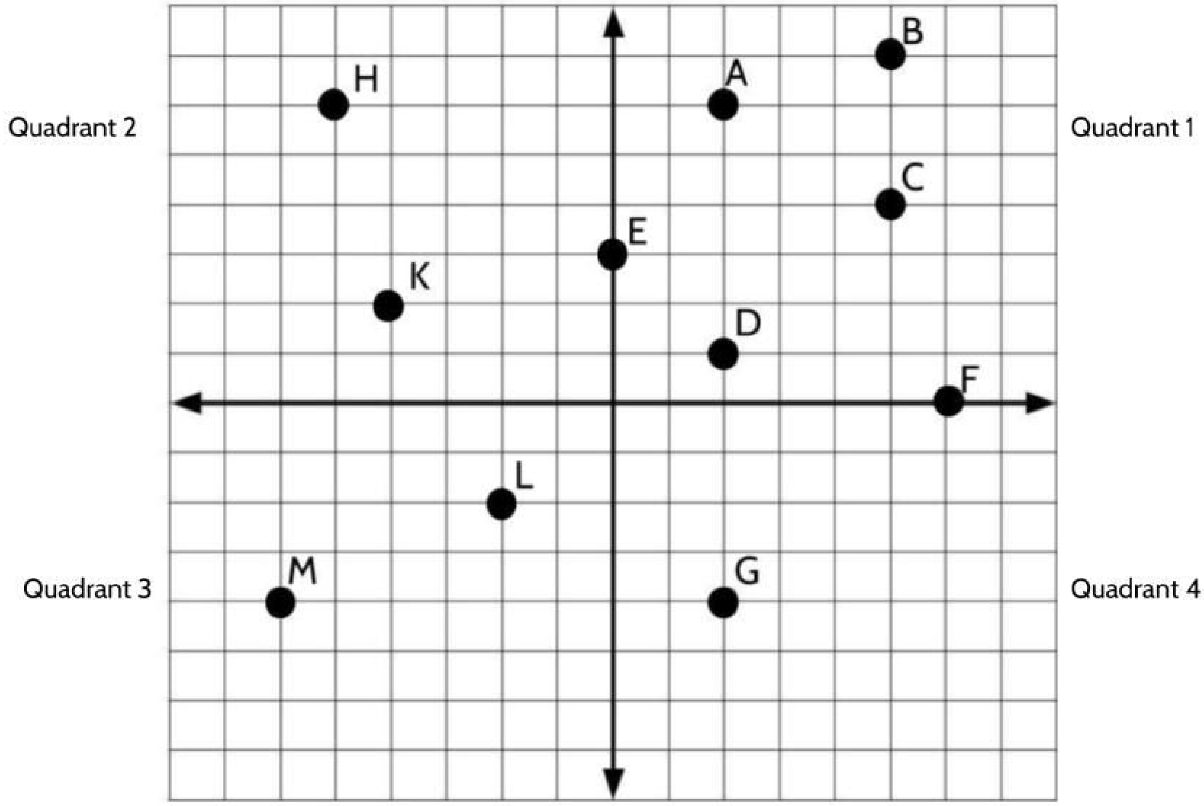


4) Complete the following statements:

Ex: Point (5, 4) is in Quadrant 1 because I start in the origin and move 5 units to the right / left and then I move 4 units up / down.

a) Point (7,-3) would be in Quadrant ____ because I start in the origin and move ____ units to the right / left and then I move ____ units up / down.
b) Point (-3, -6) would be in Quadrant ____ because I start in the origin and move ____ units to the right / left and then I move ____ units up / down.
c) Point (16, 25) would be in Quadrant ____ because I start in the origin and move ____ units to the right / left and then I move ____ units up / down.
d) Point (-10, 3) would be in Quadrant ____ because I start in the origin and move ____ units to the right / left and then I move ____ units up / down.

5) Write the name of each point next to the ordered pairs provided below:



Point D is at **(2, 1)**	
(-6, -4)	**(6, 0)**
(-2, -2)	**(0, 3)**
(-5, 6)	**(5, 4)**
(-4, 2)	**(2, 6)**
(2, -4)	**(5, 7)**

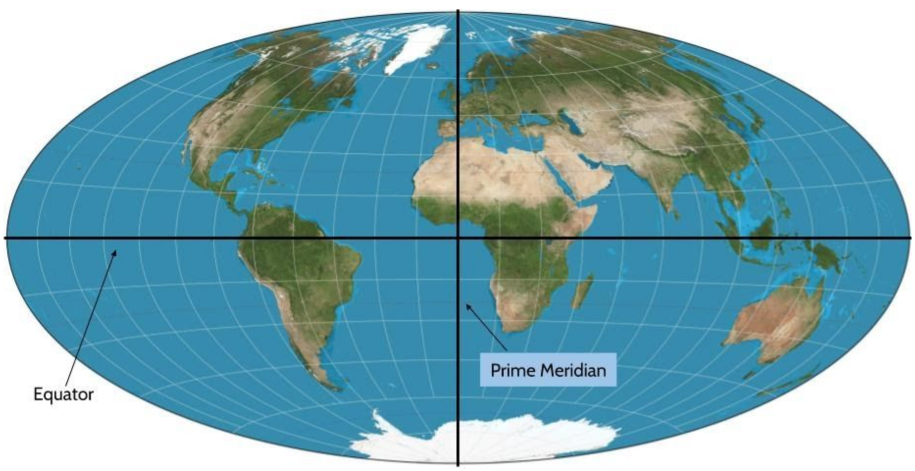
Longitude Lines and Latitude Lines

There are many examples of coordinate systems where two pieces of information are used to pinpoint an exact location. We've already seen it at work in graphs and chessboards. You may have also seen a coordinate system when looking at maps.

There are two imaginary lines drawn on the Earth.

The equator is drawn around the middle of the Earth to divide it into the northern and southern hemispheres (halves). Latitude lines are horizontal lines used to describe how far north or south locations are from the equator. The Equator is the latitude line which is 0° North/South.

The Prime Meridian is drawn from the top of the Earth to the bottom, to divide the Earth into eastern and western hemispheres (halves). Longitude lines are vertical lines used to describe how far to the east and how far to the west locations are from the Prime Meridian. The Prime Meridian is 0° West/East



For example, in the center of New York State, the city of Utica is located at:

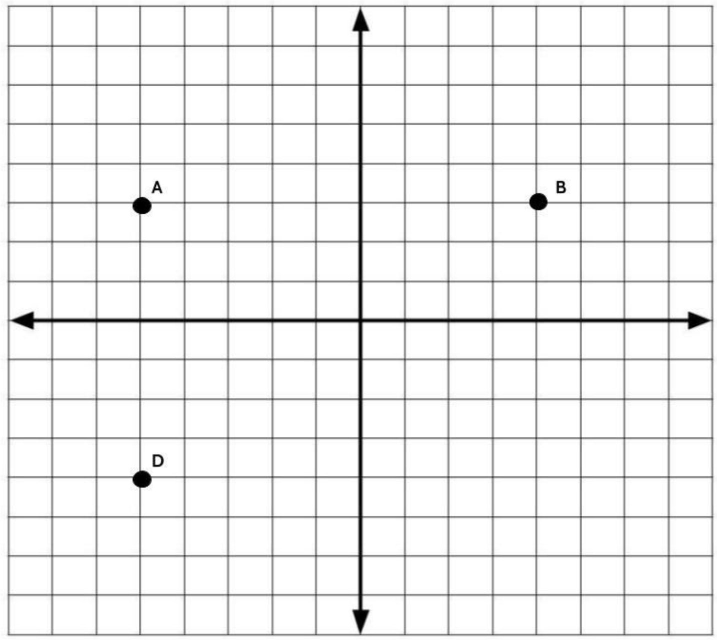
Latitude: 43.100903° and Longitude -75.232666°

That means Utica is about 43° to the north of the equator and about 75° to the west of the Prime Meridian.

Go to https://www.latlong.net/ to find the exact coordinates (latitude, longitude of your hometown.

Shapes on the Coordinate Plane

Now that we've had some practice with naming and plotting (placing) points on a coordinate plane, let's use what we've learned to make a few shapes.



A rectangle is a four-sided shape with four right (90°) angles. In a rectangle, opposite sides are both equal and parallel.

Draw Point C to complete rectangle ABCD.

1) What are the coordinates of the ordered pair for Point C?

   Point C: (,)

2) A square is a special type of rectangle.

   A square is a rectangle where all four sides are equal in length. On the coordinate grid, draw four points to create your own square.

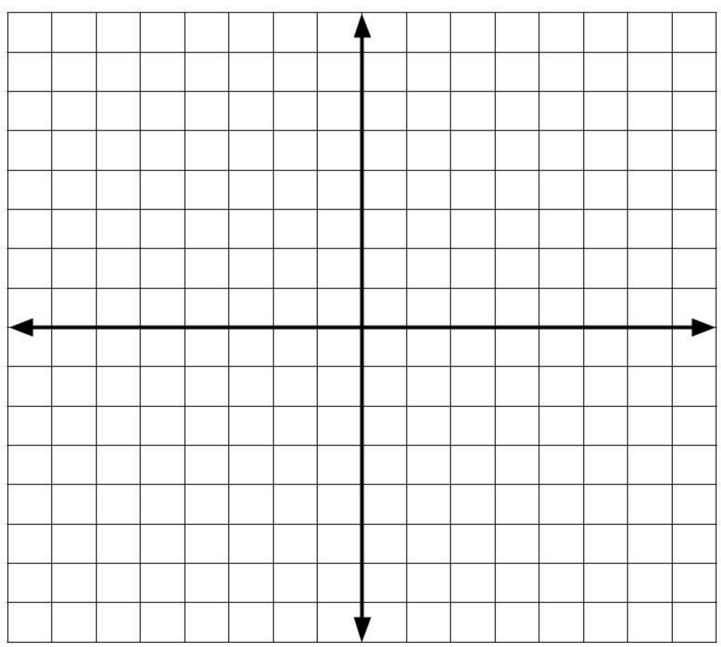
   Label the four points E, F, G, and H. Write the coordinates for each point below.

   Point E: (,)

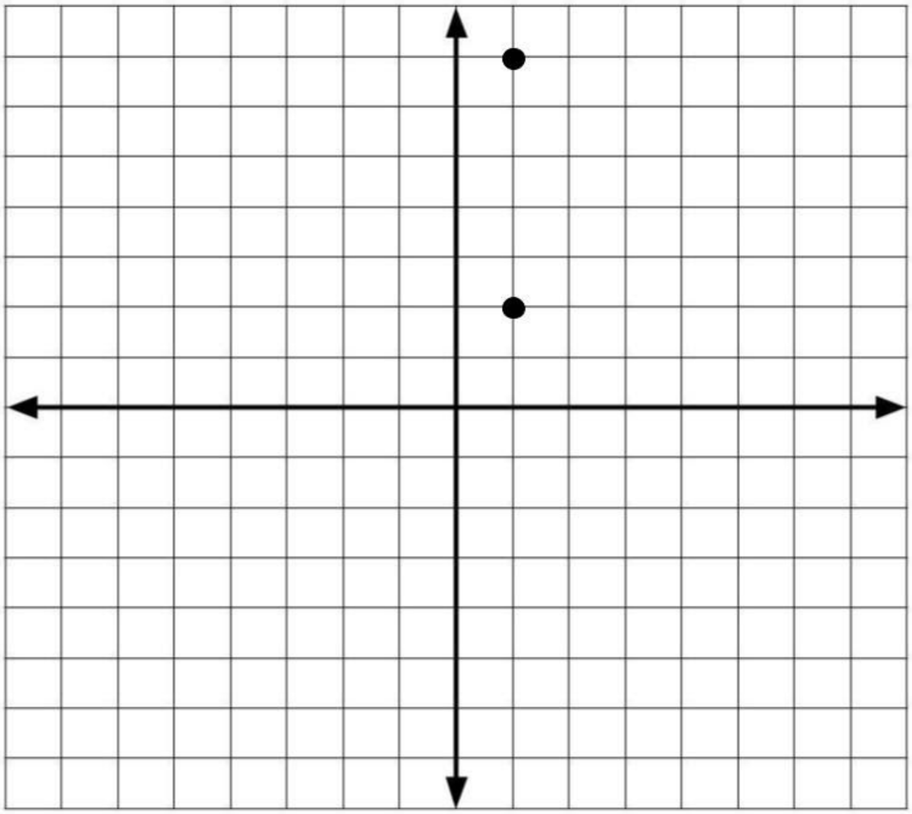
   Point F: (,)

   Point G: (,)

   Point H: (,)



3)



Which two sets of ordered pairs can be used to make a square with the two points on the coordinate grid?

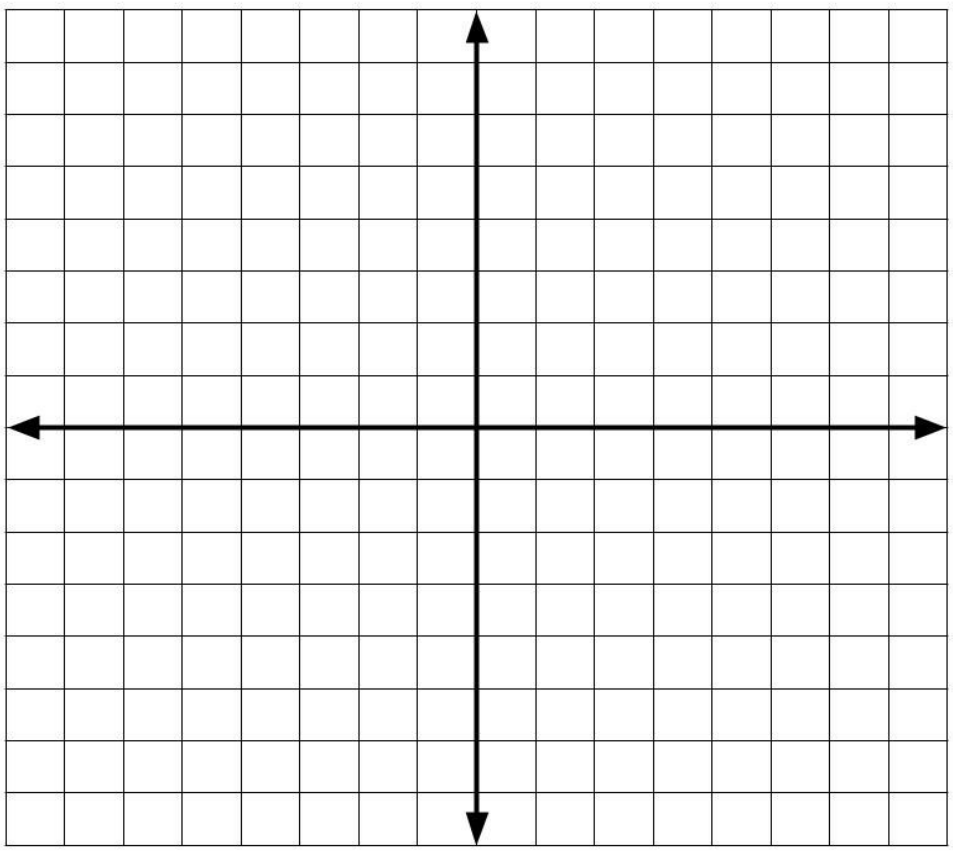
A. (7, -7) and (7, -2)
B. (6, 7) and (6, 2)
C. (-2, -2) and (-2, -7)
D. (-4, 2) and (-4, 7)

4) Find the perimeter of the rectangle that is formed by these four points:

(3, 2)   (5, 2)   (5, -5)   (3, -5)

HINT: The perimeter of any shape can be found by adding up the length of each of its sides.

The perimeter of the rectangle is _______________.



Lines on the Coordinate Plane

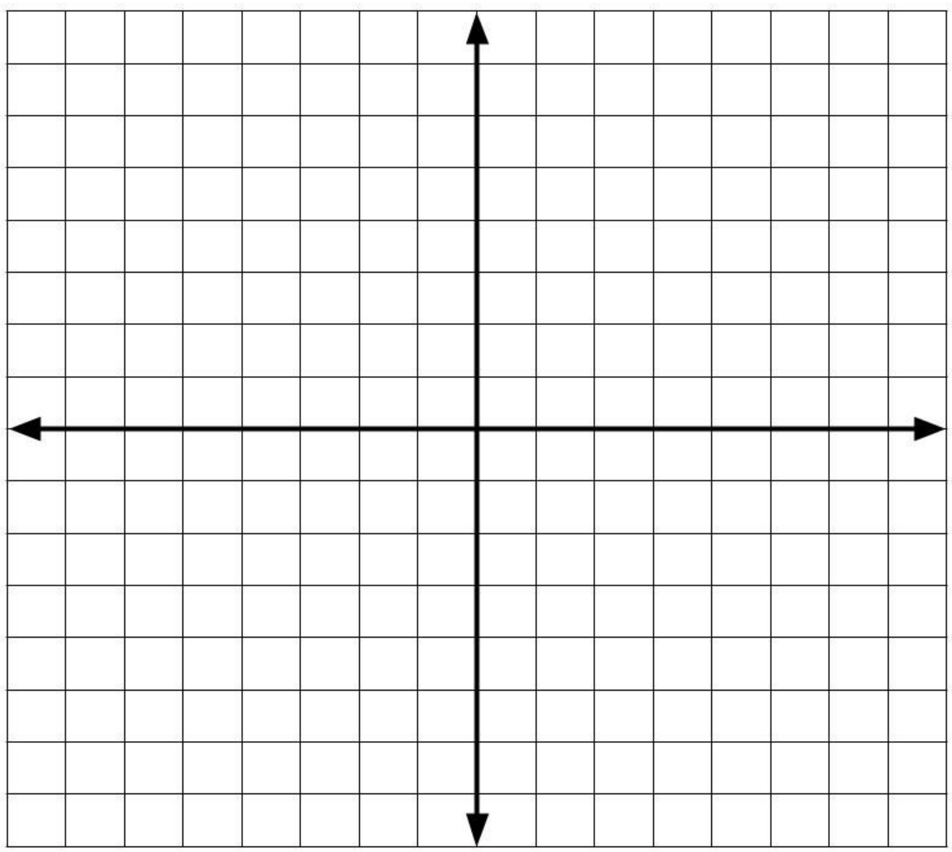
Something we will be working on in the next section is how to reflect points and shapes across different lines on the coordinate plane.

We've spent some time thinking about the two lines that define the coordinate plane - the x-axis and the y-axis. When working with rigid transformations you might be asked to reflect a shape about either.

Let's focus on a few other lines and begin with plotting some points.

5) Plot the following points on this coordinate plane:

(-7,-7), (-5,-5), (-3,-3), (-1,-1), (0,0), (2,2), (4, 4), (6,6), (8,8)

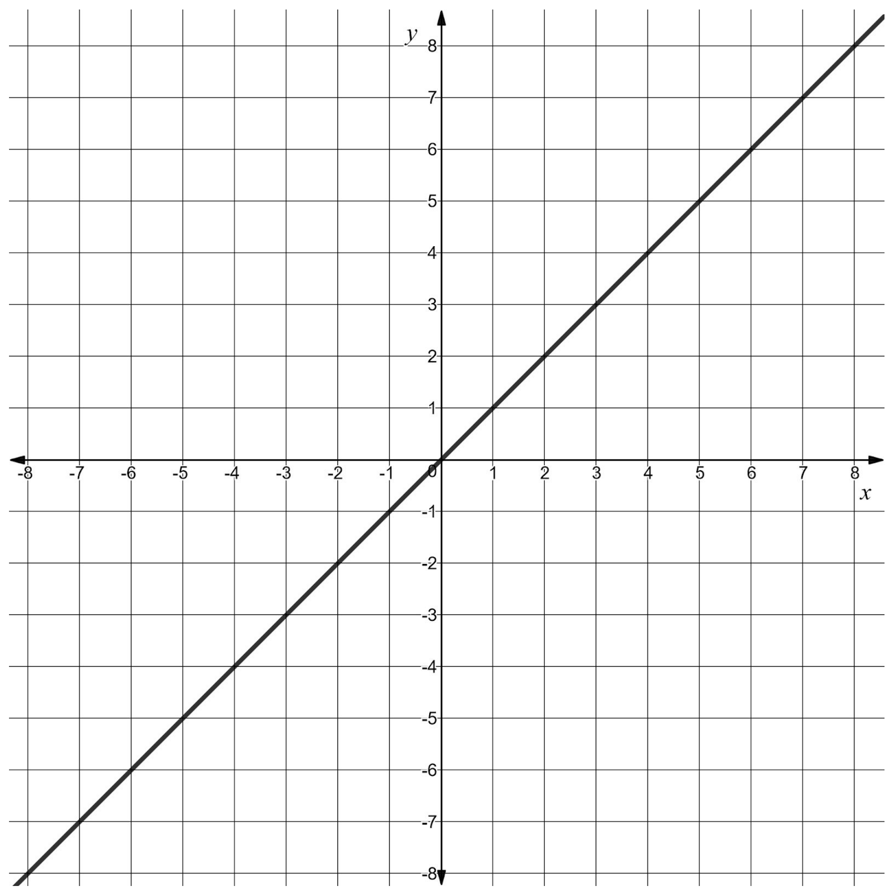


What do you notice about these points?

One thing you might have noticed is that the x and the y coordinates for each ordered pair are the same. You may also have noticed that the points form a line. In fact, if we continued plotting points, the line would continue.
Lines on the coordinate plane are named by an equation that can be used to construct the line. In this case, we call the line:

$$y = x$$

y = *x* can be read as "y equals x" and it refers to the line that cuts the coordinate plane in half, straight through the origin, where every ordered pair has the same x and y coordinates. The x value and the y value are the same at every point along the line.



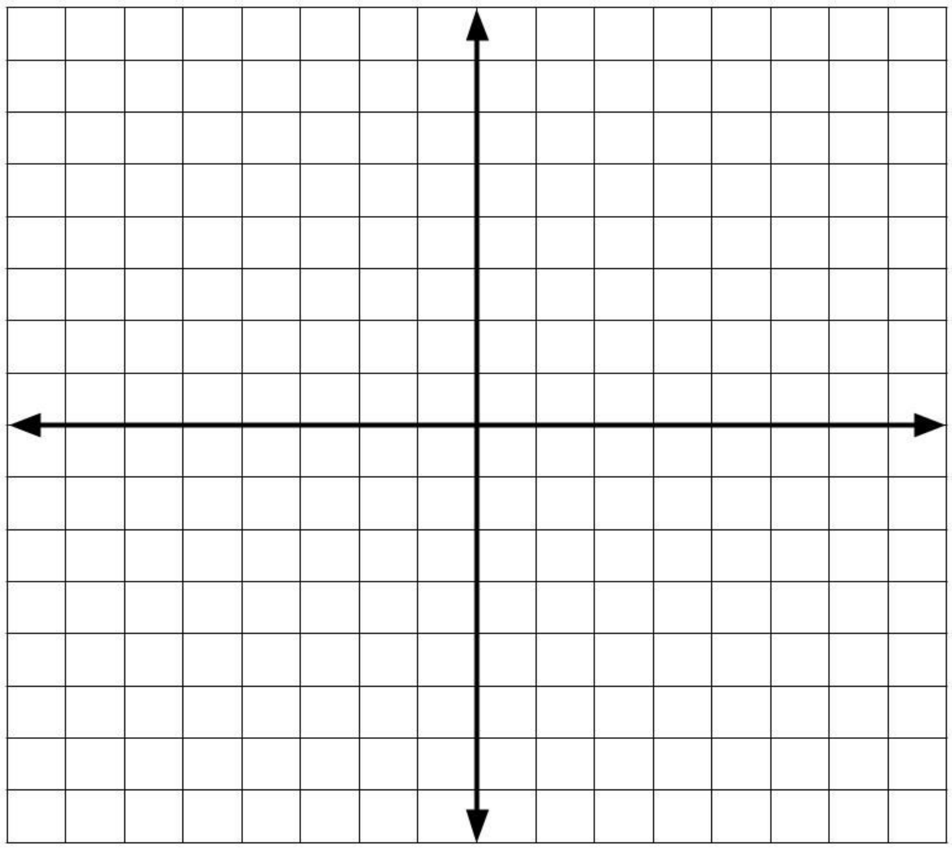
6) Circle any of the following points that line $y = x$ would pass through if we continued the line.

   (-76, -76) (16, -16) (3, 3) (24, 24) (-9, -9) (5,-5)

   How do you know?

7) Plot the following points on this coordinate plane:

(3, 6), (-2, 6), (6, 6), (-5, 6), (7, 6), (-4, 6), (1, 6), (-7, 6)



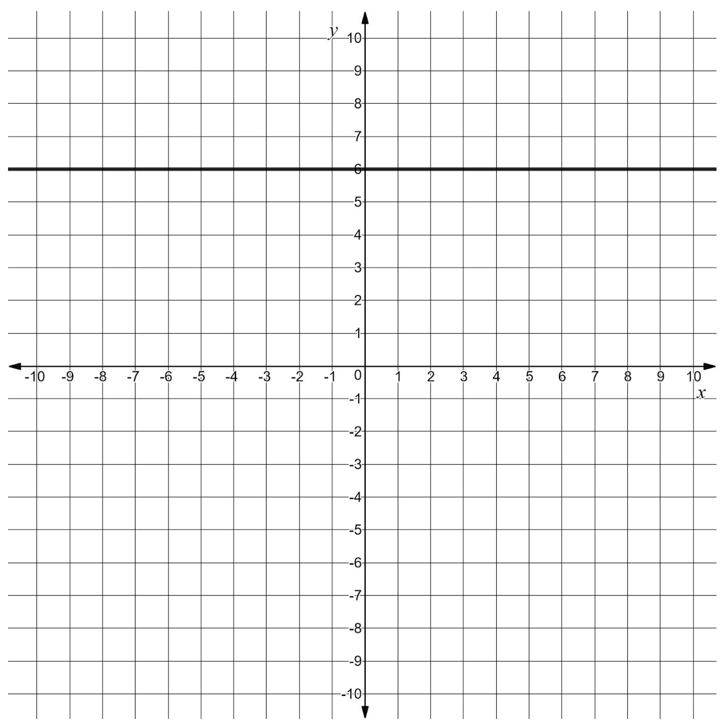
What do you notice about these points?

One thing you may have noticed is that the y coordinate in each ordered pair is always 6. You may have also noticed these points form a horizontal line.

The name for this line is:

$$y = 6$$

y = 6 can be read as “y equals 6”. y = 6 refers to the line where every ordered pair has a 6 for the y coordinate. You may also have noticed that this line is parallel to the x-axis. It is parallel because the two lines (the x-axis and y=6) are the same distance apart at any point along the line. They will always be 6 units apart.

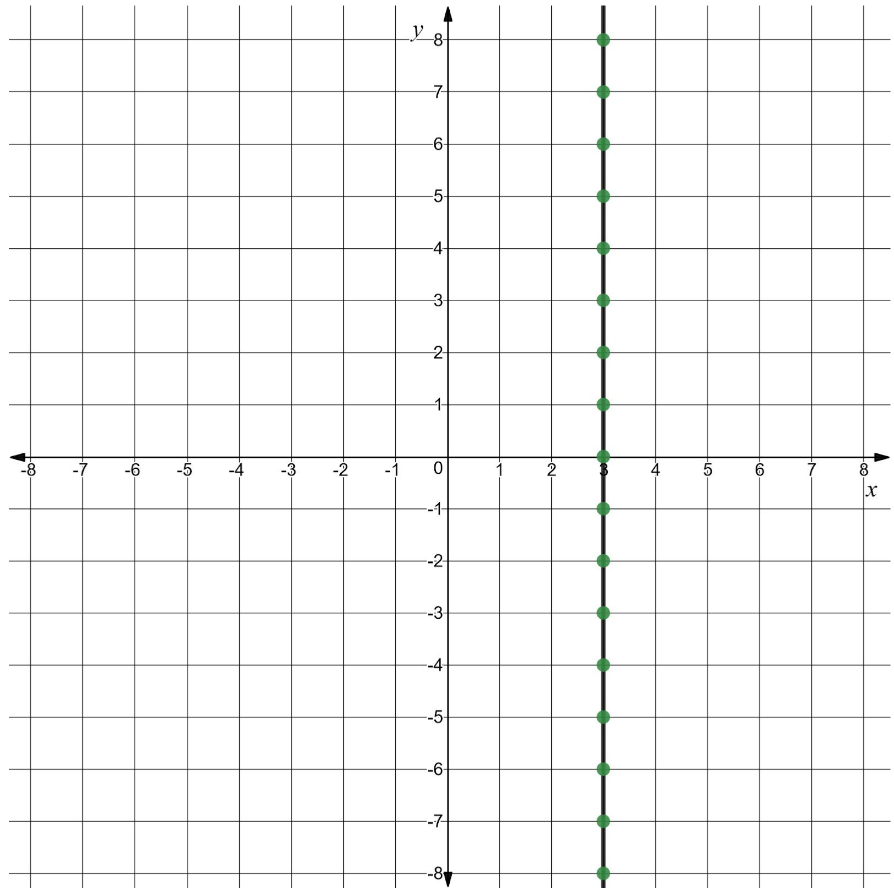


8) Circle any points that line y = 6 would pass through if we continued the line.

(32, 6) (-19, 6) (6, 12) (4, -6)

How do you know?

Use this coordinate plane to answer question #9.



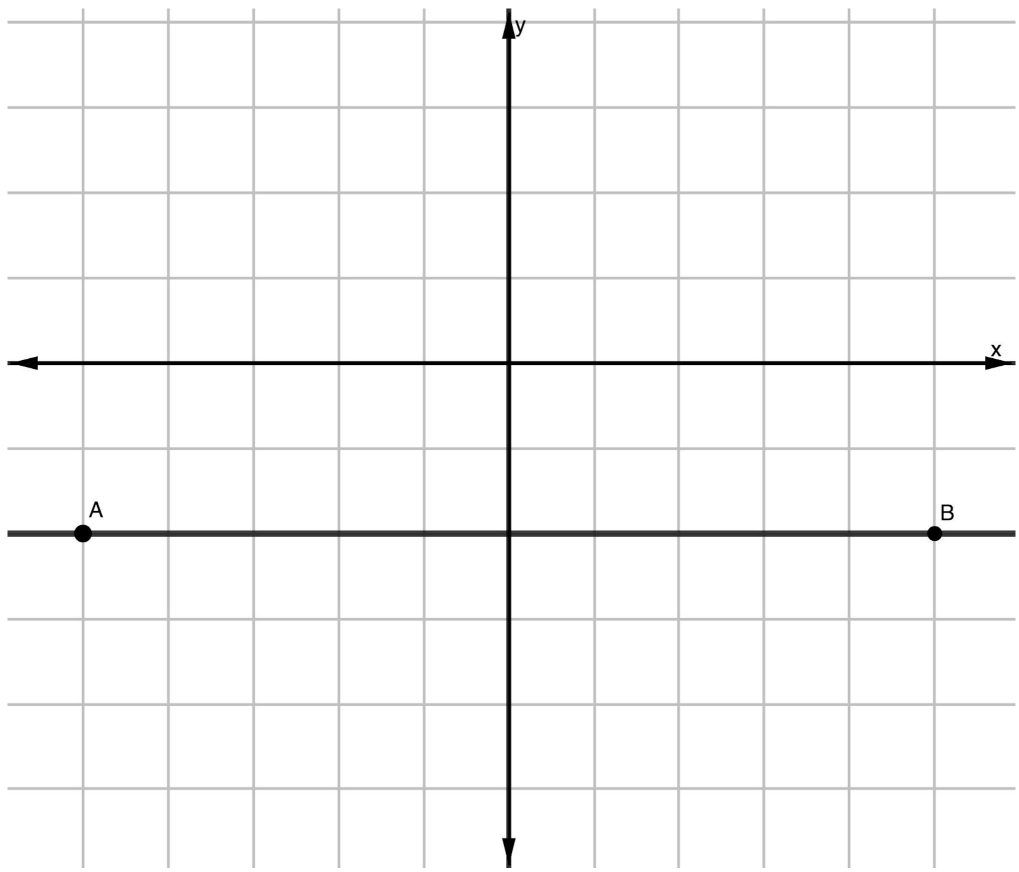
9) Fill in the blanks.

- (____, ____), (____, ____), (____, ____), and (____, ____) are all examples of points on this line.
- All points on the line have _____ as the value of the _____ coordinate.
- This line is parallel to the _____-axis, because each point is _____ units away from that axis.
- I think the equation that names this line is ___________ because ____________________

  ____.

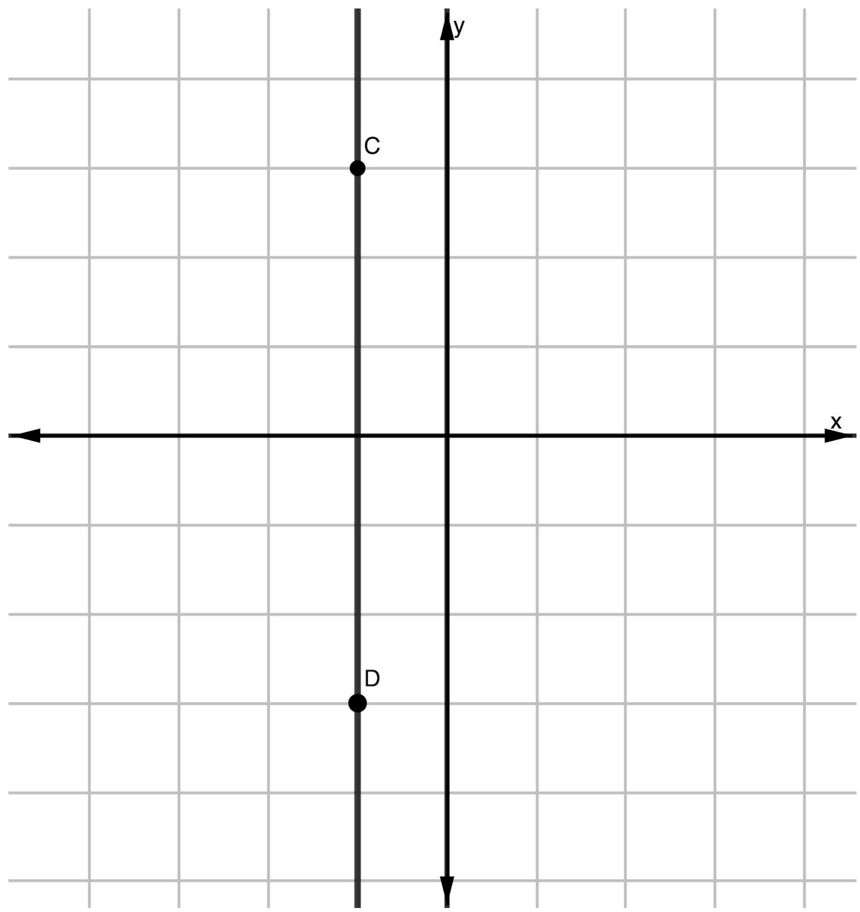
10) What is the name of the equation for line AB?

The equation for Line AB is ___________________



11) What is the name of the equation for the line CD?

The equation for Line CD is ___________________

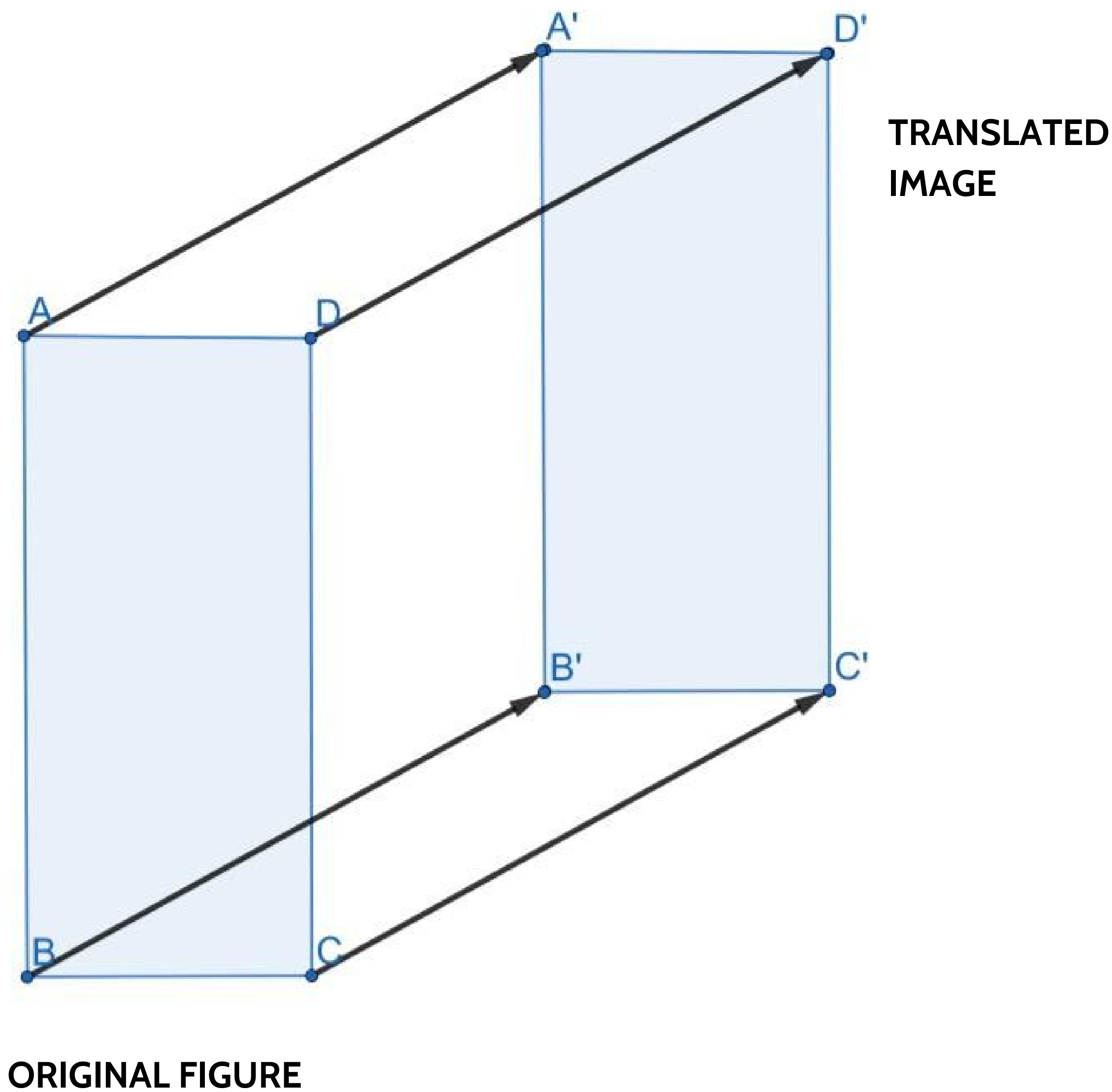


How Math is Written

In Rigid Transformations: Shapes on a Plane Part 2, you will be learning how to use rigid transformations on the coordinate plane.

One key idea in rigid transformations is that when we are sliding, turning, or flipping figures, we are sliding, turning, and flipping *every point* in that figure.

Consider the original figure on the left. We call this Rectangle ABCD.



We refer to shapes by the name of the shape (rectangle, triangle, square, etc) followed by the letter given to each point.

Notice how we slide each point the same distance in the same direction to create its image.

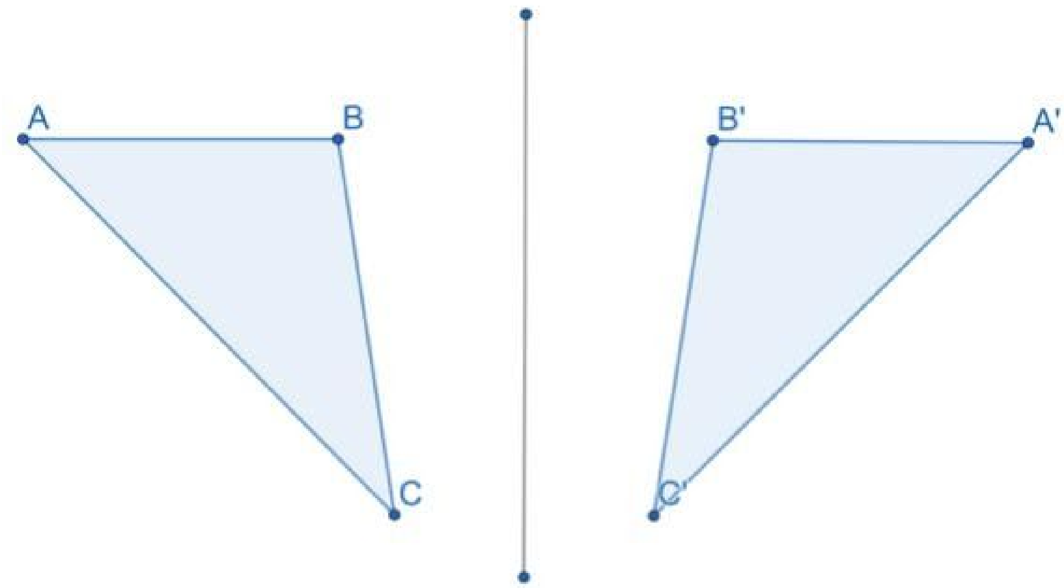
Because of this idea that each point on the original figure has a corresponding point in the image, we can say: **Rectangle ABCD *maps* onto its image Rectangle A'B'C'D'.**

Sometimes we name the corresponding points in the image produced by a rigid transformation by using a tick mark. This can help keep track of the corresponding points.

In the slide shown above:

- Point A maps to Point A'
- Point B maps to Point B'
- Point C maps to Point C'
- Point D maps to Point D'

One other important symbol is the symbol for congruence, **≅.**
In our example here, we can say Rectangle ABCD ≅ Rectangle A'B'C'D'.



Original Figure **Image**

1) What kind of transformation is this? How do you know?

2) What is the name of the original figure?

3) Fill in the blanks
   - Point A maps onto Point ____.
   - Point B ____________ onto Point B'.
   - Point C ___________ onto Point _____.
   - △ABC is ____________________ to △A'B'C' because __________

     ____

Answer Keys

Congruence and Rigid Transformations - Answer Key

1) Two shapes that are congruent to each other:

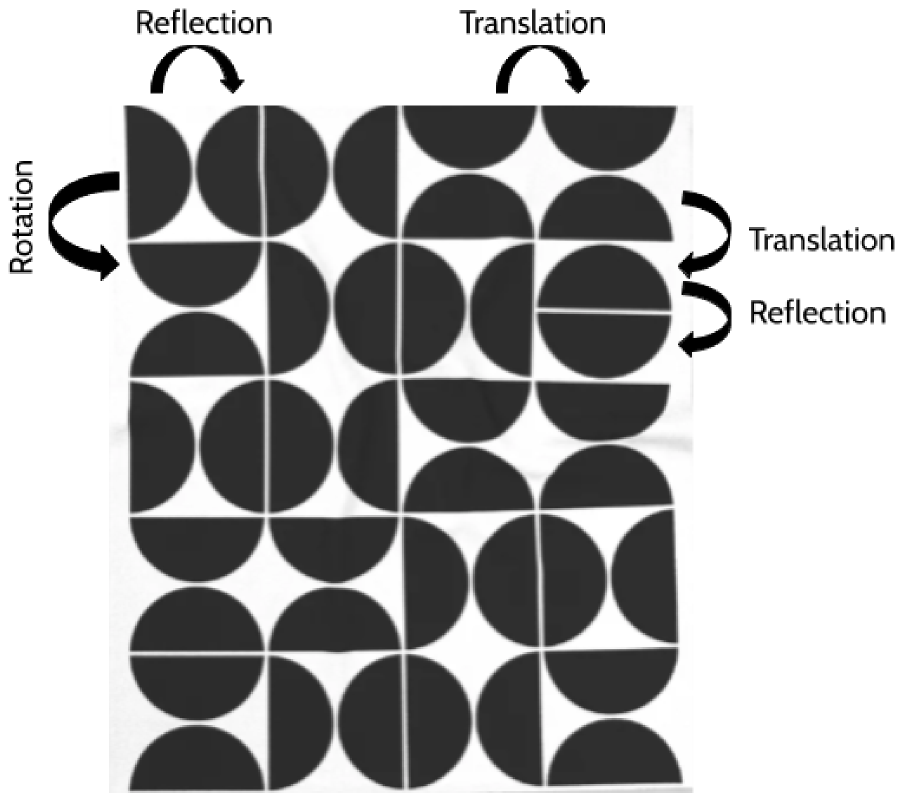
    - **Always** have the same number of side.
    - **Always** have the same shape.
    - Are **always** the same size.
    - Can **always** fit exactly over each other.

    Two shapes that are not congruent to each other:

    - **Sometimes** have the same number of sides.
    - **Sometimes** have the same shape.
    - **Sometimes** are the same size.
    - Can **never** fit exactly over each other.

2) Choice A. Slide. The Pac-Man has slid across the frame.
3) Choice B. Turn. The Pac-Man has turned.
4) Choice C. Flip. The Pac-Man has flipped.
5) This is a rotation. We can turn the pictures to fit exactly over one another.
6) This is a reflection. The two pictures are mirror images of each other. We can flip the pictures to fit exactly over one another. If you fold the page you can fit the pictures exactly over each other.
7) This is a translation. We can slide the pictures to fit exactly over one another.

8) There are many different examples of all three rigid transformations - rotation, reflection, and translation in this blanket. Here are a few examples:



9) There are countless possible quilts you can design by sliding, and reflecting the your design.

10) There are several ways to use the vocabulary to describe what is going on in the M.C. Escher design. Here are some examples:

   - All the horsemen are **congruent** to each other. They are the same size and shape.
   - The white horsemen are **translations** of each other. You can **slide** one of the white horses and it would exactly cover any of the other white horses.
   - The black horseman are also **translations** of each other.
   - The white and black horsemen are **reflections** of each other. You can flip one and it would exactly cover the other.
   - There are no **rotations** or **turns**.
   - It would take a **reflection/flip** and a **translation/slide** to cover a white horseman with a black horseman.

Fruit Graph and Chessboard - Answer Key

1) There are many things you might notice about my fruit preferences. Some examples are:
    - I think pineapples are both the most tasty and most difficult fruits.
    - I think green apples are the least tasty fruit on the list.
    - I think cherries are more difficult to eat than oranges.
    - Bananas are in the middle of both of my lists.
2) I think watermelons are tastier than cherries. We can see that watermelons are higher on the tasty/untasty line. They are closer to the word “tasty” than the cherries.
3) I think watermelons are the more difficult fruit to eat. We can see that watermelons are more to the left on the Difficult/Easy line. They are closer to the word “difficult” than the cherries.
4) Both of these fruits are on the left side of the Tasty/Untasty line because I think they are both difficult to eat. The Tasty/Untasty line divides the graph into fruits that are difficult to eat and fruits that are easy to eat. Any fruit to the right of the Tasty/Untasty line is easy to eat (in my opinion). Any fruit to the left of the Tasty/Untasty line is difficult to eat.
5) Both of these fruits are above the Difficult/Easy line because I think they are both tasty. The Difficult/Easy line divides the graph into fruits that are tasty and fruits that are untasty. Any fruit to the above the Difficult/Easy line is tasty (in my opinion). Any fruit below the Difficult/Easy line is untasty.
6) Pineapples, watermelons, cherries, and oranges are all in the tasty and difficult section of the graph. Bananas are tasty, but they are right on the line between difficult and easy.
7) There are several possible things to say about my opinion on strawberries. Some examples are:
    - I think they are the easiest fruit to eat on the list.
    - They are the second tastiest (second only to pineapple).
    - Of the fruit I think is easy to eat, strawberries are the tastiest.
8) There are several possible things to say about my opinion on apples. Some examples are:
    - I think red apples are tastier than green apples. We can tell because the red apples are above the Difficult/Easy line and the green apples are below.

  - I think green apples and red apples are the same in terms of how easy they are to eat. We can tell because they are the same distance to the right of the Tasty/Untasty line.
  - Red apples are the third tastiest fruit and the third easiest to eat.

9) There are several possible things to say about my opinion on bananas. Some examples are:
  - I think bananas are tasty.
  - Bananas are right on the line Tasty/Untasty line, between Difficult and Easy. That means I don't think they are easy or difficult. They are in the middle.
  - I think bananas taste better than cherries, grapes, oranges, lemons, and green apples.
10) My opinion puts the lemon at the place where the two lines cross. That means it is between tasty and untasty and between difficult to eat and easy to eat. You might say that I don't really like or dislike lemons. I don't have much opinion about them either way.
11) ♕b3
12) ♖e6
13) ♗c6
14) ♞d4

Note: Your answers for question 15-18 can be in any order.

15) ♟f5
16) ♟f3
17) ♟h5
18) ♟h3
19) The coordinates of the eight positions of the knight are:

f7 g6 g4 f3 d3 c4 c6 d7

(Your answers do not have to be listed in this order.)

The Coordinate Plane - Answer Key

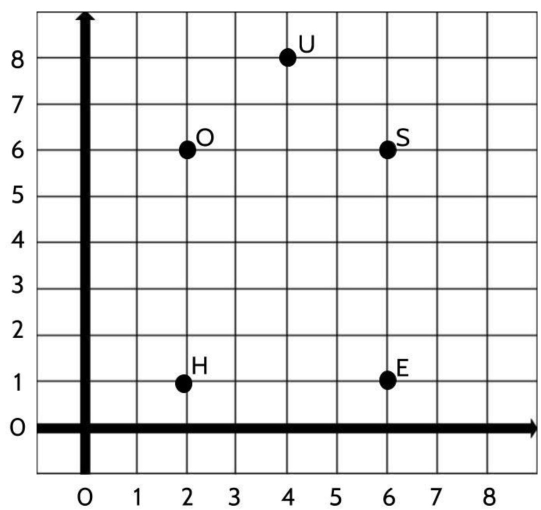
1) Point A: (2,8)

   Point B: (6,8)

   Point C: (4,5)

   Point D: (4,1)

2)



3) Choices A and E are true.

   Choice B is false because Point M and Point H are 7 units apart (not eight).

   Choice C is false because Point A and Point T have the same y coordinate, but they have different x coordinates.

   Choice D is false because the ordered pair for Point M is (1, 2)

4) Point (7, -3) is in Quadrant 4 because I start in the origin and move 7 units to the right and 3 units down.
   Point (-3, -6) is in Quadrant 3 because I start in the origin and move 3 units to the left and 6 units down.
   Point (16, 25) is in Quadrant 1 because I start in the origin and move 16 units to the right and 25 units up.

Point (-10, 3) is in Quadrant 2 because I start in the origin and move 10 units to the left and 2 units up.

5) Point M: (-6,-4)

   Point L: (-2, -2)

   Point H: (-5, 6)

   Point K: (-4,2)

   Point G: (2, -4)

   Point F: (6,0)

   Point E: (0,3)

   Point C: (5,4)

   Point A: (2,6)

   Point B: (5,7)

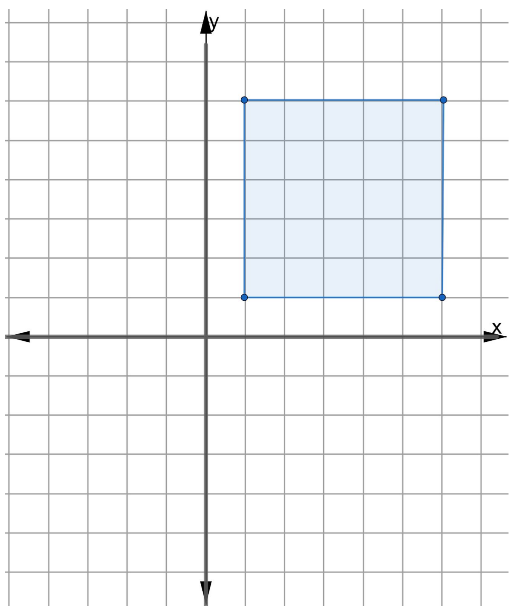
Take a look at Point E and Point F. We can read the coordinate for Point E as moving 0 units to the right and 3 units up. Point F can be read as moving 6 units to the right and 0 units up.The value of x will be zero if the point is on the y-axis. Similarly, the y value in the ordered pair will be zero when the point is on the x-axis.

Shapes and Lines on the Coordinate Plane - Answer Key

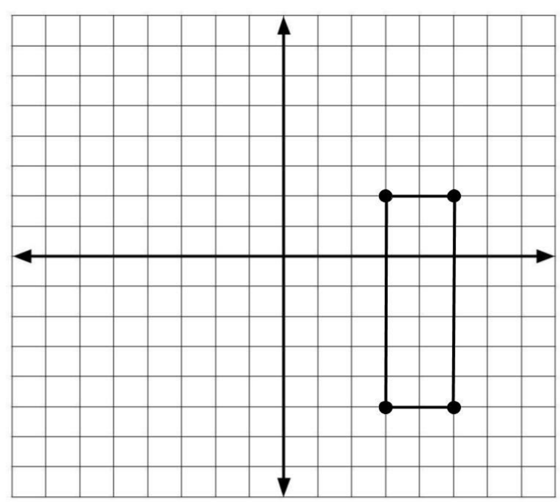
1) Point C is (4, -4)
2) There are many different ways to plot four points to form a square on the coordinate plane. There are two things to pay attention to:

   1 - The points should be in order, clockwise or counterclockwise, CDEF.

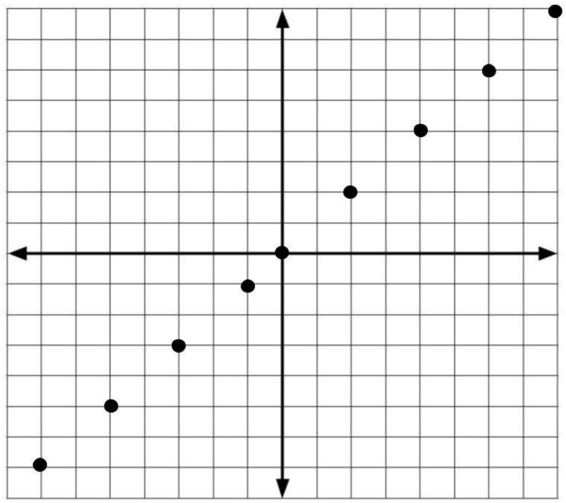
   2 - The distance of each side of the square should be the same.
3) The two given points are 5 units away from each other. Both choices B and D would form the remaining sides of a square.



4) The perimeter of the rectangle would be 18 units.

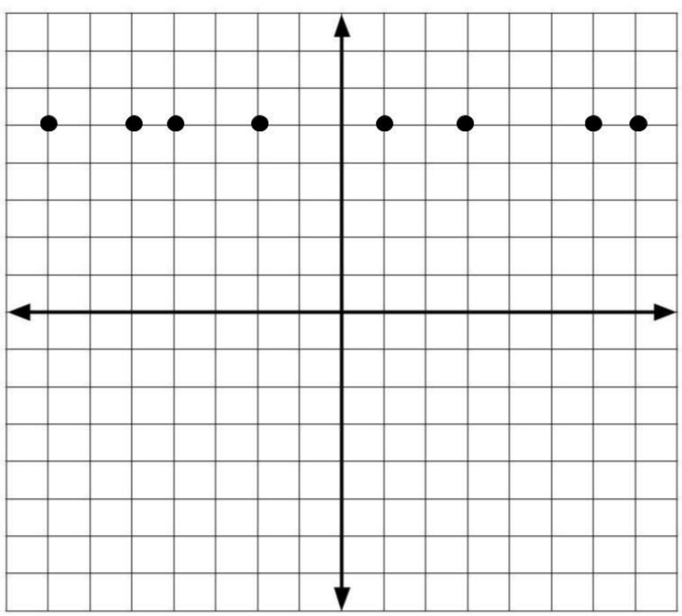


5)



6) $y = x$ would pass through four of the points: (-76, -76), (3, 3), (24,24) (-9,-9). We can tell because the x and y coordinate is the same. In the ordered pairs (16, -16) and (5, -5) the x and y values are not the same. (16, -16) and (5, -5) would be in Quadrant 4. $y = x$ goes through Quadrants 1 and 3.

7)



8) Line $y = 6$ would pass through (32, 6) and (-19, 6). Both of those points have 6 as the y coordinate.

9)
- There are many correct answers. (3, 0), (3 ,3), (3, -2), and (3, -4) are all examples of points on this line.
- All points on the line have 3 as the value of the x coordinate.
- This line is parallel to the y-axis, because each point is 3 units away from that axis.
- I think the equation that names this line is $x = 3$ because it refers to a line where every ordered pair has a 3 for the *x* coordinate.

10) The equation of the line is y = -2. The y coordinate for every ordered pair on the line is -2. The line is two units below the x-axis.

11) The equation of the line is x = -1. The x coordinate for every ordered pair on the line is -1. The line is 1 unit to the left of the y-axis.

How Math is Written - Answer Key

1) This is a reflection or a flip.
2) The original figure can be called Triangle ABC or △ ABC.
3) Point A maps onto Point **A'.**

   Point B **maps** onto Point B'.

   Point C **maps** onto Point **C'.**

   △ABC is **congruent** to △A'B'C'.

There are different answers to explain why △ABC is congruent to △A'B'C'. Some possible answers are:

- △A'B'C' was produced by a rigid transformation and any image produced by a rigid transformation is congruent to the original shape.
- Each point on △ABC maps to the points in △A'B'C'.
- △ABC and △A'B'C meet our definition of congruent. They are the same size and shape. Also, if you moved them on top of one another, one of them would exactly cover the other one.