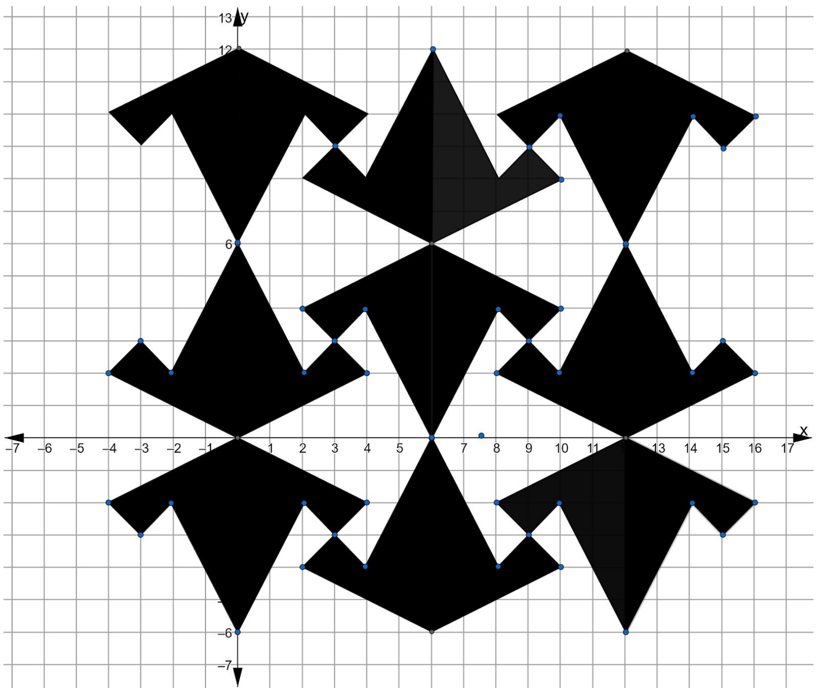
Rigid Transformations: Shapes on a Plane

Fast Track Math GRASP Packet

Part 2



Based on a detail of tile work from the Alhambra Palace in Granada, Spain

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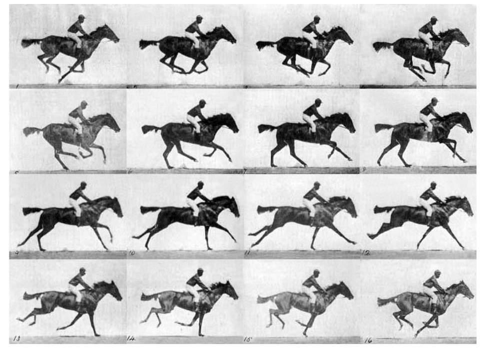
Introduction



Movies create the illusion of movement by showing us 24 images per second. When the human eye processes 24 images per second it is interpreted in our brains as movement.

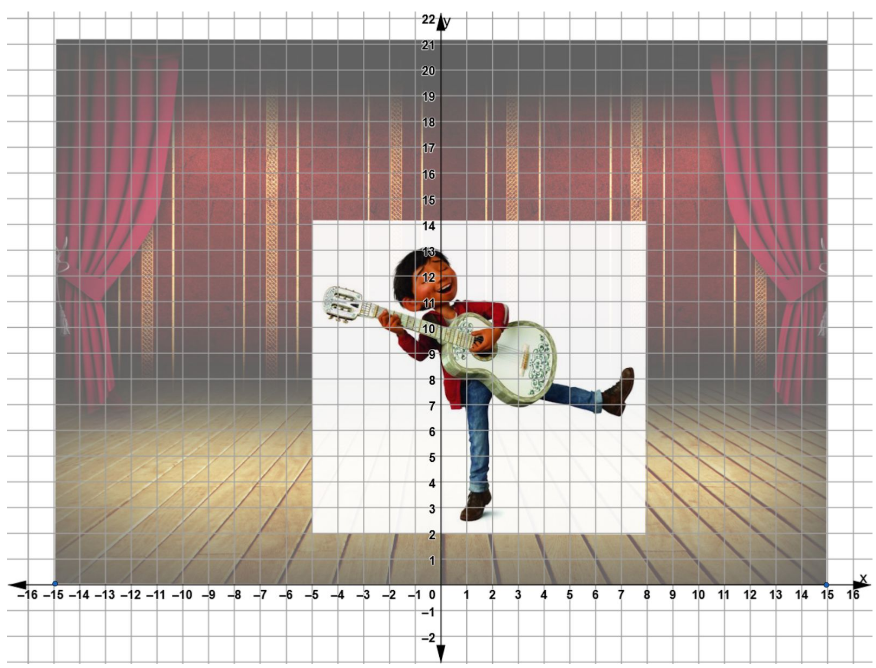
Computer animated films work the same way as other films in the sense that they are a series of single images played together quickly.

Question: Consider the recent film, Coco (2017) created by the computer animation studio, Pixar. That movie is 105 minutes long. If movies are created showing 24 images per second, how many images would you expect it took to make Coco?

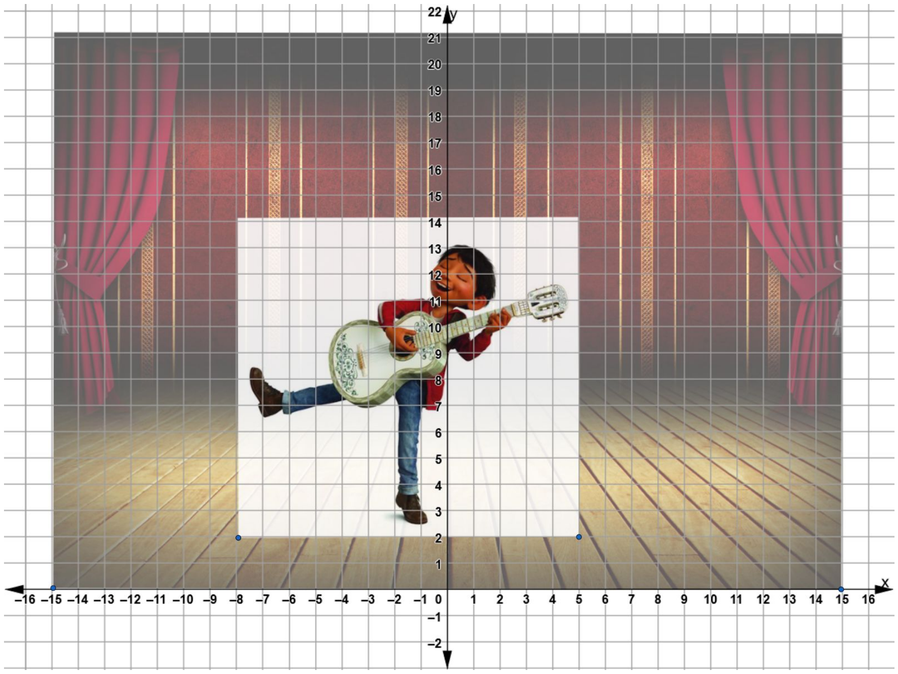
In the introduction, we looked at some examples of rigid transformations in art that were done by hand. In this section, we are going to be looking at the skills needed to use computers to make rigid transformation. The coordinate grid allows human artists to communicate their ideas to computers. Rigid transformations on the coordinate grid are at the heart of computer animation and video game design.

One of the main characters of the movie Coco is a young boy named Miguel who loves music and is always singing and practicing his guitar.

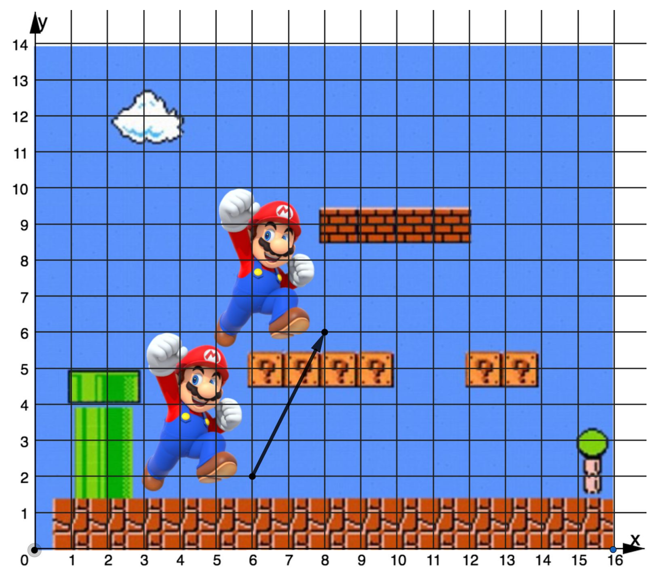
Consider this image of Miguel playing his guitar on stage.



The art team at Pixar can use coordinates to create a reflection of that image across the y-axis, making it appear as though Miguel has moved from his right foot to his left foot.



Rigid transformations on the coordinate grid are also used in the design of video games. On this coordinate grid, the Nintendo character, Mario is being translated 2 units to the right and 4 units up.



In this section you'll be learning how to map translations, reflections, and rotations on the coordinate grid. You'll also have an opportunity to use rigid transformations on the coordinate grid to design and create patterns.

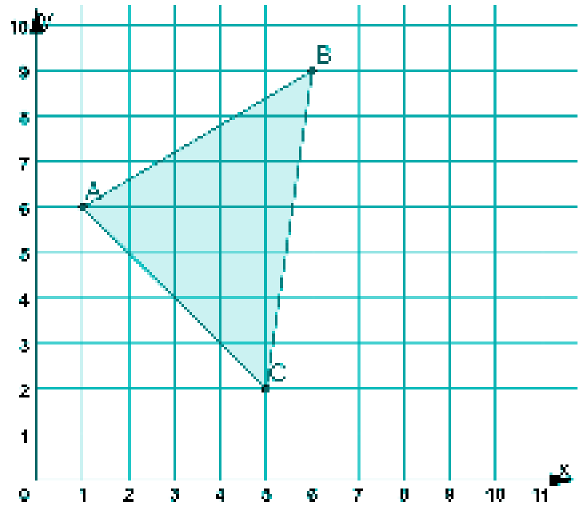
Answer: If Coco is 105 minutes long, we can multiply 105 minutes x 60 seconds for each minute to see that 105 minutes is 6,300 seconds. If film uses 24 frames per second, that's 24 images for each second of the film. 6,300 seconds x 24 images is 151,200. So Pixar had to create 151,200 images to make Coco.

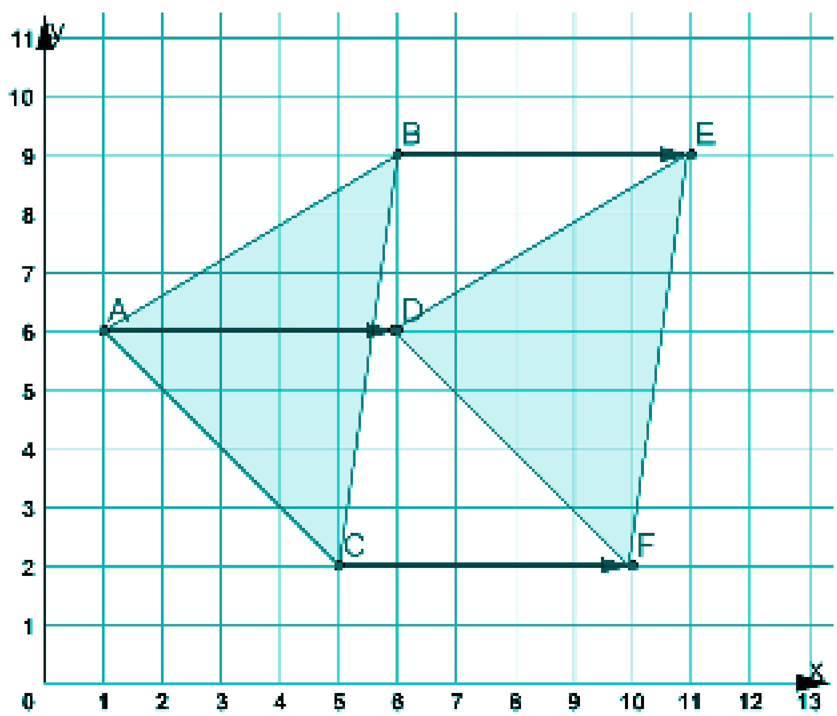
Translations on the Coordinate Plane

Translations involving sliding a point or shape to another location without turning it. As with all rigid transformations, any images created by translation are congruent to the original figure.

One thing you can do with translations on the coordinate grid is take a shape and translate it to make another congruent shape.

Let’s take △ABC.

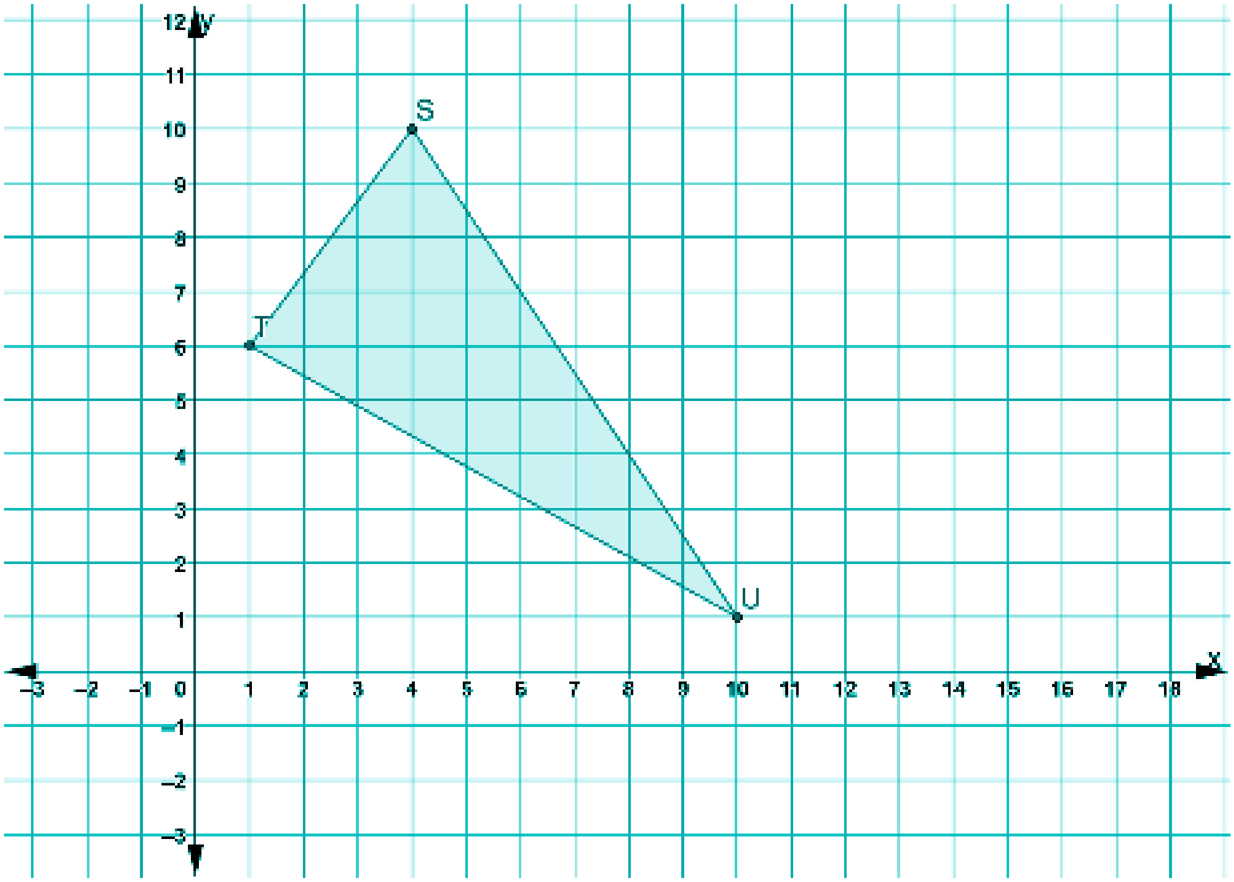




If we slide each point in △ABC 5 units to the right, we make △DEF. △DEF is congruent to △ABC.

Now you try.

1) Translate △STU 6 units to the right to make △S'T'U'.



What are the coordinates for:

Point S (,)

Point T (,)

Point U (,)

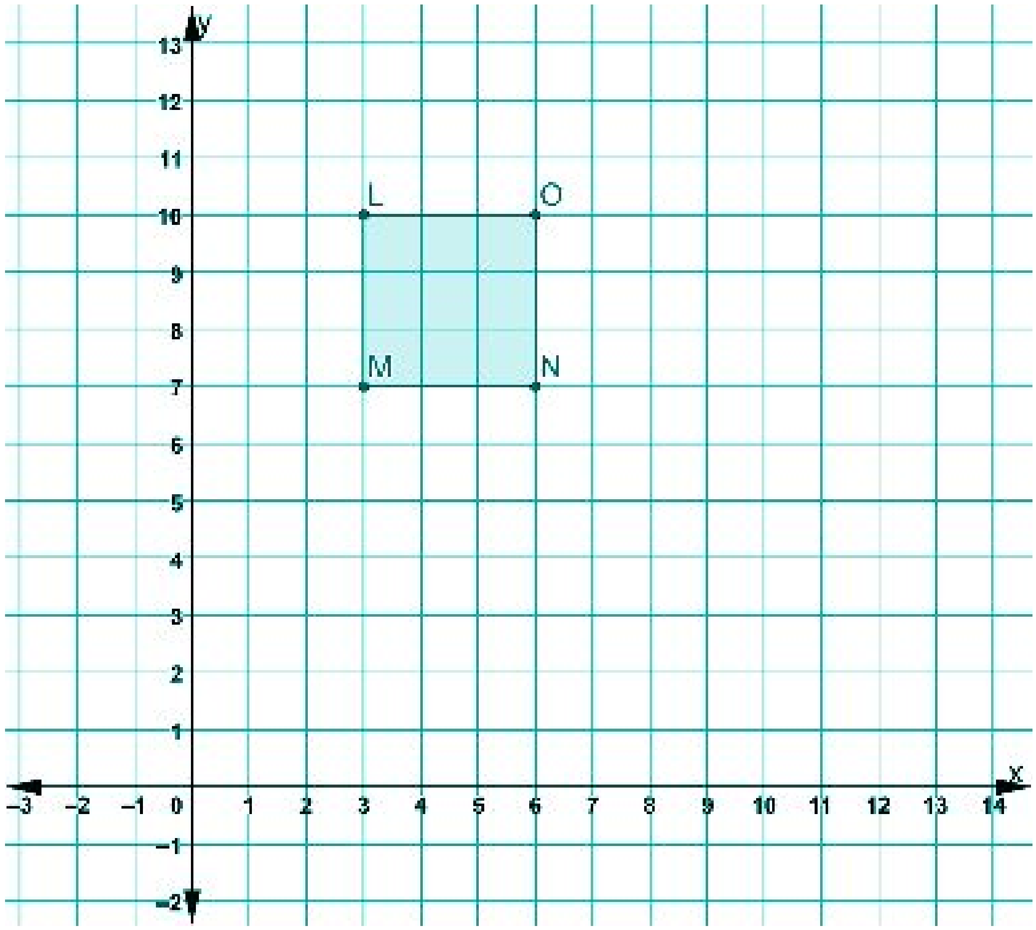
What are the coordinates for:

Point S' (,)

Point T' (,)

Point U' (,)

2) Translate Rectangle LMNO 4 units to the right and 4 units down. Label the points of the translated image L', M', N', O'.



What are the coordinates for:

Point L (,)

Point M (,)

Point N (,)

Point O (,)

What are the coordinates for:

Point L' (,)

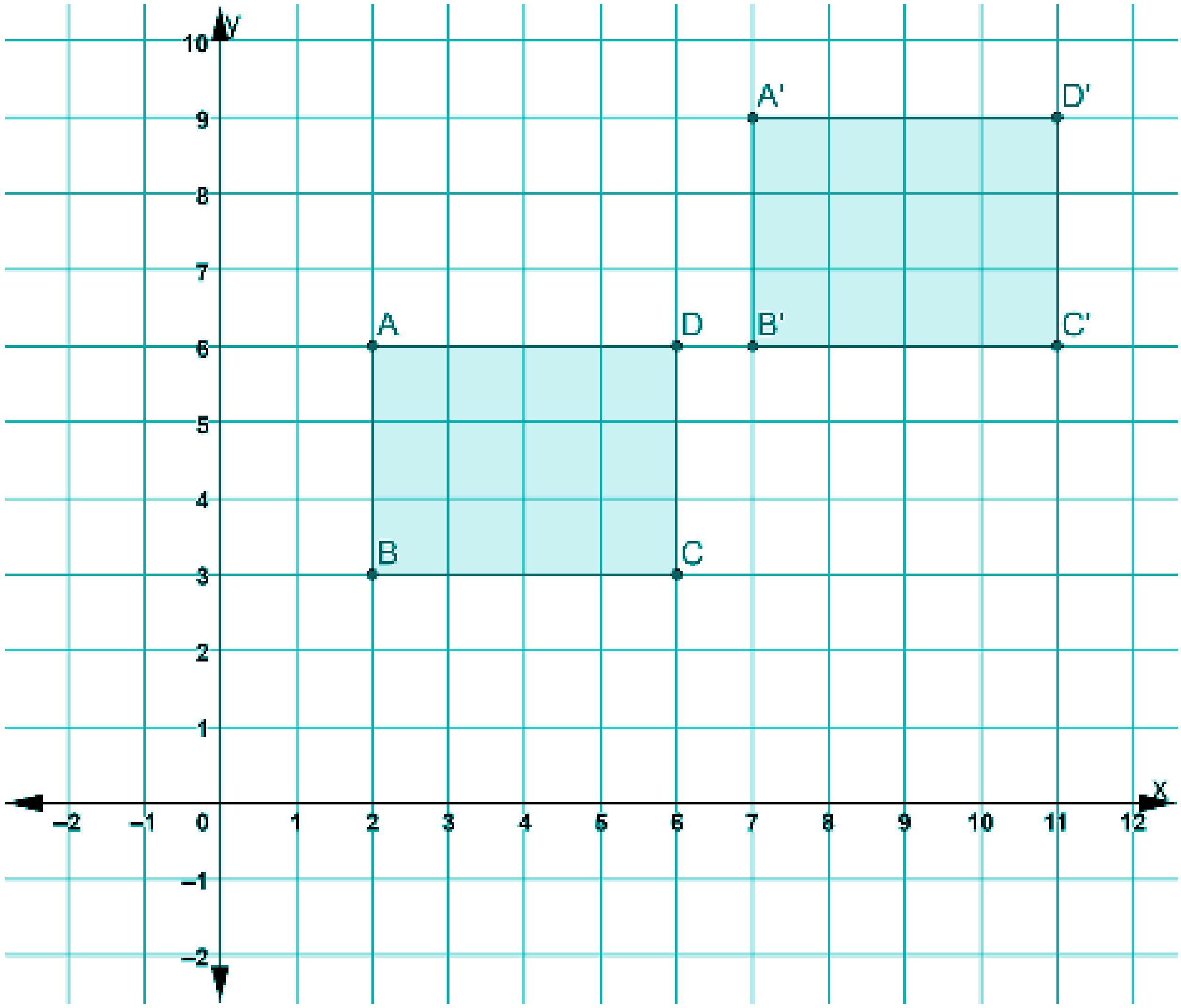
Point M' (,)

Point N' (,)

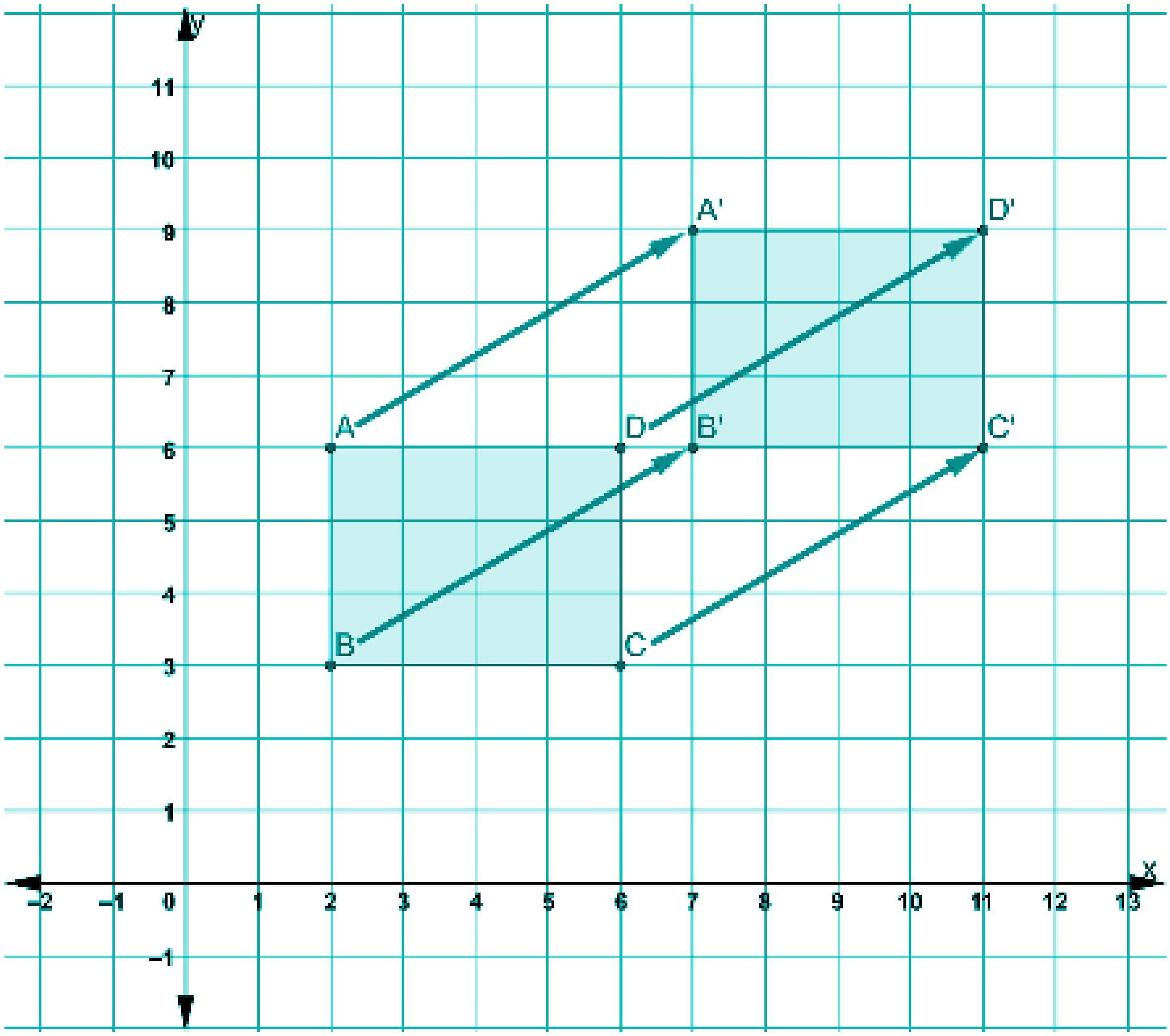
Point O' (,)

Once we understand how translations work, we can look at two congruent figures that have been created by a translation and figure out what the translation was.

Rectangle ABCD maps onto rectangle A’B’C’D’.



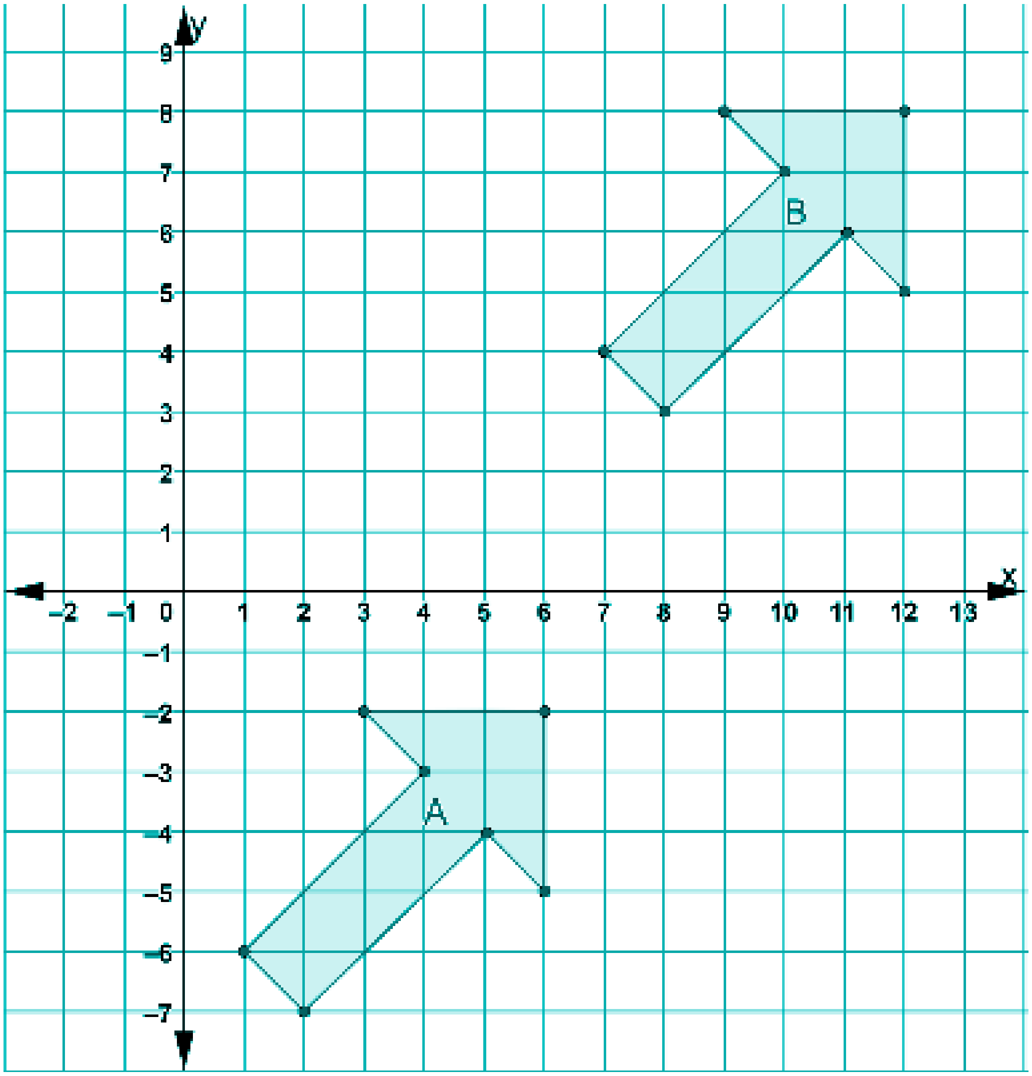
We can tell it is a translation because we can move the figures on top of each other with a slide. But how can we describe what the translation is? How do we get from our original rectangle to its translated image?



The first step when describing a translation is to be clear about which figure is the original figure. In this example, rectangle ABCD is the original and rectangle A'B'C'D' is its translated image.

The next step is to choose one point on the original figure and find the translated point on the image. For example, point C maps onto point C'. To get from point C to point C', we move 5 units to the right and three units up. If you count the units to move from point A to point A', you will see it is the same. In fact, you could use any point on the original figure to see how it moves onto its translated image. In the coordinate grid above, each point in rectangle ABCD has been translated 5 units to the right and 3 units up to make congruent rectangle A'B'C'D'. Note that when we write the directions for a translation, we give the horizontal (right/left) directions first and then the vertical (up/down).

Now you try.



3) Which translation can be used to map figure A to figure B?
   A) A translation 2 units to the right and 5 units up
   B) A translation 2 units to the left and 5 units down
   C) A translation 6 units to the left and 10 units down
   D) A translation 6 units to the right and 10 units up

Creating Designs with Translations on the Coordinate Plane

4) Textile designers can use translations to create designs and patterns in clothing, fabric, tiles, etc.

    a) On the coordinate plane on the next page, translate each point of the original figure 2 units to the right and 6 units up. One point has already been done.

        Label the translated image, “Figure B”.

    b) Now translate Figure B two units to the right and six units down.

        Label this image, Figure C.

    c) Now translate Figure C two units to the right and six units up.

        Label this image Figure D.

    d) Next translate Figure D four units to the right.

        Label this new image Figure E.

    e) Next translate Figure E two units to the left and 6 units down.

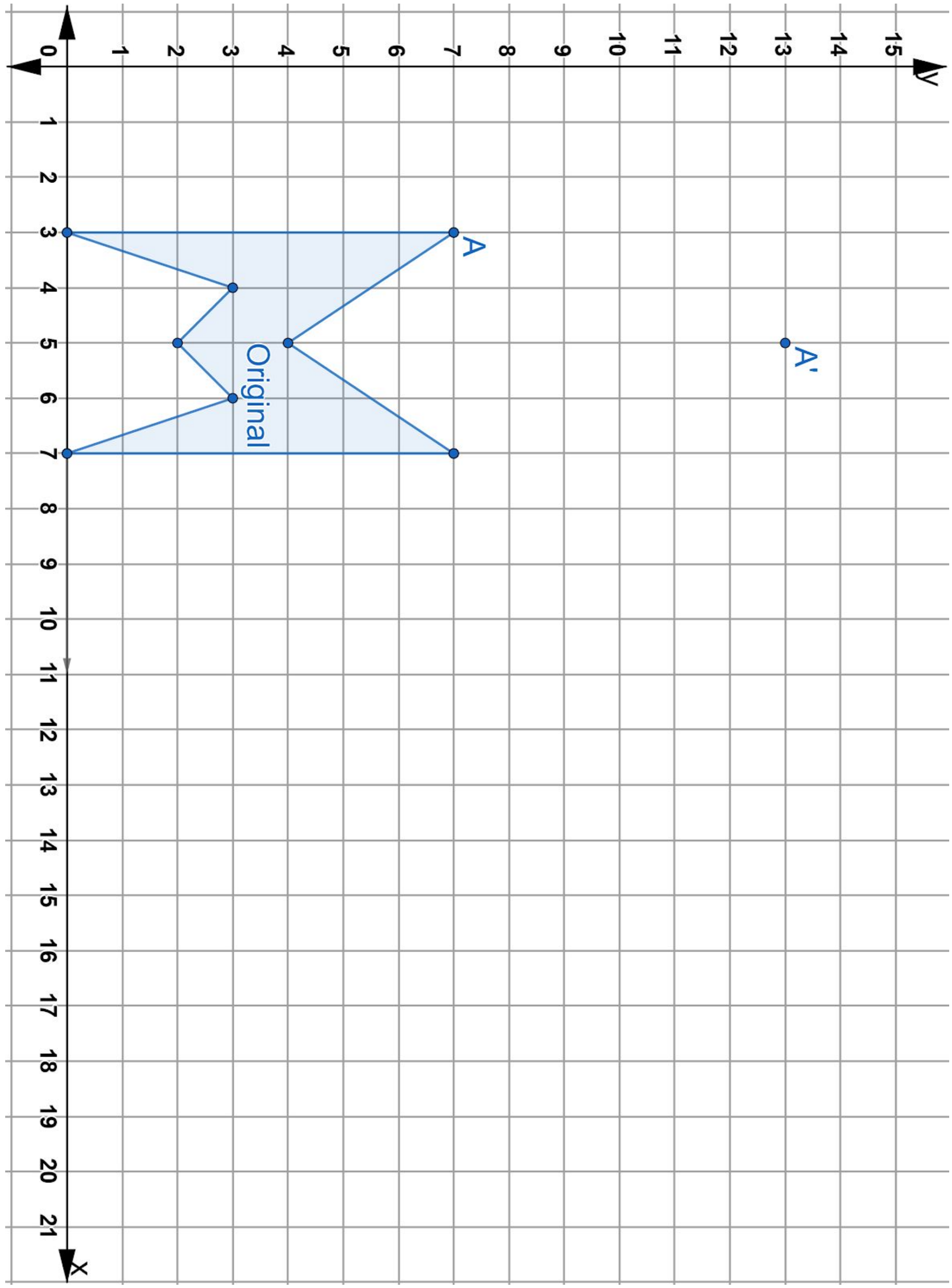
        Label this new image Figure F.

    f) Finally translate Figure F four units to the right.

        Label this new image Figure G.

You should end up with seven congruent figures.

Feel free to color the figures in before moving on

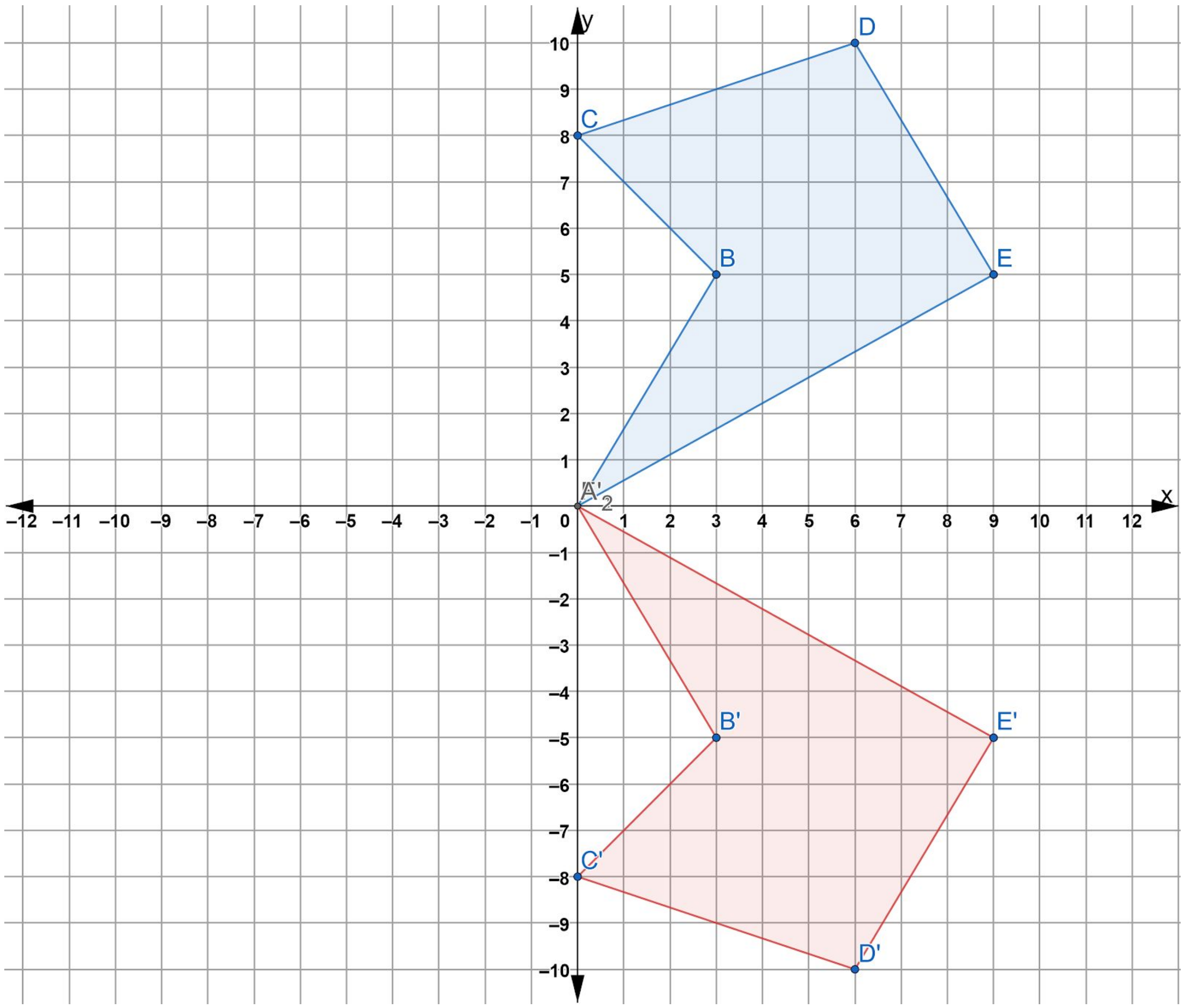

Original
A
A'
x
y

Reflections on the Coordinate Plane

Remember that reflections involve flipping a point or a shape about a line of reflection. You will now learn to use the coordinate plane to do two things:

(1) use a line of reflection to create an image of a shape, and
(2) to use an original figure and an image to find the line of reflection.

The coordinate plane below shows original figure ABCDE being reflected about the x-axis to create image A'B'C'D'E'. If you folded this paper along the x-axis, the two shapes would exactly fit over one another. In this case, the x-axis is the line of reflection.



Write three things you notice on the coordinate grid.

1) Use the coordinate plane on the previous page to complete the following statements:

   a) Point A and Point A' are both 0 units away from the line of reflection.

   b) Point B and Point B' are both ____ units away from the line of reflection.

   c) Point C and Point C' are both ____ units away from the line of reflection.

   d) Point D and Point D' are both ____ units away from the line of reflection.

   e) Point E and Point E' are both ____ units away from the line of reflection.

Note that each point in the original figure is the same distance from the line of reflection as the corresponding point in the image. For example, point B is 5 units above the line of reflection and point B' is 5 units below the line of reflection. Another way to look at this is to say that points B and B' are 10 units away from each other, and the line of reflection must be half way between them.

We can measure the distance between each point in the figure to the line of reflection. The image of each point will have to be the same distance away from the line of reflection.

2) If you reflected point B' across the y-axis, the reflected point B" would appear in Quadrant 3. What are the coordinates of point B"? How do you know?

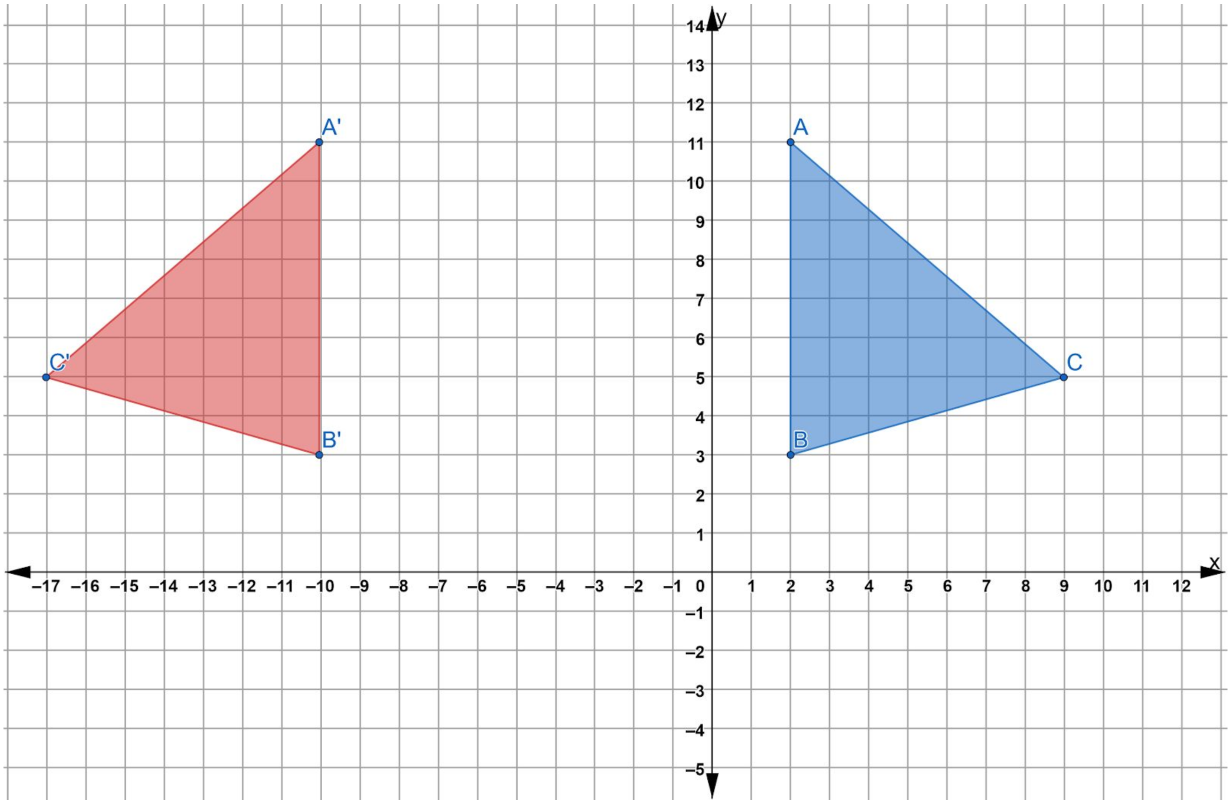
3) Reflect the remaining points of the shape in Quadrant 4 across the y-axis on the coordinate plane.

   What are the coordinates of point E"?

4) Now reflect figure A"B"C"D"E" over the x-axis. Your new image will be figure A'"B'"C'"D'"E'" and should appear in Quadrant 2.

   What are the coordinates of point D'"?

5) Figure $\triangle ABC$ below has been reflected about a line of reflection resulting in image $\triangle A'B'C'$. Draw the line of reflection and write the equation for the line on the coordinate plane below.



HINT: You can start by finding the distances between the points in the original figure and the points on the image.

6) What do you want to remember about reflecting figures and points across lines of reflection on the coordinate grid?

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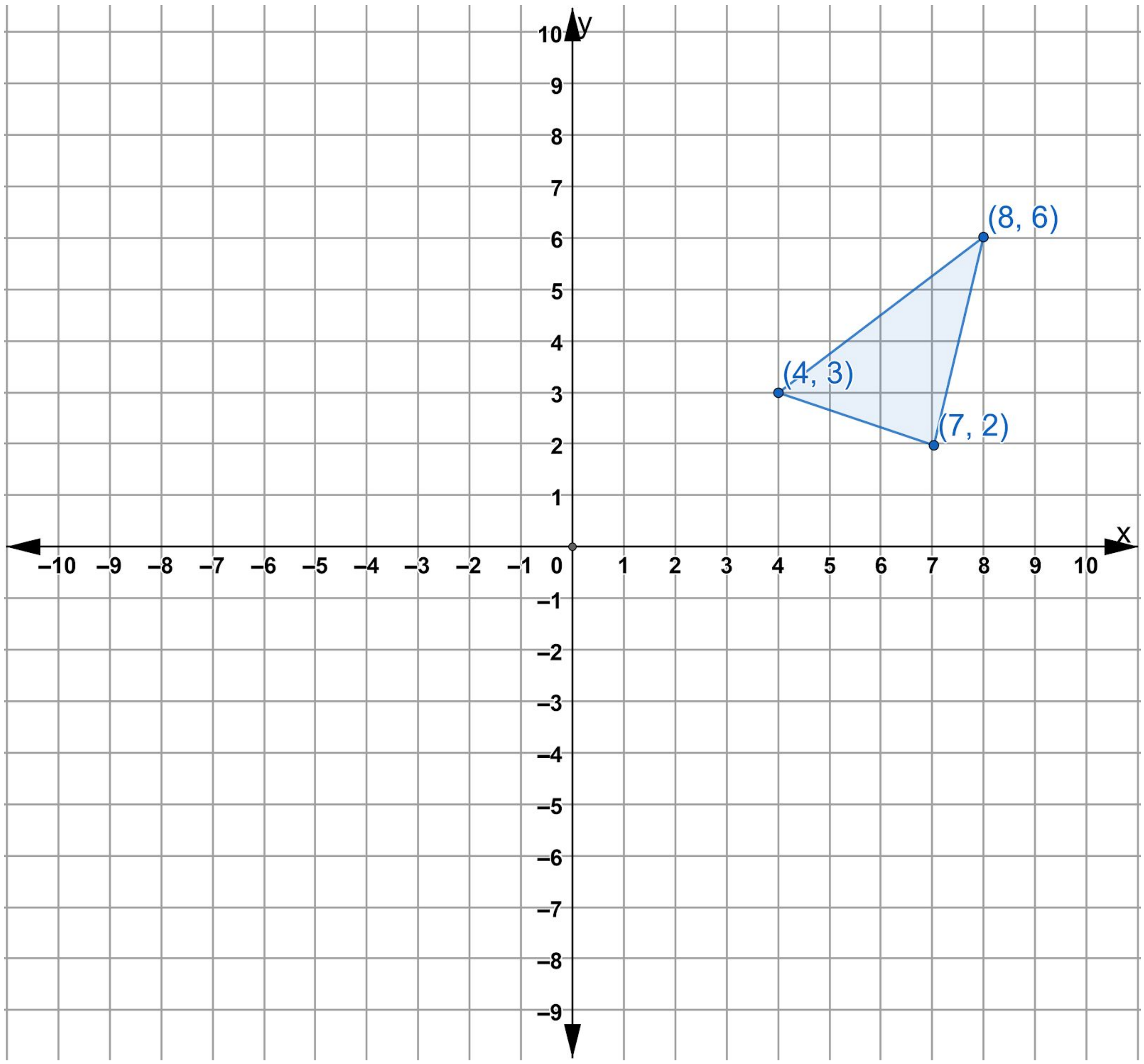
This next activity will help you see what happens to points and shapes when they are reflected across the x-axis, the y-axis, or the line *y=x.*

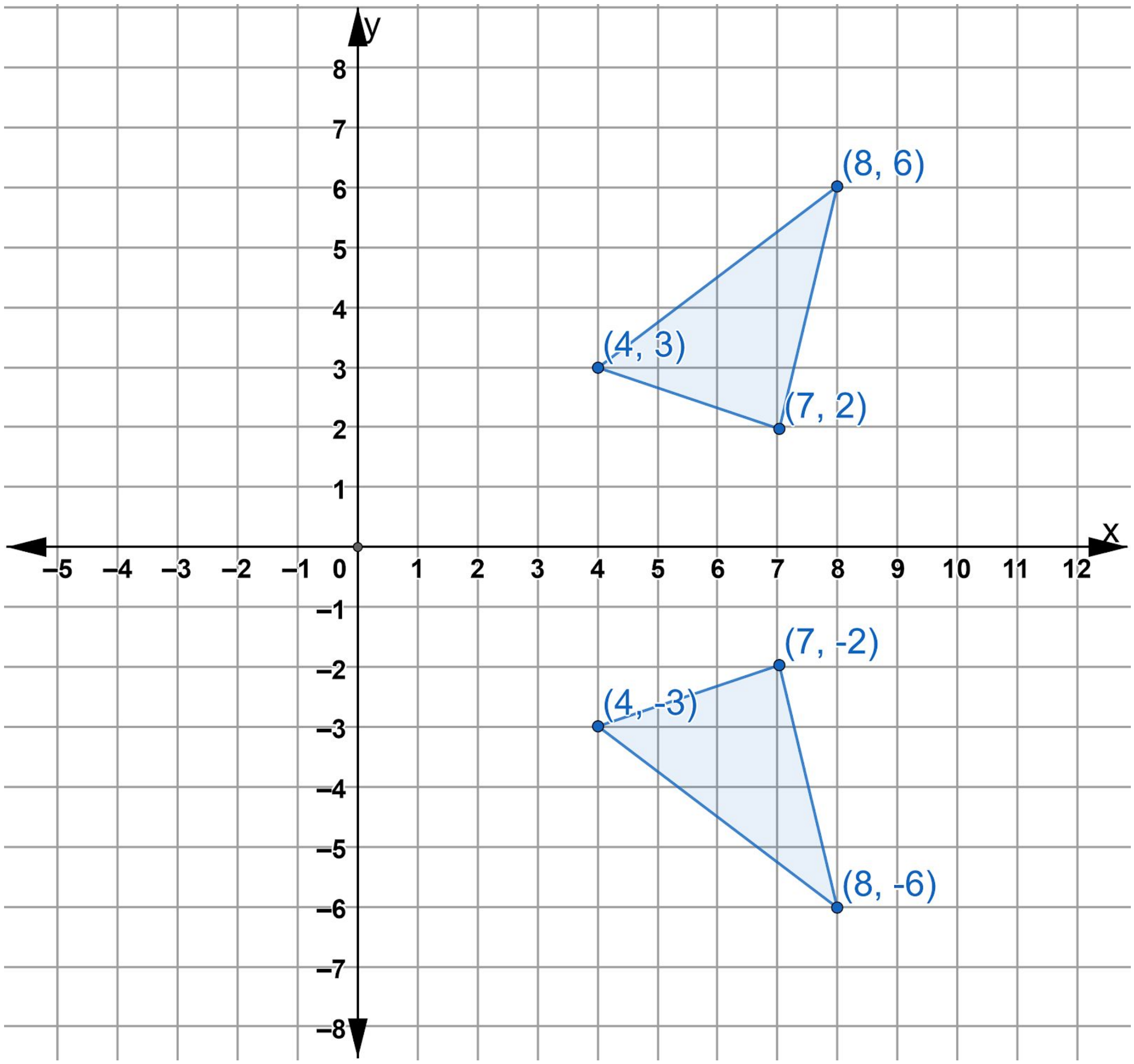
Consider the triangle on the coordinate grid below.



Reflect the triangle across the x-axis (from Quadrant 1 into Quadrant 4). Write the ordered pair next to each reflected point.

Then reflect the original triangle across the y-axis (from Quadrant 1 into Quadrant 2). Write the ordered pair next to each reflected point.

Let's focus on the reflection over the x-axis first.



Notice the coordinates of the reflected image.

Original Triangle	Corresponding Points in Reflected Image
(4,3)	(4,-3)
(8,6)	(8,-6)
(7,2)	(7,-2)

How are they similar to the original coordinates? ______________________________

____

____

How are they different? ____

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____

The Relationship Between Points Reflected Over the X-axis

When the triangle is reflected over the x-axis, what is similar is the x coordinates and what is different are the y coordinates. The x coordinates remain the same and the y coordinates of the image become the opposite of the y coordinates in the original figure.

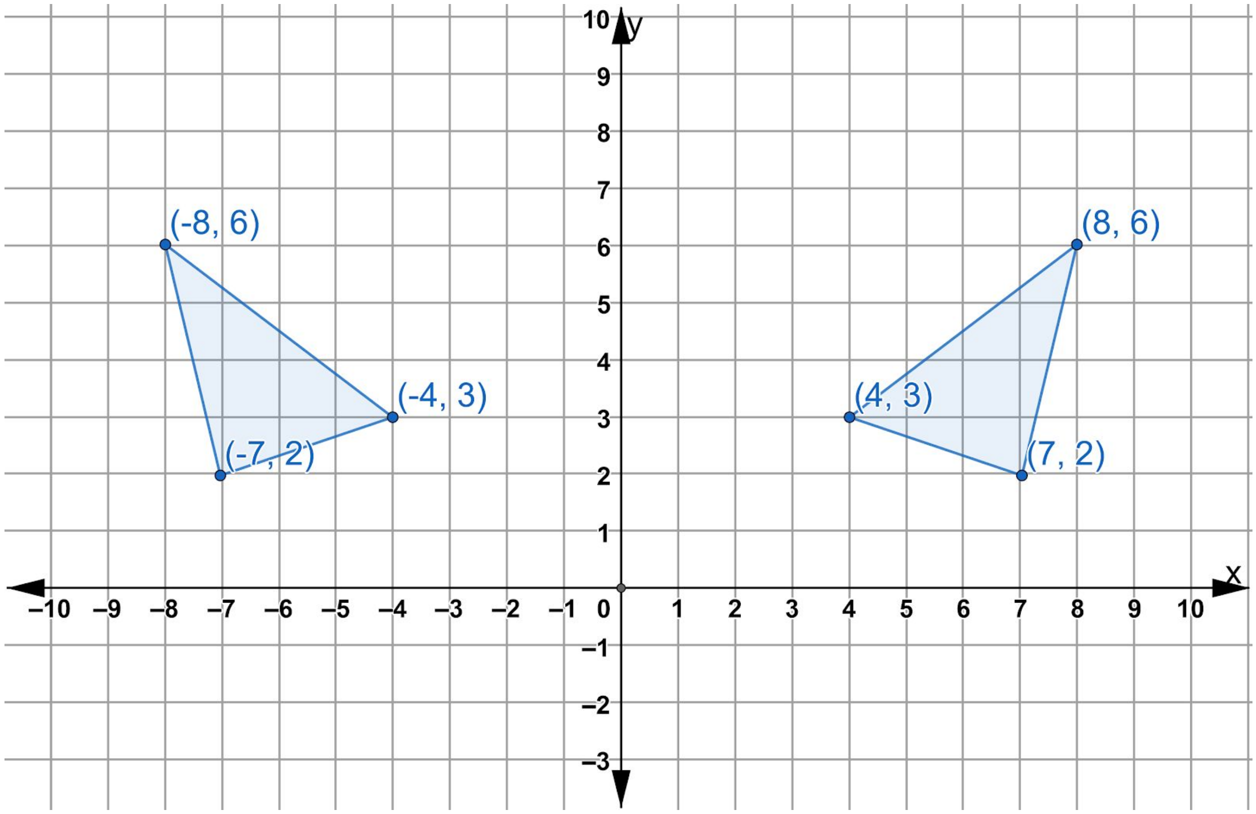
The original triangle here is in Quadrant 1. When we reflect it over the x-axis, it flips into Quadrant 4. Let's look at each specific point in the original triangle.

(7,2) is 7 units to the right of the origin and 2 units above the x-axis. If we reflect it across the x-axis, it will still be 7 units to the right of the origin. But since the original point is 2 units above the x-axis, the image point needs to be 2 units below the x-axis.

The same will be true for (8,6). Since (8,6) is 6 units above the x-axis, then its image needs to be 6 units below the x-axis. So (8,6) reflected over the x-axis becomes (8,-6).

(4,3) is 3 units above the x-axis, so its image reflected across the x-axis would be 3 units below the x-axis. (4,3) reflected over the x-axis becomes (4,-3).

Let's see what happens when we reflect the triangle over the y-axis.



Now the y coordinates stay the same and the x coordinates in the image become the negative of the x coordinates in the original. (4,3) becomes (-4,3). (8,6) becomes -8,6). (7,2) becomes (-7,2)

7) Describe what is happening to each point to explain why this happens.

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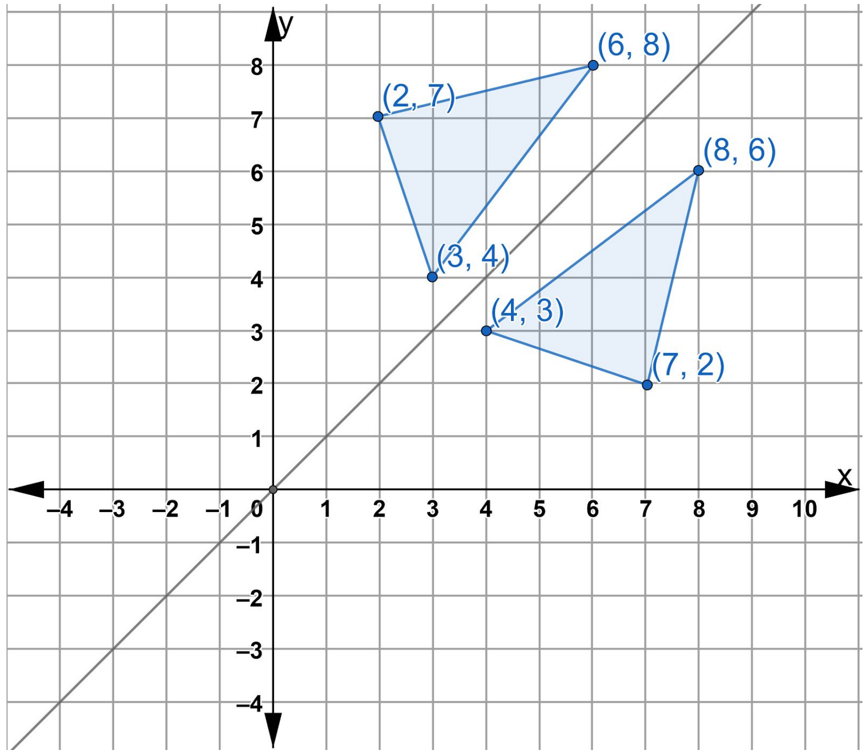
Let's look at one more reflection. This time let's see what happens when we reflect our original triangle over the line *y=x*.



What relationship do you see between the original points and their images reflected over *y=x*? ____

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____

____

____

____

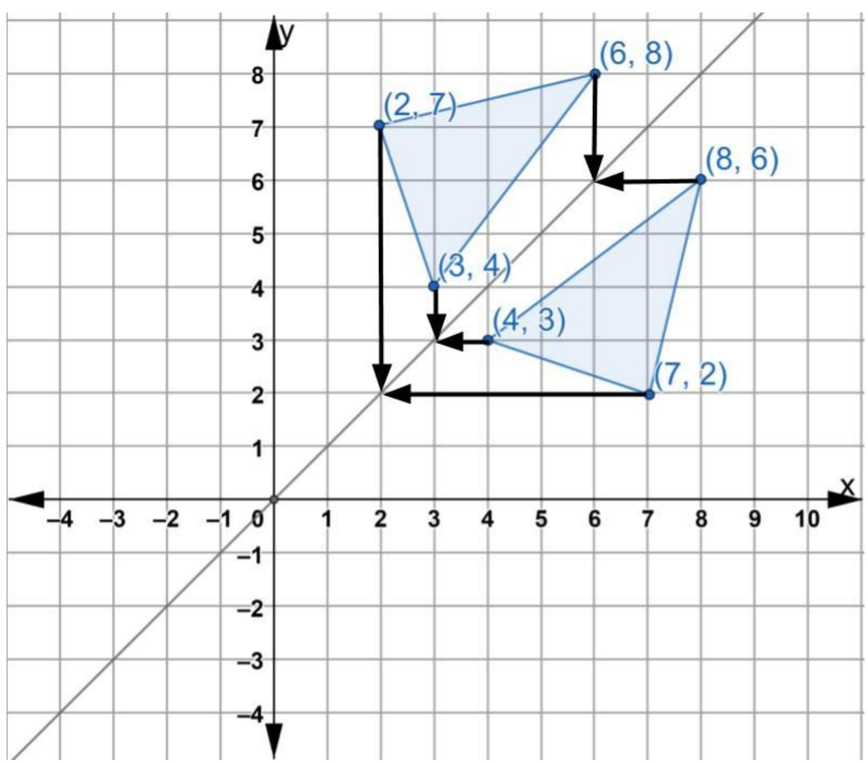
The Relationship Between Points Reflected Over the Line *y=x*

When a point or a shape is reflected across the line *y=x* the coordinates of the x and y coordinates switch.

Point (4,3) becomes (3,4), point (8,6) becomes (6,8) and point (7,2) becomes point (2,7).

To help us see why, let's think about how we can figure out the distance between each point and the line of reflection *y=x*.

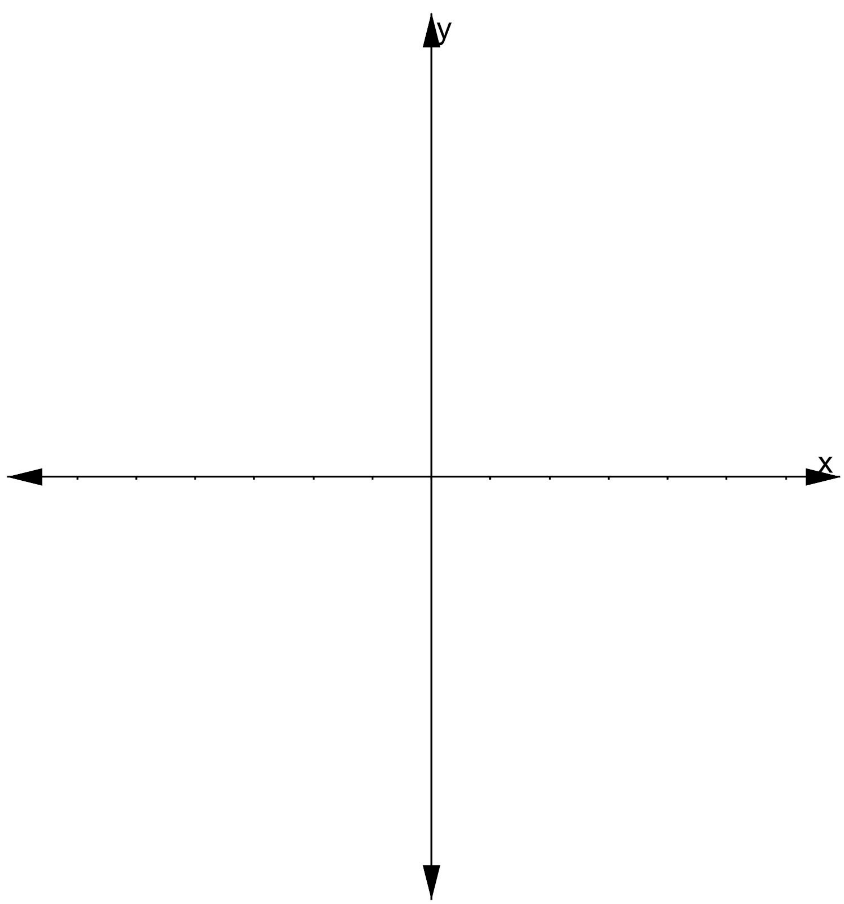
The line *y=x* is a diagonal line, so it is a little different when we want to find the distance between the points and the line of reflection. One strategy you can use is shown in the diagram below. To move from point (7,2) to the line of reflection, we move 5 units to the left. To move from the image point, we need to move 5 units down, forming an L shape with arms of equal length. You can do the same with the other points. To move from point (8,6) to the line of reflection, we move two units to the left and the two units up. So the x coordinate will go down by 2 and the y coordinate will go up by 2.



Let's practice what we've learned about reflecting shapes across the x-axis, the y-axis and the line *y*=*x*.

8) Use what you know about reflections to fill in the table below. A coordinate grid is provided to help you sketch out your ideas.

Original Figure	**(7,6)**	**(10,6)**	**(10,1)**	**(7,1)**
Reflected over the x-axis				
Reflected over the y-axis				
Reflected over *y*=*x*				



One last thing we will look at in this section is what happens when a figure goes through a sequence of reflections. In other words, what can we notice about a figure that is reflected more than one time?

9) On the coordinate plane on the next page, plot figure ABCDE.

   A (2,1) B(2,7) C(9,7) D(5,1) E(5, 3)

   Imagine two different sequences of reflections:

   Sequence 1 - Figure ABCDE is reflected about the x-axis and the resulting image is reflected about the y-axis, or

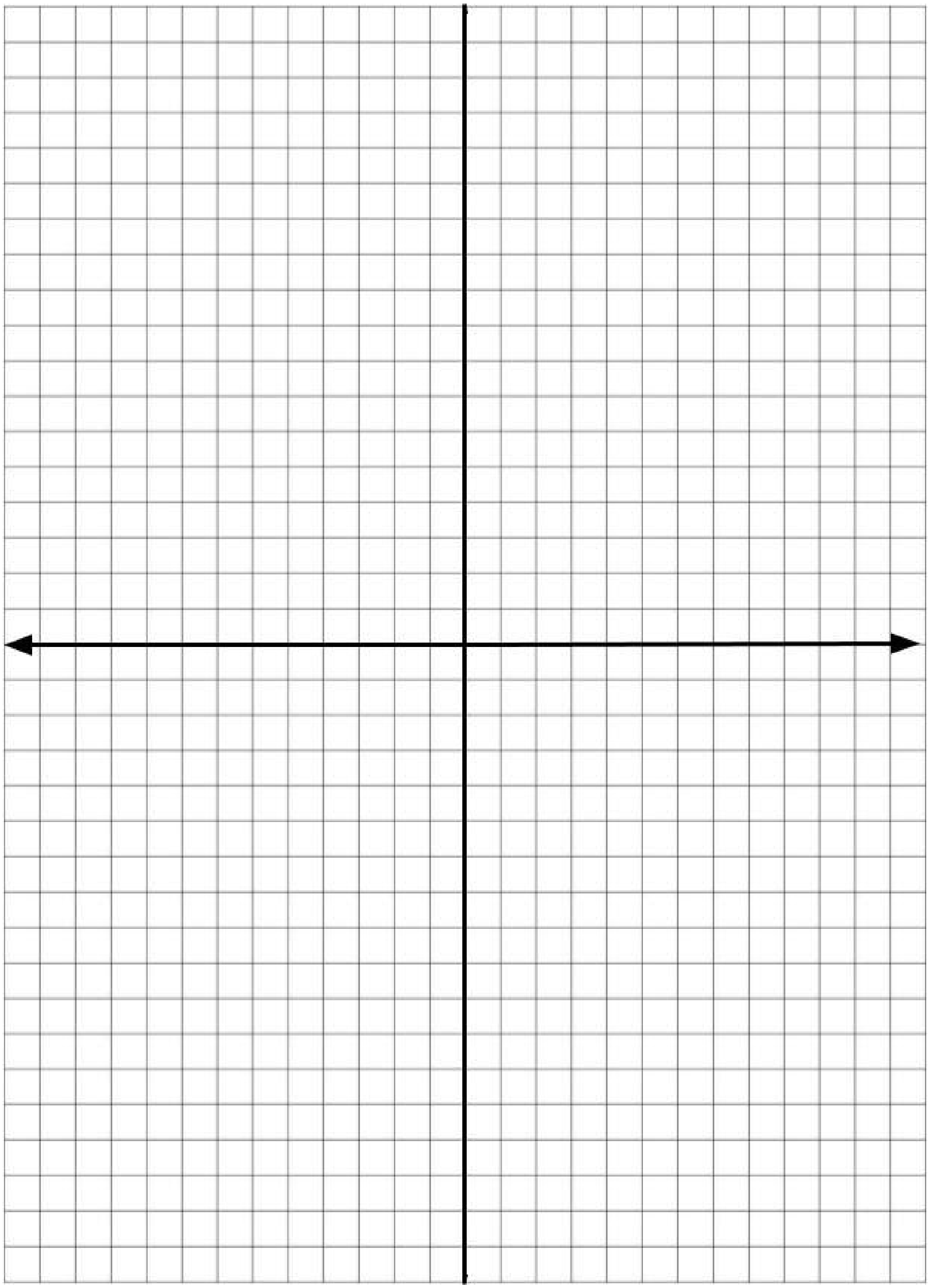
   Sequence 2 - Figure ABCDE is reflected over the y-axis first and then the resulting image is reflected over the x-axis.

   Would the final image in each sequence be different or the same?

   What do you think will happen? (Circle the statement you agree with)

   I think the final figures will be the same. **I think the final figures will be different.**

   Now use the coordinate plane on the next page to try out both sequences!

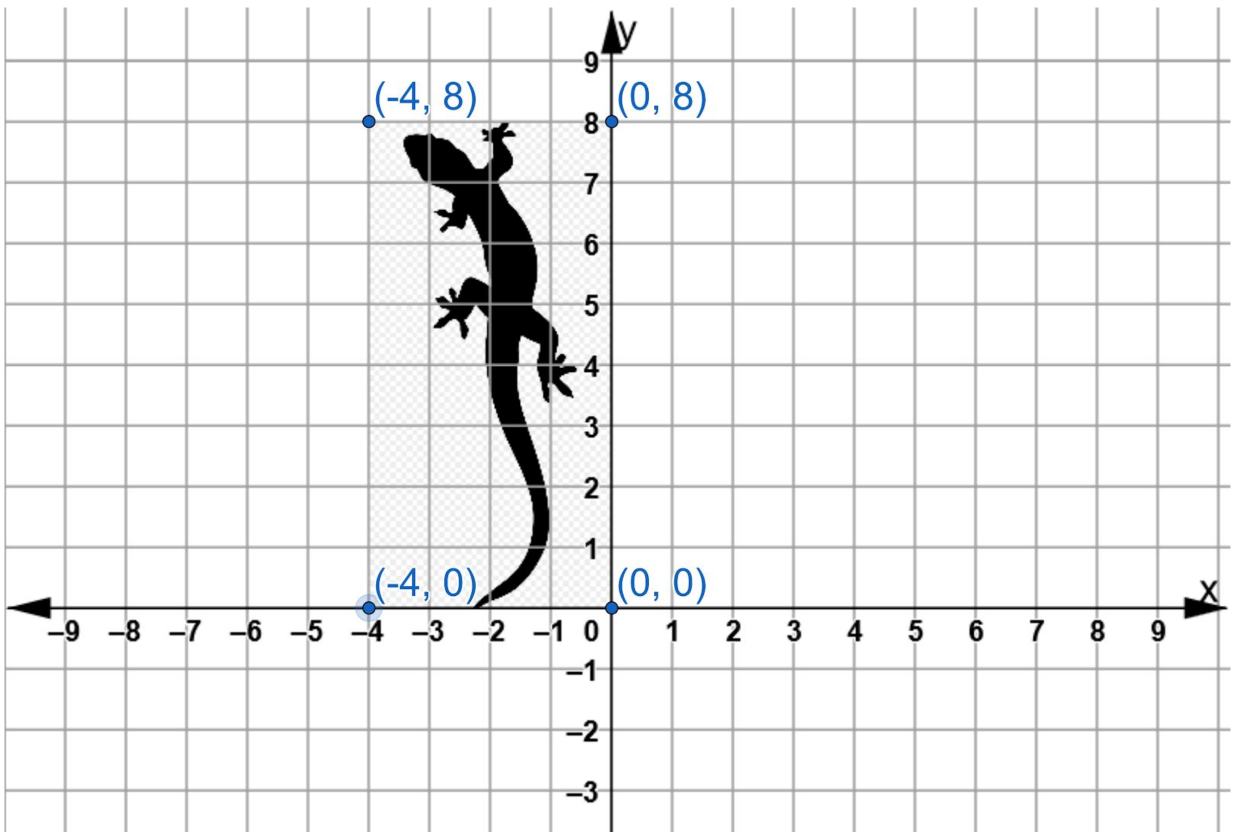


Rotations on the Coordinate Plane

Rotations are the third type of rigid transformation. In a rotation, all of the points in the original figure rotate, or turn the same number of degrees around a fixed point. As with all rigid transformations, the rotated image is congruent to the original figure.

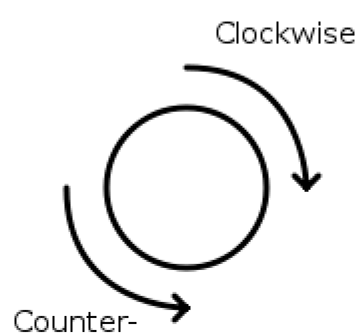
In this section, we will explore rotations on the coordinate plane, using the origin as the fixed point of rotation.

We have placed an image of a lizard in Quadrant 2 of this coordinate plane. We used a rectangle to make an outline around the lizard.

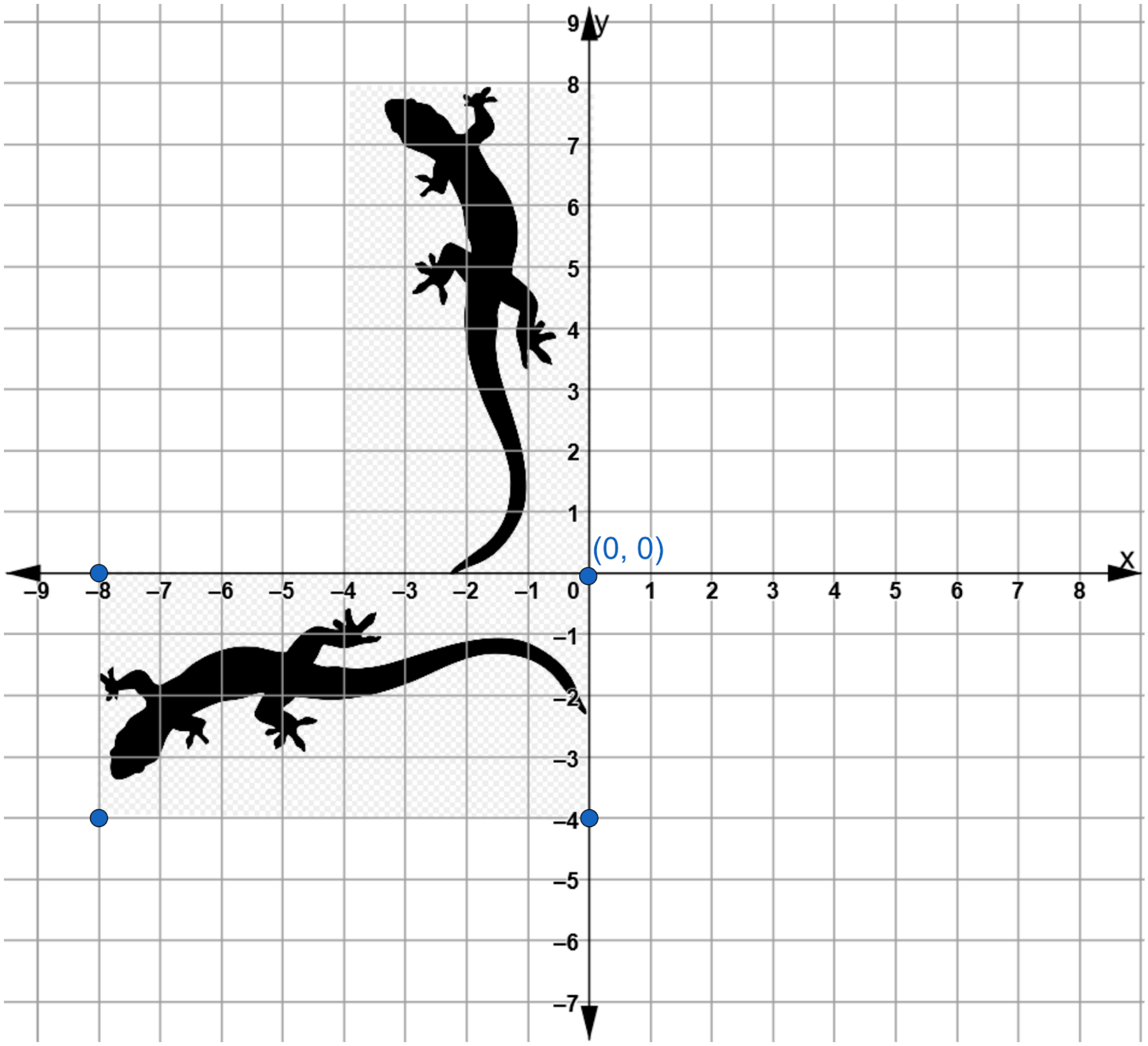


On the next few pages we will rotate this image counterclockwise around the origin.

Remember, a clockwise turn is a turn that moves in the same direction as the hands of a clock. A counterclockwise turn is a turn in the opposite direction.



The coordinate grid below shows the original lizard in Quadrant 2 and a congruent image in Quadrant 3. This new image was created by transforming the original lizard through a rotation of 90° counterclockwise around the origin.

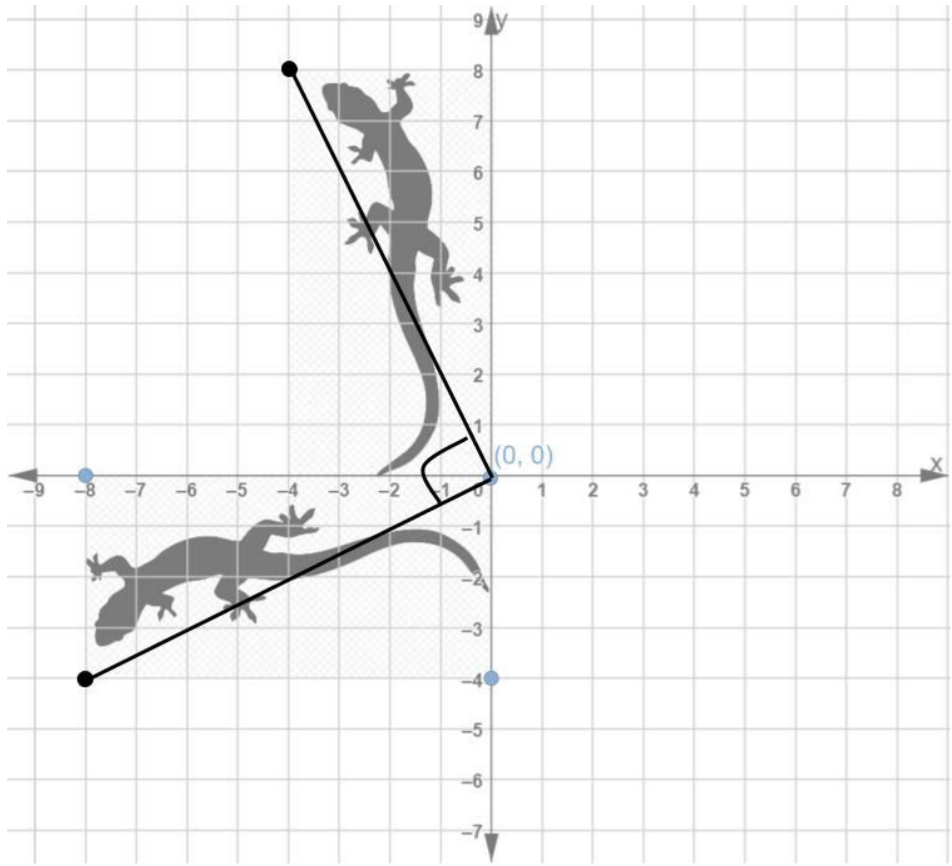


Can you see the 90° angle? It helps if you focus on two corresponding points -- one from the original lizard and one from the rotated image. Then draw a line from each point to the point of rotation, which in this case was the origin.

Because a rotation means we turn every point in the original figure the same number of degrees, this will work with any pair of corresponding points.

Let's focus on the corner of the rectangle that is closest to the lizard's mouth. When we draw a straight line from each corresponding point to the point of rotation, we can see the 90° angle.

Now you try.



1) Draw a straight line from any of the other corresponding points to the origin. The two lines should form a 90° angle.

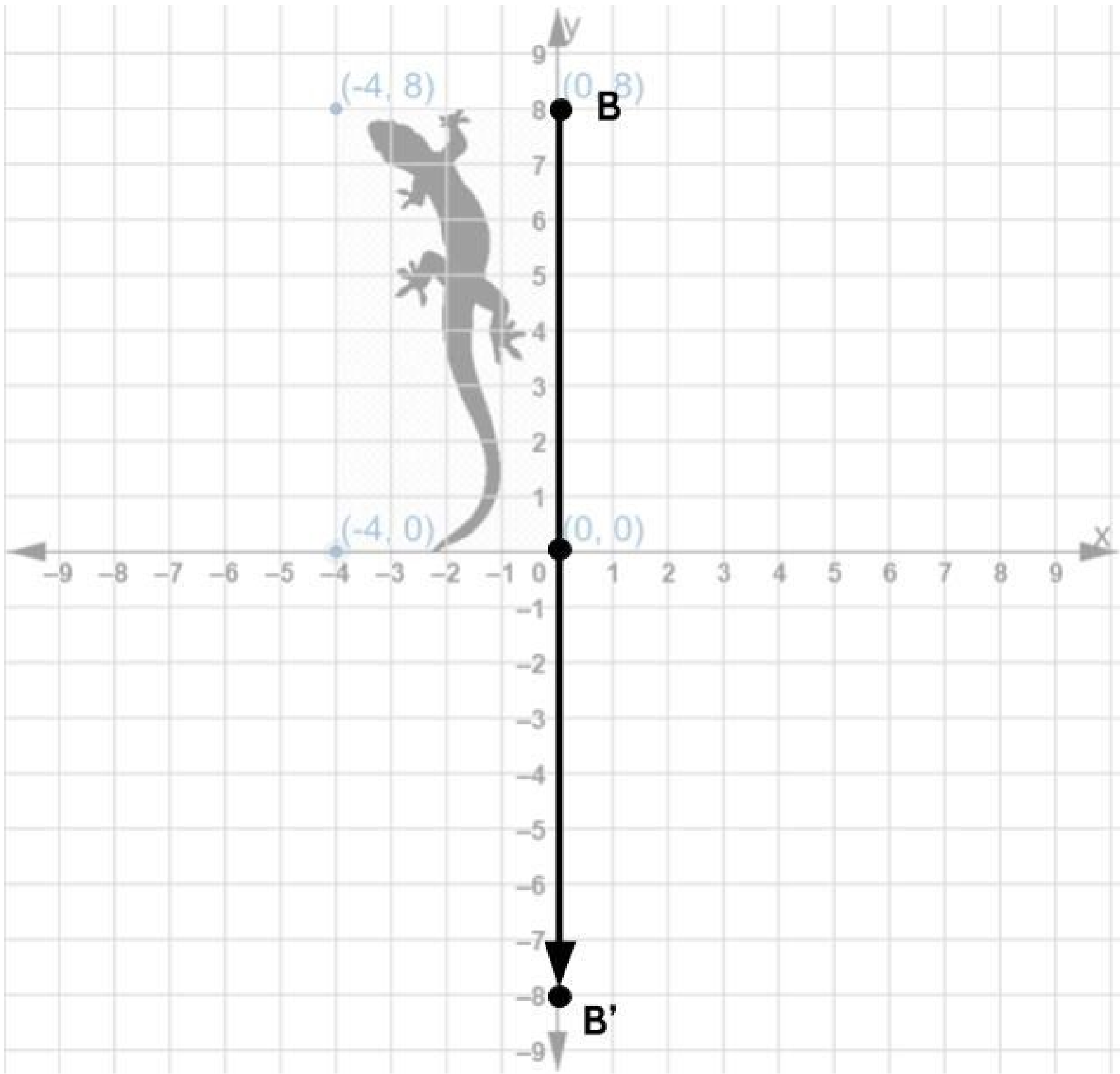
If we return to our original lizard and rotate it 180° counterclockwise around the origin, where do you think the rotated image will appear?

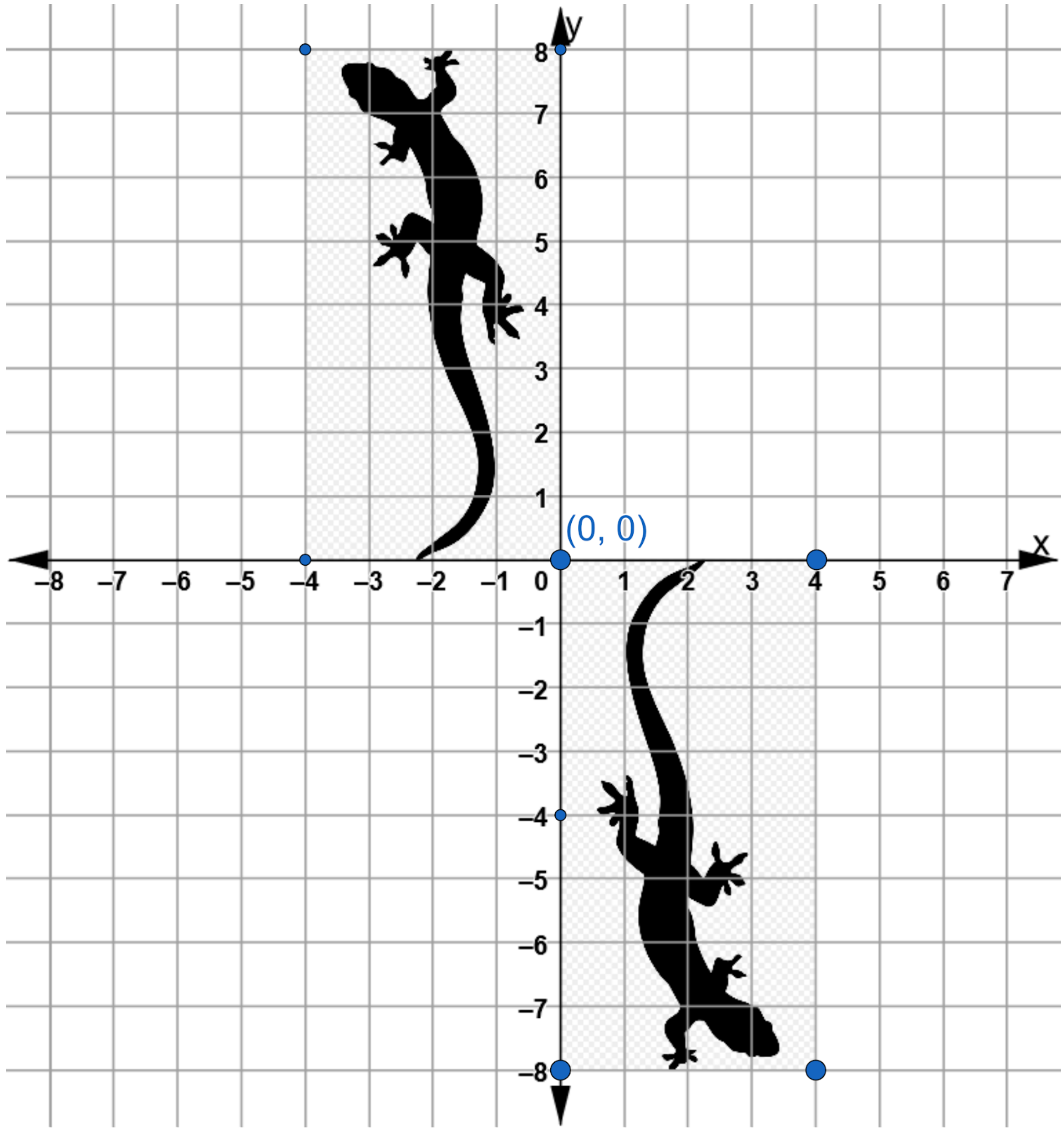
We can imagine it the 180° turn by making another 90° turn. Two 90° turns is the same as one 180° turn.

Here's another way to figure out each corresponding point on the image rotated 180°. Draw a straight line from a point on the rectangle to the origin. Then continue the line straight from the origin until your second line is the same length as your first line.

For example, start at Point B and draw a straight line to the point of rotation. Then continue to draw until you have doubled your first line. Since Point B is 8 units away from the origin, point B' must be another 8 units away, in a straight line.

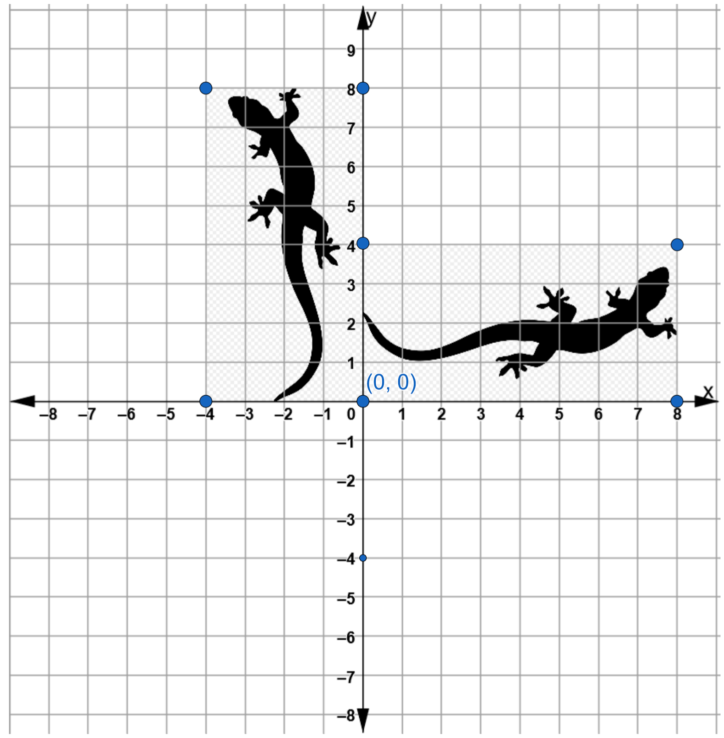
2) Map the remaining two points (-4, 8) and (-4, 0) to their corresponding points on the image we would get if we rotated the original lizard 180° around the origin.



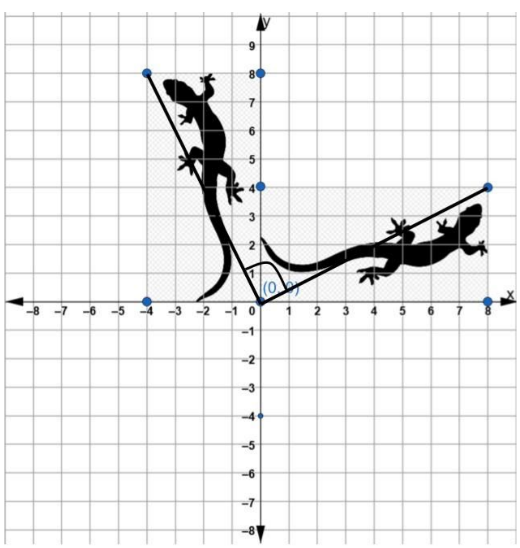


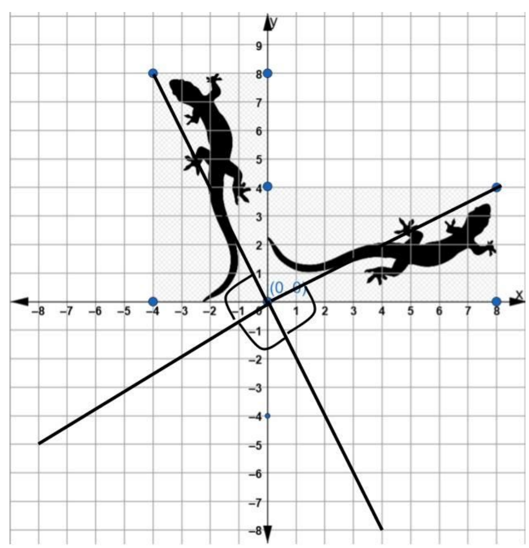
Write 3 things you notice when you compare the original lizard with its image rotated 180 degrees counterclockwise around the origin.

How would you describe the rotation of the original lizard around the origin on the coordinate grid to the right?



There are actually two ways to describe this rotation. .



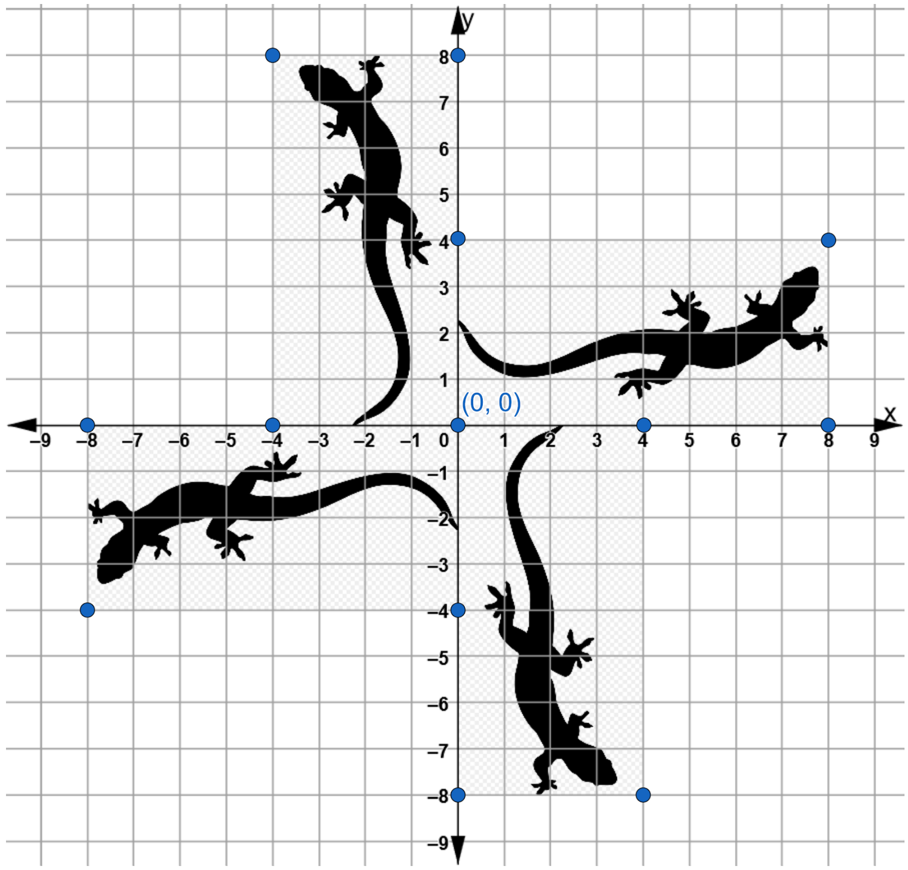


A rotation of 90° *clockwise* around the origin.

A rotation of 270° (or three 90° turns) *counterclockwise* around the origin.

Direction is important when describing rotation.

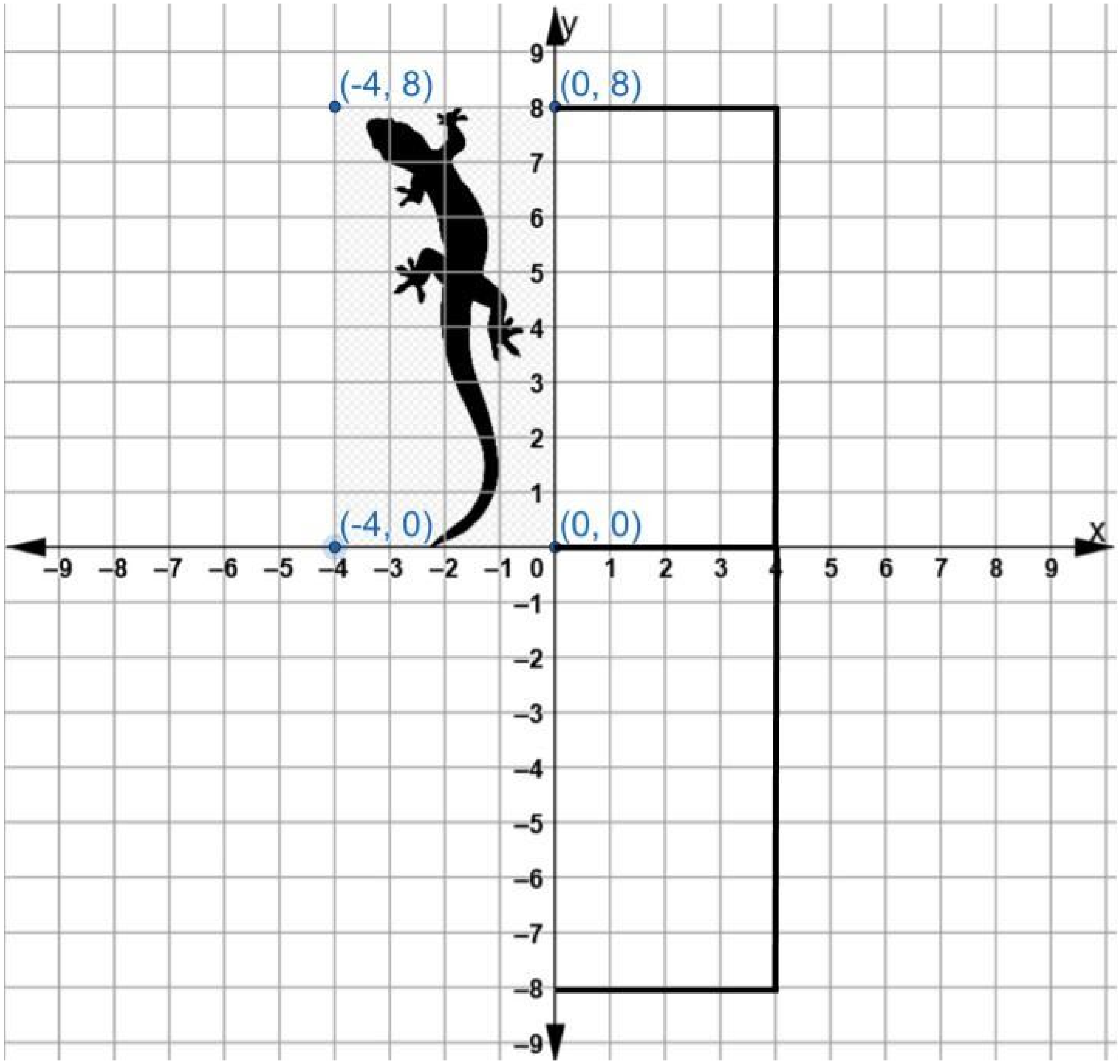
Here is the original lizard and all three **counterclockwise** rotations around the origin on one coordinate grid.



3) The coordinates for the rectangle around the lizard are recorded in the table below. Fill in the coordinates for the corresponding coordinate points for each of the 3 rotations.

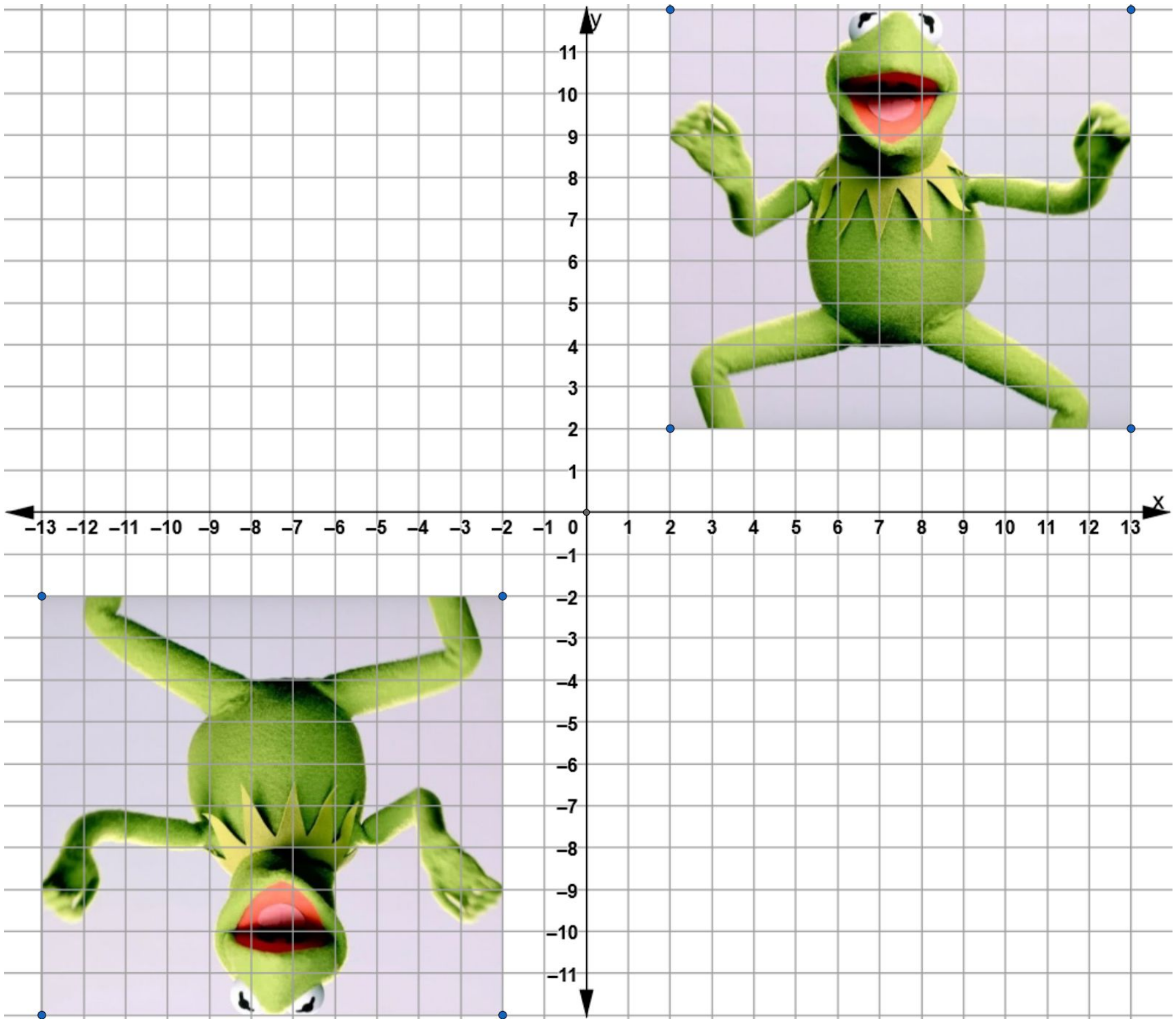
Coordinate Points in Original Figure	**Coordinate Points for Image rotated 90° counterclockwise**	**Coordinate Points for Image rotated 180° counterclockwise**	**Coordinate Points for Image rotated 270° counterclockwise**
(0,0)			
(0,8)			
(-4,8)			
(-4,0)			

Let's do one final activity with our lizard. Draw an image of the lizard reflected over the y-axis. Then reflect that image about the x-axis.



What do you notice?

4) The image of Kermit the Frog in Quadrant 1 of this coordinate grid maps onto the image in Quadrant 3.

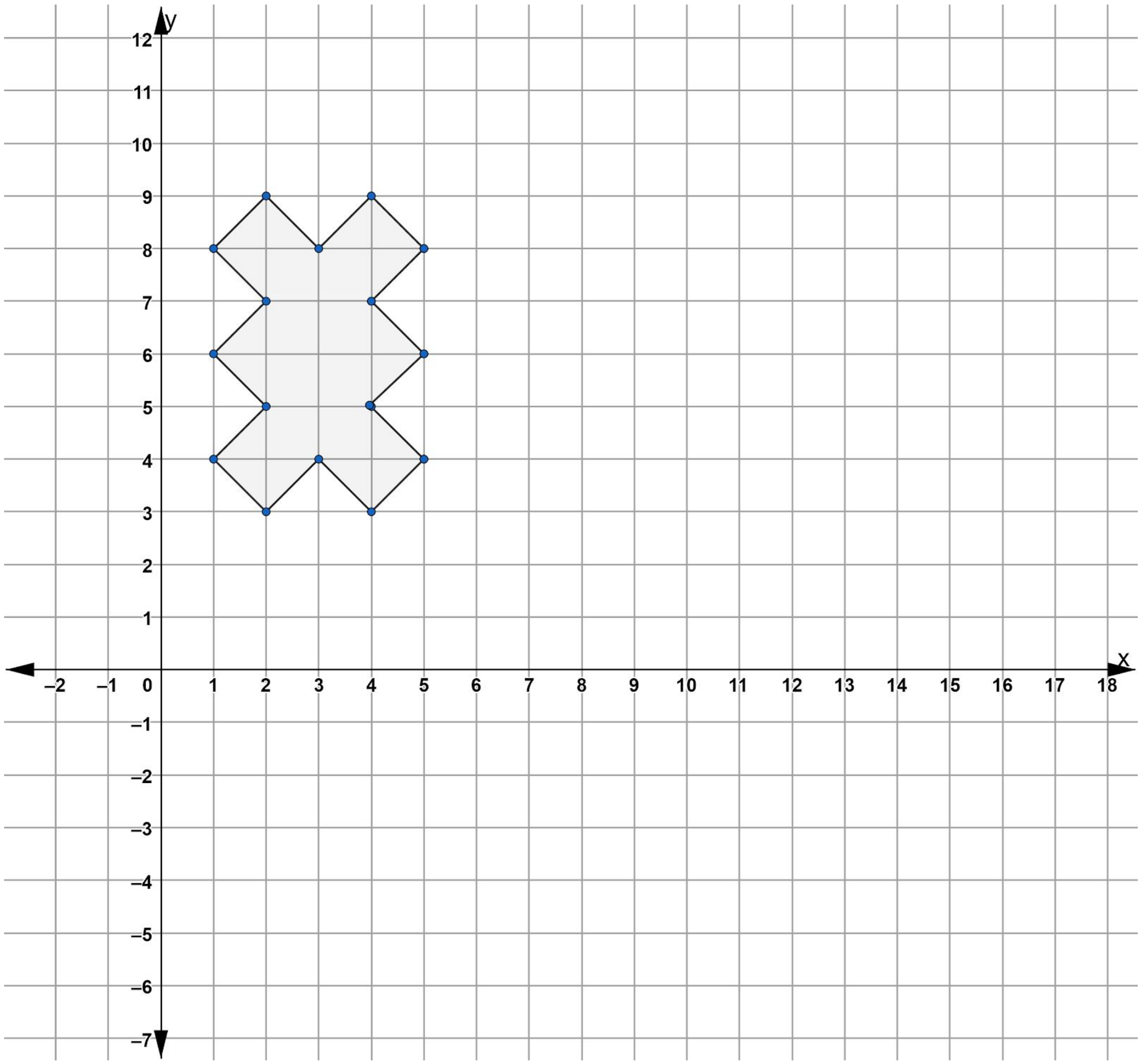


Which of the following statements are possible transformations that map the original Kermit onto the image? *(Note: There may be more than one possible answer)*

A) A rotation of 90° clockwise about the origin.
B) A rotation of 180° counterclockwise about the origin.
C) Two rotations of 90° clockwise about the origin.
D) A reflection over the y-axis followed by a reflection over the x-axis

5) Remember a tessellation is an image that includes repeated congruent figures that cover a surface without gaps or overlapping We can create tessellations by rotating figures on a coordinate grid.

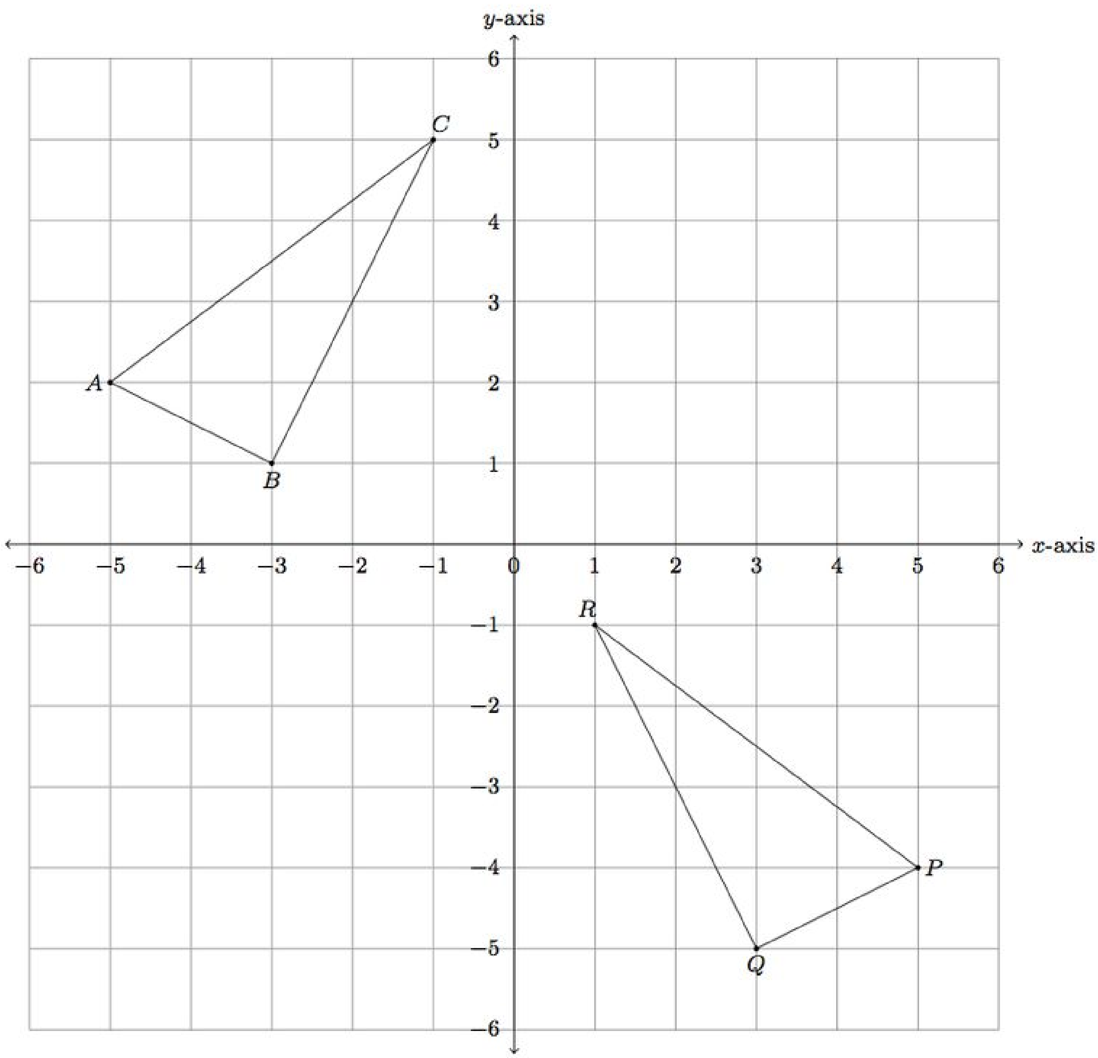
Consider the figure below. Create three congruent figures by rotating the figure 90°, 180° and 270° counterclockwise. Use the point (5,4) as the point of rotation. Shade or color in each figure as you draw them.



Sequences of Transformations

So far, we have looked at what happens to when we reflect figures across a line of reflection, when we rotate figures around a point of rotation, and when we translate figures. In this section, we are going to use what you have learned to see what happens when we use combinations of rigid transformations.

△ABC and △PQR are congruent.



There is no single translation that would map ABC to PQR. There is no single reflection or single rotation that would do it either. Triangle ABC can be mapped to Triangle PQR using a sequence of *two* rigid transformations.
One place to start when trying to figure out a sequence of transformations is look at which points in the original figure map to which points in the image.

By looking at the triangle you might be able to tell that point C maps to point R.

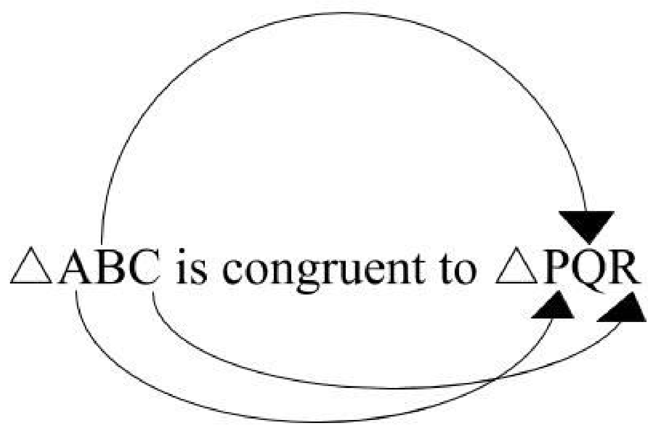
But which point in Triangle PQR corresponds to point A?

And which point in Triangle PQR corresponds to point B?

You might be able to see that point B maps to point Q and point A maps to point P also by looking at the figures, but there is another way.

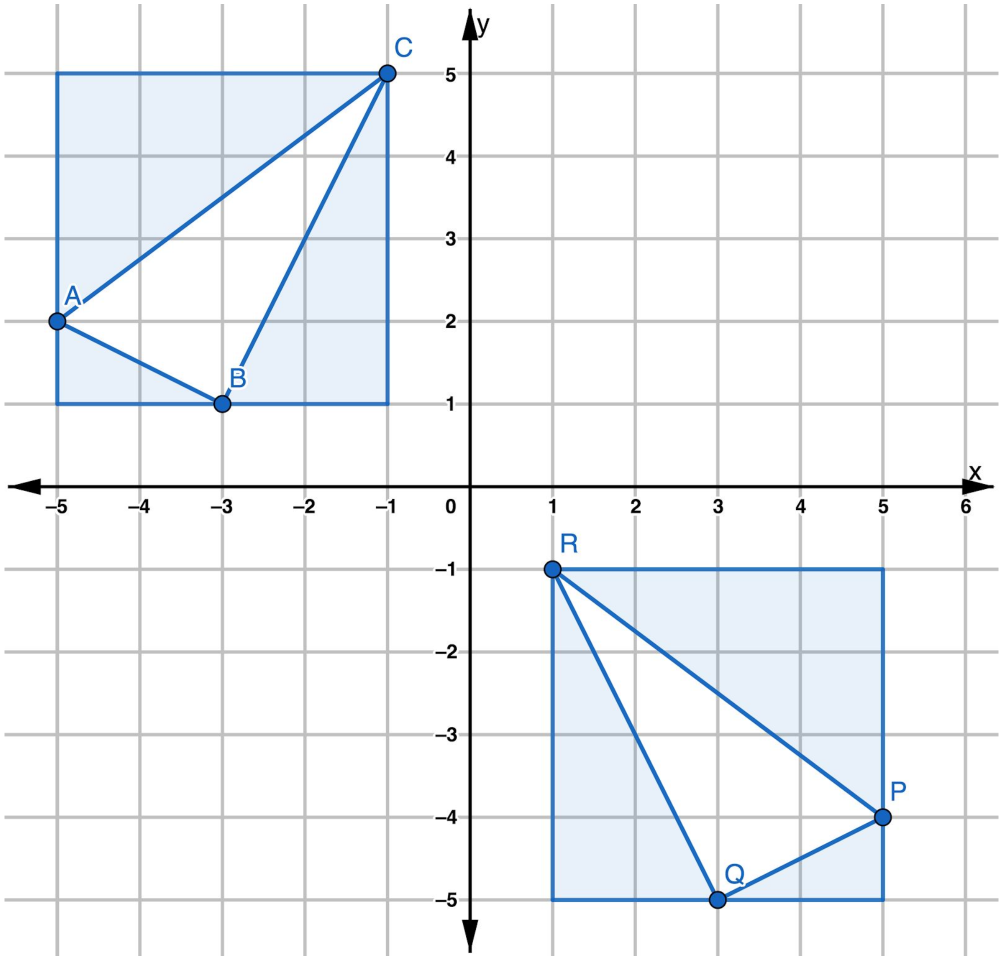
△ABC and △PQR are described as congruent. As you have already learned, that means that the two figures are the same size and shape. It means that you can map one to fit exactly over the other using rigid transformations. But there is also a clue in the way the names of the triangles (or any other shape) are written.

△ABC and △PQR means that point A corresponds to point P, point B corresponds to point Q and point C corresponds to point R. When figures are described as congruent, the order of the points in the name of one figure must match the order of the corresponding points in the other figure.



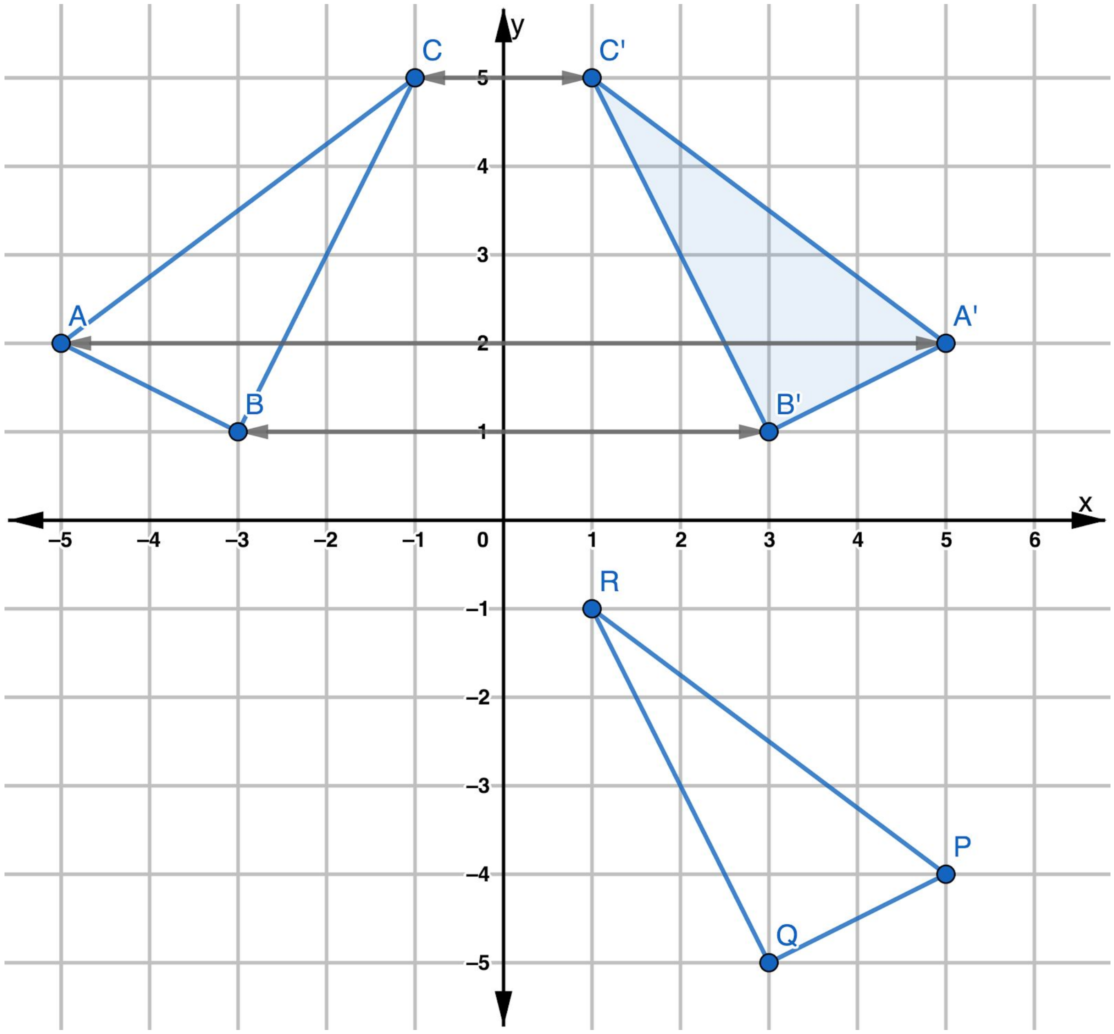
Once you know which points in the original figure map to the points in the image, try different sequences to get from each point to its corresponding point.

We shaded around each triangle to help you think about the transformations.



What do you notice?

If we reflect △ABC across the y-axis, we end up with △A’B’C’.



If we slide Points A’, B’, and C’ down 6 units, the triangle will exactly fit over △PQR.

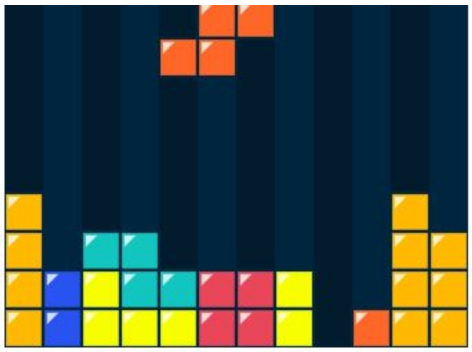
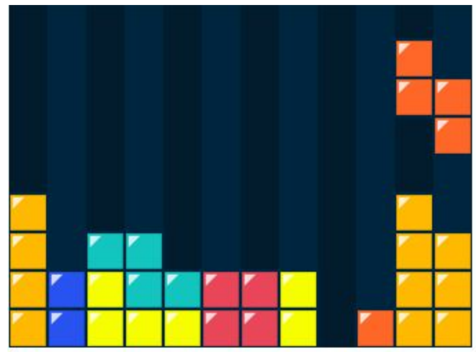
So △ABC can be mapped to △PQR using the following sequence of transformations:

- Step One: A reflection across the y-axis.
- Step Two: A translation of 6 units down.

You will have an opportunity to practice working with sequences of transformations on the coordinate grid in the Practice Questions section of this packet.

The video game Tetris uses sequences of rigid transformations. Tetris can be described as a moving puzzle. Players arrange a series of falling shapes to form horizontal rows. Each shape can be moved by a sequence of rigid transformations (rotation and translation).

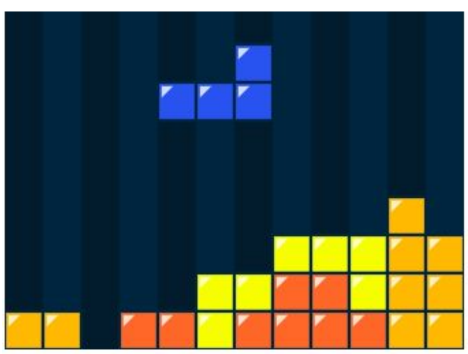
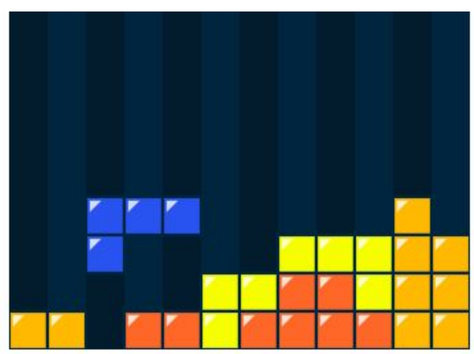
1)

Screen 1	Screen 2
	

Which statement below describes a sequence of transformations that would move the falling shape from its position in Screen 1 to its position in Screen 2?

A. A clockwise rotation of 180°, a translation of 5 units to the right, a translation of 1 unit down
B. A clockwise rotation of 90°, a translation of 5 units to the right, a translation of 1 unit down

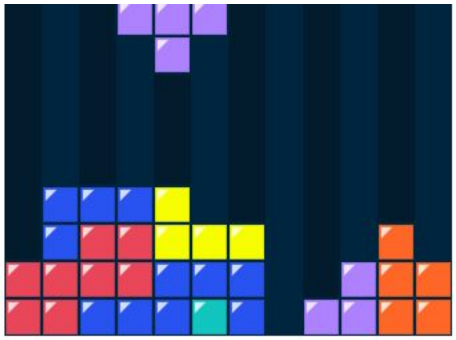
2)

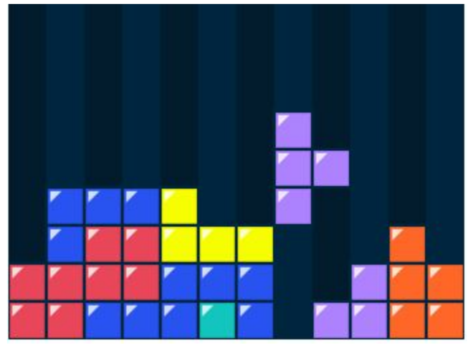
Screen 1	Screen 2
	

Which statement below describes a sequence of transformations that would ***not*** move the falling shape from its position in Screen 1 to its position in Screen 2?

A. A counterclockwise rotation of 180°, a translation of 2 units to the left, a translation of 3 units down.
B. A clockwise rotation of 180°, a translation of 2 units to the left, a translation of 3 units down.
C. A clockwise rotation of 90°, a translation of 4 units to the left, a translation of 3 units down.

3) **Screen 1** **Screen 2**

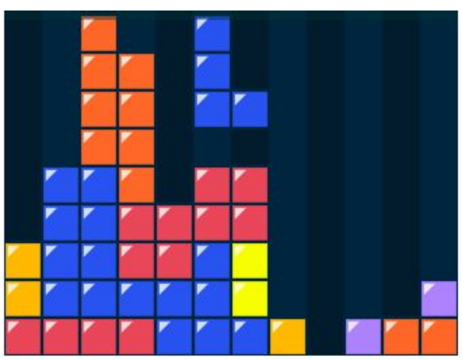


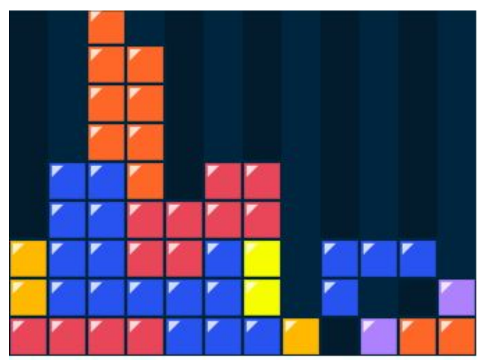


Which statement below describes a sequence of transformations that would move the falling shape from its position in Screen 1 to its position in Screen 2?

A. A clockwise rotation of 90°, a translation 4 units to the right, a translation 3 units down.
B. A clockwise rotation of 180°, a translation 4 units to the right, a translation 3 units down.
C. A counterclockwise rotation of 90°, a translation 4 units to the right, a translation 3 units down.
D. A counterclockwise rotation of 180°, a translation 4 units to the right, a translation 3 units down.

4) **Screen 1** **Screen 2**

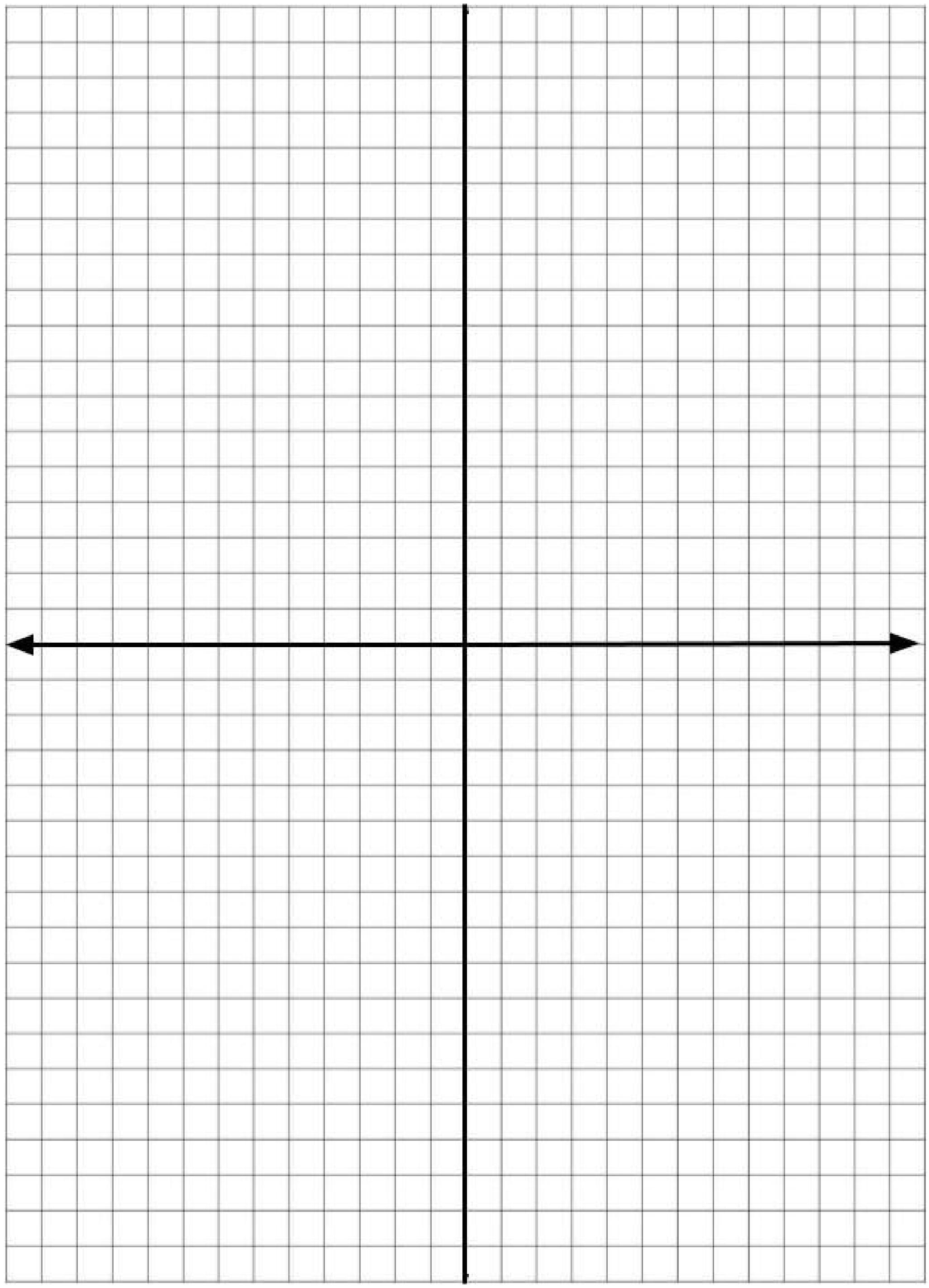




Which statement below describes a sequence of transformations that would move the falling shape from its position in Screen 1 to its position in Screen 2?

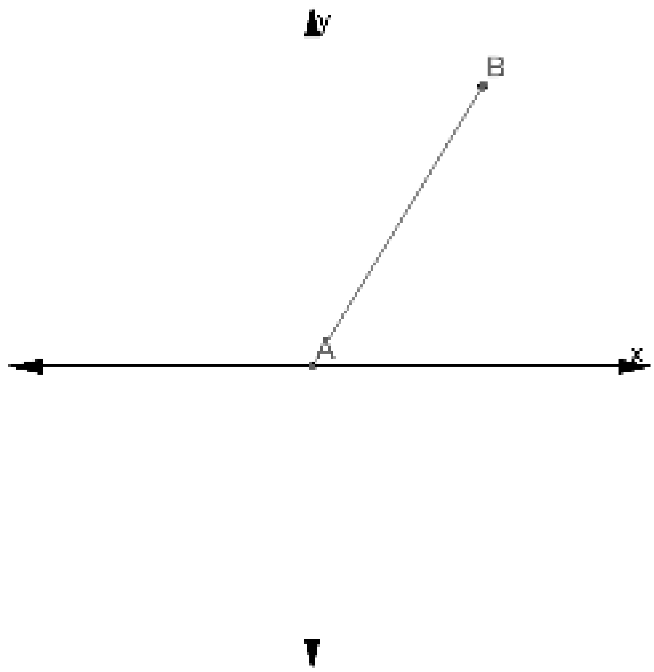
A. A clockwise rotation of 90°, a translation 2 units to the right, a translation 4 units down.
B. A counterclockwise rotation of 90°, a translation 2 units to the right, a translation 4 units down.
C. A clockwise rotation of 90°, a translation 2 units to the left, a translation 4 units down.

5) A good way to help practice a sequence of rigid transformations is to create your own sequence. On the grid below, draw a shape that has between 5 to 8 sides with no curved edges. Label the shape Figure 1. Transform your shape through 3 rigid transformations of rotation, translation, and reflection — in any order. Label your 3 transformations Figure 2, Figure 3, and Figure 4.



Test Practice Questions

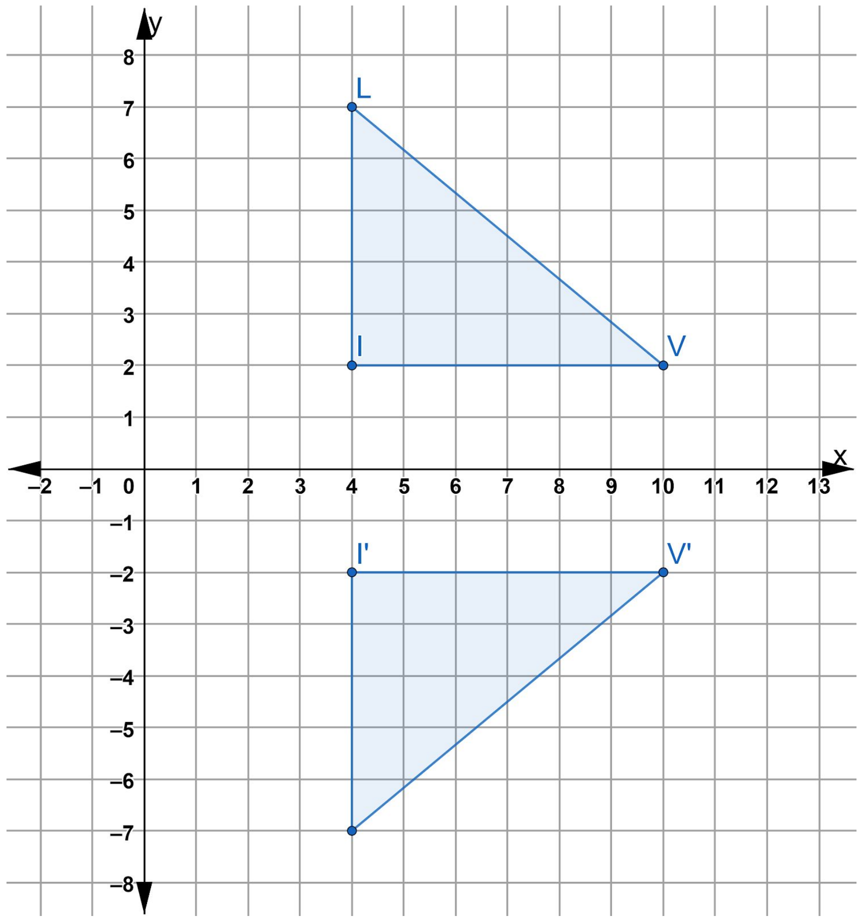
1) Which <u>two</u> transformations of $\overline{AB}$ would result in an image parallel to $\overline{AB}$?



A. translation of 2 units down
B. reflection over the x-axis
C. reflection over the y-axis
D. clockwise rotation of 90° about the origin
E. translation of 4 units to the right

Explain your answer:

2) Consider △LIV and its image △L'I'V' on the coordinate plane.



Which transformation maps △LIV to △L'I'V'?

A) translation 4 units down

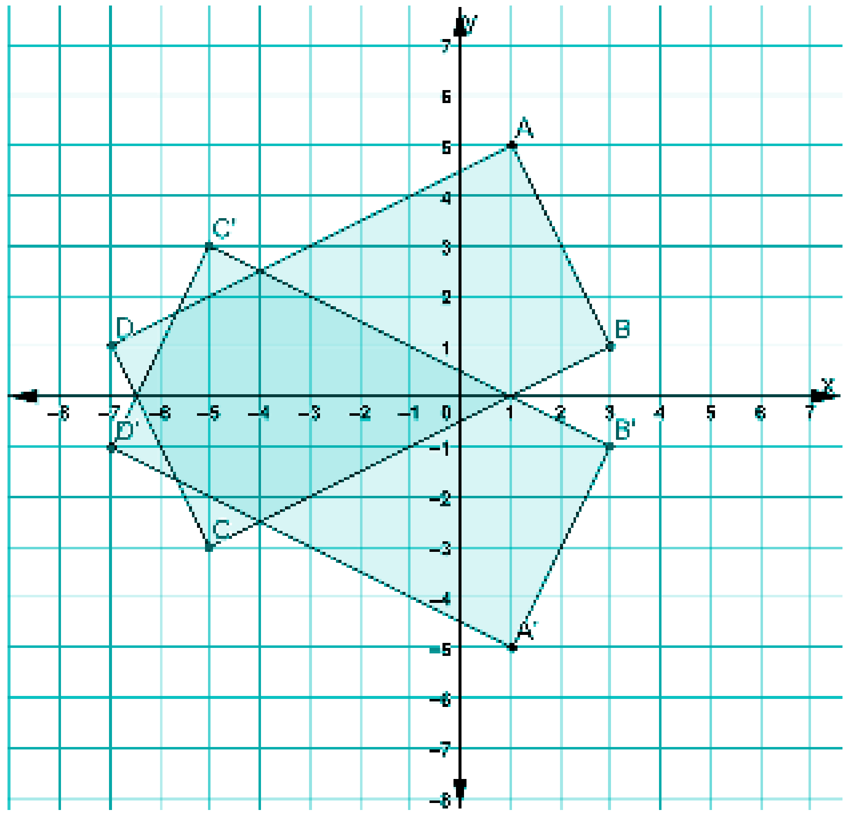
B) reflection about the x-axis

C) translation 14 units down

D) reflection about the line *x*=1

Explain your answer:

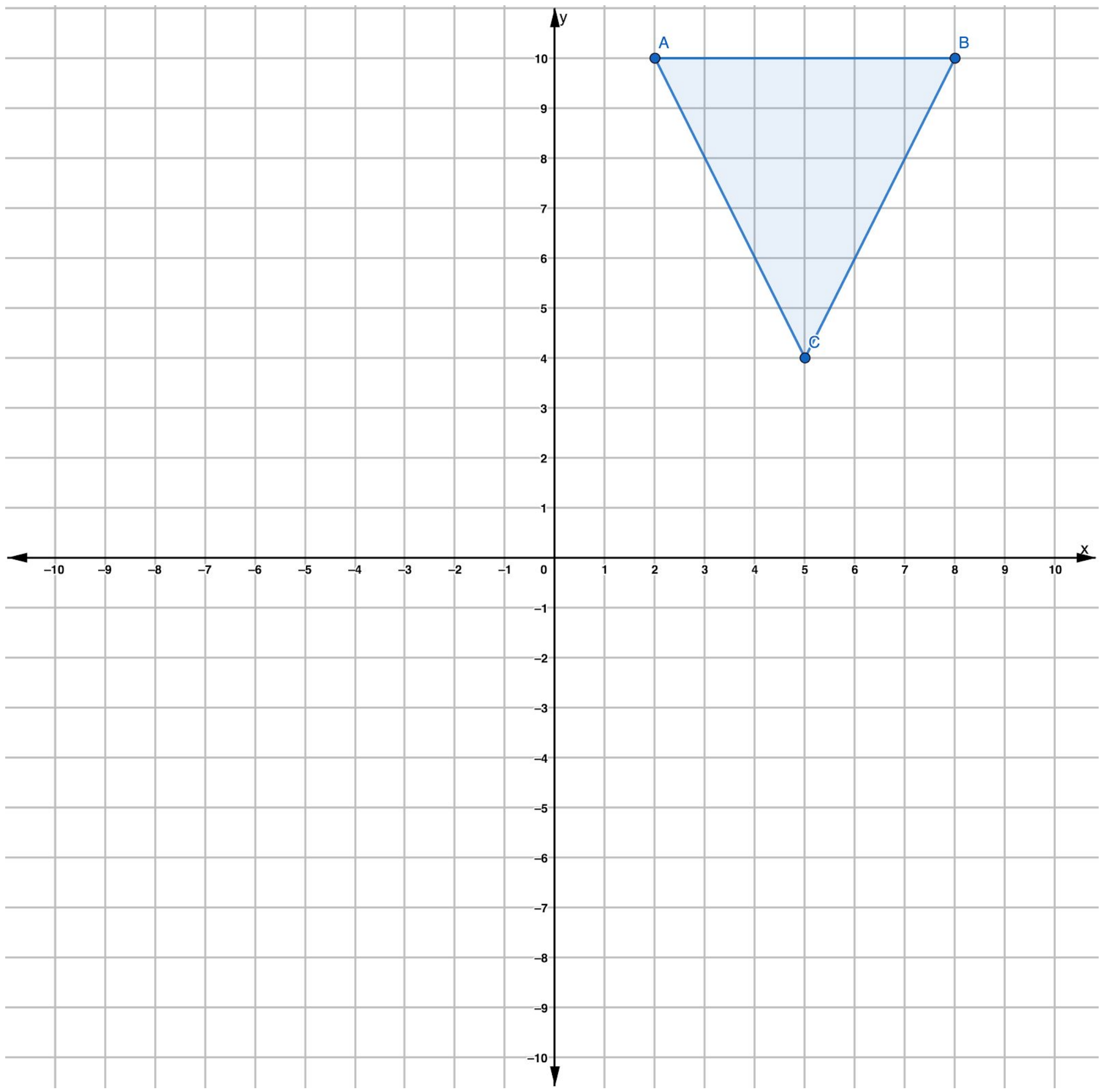
3) On the set of axes below, rectangle ABCD can be proven congruent to rectangle A'B'C'D' using which transformation?



A) reflection over the x-axis
B) reflection over the y-axis
C) rotation about the origin
D) translation

Explain your answer:

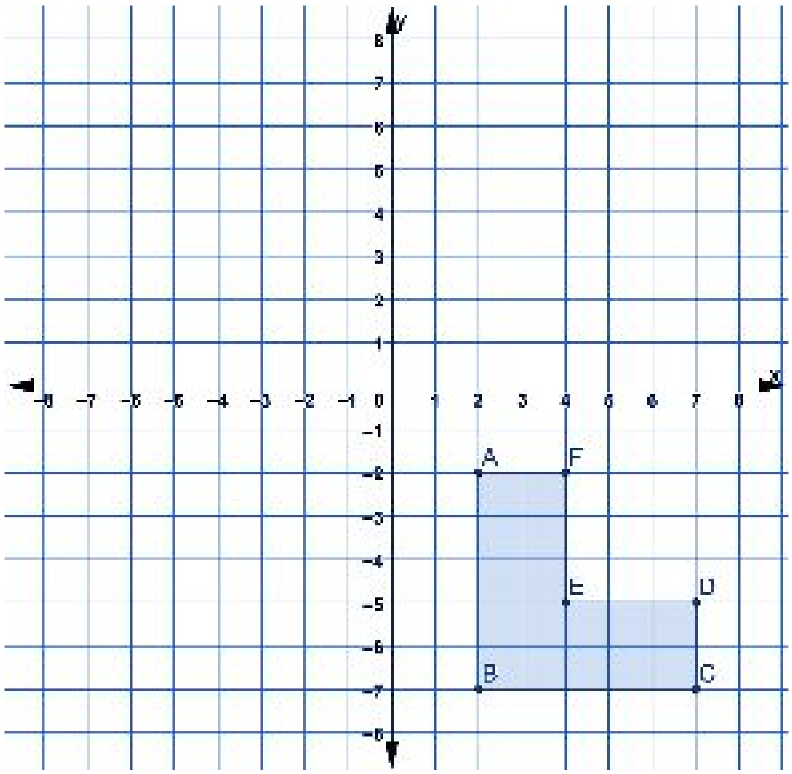
4) Which transformation on $\triangle ABC$ would result in a triangle that is <u>not</u> congruent to $\triangle ABC$?



A) reflection about the x-axis
B) translation 4 units to the left and 6 units down
C) dilation by a scale factor of 2
D) rotation of 180° clockwise about the origin

Explain your answer:

5) If the figure on the coordinate plane is translated 4 units to the right and 4 units down, what would the coordinates of point C' be?



A) (7,-7)

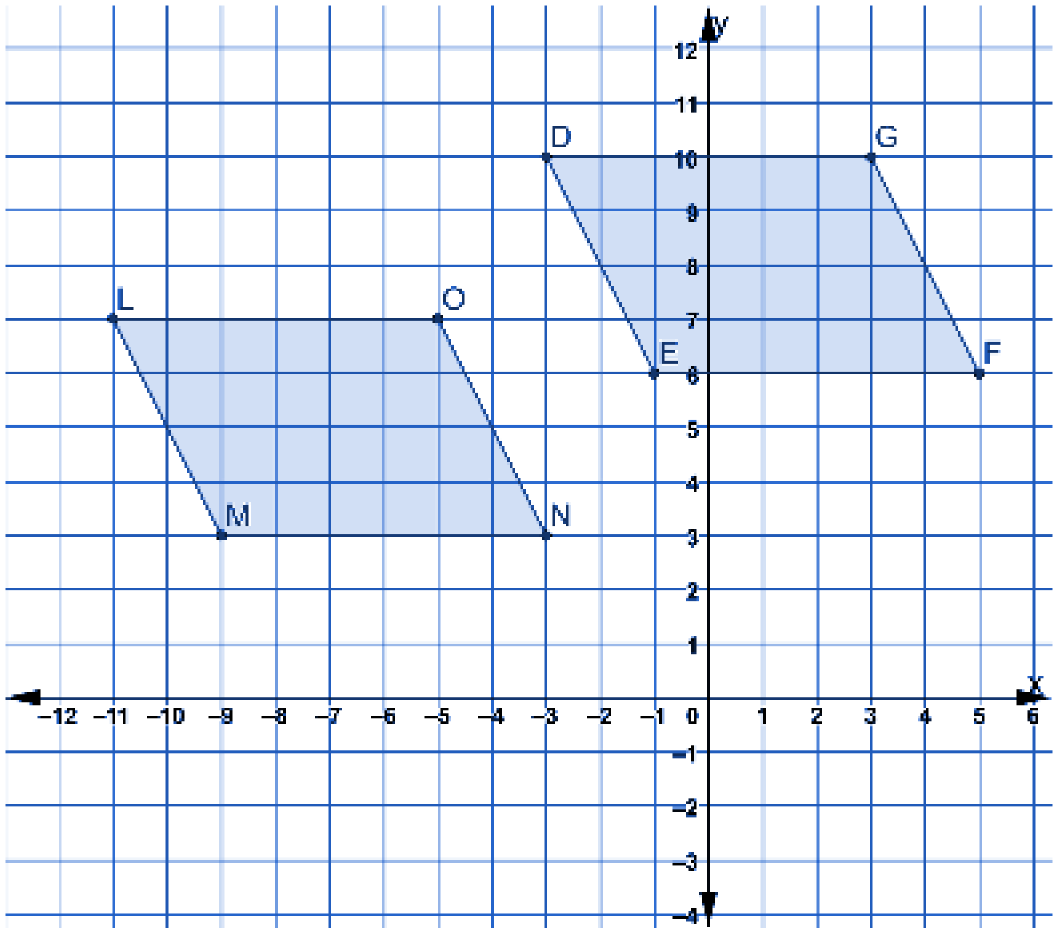
B) (7,-11)

C) (11,-7)

D) (11,-11)

Explain your answer:

6) Consider parallelogram LMNO on the coordinate plane.

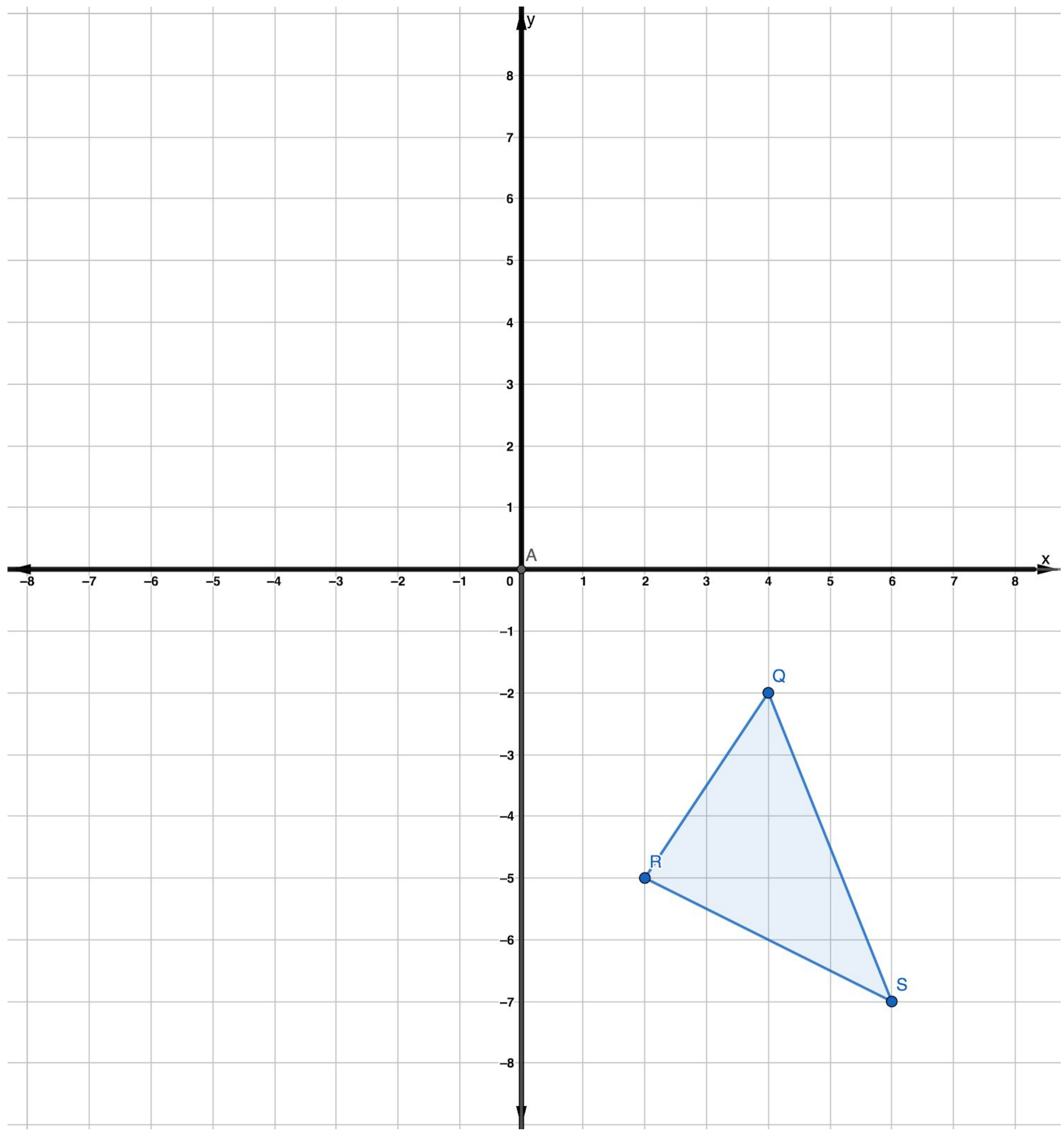


Which translation could have resulted in the congruent image, DEFG?

A) translation of 2 units left and 3 units down
B) translation of 2 units right and 3 units up
C) translation 8 units to the right and 3 units up
D) translation of 8 units left and 3 units down

Explain why answer B is incorrect:

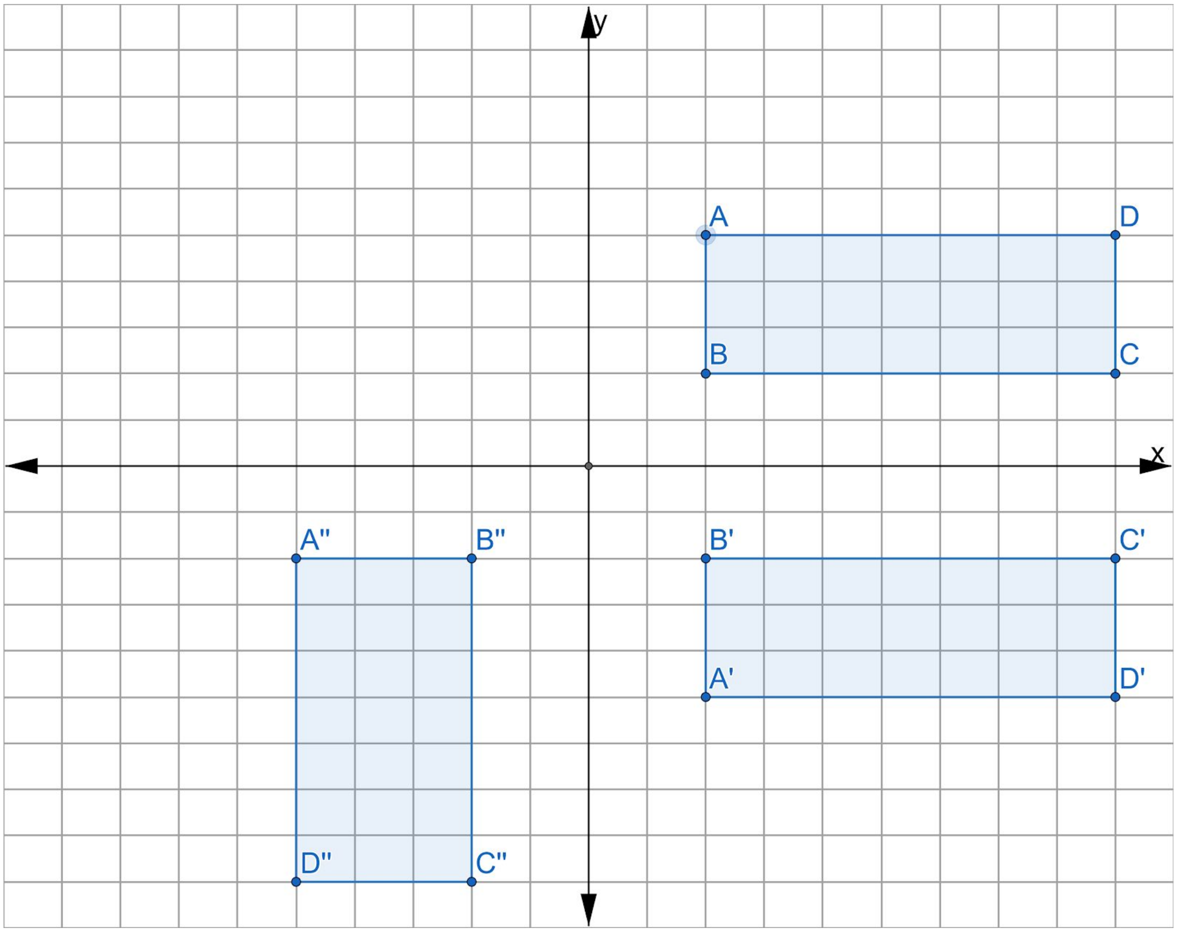
7) Triangle QRS (shown) maps on to triangle Q'R'S' (not shown) by a certain translation.



If two of the vertices of triangle Q' are (1, 7) and S' (3, 2), which would be the coordinates of R'?

A) (-1, 4)
B) (-7, -2)
C) (5, 4)
D) (-2, 5)

8) A sequence of transformations maps rectangle *ABCD* onto rectangle *A"B"C"D"*, as shown in the diagram below.

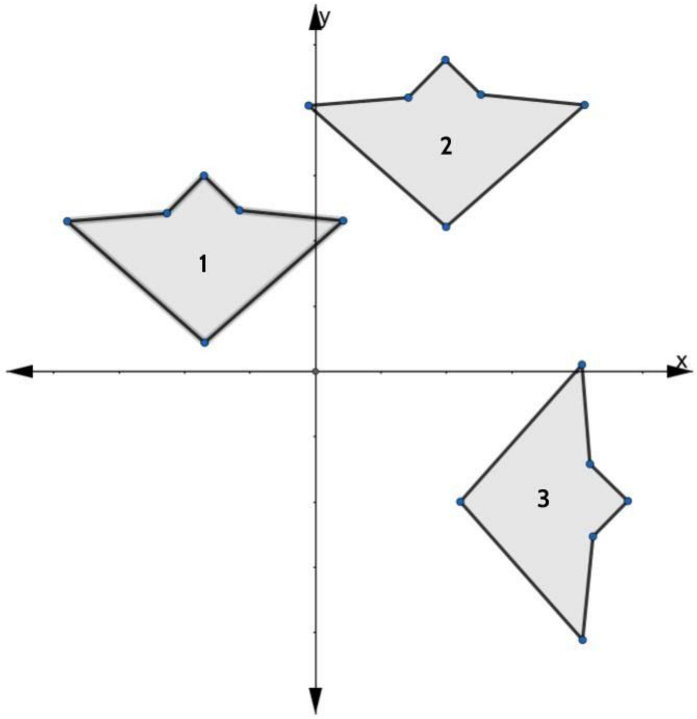


Which sequence of transformations maps ABCD onto A'B'C'D' and then maps A'B'C'D' onto A"B"C"D"?

A) a reflection followed by a rotation
B) a reflection followed by a translation
C) a translation followed by a rotation
D) a translation followed by a reflection

Explain why answer C is <u>incorrect</u>:

9) In the diagram below, congruent figures 1, 2, and 3 are drawn.

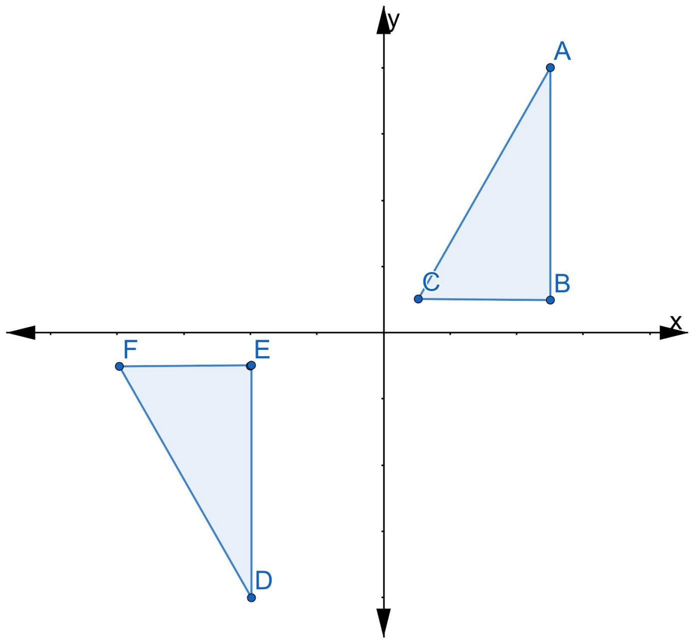


Which sequence of transformations maps figure 1 onto figure 2 and then figure 2 onto figure 3?

A) a reflection followed by a translation
B) a rotation followed by a translation
C) a translation followed by a reflection
D) a translation followed by a rotation

Explain your answer:

10) In the diagram below, $\triangle ABC \cong \triangle DEF$



Which sequence of transformations maps ABC onto DEF?

A. reflection over the y-axis followed by a translation
B. reflection over the x-axis followed by a translation
C. counterclockwise rotation of 90° about the origin followed by a translation
D. counterclockwise rotation of 180° about the origin followed by a translation

Explain your answer:

Final Project

On the front cover of this packet is a detail of tile work from the Alhambra Palace in Granada, Andalusia, Spain.

You can recreate this image using everything you have learned about rigid transformation.

1. Plot the following points on the coordinate grid on the next page and shade in the figure. This is your original figure.

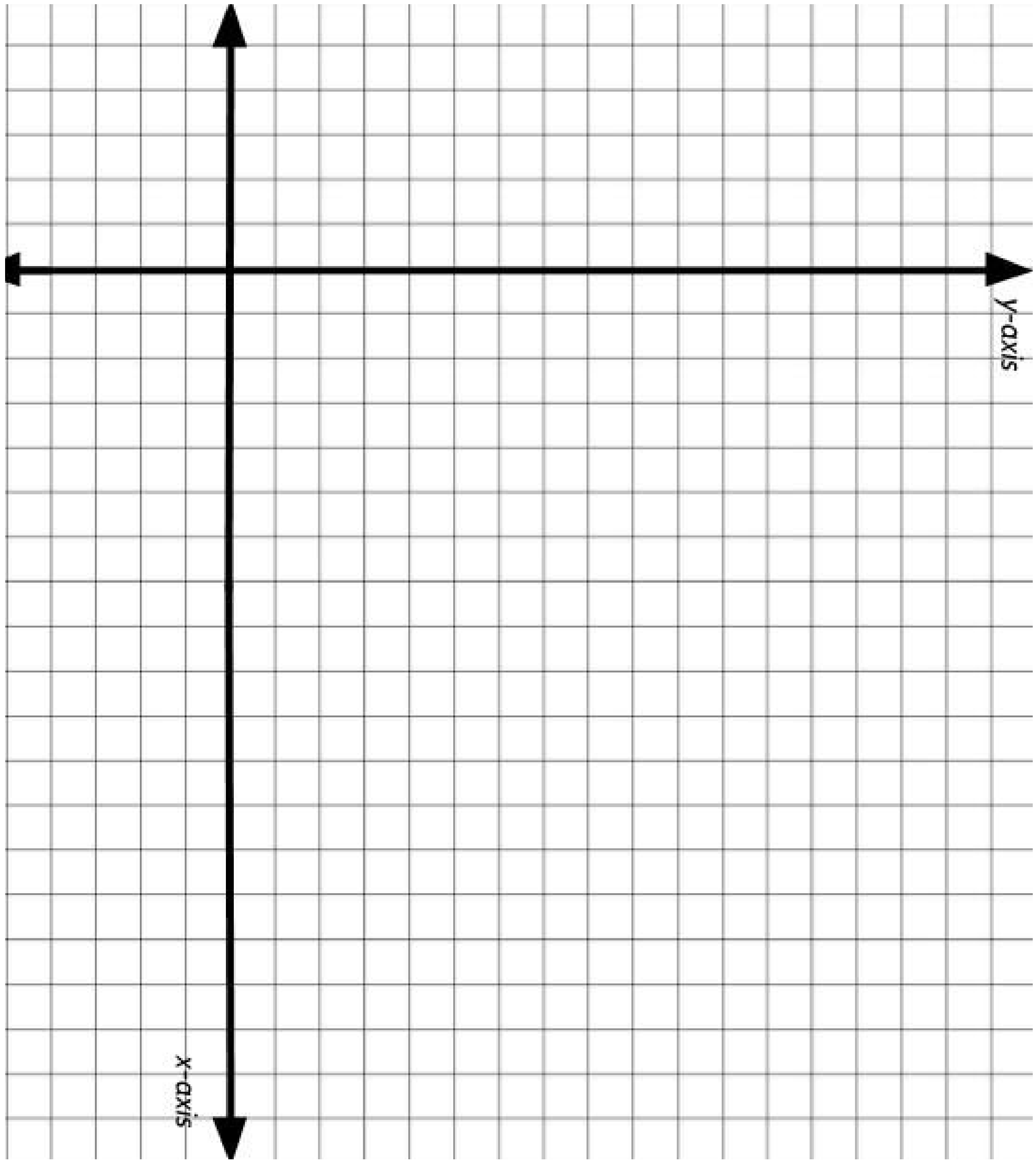
(0,0) (-4,2) (-3,3) (-2,2) (0,6) (2,2) (3,3) (4,2)

2. Reflect the original figure about the line y = 6. Call this figure 2.
3. Reflect the original figure about the x-axis. Call this figure 3.
4. Rotate the original figure 180° clockwise around point (3,3). Call this figure 4.
5. Reflect the image you create in step 4 about the line y = 6. Call this figure 5.
6. Reflect the image you created in step 4 over the x-axis. Call this figure 6.
7. Rotate figure 5 180° clockwise around the point (9,9). Call this figure 7.
8. Reflect figure 7 about the line y = 6. Call this figure 8.
9. Rotate figure 6 180° clockwise about point (9,-3). Call this figure 9.
10. Shade in figures 1 through 9.

Can you continue the rigid transformations until the grid is covered?

If you'd like to see what steps 1-9 look like on the coordinate plane,

go to http://bit.ly/RigidTransformations_FinalProject


y-axis
x-axis

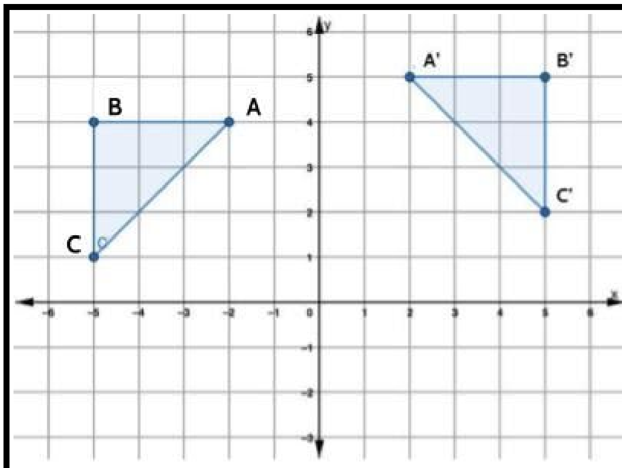
The Language of Rigid Transformations

Matching Pictures and Descriptions

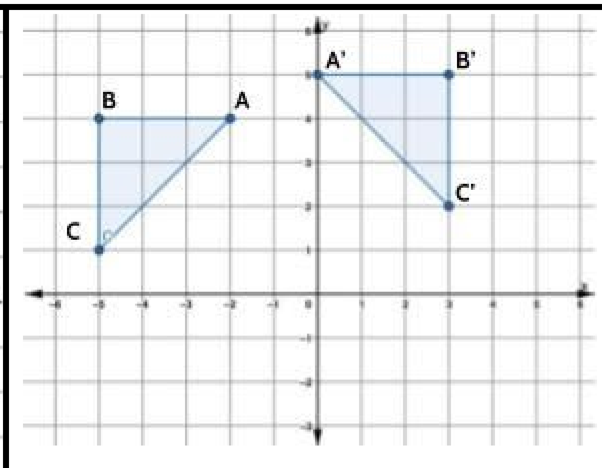
Match each sentence with the picture it describes. Write the letter of that sentence in the blank box below each picture. You will have one sentence left. Write that letter in the box for #4 and complete the picture to match that description.

Sentences

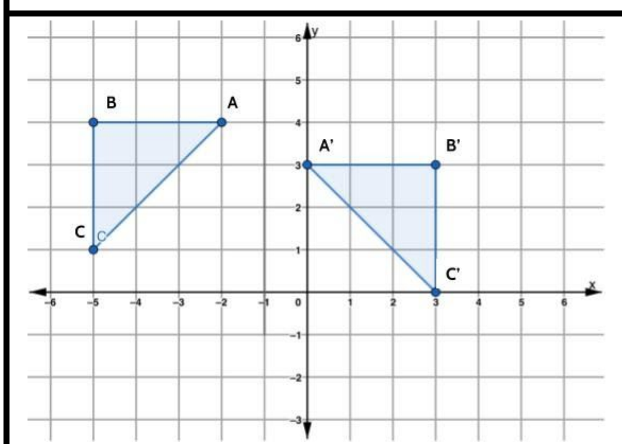
A) a reflection about the y-axis and a translation 1 unit down
B) a reflection about the line x = -1 and a translation 1 unit down
C) a reflection about the line x = -1 and a translation 1 unit up
D) a reflection about the y-axis and a translation 1 unit up



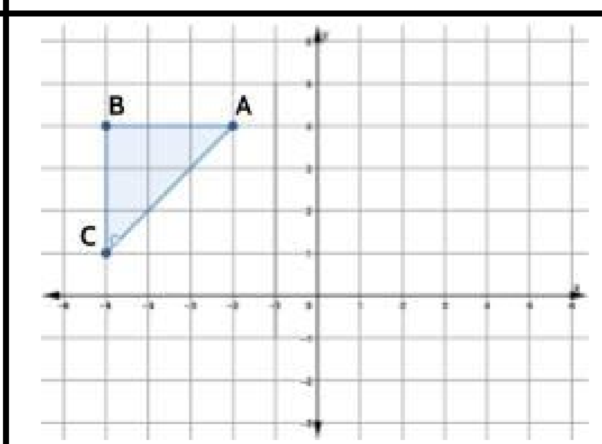
1) The transformation above matches with sentence ____



2) The transformation above matches with sentence ____



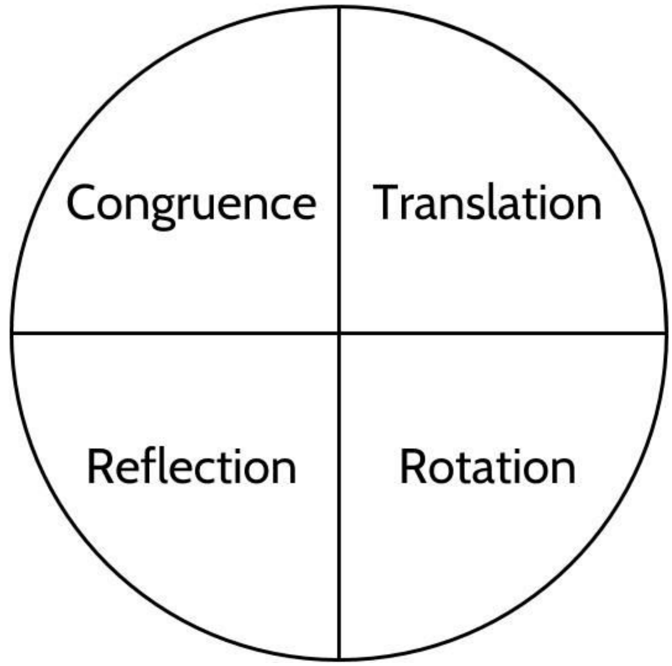
3) The transformation above matches with sentence ____

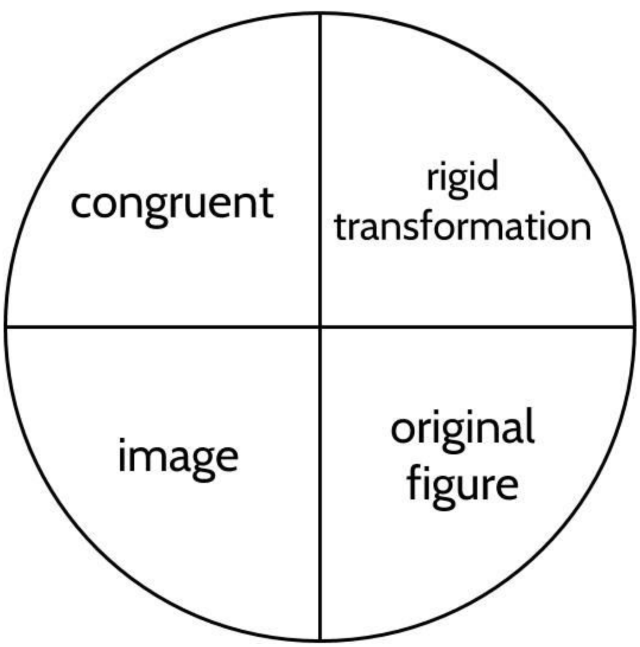


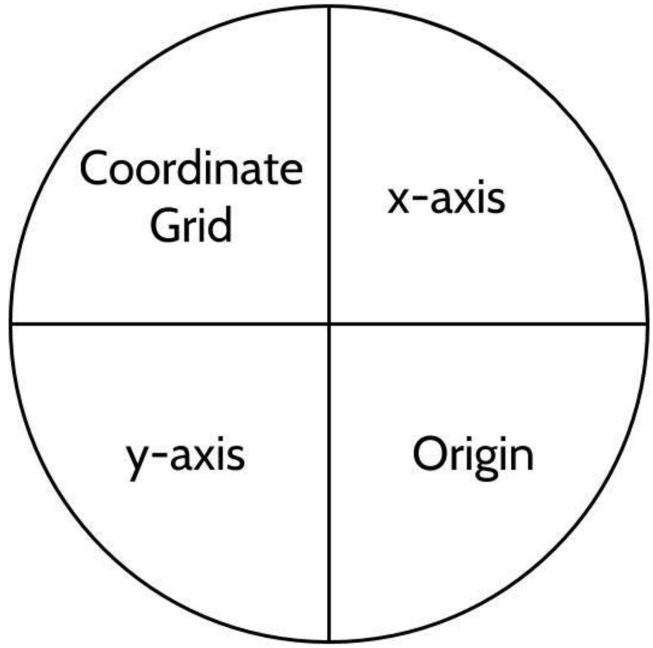
4) The transformation above matches with sentence ____

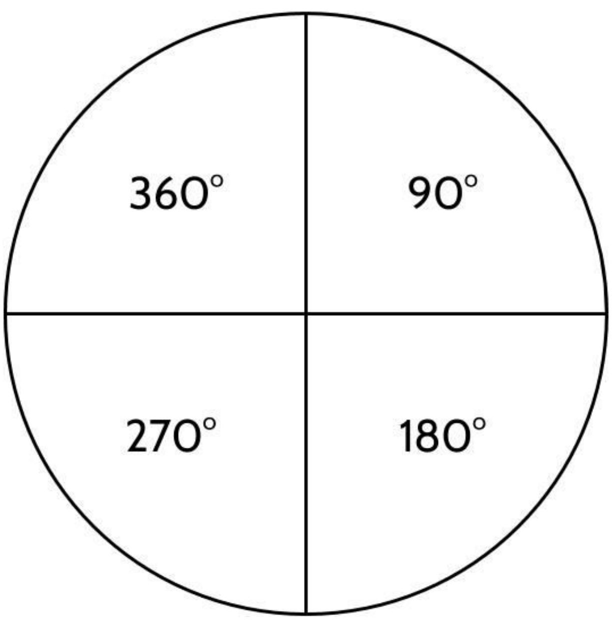
Concept Circle

5) Explain the four words in each circle and the connections you see between them.






Coordinate Grid
x-axis
y-axis
Origin


360°
90°
270°
180°

Rigid Transformations and You

6) Draw at least one example of each of the following from your life:

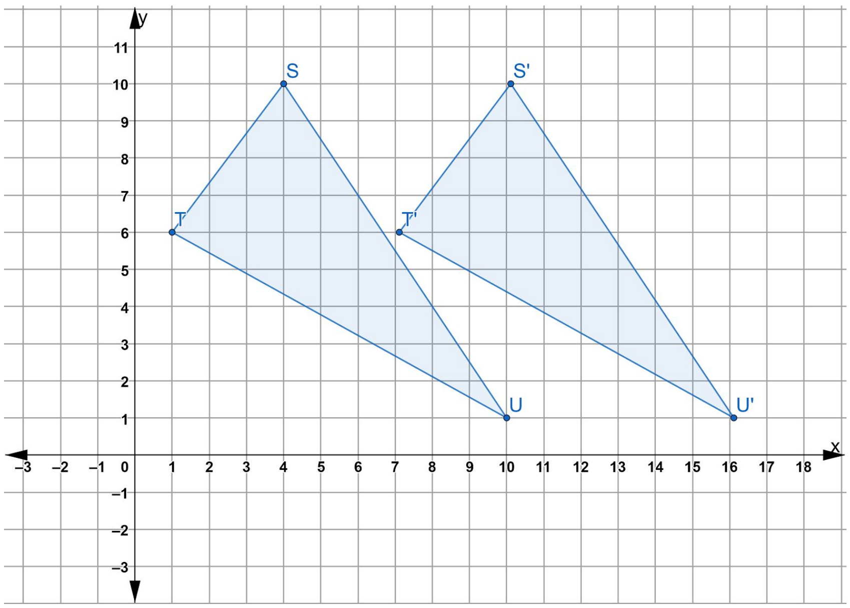
   - An image or pattern created by translation
   - An image or pattern created by reflection
   - An image or pattern created by rotation.

   Label your pictures with as many vocabulary words as you can.

Answer Keys

Translations on the Coordinate Plane - Answer Key

1)



Point S (4,10)

Point T (1,6)

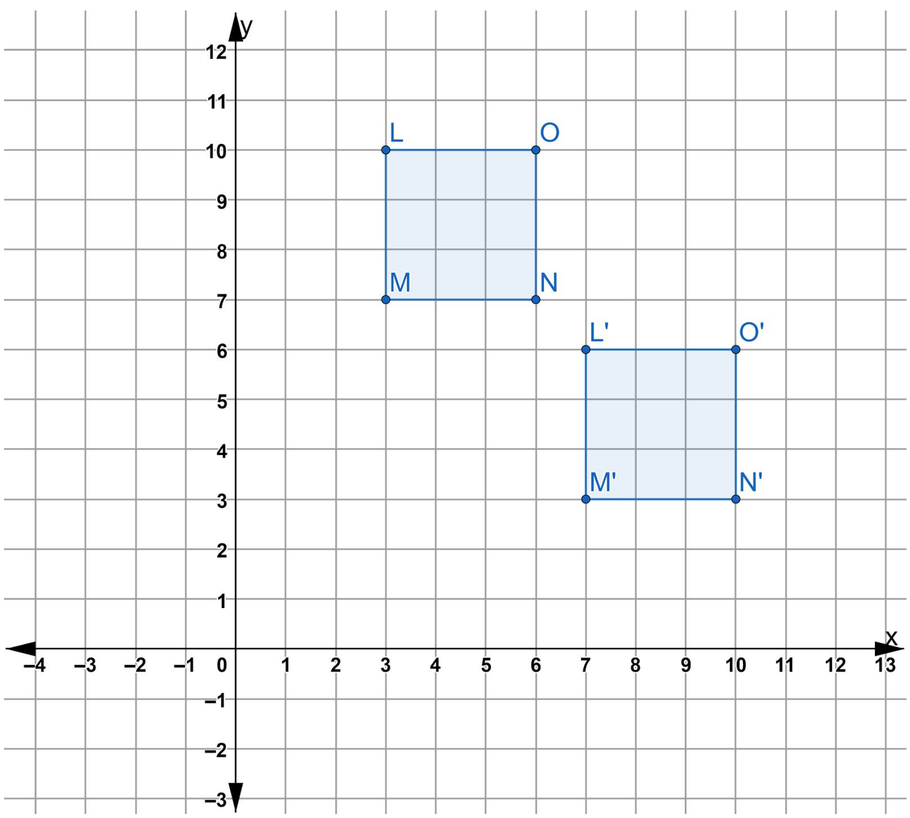
Point U (10,1)

Point S' (10,10)

Point T' (7,6)

Point U' (16,1)

2)

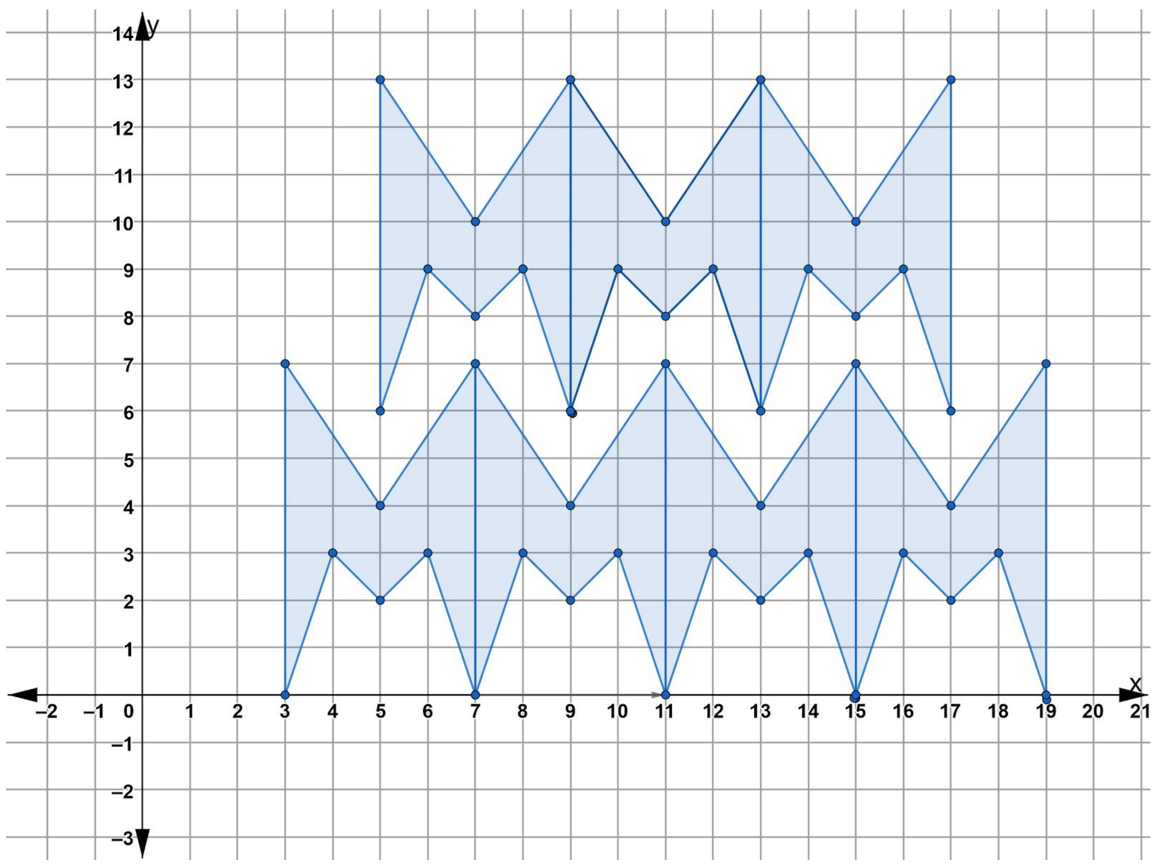


3) Choice D.

   Choice A is a common mistake if you do not map the point on figure A to the same point on figure B. Choice A maps the point at the tip of the original arrow to the back of figure B.

   Choice C is another common mistake if you confuse which is the original figure and which is the translated image. Choice C is the translation that maps Figure B onto Figure A. The question is asking about how we slide figure A onto Figure B.

4) The design built by those translations should look like this.



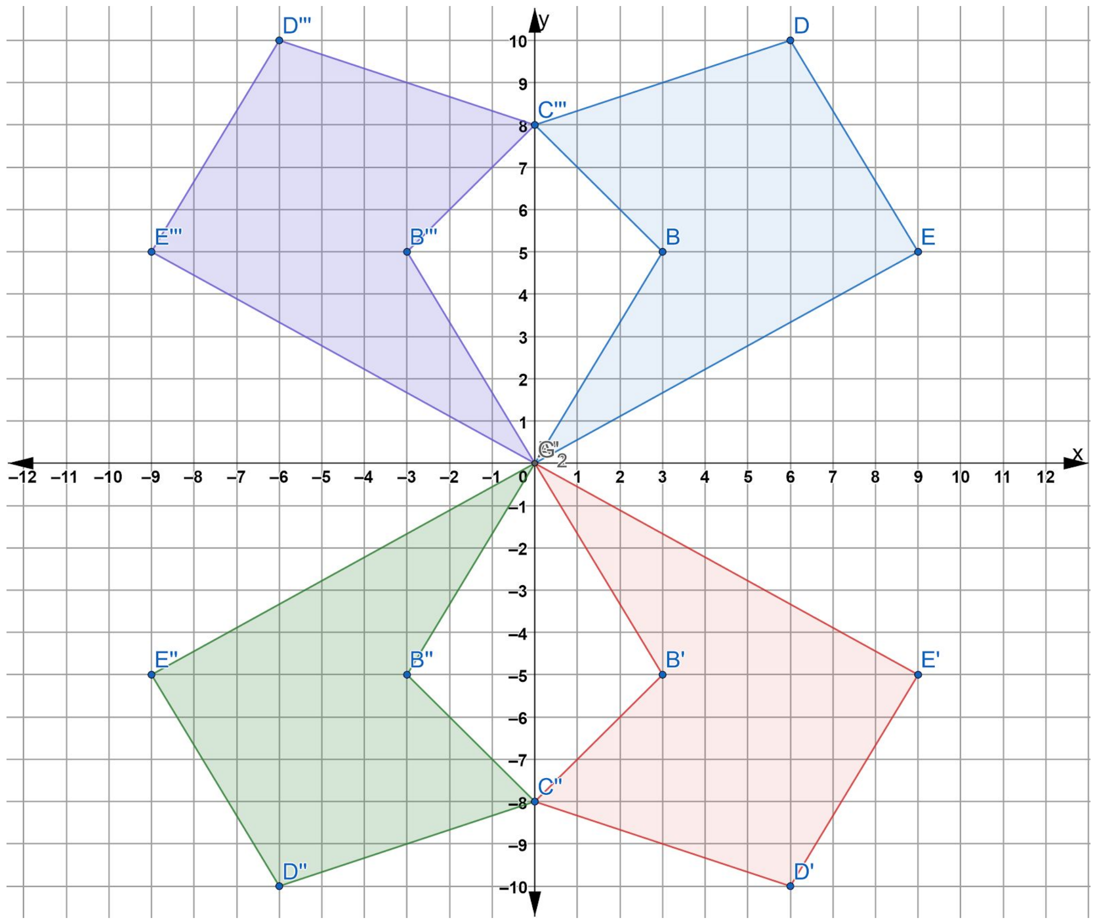
Reflections on the Coordinate Plane - Answer Key

1)
   a) Point A and Point A' are both 0 units away from the line of reflection.
   b) Point B and Point B' are both 5 units away from the line of reflection.
   c) Point C and Point C' are both 8 units away from the line of reflection.
   d) Point D and Point D' are both 10 units away from the line of reflection.
   e) Point E and Point E' are both 5 units away from the line of reflection.

2) Point B" would be (-3, -5). B' is at (3, -5). From B', we have to move 3 units to get to the line of reflection. Since B" has to be the same distance away from the line of reflection, we need to move another 3 units. That brings us to the point (-3, -5).

3) The ordered pair for point E" is (9, -5).

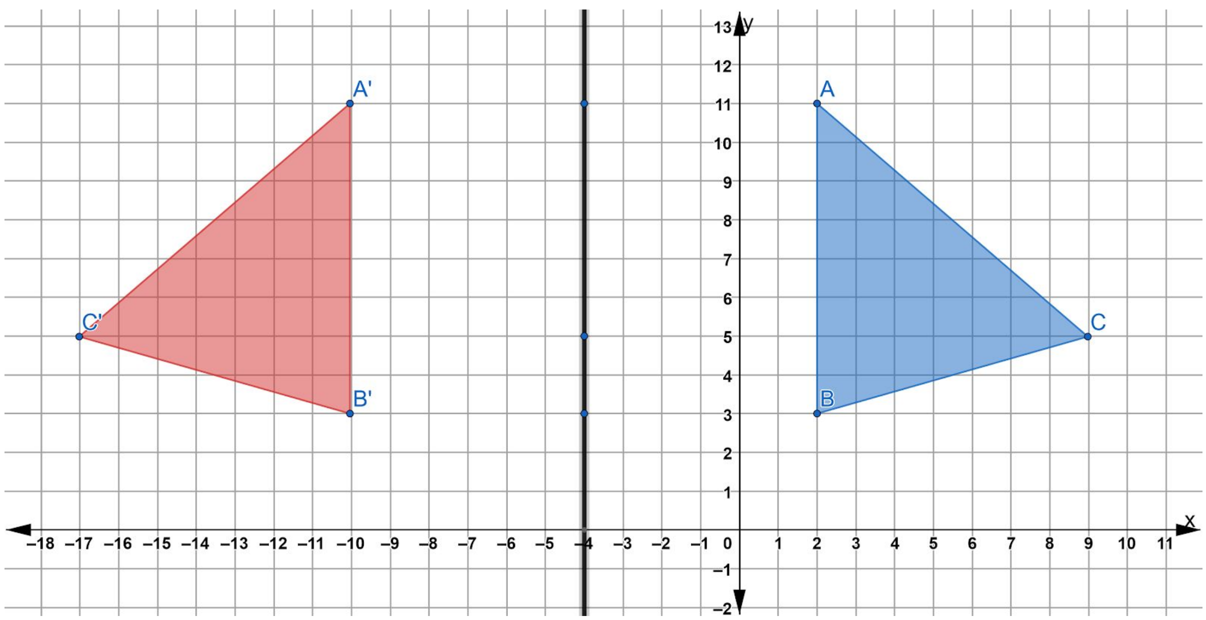
4) The ordered pair for point D"' is (-6, 10). This is what your coordinate grid should look like after you complete the reflections in questions 3 and 4.



5) If we count the distance between point A and point A’, we find that they are 12 units apart. The line of reflection needs to halfway between that, which means the line of reflection needs to be 6 units from either point. The same is true of points B and B’. Points C and C’ are 26 units away which means the line of reflection has to be 13 units away from each point.

Original Figure	Halfway Point	Image
A	(-4, 11)	A’
B	(-4, 3)	B’
C	(-4, 5)	C’

We can see that the three halfway points form a line. We can also see that they all have an x coordinate of -4. So the line of reflection is the line **x = -4**



6) When we are reflecting a figure across a line of reflection, the line of reflection is like a mirror. Corresponding points are the same distance away from the line of reflection.

7) The original triangle here is in Quadrant 1. When we reflect it over the y-axis it flips into Quadrant 2. (7,2) is 7 units to the right of the y-axis. If we reflect it across the y-axis, it will need to be 7 units to the left of the y-axis.

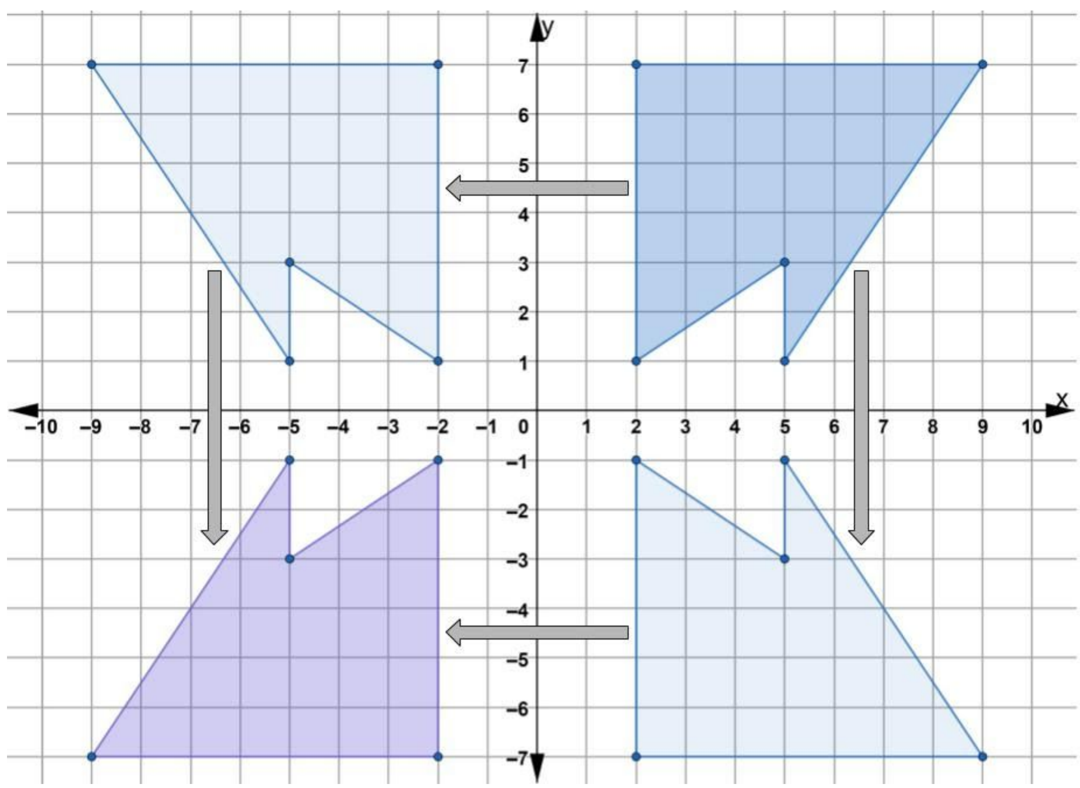
The same will be true for (8,6). Since (8,6) is 8 units to the right of the y-axis, then its image needs to be 8 units to the left of the y-axis. So (8,6) reflected over the y-axis becomes (-8,6).

(4,3) is 4 units to the right of the y-axis, so its image reflected across the y-axis would be 4 units to the left of the x-axis. (4,3) reflected over the y-axis becomes (-4,3).

8)

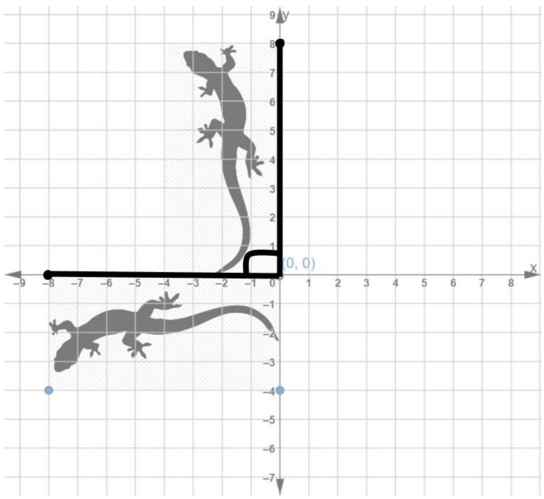
Original Figure	(7,6)	(10,6)	(10,1)	(7,1)
Reflected over the x-axis	*(7,-6)*	*(10,-6)*	*(10,-1)*	*(7,-1)*
Reflected over the y-axis	*(-7,6)*	*(-10,6)*	*(-10, 1)*	*(-7,1)*
Reflected over *y=x*	*(6,7)*	*(6,10)*	*(1,10)*	*(1,7)*

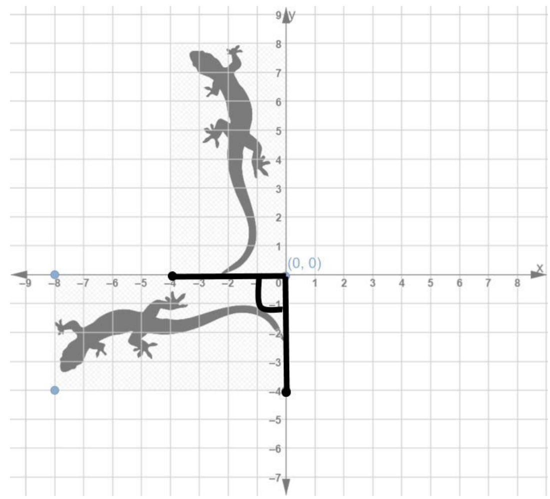
9) If you reflect ABCDE over the x-axis and then reflect the resulting figure over the y-axis you will end up with an image in the same place as you would if your reflected ABCE over the y-axis first and then over the x-axis.



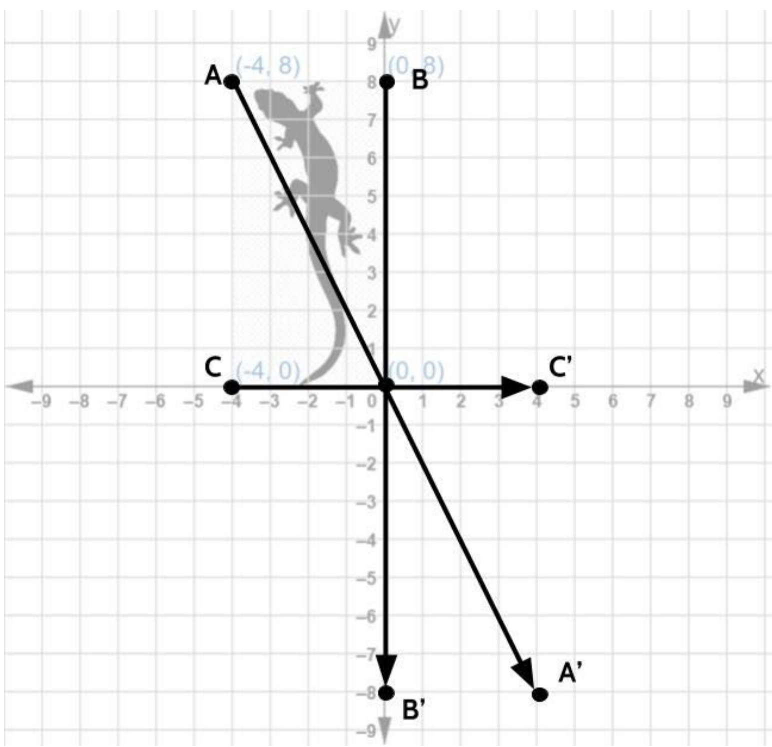
Rotations on the Coordinate Grid - Answer Key

1)





2)

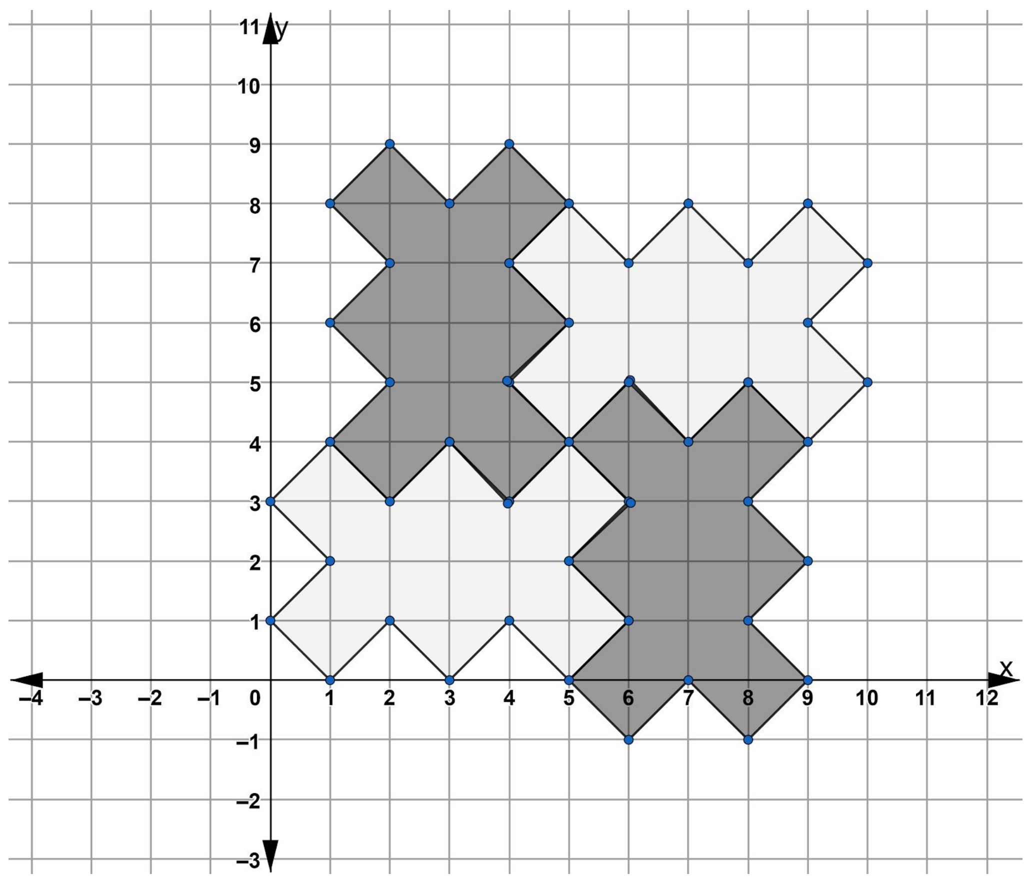


3)

Coordinate Points in Original Figure	Coordinate Points for Image rotated 90° counterclockwise	Coordinate Points for Image rotated 180° counterclockwise	Coordinate Points for Image rotated 270° counterclockwise
(0,0)	(0,0)	(0, 0)	(0, 0)
(0.8)	(-8, 0)	(0, -8)	(8,0)
(-4,8)	(-8, -4)	(4, -8)	(8, 4)
(-4,0)	(0, -4)	(4,0)	(0,4)

4) Choices B, C and D are all transformations that map the original Kermit to the image.

5) When you turn the original figure with rotations of 90, 180, and 270 counterclockwise about point (2,3) you will end up with a tessellation.

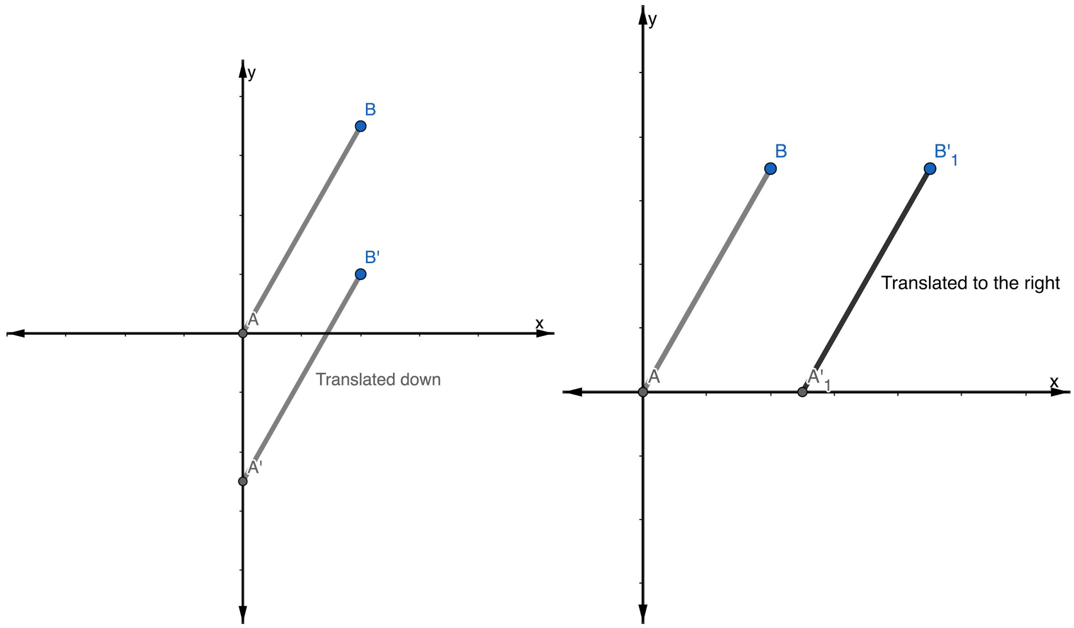


Sequences of Transformations - Answer Key

1) Choice B
2) Choice C
3) Choice C
4) Choice A
5) There are infinite possible sequences you can draw. Be creative!

Test Practice Questions - Answer Key

1) Choice A and E. Parallel lines are lines that remain the same distance away from each other and never touch. If you slide every point on a line down or to the right, each point will be the same distance away from each other and the lines will be parallel.



2) Choice B.

   We can see that △LIV has been flipped, so we can tell that it is a reflection and not a translation. That narrows the options down to choice B or D. To identify the line of reflection, we can measure the distance between the points in the original figure to those in the image. For example, point V is 4 units away from point V'. We know the line of reflection needs to be halfway between them, which means the line of reflection in this transformation is the x-axis.

3) Choice A, a reflection over the x-axis.

   The x-axis is halfway between each point in the original figure and in the reflected image. Because the x-axis goes through rectangle ABCD part of rectangle A'B'C'D' overlaps with the original rectangle.

4) Choice C.

   Reflections, rotations and translations are all examples of rigid transformations. Rigid transformations are ways of moving figures without changing their shape or size.

Dilations change the size of a figure, which means dilations **never** produce congruent figures. A dilation of a scale factor of two would make △ABC twice as big.

5) Choice D.

   This problem is a little trickier because point C' is outside the coordinate plane that is provided. One strategy you might use to answer questions like this one is to sketch some lines to continue the coordinate plane. As a reminder, you are entitled to have graph paper during your HSE exam. If you are not given graph paper, you should request some from the person administering the exam. Another strategy is to write out the coordinates you are being asked to transform - in this case, point C (7,-7). If you need to slide the figure 4 units to the right, then each x coordinate will increase by 4. If you need to slide the figure 4 units down, then the y coordinates will need to decrease by 4. So (7, -7) would slide to (11, -11).

6) Choice C, a translation 8 units to the left and 3 units down.

   We can tell Choice B is incorrect by looking at the corresponding points. Point N on the original figure maps to point F on the image, but moving 2 units to the right and 3 units up would map point N to point E.

   Choice D is the translation that would take us from image LMNO to original figure DEFG. Choice D is a common mistake if you confuse which is the original figure and which is the translated image.

7) Choice A, (-1, 4).

   If point Q (4, -2) maps to point Q' (1, 7) then triangle QRS maps to triangle Q'R'S' by a translation of 3 units left and 9 units up.

8) Choice A, a reflection followed by a rotation.

   Choice C is a very compelling answer. It appears like rectangle ABCD slides down and then rotates. Only by looking at the names of the points can you see that A'B'C'D' is made by reflecting rectangle ABCD over the x-axis.

9) Choice D, a translation followed by a rotation.

10) Choice B, reflection over the x-axis followed by a translation.

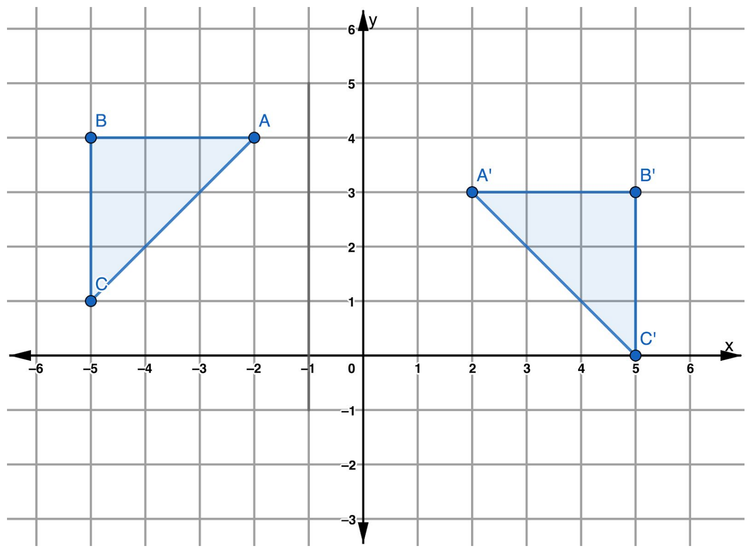
The Language of Rigid Transformations - Answer Key

1) The transformation above can be described as a reflection about the y-axis and a translation 1 unit up. (Sentence D)

2) The transformation above can be described as a reflection about the line x = -1 and a translation 1 unit up. (Sentence C)

3) The transformation above can be described as a reflection about the line x = -1 and a translation 1 unit down. (Sentence B)

4) Sentence A is the one that does not have a corresponding picture. A reflection of the original about the y-axis and a translation 1 unit down would look like this:

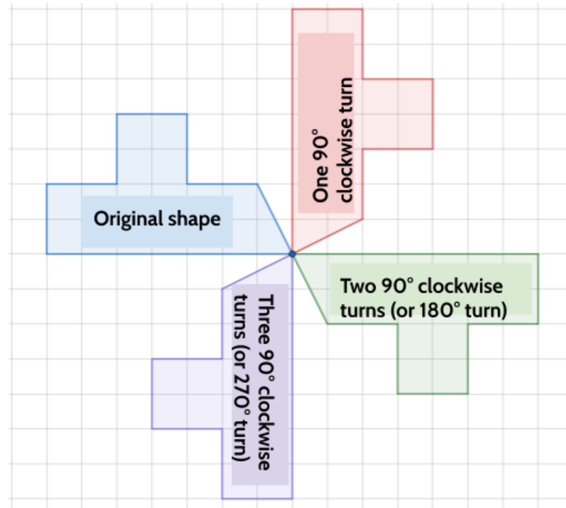


5) Each paragraph should use the 4 vocabulary words in the circle. Be creative. There is no wrong way to do this activity!

6) Take your time with this activity. Look around and describe what you see. Explain what rigid transformations are to someone else and have them help you find examples. This is an opportunity to practice all the vocabulary and math skills you have learned.

Vocabulary Review

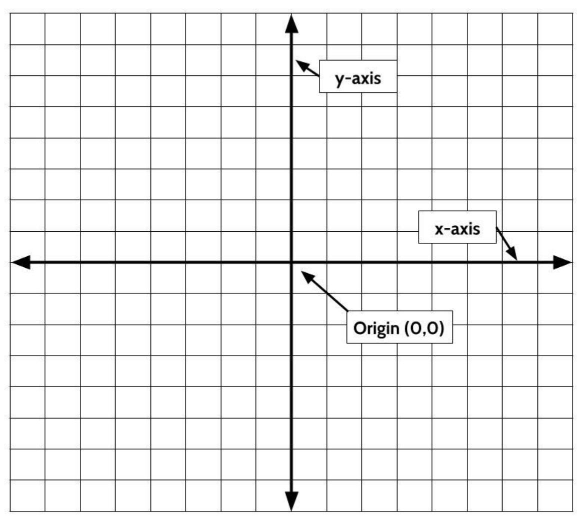
90 degree turn: A 90° turn is a quarter turn. A figure that is turned 360° turns in a complete circle and returns to its original position. In the diagram to the right, the original shape is being turned clockwise 90°**.** (See *ROTATION, ROTATIONAL SYMMETRY*)



clockwise / counterclockwise (adjective): clockwise means turning something in the same direction in which the hands of a clock move. Counterclockwise means turning something in the opposite direction in which the hands of a clock move. Turning a screw clockwise will tighten the screw, turning a screw counterclockwise will loosen the screw. (See *ROTATION, ROTATIONAL SYMMETRY*)

congruent (adjective): two shapes are congruent to each other if you can slide, flip or turn one figure so it fits exactly on top of the other figure. Any image that is the result of a rigid transformation is congruent to the original figure.

coordinate plane (noun): a completely flat surface formed when two straight number lines cross each other at right angles. The point where the lines intersect is called the origin and represents the zero on each axis. An exact position on the grid can be described using coordinates. The coordinate plane is also called the coordinate grid.



corresponding (adjective): If a part of the original figure matches up with a part of the image, we call them corresponding parts. The part could be an angle, point, or side, and you can have corresponding angles, corresponding points, or corresponding sides.

dilation (noun): a dilation is a *non-rigid* transformation that makes a figure larger or smaller without changing its shape. A figure that is dilated is <u>not congruent</u> to the original figure because you cannot slide, flip, or turn the image to fit exactly on top of the original figure. Dilations can be described by the term *scale factor*. In this diagram, rectangle ABCD is dilated by a scale factor of 2. That means the image is 2 times larger than the original figure. A scale factor of ½ means that the dilated image is half the size of the original figure.

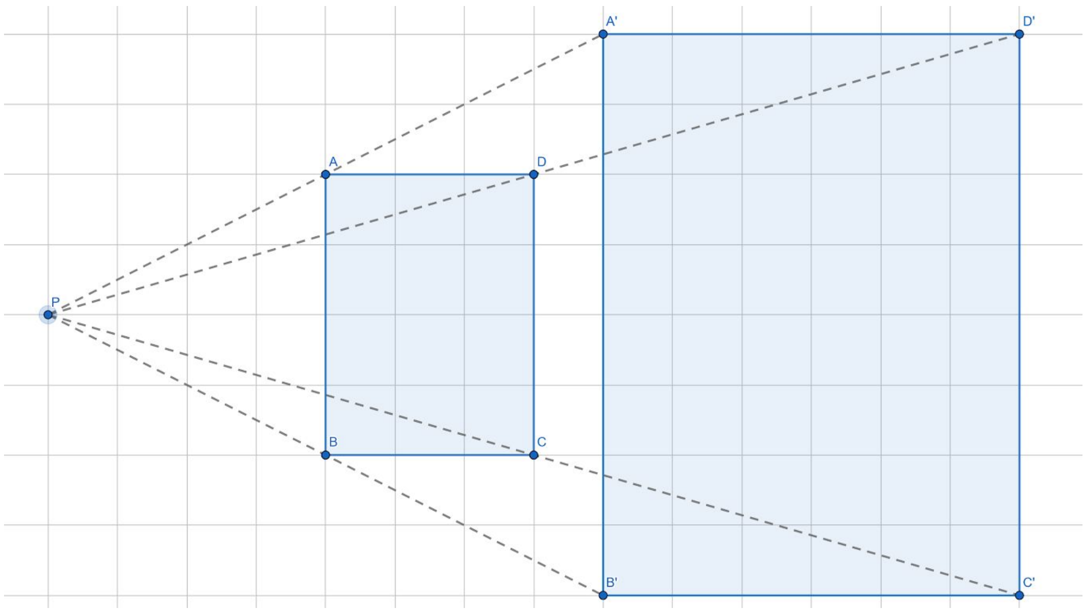


figure (noun): a shape. *A triangle is a three-sided figure.*

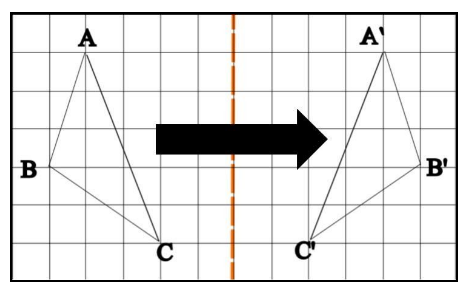


image (noun): in everyday English, an image is a picture. In the language of rigid transformations, an image specifically refers to a figure that has gone through a rigid transformation. An image is congruent to the original figure. (See *ORIGINAL)*

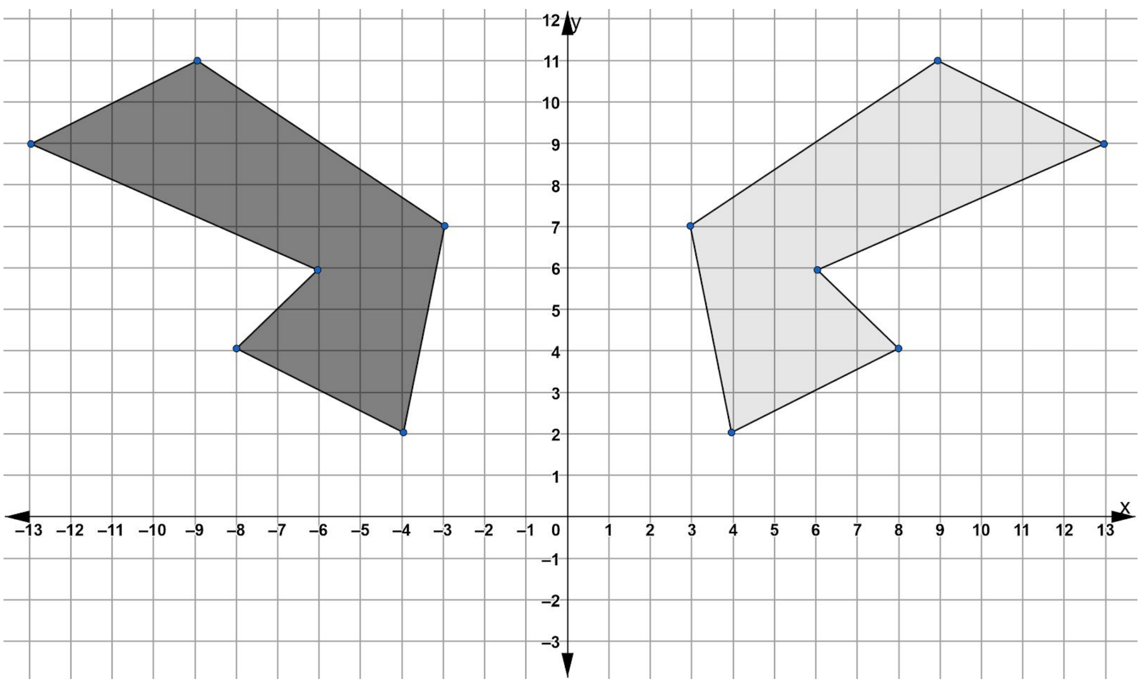
map (verb): in everyday English, map means to make a drawing of an area showing roads, rivers, cities, etc. In the language of rigid transformations, to map a figure is to move all of the points of an original figure onto an image. For example, in the diagram to the right, we use a reflection to *map* △ABC to △A'B'C'.

non-rigid transformation (noun): a transformation that does not keep the size and shape of the original figure. (See *DILATION*)

origin (noun): the point where the x-axis and the y-axis intersect on the coordinate grid. The origin has the coordinates (0,0).

original (noun): In everyday English, original means the first or earliest. In the language of transformations, original specifically refers to the figure you start with. <See *IMAGE*>

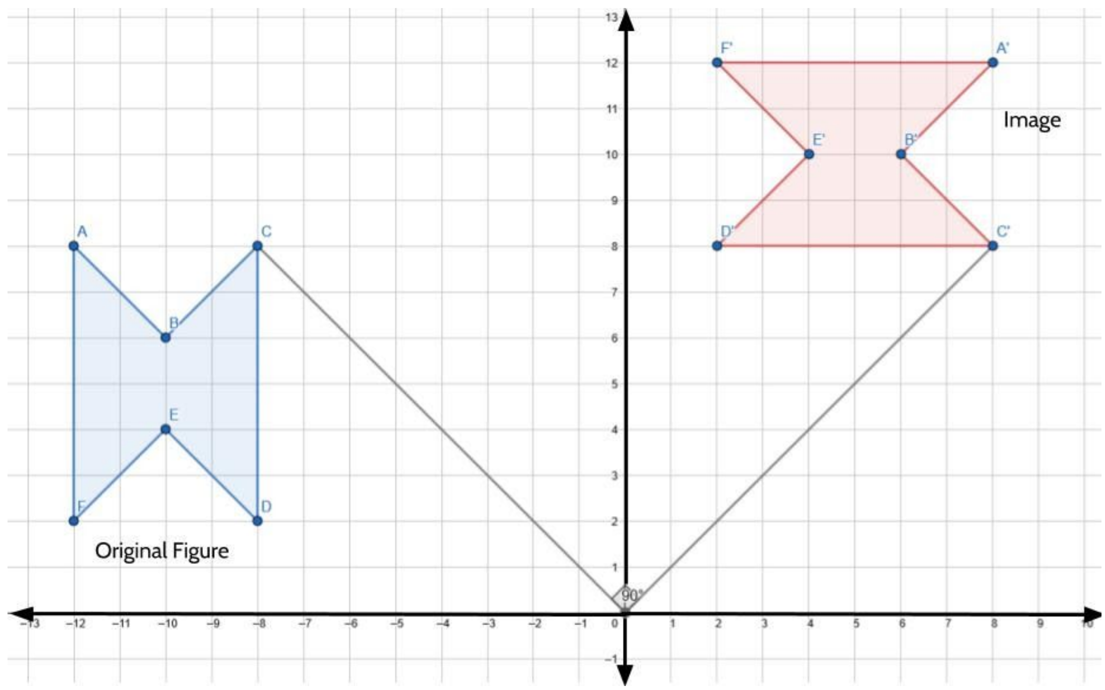
reflection (noun): A transformation that maps each point of a figure to a point in its mirror image. In the diagram below, the darker shape is reflected about the y-axis into the lighter image. The line of reflection acts as the mirror and is the halfway point between each point and its image. In this case, the y-axis is the line of reflection. So each on the darker shape is the same distance away from the y-axis as its reflected point in the lighter figure.

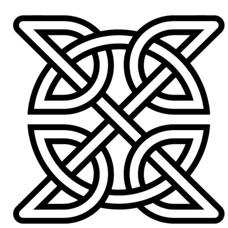


reflectional symmetry (noun): a figure or design has reflectional symmetry if you can draw a line that divides the figure (or design) into mirror images. We call the line that cuts the figure (or design) into halves, the line of symmetry or the line of reflection.

rigid transformation (noun): When you move all the points of a figure in such a way that produces an image that is identical in size and shape to the original figure. With rigid transformations, the image and original figure should fit exactly on top of each other. If two figures are congruent, that means there is a sequence of rigid transformations we could describe that would make one of the figures look like the other. (See *REFLECTION, ROTATION, TRANSLATION*)

rotation (noun): a transformation that rotates all of the points in a figure about a point. When we describe rotations, we describe the number of degrees it is turned, and whether it is turned clockwise or counterclockwise. For example, in the diagram below, the original figure was rotated 90° clockwise about the origin. Since the original figure and the image are the same size and shape, we can say they are congruent.





rotational symmetry (noun): a figure or design has rotational symmetry if it can be rotated less than 360 degrees about a point to a position where it looks the same as it did in the original position. The Celtic knot to the right has rotational symmetry because it can be rotated 180 degrees and look identical to the way it did in its original position.

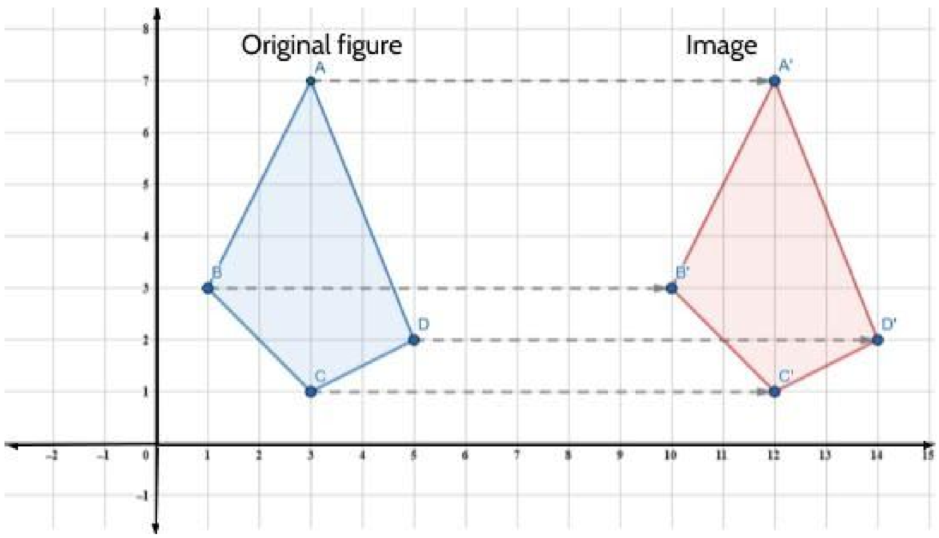
sequence (noun): a series of related events that happen in a particular order. A *sequence* of transformations would mean a figure going through multiple transformations.

step (noun): one in a series of things that are done to achieve something. If you wanted to plot a point on the coordinate grid, your first *step* would be to start in the origin. Your next *step* would be to count along the x-axis. Your third *step* would be to count along the y-axis.

symmetry (noun): Something has symmetry if it has two or more sides that are identical after a flip, slide, or turn. (See *ROTATIONAL SYMMETRY, REFLECTIONAL SYMMETRY*)

tessellation (noun): an image that includes repeated congruent figures that cover a surface without gaps or overlapping. The art of M.C. Escher often uses tessellations. Tiles in a bathroom is another example.

translation (noun): In everyday English, translation means to change speech or writing from one language to another. It has a different definition in the language of rigid transformations. A translation means sliding each point of a figure the same distance, in the same direction. In the diagram below, each point of the original figure is translated 9 units to the right. Since the original figure and the image are the same size and shape, we can say they are congruent.



x-axis (noun): the line that goes from left to right of a graph. The horizontal axis on the coordinate grid.

y-axis (noun): the line that goes from top to bottom of a graph. The vertical axis on the coordinate grid.

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