

# Rigid Transformations: Shapes on a Plane

*Fast Track Math GRASP Packet*



*Detail of tile work from the Alhambra Palace in Granada, Spain*

Version 1.6  
Released 1/16/2019



This Fast Track GRASP Math Packet was made possible through support from the New York State Education Department, Office of Adult Career and Continuing Education Services. The Fast Track GRASP Math packets use a Creative Commons license of Attribution-NonCommercial 4.0 International (CC BY-NC 4.0), which means that they can be shared, copied and redistributed in any form, as long as the document retains attribution to CUNY for their creation.



## **Table of Contents**

<b>Welcome!</b>	<b>4</b>
Vocabulary	5
Introduction	7
<b>Congruence and Rigid Transformations</b>	<b>9</b>
Translation	14
Reflection	15
Rotation	16
Congruence and Rigid Transformations - Answer Key	22
<b>The Coordinate Plane</b>	<b>24</b>
The Fruit Graph	24
The Chessboard	29
Fruit Graph and Chessboard - Answer Key	32
The Coordinate Plane	34
Plotting Points on the Coordinate Plane	36
The Coordinate Plane - Answer Key	42
Longitude Lines and Latitude Lines	44
Shapes on the Coordinate Plane	45
Lines on the Coordinate Plane	48
Shapes and Lines on the Coordinate Plane - Answer Key	55
How Math is Written	58
How Math is Written - Answer Key	61
<b>Working with Transformations on the Coordinate Plane</b>	<b>62</b>
Translations on the Coordinate Plane	64
Creating Designs with Translations on the Coordinate Plane	70

## *Rigid Transformations*

Translations on the Coordinate Plane - Answer Key	72
Reflections on the Coordinate Plane	74
Reflections on the Coordinate Plane - Answer Key	86
Rotations on the Coordinate Plane	89
Rotations on the Coordinate Grid - Answer Key	99
Sequences of Transformations	101
<b>Practice Questions</b>	<b>106</b>
Practice Questions - Answer Key	116
Final Project	118
<b>The Language of Rigid Transformations</b>	<b>120</b>
Vocabulary Review	120
Using Graphic Organizers to Learn Vocabulary	125
Matching Pictures and Descriptions	130
Concept Circle	131
Rigid Transformations and You	133
The Language of Rigid Transformations - Answer Key	134

## Welcome!

Congratulations on deciding to continue your studies! We are happy to share this study packet on the topic of rigid transformations. We hope that that these materials are helpful in your efforts to earn your high school equivalency diploma. This group of math study packets will cover mathematics topics that we often see on high school equivalency exams. If you study these topics carefully, while also practicing other basic math skills, you will increase your chances of passing the exam.

Please take your time as you go through the packet. You will find plenty of practice here, but it's useful to make extra notes for yourself to help you remember. You will probably want to have a separate notebook where you can recopy problems, write questions and include information that you want to remember. Writing is thinking and will help you learn the math.

After each section, you will find an answer key. Try to answer all the questions and then look at the answer key. It's not cheating to look at the answer key, but do your best on your own first. If you find that you got the right answer, congratulations! If you didn't, it's okay. This is how we learn. Look back and try to understand the reason for the answer. Please read the answer key even if you feel confident. We added some extra explanation and examples that may be helpful. If you see a word that you don't understand, try looking at the *Vocabulary Review* at the end of the packet.

We also hope you will share what you learn with your friends and family. If you find something interesting in here, tell someone about it! If you find a section challenging, look for support. If you are in a class, talk to your teacher and your classmates. If you are studying on your own, talk to people you know or try searching for a phrase online. Your local library should have information about adult education classes or other support. You can also find classes listed here: <http://www.acces.nysed.gov/hse/hse-prep-programs-maps>

You are doing a wonderful thing by investing in your own education right now. You have our utmost respect for continuing to learn as an adult.

Please feel free to contact us with questions or suggestions.

Best of luck!

Mark Trushkowsky ([mark.trushkowsky@cuny.edu](mailto:mark.trushkowsky@cuny.edu)) & Eric Appleton ([eric.appleton@cuny.edu](mailto:eric.appleton@cuny.edu))  
CUNY Adult Literacy and High School Equivalency Program



## Vocabulary

It is important to understand mathematical words when you are learning new topics. The following vocabulary will be used a lot in this study packet:

slide • turn • flip • congruent • reflect • rotate • translate • transformation

In this first activity, you will think about each word and decide how familiar you are with it. To start, think about the word “slide .” Which of these statements is true for you?

- I know the word “slide” and use it in conversation or writing.
- I know the word “slide ,” but I don’t use it.
- I have heard the word “slide ,” but I’m not sure what it means.
- I have never heard the word “slide ” at all.

In the chart on the next page, read each word and then choose one of the four categories and mark your answer with a ✓ (checkmark). Then write your best guess at the meaning of the word in the right column. If it’s easier, you can also just use the word in a sentence.

Here’s an example of how the row for “slide” might look when you’re done:

Word	I know the word and use the word	I know the word but don’t use it	I have heard the word, but I’m not sure what it means	I have never heard the word	My best guess at the meaning of the word (or use the word in a sentence)
slide	✓				To move slowly over a surface while continuing to touch it

Complete the table on the next page.

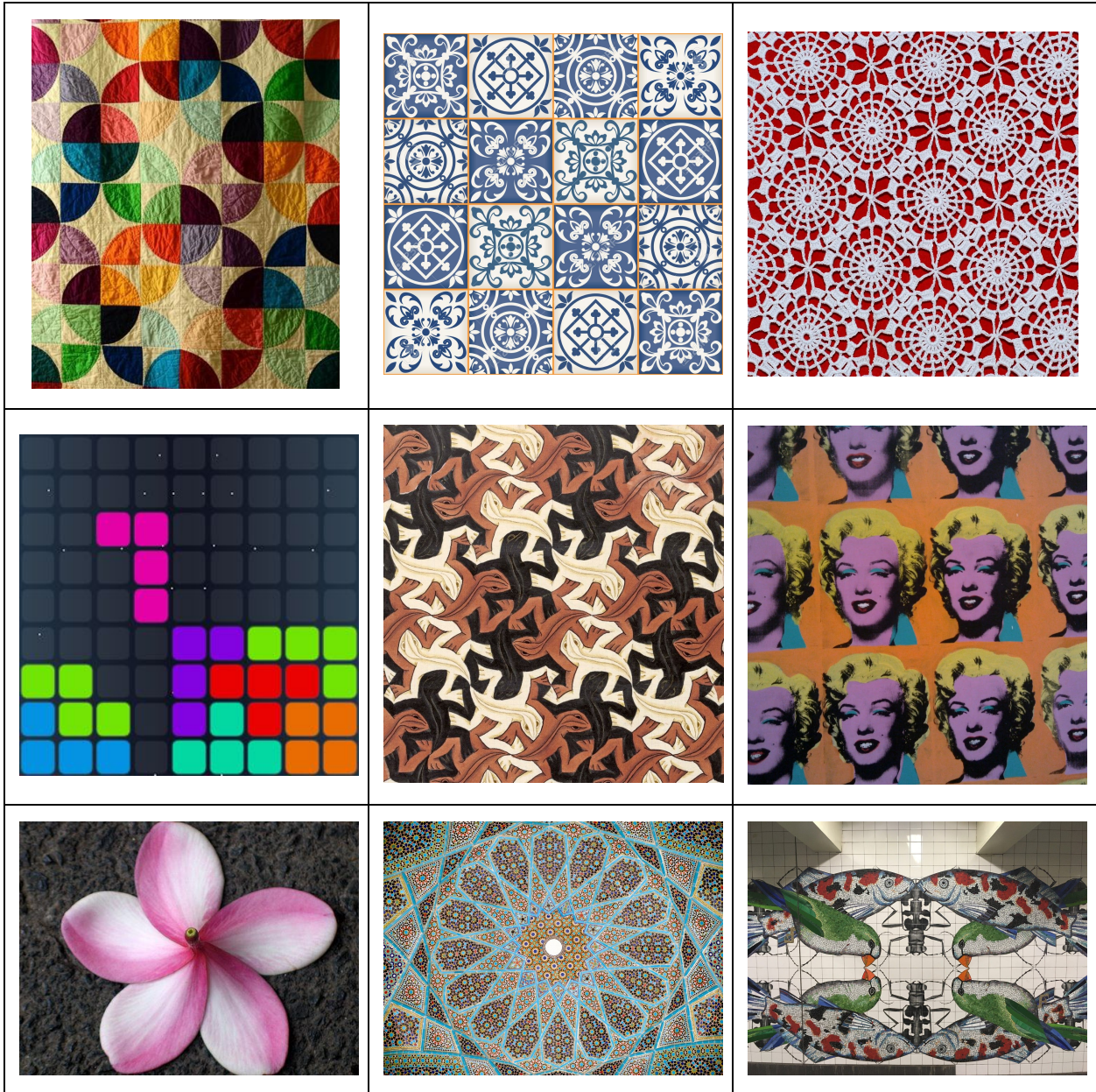
Rigid Transformations

Word	I know the word and use the word	I know the word but don't use it	I have heard the word, but I'm not sure what it means	I have never heard the word	My best guess at what the word means (or use the word in a sentence)
slide					
turn					
flip					
congruent					
reflect					
rotate					
translate					
transformation					

## Introduction

In this packet we are going to be working on a topic in geometry called rigid transformations.

We see rigid transformations everywhere. Nature, art, design, architecture, textiles, clothing, furniture, computer animation, advertisements, logos. and video games are just some of places where we see rigid transformations everyday.



## *Rigid Transformations*

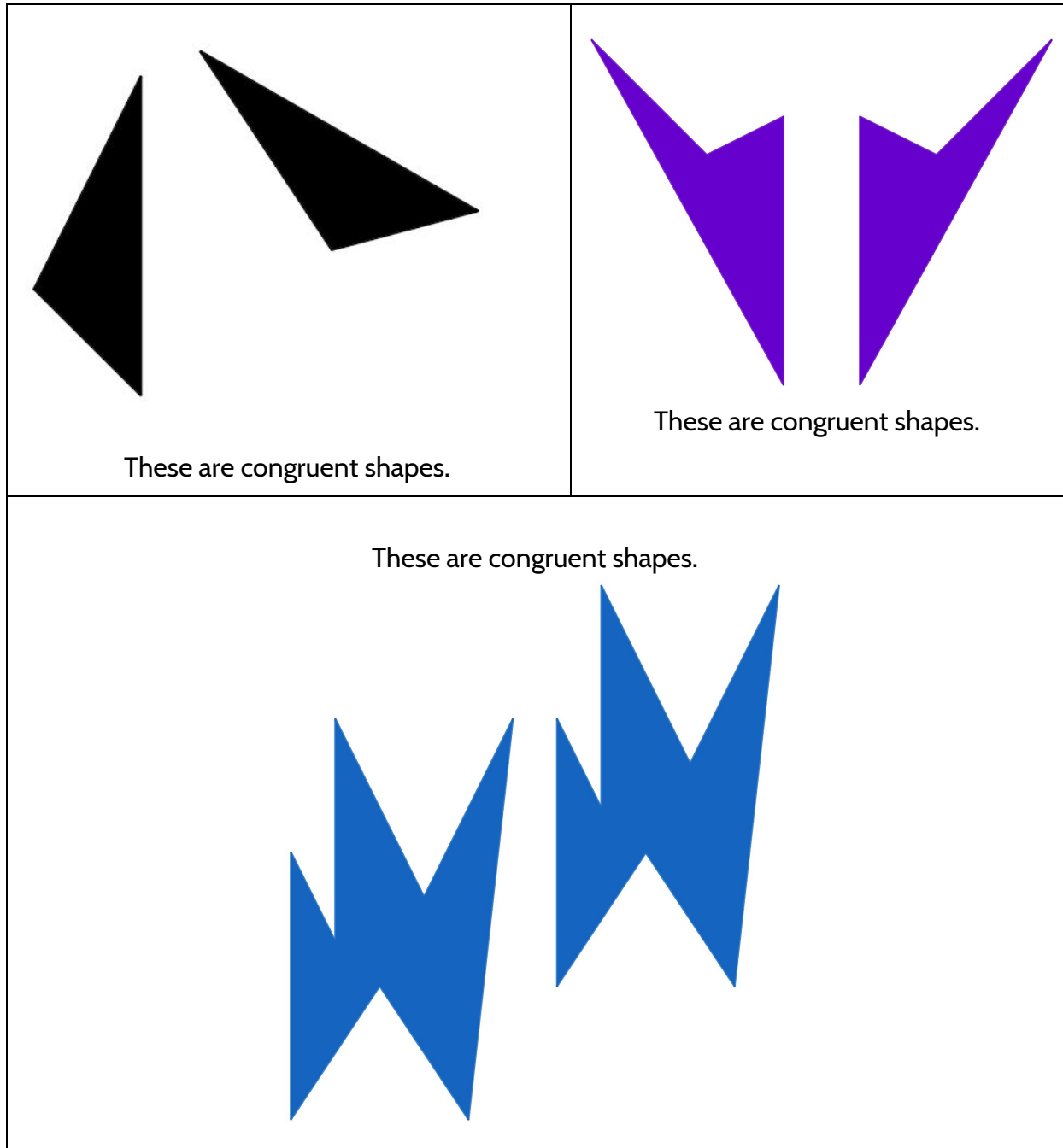
In everyday language, “rigid” and “transformation” might seem like a strange pair of words to put together. “Rigid” means “stiff, not bending or moving, very difficult to change.” On the other hand, “transformation” means to change into something else—for example, a caterpillar transforms into a butterfly.

In math, a transformation still means a change. But with rigid transformations, only some things are changing. With rigid transformations, we change the position of shapes, but we keep their size and shape the same.

In this packet you will learn about the three types of rigid transformation and how they work. You’ll also learn about the coordinate grid and how it can be used to describe rigid transformations. You will have the opportunity to practice the kinds of questions you might see on the TASC dealing with rigid transformations. You will also have a chance to use rigid transformation to identify and even create designs and patterns.


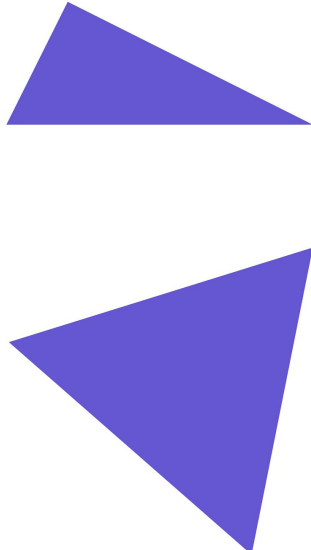
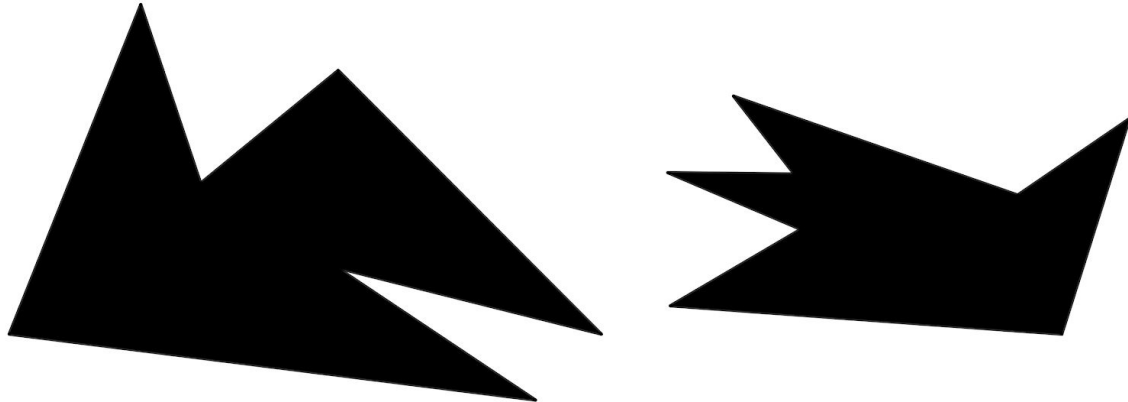
## Congruence and Rigid Transformations

The mathematics of rigid transformations begins with two or more shapes being congruent to each other.. But what does it mean for two shapes to be congruent to each other? Here are three examples of shapes that are congruent to each other.



*Rigid Transformations*

Here are three examples of shapes that are not congruent to each other:

<p>These are not congruent shapes.</p> 	<p>These are not congruent shapes.</p> 
 <p>These are not congruent shapes.</p>	

I think congruent shapes are shapes that \_\_\_\_\_  
\_\_\_\_\_

## Rigid Transformations

To help you think more about the definition of congruence, use the congruent and not congruent shapes to fill in this chart.

- Based on the examples provided, circle whether the description is sometimes true, always true, or never true.

For example:

Shapes that are congruent Sometimes / Always / Never have the same number of sides.

Two shapes that are congruent to each other...	Two shapes that are not congruent to each other...
...sometimes / always / never have the same number of sides.	...sometimes / always / never have the same number of sides.
...sometimes / always / never have the same shape.	...sometimes / always / never have the same shape.
...sometimes / always / never are the same size.	...sometimes / always / never are the same size.
...can sometimes / always / never fit exactly over each other.	...can sometimes / always / never fit exactly over each other.

We can see that if two shapes are congruent to each other, they always have the same number of sides, but that is not enough for a definition. Two shapes that are not congruent to each other can still sometimes have them same number of sides.

If we put the different characteristics together we can make a definition that is true for all of the congruent shapes and not true for the shapes that were not congruent.

**Key idea:** Congruent shapes are the same size and have the same shape. Two shapes are congruent if you can move one so that it fits exactly over the other one.

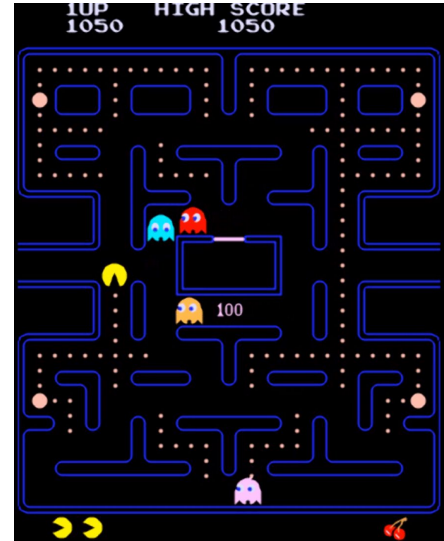


## Rigid Transformations

Rigid transformations are rules for moving shapes into different positions *without changing their shape or their size.* We call them *rigid* because they do not change their size or shape but we describe them as *transforming* because they do change their position.

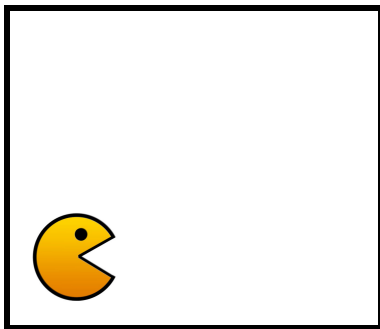
One of the most influential video games of all time was released in 1980. It was called Pac-Man. In 1981, an improved version was developed called Ms. Pac-Man. In both versions of the game, players move a character around a board as he/she ate power pellets and tried to avoid ghosts.

The computer programmers who wrote Pac-Man and Ms. Pac-Man used rigid transformations to move the characters on the video screen.

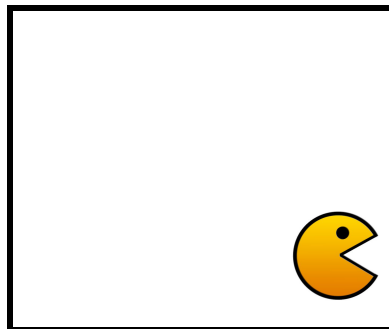


There are three types of rigid transformations. In other words, there are 3 ways we can change the position of a shape without changing its size or shape.

Let's look at the three ways that Pac-Man was programmed to move.



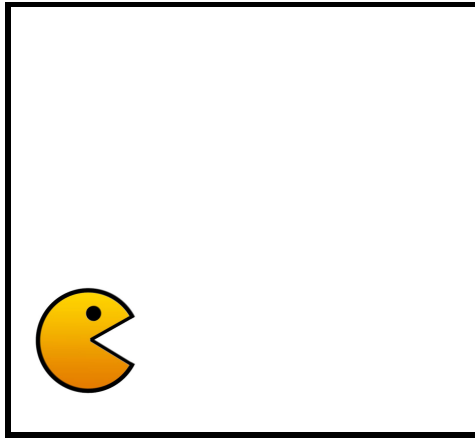
Frame 1



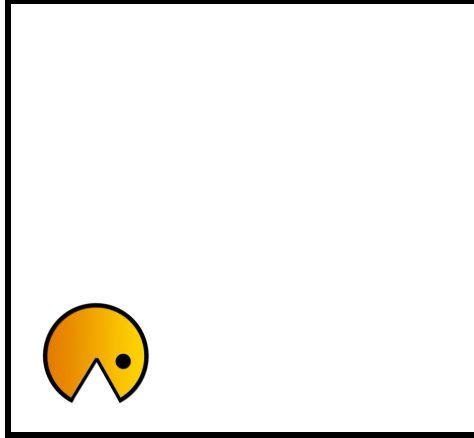
Frame 2

- 2) What language would you use to describe how Pac Man moved from frame 1 to frame 2?
- a) Slide
  - b) Turn
  - c) Flip



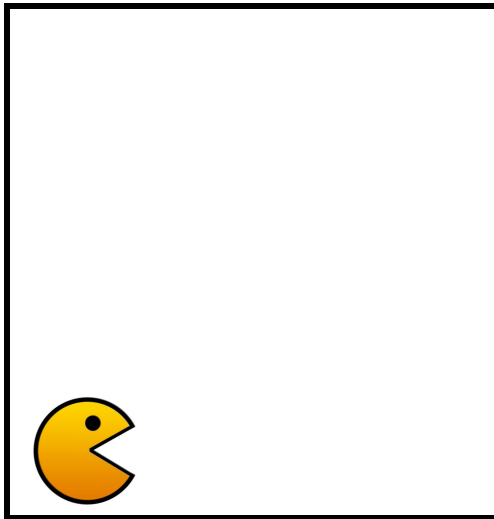


Frame 1

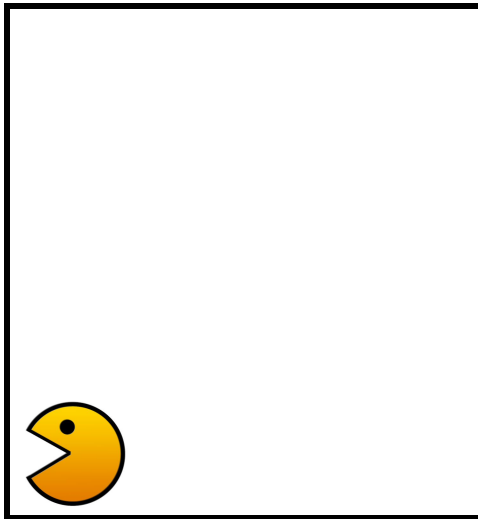


Frame 2

- 3) What language would you use to describe how Pac Man moved from frame 1 to frame 2?
- a) Slide
  - b) Turn
  - c) Flip



Frame 1

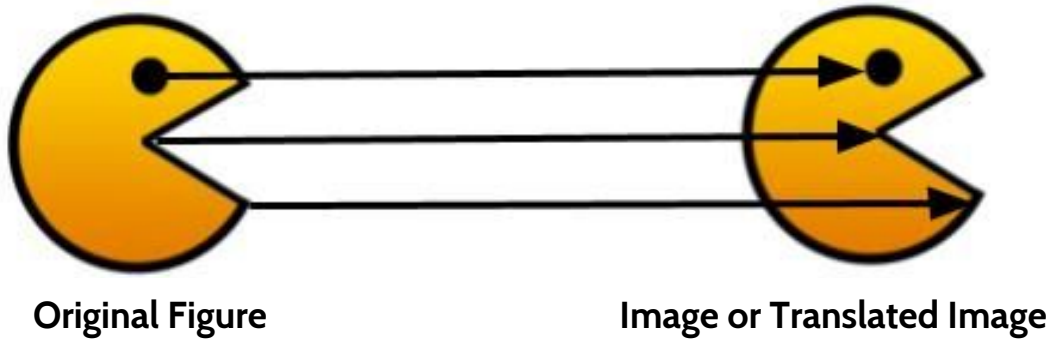


Frame 2

- 4) What language would you use to describe how Pac Man moved from frame 1 to frame 2?
- a) Slide
  - b) Turn
  - c) Flip

## Translation

The first type of rigid movement is a slide.



You can see this translation by tracing the Original Figure of Pac-Man and then sliding it along a straight line without turning it. When we slide a figure all points in that figure slide the same distance in the same direction. That is what produces the congruent image. If you cut out the translated image, and slide it over, you would see that the image fits exactly over the original Pac-Man

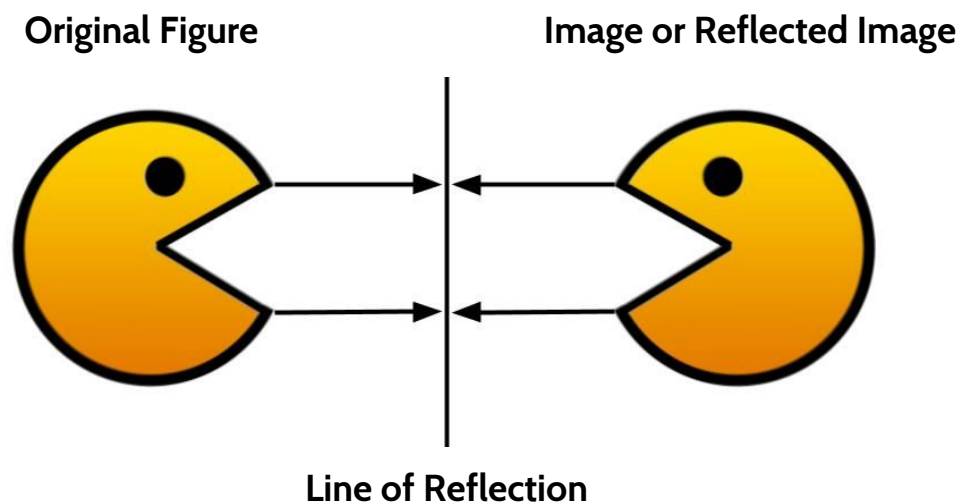
**Key Idea:** The rigid transformation of sliding a figure is called **translation**.

## Reflection

The second type of rigid transformation is the flip. This is the rigid transformation you probably see every morning when you are brushing your teeth. This type of rigid transformation produces a “mirror image”.

**Key Idea:** The rigid transformation of flipping a figure is called **reflection**.

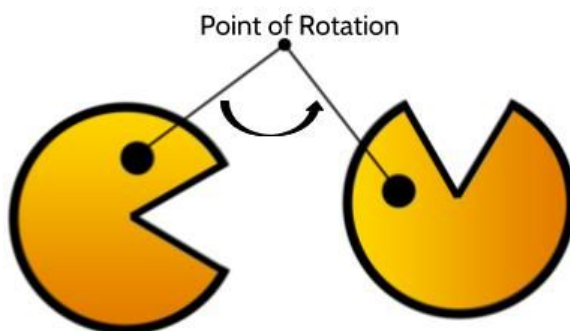
In geometry, when we reflect figures we use a **line of reflection**, which is the halfway point between the original figure and its reflected image. If you fold this page at the line of reflection, you will see that the reflected image fits exactly over the original Pac-Man.



## Rotation

The third type of rigid transformation is a turn. Here we are turning Pac-Man around the fixed point, which we call the point of rotation. When we turn a figure, we turn, or rotate all the points in that figure the same number of degrees around the point of rotation. If you drive, when you make a turn, the fixed point is the spot outside the car that your body moves towards as you turn the car.

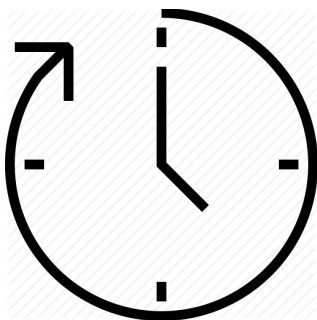
### Original Figure      Image or Rotated Image



**Key Idea:** The rigid transformation of turning a figure is called **rotation**.

A rotation requires three things: (1) the number of degrees the figure is turned, (2) the direction of the turn (is it a clockwise or counterclockwise turn), and (3) a point of rotation. The Pac-Man above was rotated  $90^\circ$  counterclockwise around the point of rotation.

Note: When we talk about the point of rotation, we can say “a rotation around the point of rotation.” We can also say the figure was rotated about the point of rotation. You will encounter both ways of saying it on your high school equivalency exam and you will have a chance to practice both as you work through this packet.



Clockwise turns move in the same direction that the hands move around a clock. We open most doorknobs by turning them clockwise. We screw in most light bulbs by turning them clockwise.

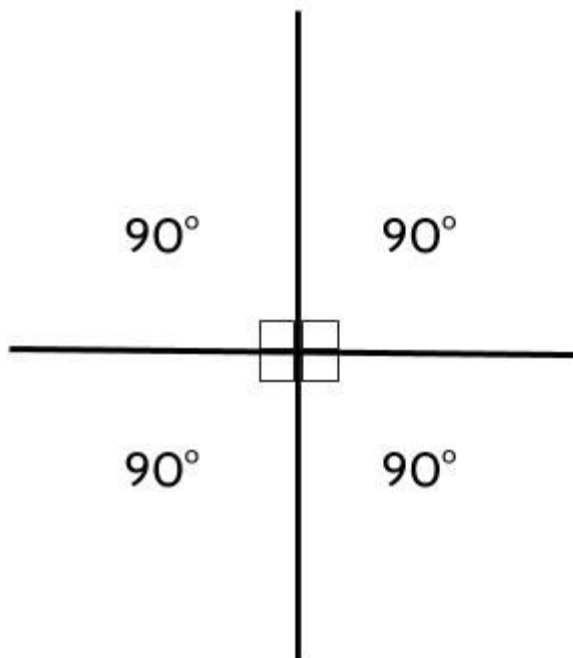
## Rigid Transformations

Counterclockwise turns move in the opposite direction from clockwise. When we unscrew a light bulb, we turn it counterclockwise.

In this packet, we will focus on turns of 90 degrees, 180 degrees and 270 degrees.

The missing number in all of this is  $360^\circ$ . If you watch competitive skiing or skateboarding you have probably heard the announcers refer to an athlete doing a  $360^\circ$ . That means the turned their bodies in a full circle.

If turning in a full circle is  $360^\circ$ , we can imagine dividing that full-circle turn into four  $90^\circ$  turns. Two of those  $90^\circ$  turns is  $180^\circ$ . Three of those  $90^\circ$  is  $270^\circ$ . Four of those turns is  $360^\circ$

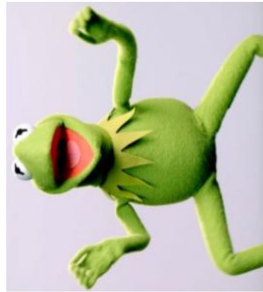


Stretch your left hand out straight in front of you. Stretch your right arm straight out from your right side. Line your head up with the arm in front of you and turn until your head is lined up with the arm out at your side. You just turned your head  $90^\circ$  (also written as  $90^\circ$ ).

Now stretch both arms straight out from your sides. Line your head up with one arm and turn it until it is lined up with the other arm. You just turned your head  $180^\circ$ . Notice that your head is facing in the opposite direction from where you started.

*Rigid Transformations*

Three rigid transformations are shown below. Label the **reflection**, the **translation**, and the **rotation**. Explain how you know.



5)



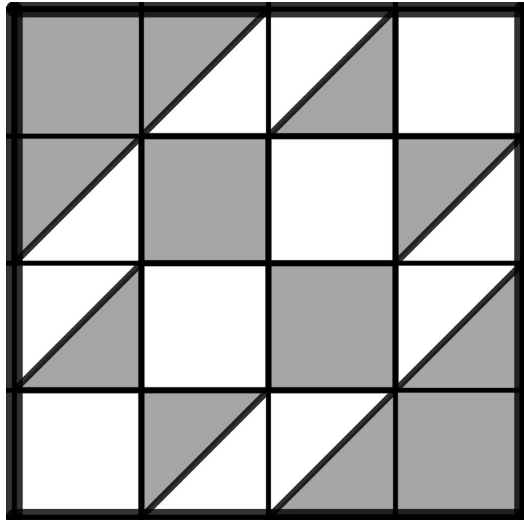
6)



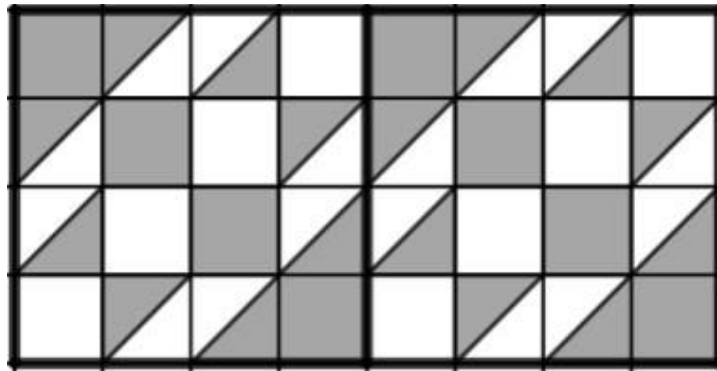
7)

## Create Your Own Quilt

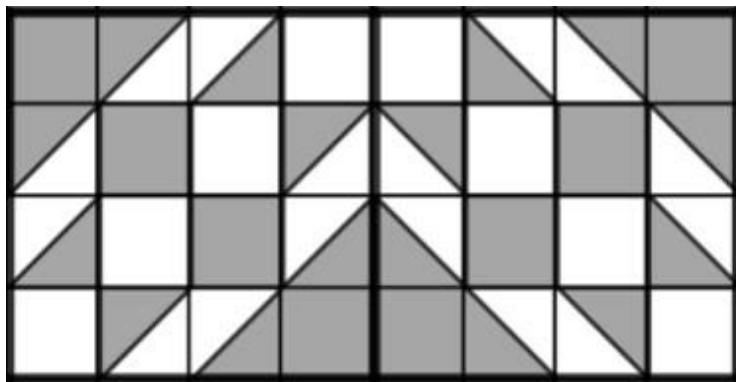
Rigid transformations can often be seen in quilt design.



Let's say we wanted to repeat this pattern in a quilt using rigid transformations.



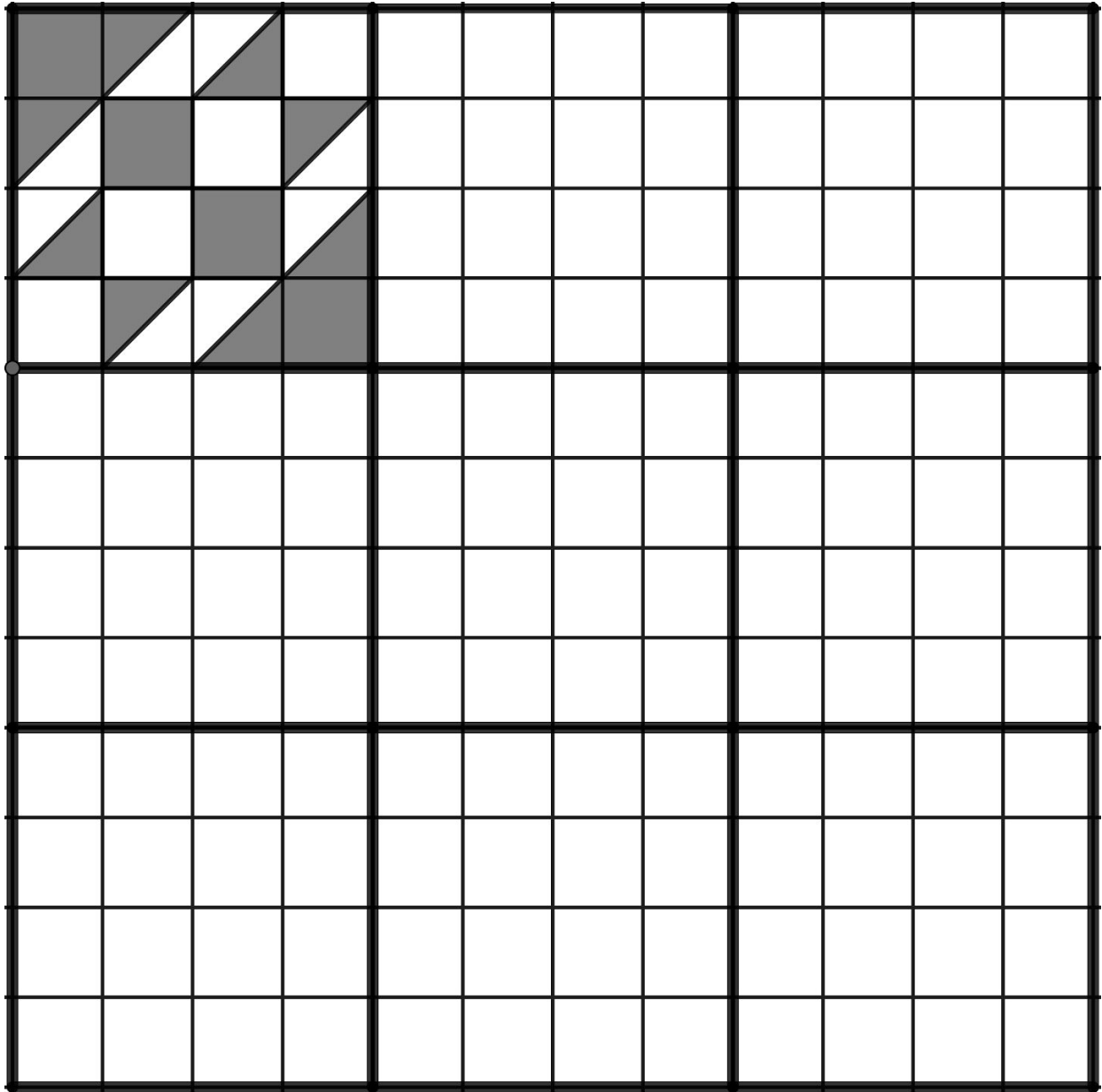
We could slide it with a translation.



We could flip it with a reflection.

*Rigid Transformations*

- 8) The quilt below is divided up into 9 sections. In the top left corner is our original pattern. Move the original design into each section with a rigid transformation. For this activity, you can either slide the original design with a translation, or flip it with a reflection.





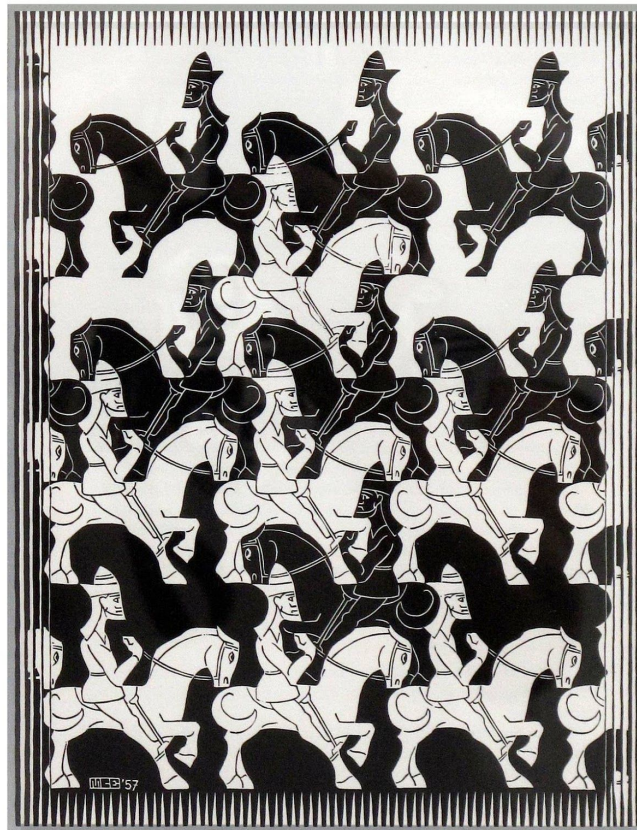
## **Tessellations**

A tessellation is an image that includes repeated congruent figures that cover a surface without gaps or overlapping. The designs of Dutch artist, MC Escher often used tessellations.

Below is a print by MC Escher called Horsemen. It is an example of a tessellation.

- 9) Let's practice using some of our new vocabulary. Describe what you see and try to use some of the following words:

**Slide Turn Flip Congruent Reflection Translation Rotation**



## Congruence and Rigid Transformations - Answer Key

1) Two shapes that are congruent to each other:

- **Always** have the same number of side.
- **Always** have the same shape.
- Are **always** the same size.
- Can **always** fit exactly over each other.

Two shapes that are not congruent to each other:

- **Sometimes** have the same number of sides.
- **Sometimes** have the same shapes.
- **Sometimes** are the same size.
- Can **never** fit exactly over each other.

2) Choice A. Slide. The Pac-Man has slid across the frame.

3) Choice B. Turn. The Pac-Man has turned.

4) Choice C. Flip. The Pac-Man has flipped.

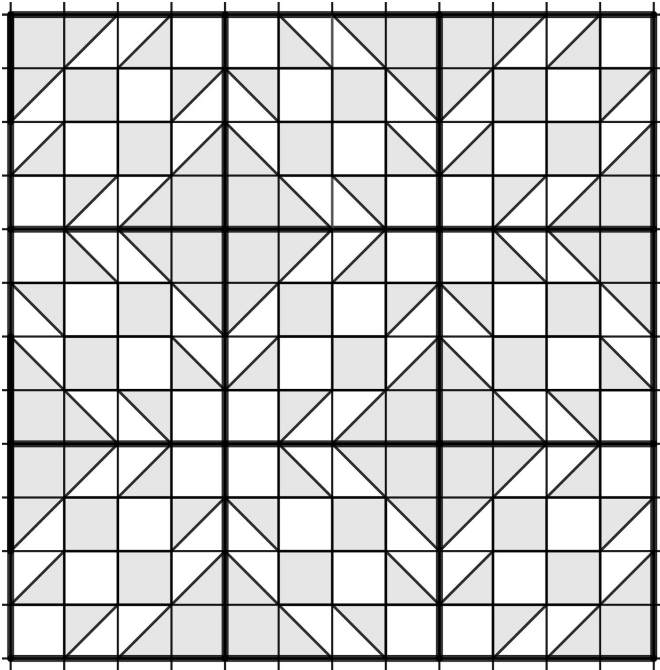
5) This is a rotation. We can turn the pictures to fit exactly over one another.

6) This is a reflection. The two pictures are mirror images of each other. We can flip the pictures to fit exactly over one another. If you fold the page you can fit the pictures exactly over each other.

7) This is a translation. We can slide the pictures to fit exactly over one another.

## Rigid Transformations

- 8) There are countless possible quilts you can design rotating, sliding, and reflecting the given patterns. Here is an example of a quilt design that uses only reflections of the original pattern.



- 9) There are several ways to use the vocabulary to describe what is going on in the MC Escher design. Here are a some examples:
- All the horsemen are **congruent** to each other. They are the same size and shape.
  - The white horsemen are **translations** of each other. You can **slide** one of the white horses and it would exactly cover any of the other white horses.
  - The black horsemen are also **translations** of each other.
  - The white and black horsemen are **reflections** of each other. You can flip one and it would exactly cover the other.
  - There are no **rotations** or **turns**.
  - It would take a **reflection/flip** and a **translation/slide** to cover a white horseman with a black horseman.











## The Coordinate Plane

The computer programs that are used to design the images for the computer animation in movies, TV shows, and video games all use something called *coordinate graphing systems*. Coordinate graphing systems can be used to communicate two or more pieces of information at once. In addition to it being useful, it is definitely something you will see on the high school equivalency exam, so it is good to have a strong foundation. In this section you will learn how to use coordinates to prepare for your work on rigid transformations.

## The Fruit Graph

Think about your favorite fruit. There are many attributes of fruit we could think about. For now, we are going to focus on how tasty a fruit is and how easy it is to eat.

Here are ten fruits:

Pineapples	Red Apples	Green Apples	Bananas	Grapes
				
Cherries	Strawberries	Watermelons	Oranges	Lemons
				

Which of these fruits do you like the best? \_\_\_\_\_

Explain why you like that fruit best.

## *Rigid Transformations*

I ranked some fruits by how tasty they are to me (with 1 being the tastiest and 10 being the least tasty):

1. Pineapples
2. Strawberries
3. Red Apples
4. Watermelon
5. Bananas
6. Cherries
7. Oranges
8. Grapes
9. Lemons
10. Green Apples

Here are the same fruits, ranked in order of how difficult I think they are to eat (with 1 being the most difficult to eat and 10 being the easiest to eat):

- |                |                         |
|----------------|-------------------------|
| 1.             | Pineapples              |
| 2.             | Watermelons             |
| 3.             | Cherries                |
| 4.             | Oranges                 |
| 5.             | Lemons                  |
| 6.             | Bananas                 |
| 7 and 8 (tie). | Green Apples/Red Apples |
| 9.             | Grapes                  |
| 10.            | Strawberries            |

1) Write four things you notice about my fruit preferences.

- a)
- b)
- c)
- d)

With the lists above, you can make plenty of statements about my fruit preferences, but there is still a lot you can't see. You can see that I like cherries more than green apples, but you don't know how much more. You can tell I think pineapples are more difficult to eat than cherries, but you don't know how much more difficult.

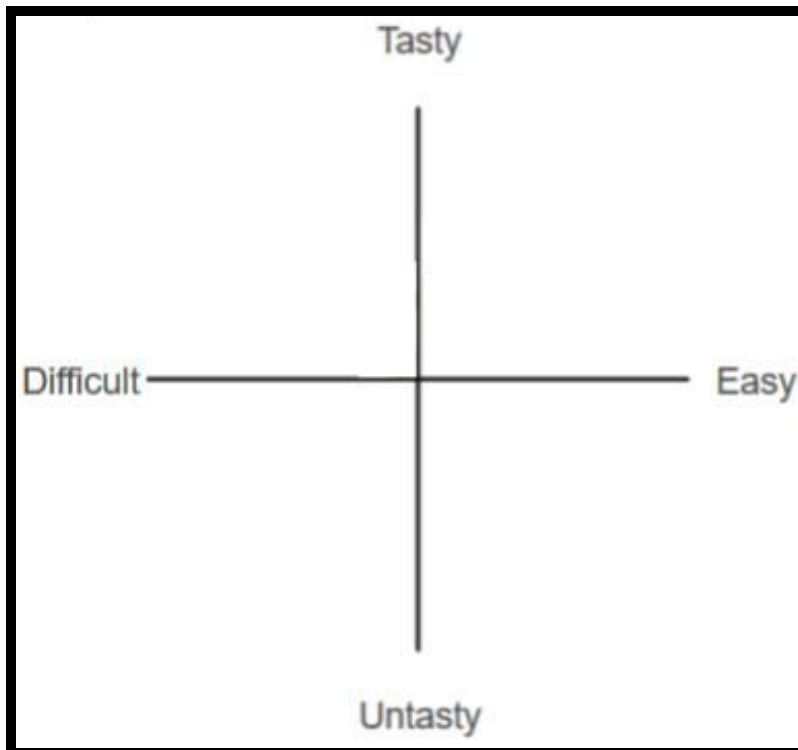
## Rigid Transformations

Here is where a coordinate graph can help us look at both pieces of information - tasty/untasty and difficult/easy - at the same time.

Here's an example of a coordinate graph.

We draw two lines - one horizontal (across) and one vertical (up and down). For this graph, the horizontal line shows how difficult/easy the fruit is to eat. The vertical line shows how tasty/untasty the fruit is.

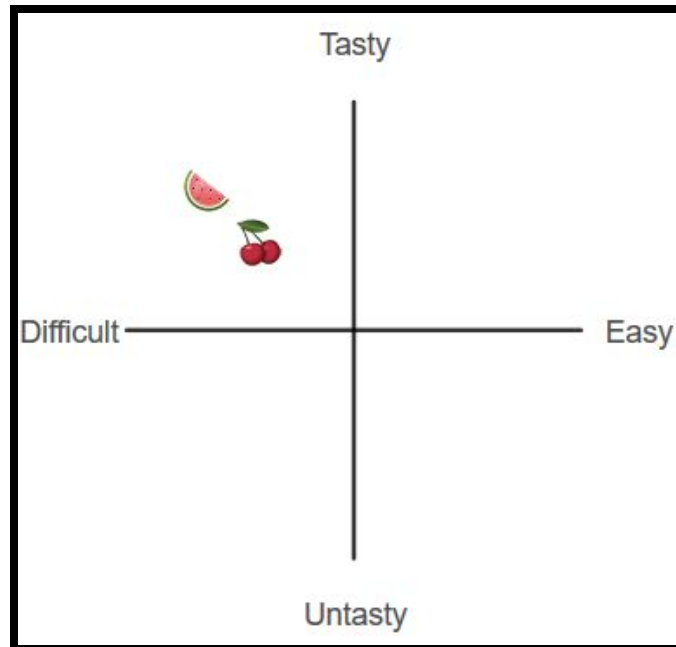
It is important to pay attention to is the place where the two lines cross.



Let's put some fruits on this coordinate graph and learn more about how it works.

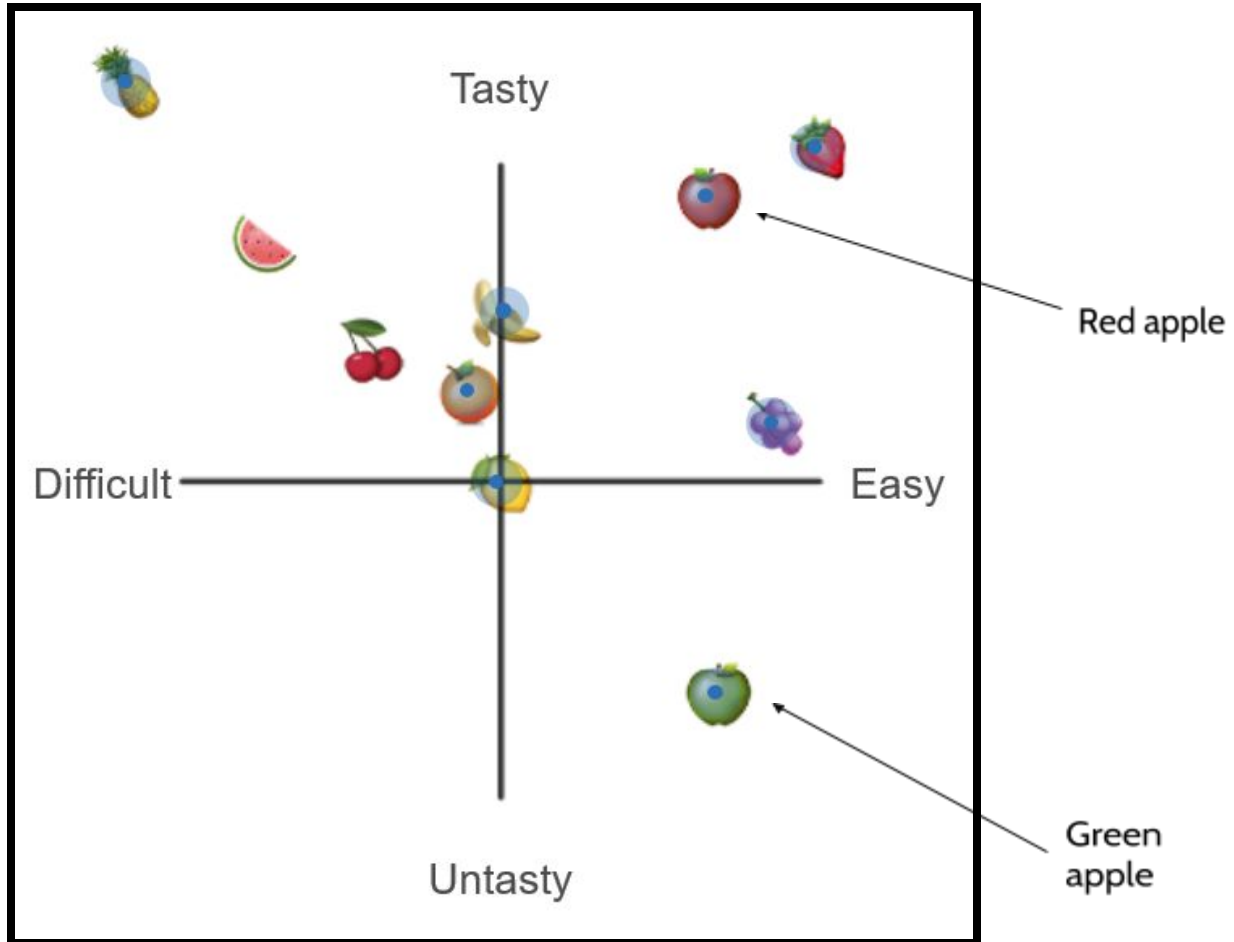
## Rigid Transformations

Let's start with cherries and watermelons. Here are what those two fruits might look like if we made a coordinate graph of my fruit preferences.



- 2) Which of these two fruits do I think is tastier? How do you know?
- 3) Which of these two fruits do I think is more difficult? How do you know?
- 4) Why are both of these fruits on the left side of the Tasty/Untasty line?
- 5) Why are both of these fruits above the Difficult/Easy line?

Let's see what it looks like with all of the fruits on the coordinate graph.

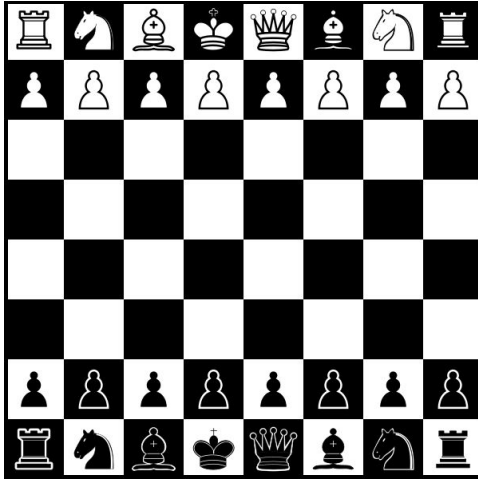


- 6) Which fruits do I think are tasty and difficult to eat?
- 7) Write everything you can about my opinion on strawberries.
- 8) Write everything you can about my opinion on apples (green and red).
- 9) Write everything you can about my opinion about bananas.
- 10) The lemon is at the place where the two lines cross. What does that mean about my opinion about lemons?



## The Chessboard

In the fruit activity, we looked at how a coordinate graph can be set up by drawing a horizontal and vertical line that cross. Now we are going to look at the way grids can help us make precise statements about location on the coordinate grid.



Chess is said to have its roots in Eastern India in the 6th century in a game called chaturanga. It spread to Persia and came to be known as shatranj. The game reached Europe around the 9th century, and by the 15th century the game was essentially played the same way it is today.

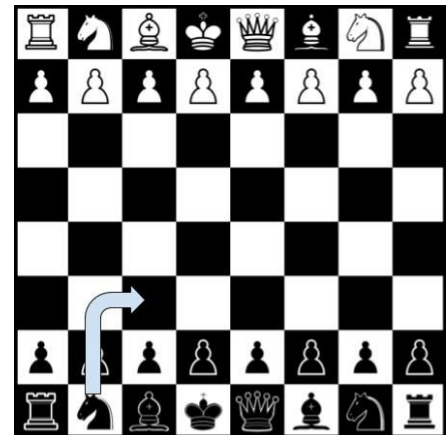
Chess has a long history and has spent more than a thousand years becoming the game we know today.

Fans of chess wanted a way to record each move so they could study strategies. They also wanted to save dramatic games.

It is possible to recreate chess games that took place hundreds of years ago. Gioachino Greco was an Italian chess player in the 17th century. He is responsible for some of the earliest recorded chess games, including one he played in the year 1619 with an amateur player.

In those days they would write out a full sentence for each move. The opening move on the board to the right might have been recorded as: *“The black king commands his own knight into the third house before his own bishop.”*

A chess game can involve many moves. In the average game, each player will make about 40 moves each. Writing a full sentence for each of those moves would take up a lot of time (and paper!). Over the years, the notation for chess has changed. The goal was to make it easier and more efficient to keep track of moves.



Today, the move on the board would be written as: ♞c3.


How does that work?

## Rigid Transformations

We use letter and numbers together to identify each square on the chessboard.

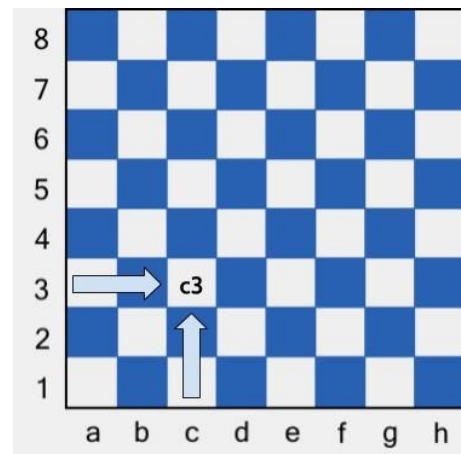


The letters name the columns (up and down) and the numbers name the rows (across). If you have used computer spreadsheets, the columns and rows are also labeled with letters and numbers.

 c3 means the piece moves into the square in the c column and the 3 row.

To name each square, we say the letter first and then the number. We start in the bottom left corner. First we go across and then we go up.

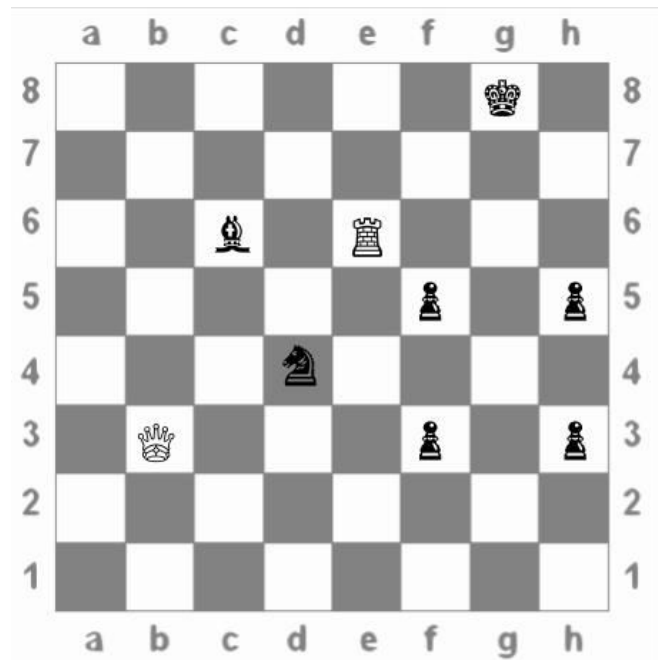
This way of naming the exact location on the board is an example of how we use **coordinates** on a coordinate graphing system. Coordinates are a pair of numbers or letters showing the exact position of a point on a line, a map, a graph, or in this case, a chessboard.



In the example on the left, we can say the knight is being moved into the square at the coordinates, **c3**.

## Rigid Transformations

Let's practice. There are nine chess pieces on the board below. Write the coordinates for the location of each chess piece. For example, the ♔ is on g8.




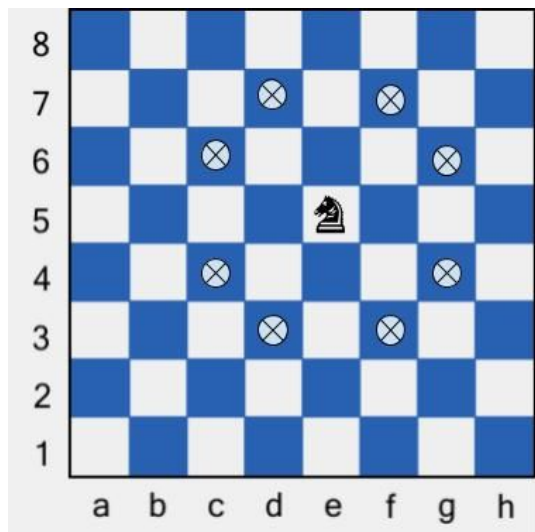
11)	♔
12)	♖
13)	♗
14)	♘
15)	♙
16)	♚
17)	♛
18)	♜

19) In chess, the Knight (or Horse) piece moves like a translation.

To figure out where your knight can move:

- slide two spaces horizontally and one space vertically, or
- slide one space horizontally and two spaces vertically.

The Knight on the board to the right is on **e5**. From that position the knight can move to eight possible squares, shown by the .



On the 8 lines below, write the coordinates for each square where the Knight can move.

\_\_\_\_\_

## Fruit Graph and Chessboard - Answer Key

- 1) There are many things you might notice about my fruit preferences. Some examples are:
  - I think pineapples are both the most tasty and most difficult fruits.
  - I think green apples are the least tasty fruit on the list.
  - I think cherries are more difficult to eat than oranges.
  - Bananas are in the middle of both of my lists.
- 2) I think watermelons are tastier than cherries. We can see that watermelons are higher on the tasty/untasty line. They are closer to the word “tasty” than the cherries.
- 3) I think watermelons are the more difficult fruit to eat. We can see that watermelons are more to the left on the Difficult/Easy line. They are closer to the word “difficult” than the cherries.
- 4) Both of these fruits are on the left side of the Tasty/Untasty line because I think they are both difficult to eat. The Tasty/Untasty line divides the graph into fruits that are difficult to eat and fruits that are easy to eat. Any fruit to the right of the Tasty/Untasty line is easy to eat (in my opinion). Any fruit to the left of the Tasty/Untasty line is difficult to eat.
- 5) Both of these fruits are above the Difficult/Easy line because I think they are both tasty. The Difficult/Easy line divides the graph into fruits that are tasty and fruits that are untasty. Any fruit above the Difficult/Easy line is tasty (in my opinion). Any fruit below the Difficult/Easy line is untasty.
- 6) Pineapples, watermelons, cherries, and oranges are all in the tasty and difficult section of the graph. Bananas are tasty, but they are right on the line between difficult and easy.
- 7) There are several possible things to say about my opinion on strawberries. Some examples are:
  - I think they are the easiest fruit to eat on the list.
  - They are the second tastiest (second only to pineapple).
  - Of the fruit I think is easy to eat, strawberries are the tastiest.
- 8) There are several possible things to say about my opinion on apples. Some examples are:
  - I think red apples are tastier than green apples. We can tell because the red apples are above the Difficult/Easy line and the green apples are below.

## Rigid Transformations

- I think green apples and red apples are the same in terms of how easy they are to eat. We can tell because they are the same distance to the right of the Tasty/Untasty line.
  - Red apples are the third tastiest fruit and the third easiest to eat.
- 9) There are several possible things to say about my opinion on bananas. Some examples are:
- I think bananas are tasty.
  - Bananas are right on the line Tasty/Untasty line, between Difficult and Easy. That means I don't think they are easy or difficult. They are in the middle.
  - I think bananas taste better than cherries, grapes, oranges, lemons, and green apples.
- 10) My opinion puts the lemon at the place where the two lines cross. That means it is between tasty and untasty and between difficult to eat and easy to eat. You might say that I don't really like or dislike lemons. I don't have much opinion about them either way.
- 11) ♖b3
- 12) ♗e6
- 13) ♘c6
- 14) ♙d4

Note: Your answers for question 15-18 can be in any order.

- 15) ♚f5
- 16) ♛f3
- 17) ♜h5
- 18) ♝h3

- 19) The coordinates of the eight positions of the knight are:

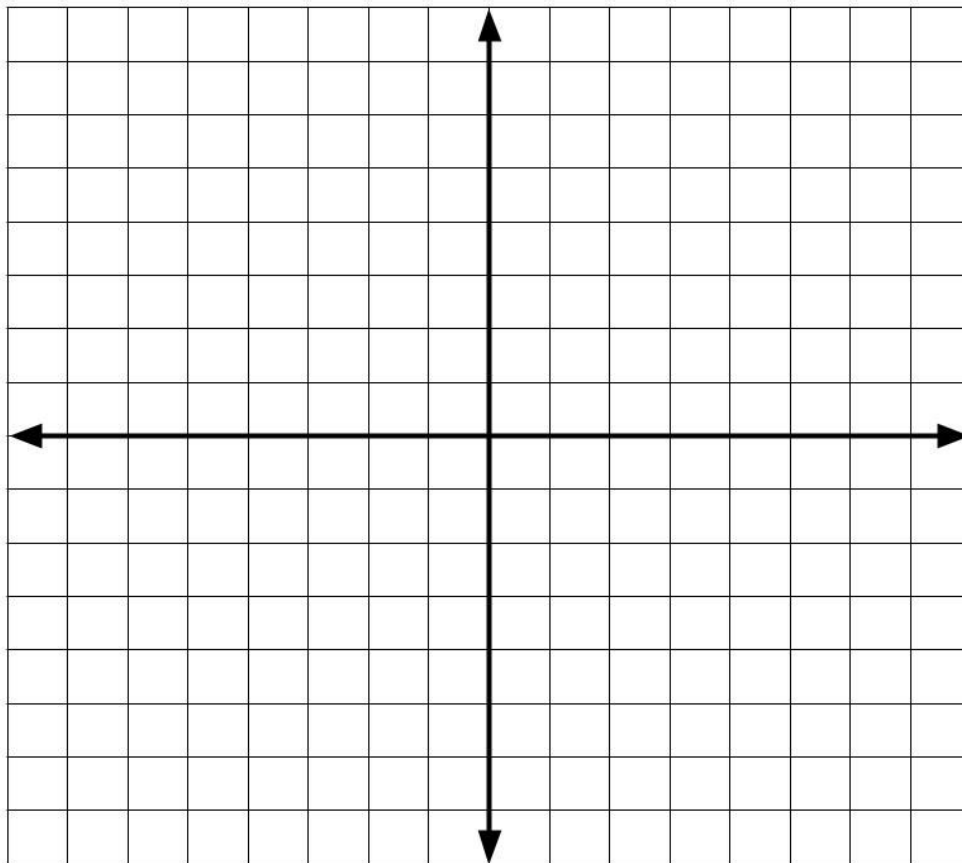
f7 g6 g4 f3 d3 c4 c6 d7  
(Your answers do not have to be listed in this order.)

## The Coordinate Plane

We are now going to review some of the important concepts you've been working on.

This is a coordinate plane.

There are some differences between this and the fruit graph and the chessboard, but let's focus on how they are similar. They all share some important features that are worth noticing.



How is this grid similar to the fruit graph or the chessboard?

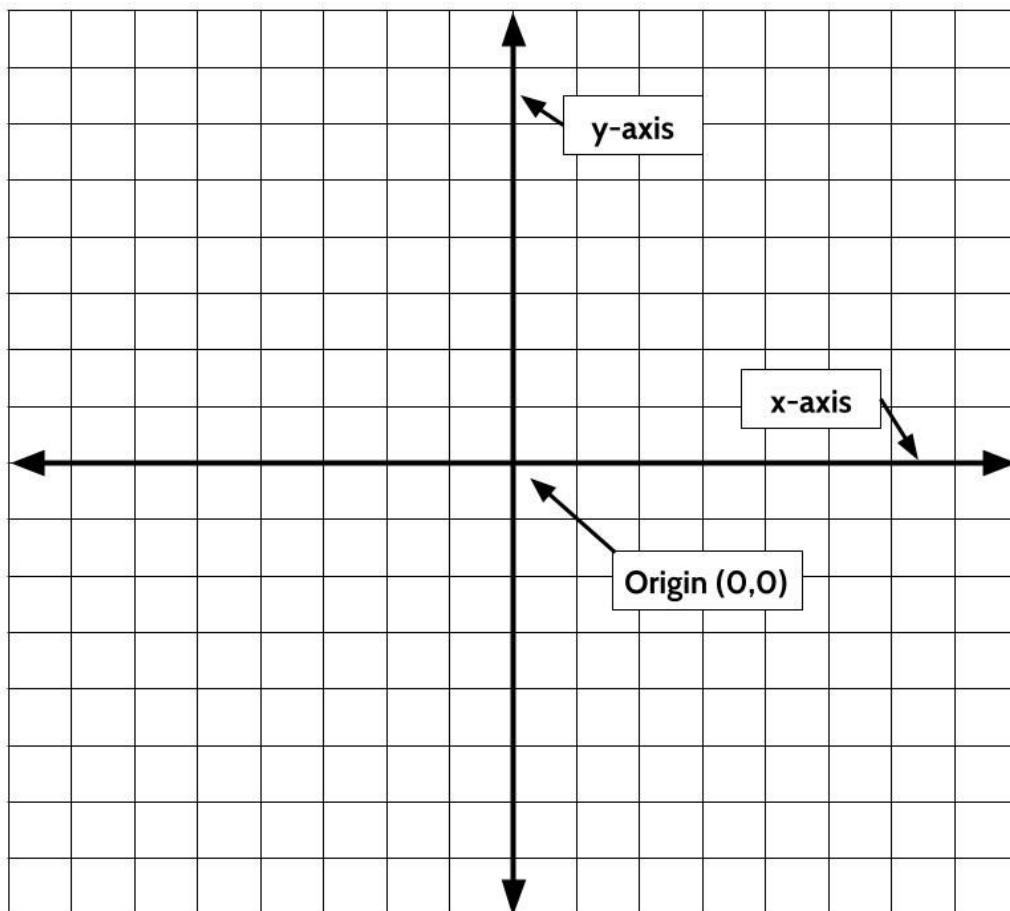
## Rigid Transformations

One similarity is that all three are formed by a horizontal line and a vertical line that go across each other at right angles.

- In the fruit graph, those lines were the Difficult/Easy line and the Tasty/Untasty line.
- On the chessboard, those lines were made by the letters going across and the numbers going up.

When we talk about coordinate planes in general, we can use the names **x-axis** and **y-axis** to talk about those two lines that cross. The x-axis is always the horizontal line. The y-axis is always the vertical line.

Remember how the lemon was in the center of the fruit graph? On the coordinate plane, the point where the x-axis and the y-axis cross is called the **origin**. The origin is the place where we are at 0 on the y-axis and 0 on the x-axis. We write the combination of those two coordinates as  $(0,0)$ .



## Plotting Points on the Coordinate Plane

In chess and on the coordinate plane we use coordinates to describe an exact location.

For now, let's focus on the top right part of the coordinate plane. We call this section Quadrant 1.

This now resembles the 8 by 8 chessboard. On the chessboard, we used letters across the bottom to tell us how many squares to move to the right. Then we used the numbers up the side to tell us how many squares to move vertically.

The coordinate plane has two important differences.

The first difference is we use numbers for both directions. The numbers along the x-axis tell us how to move horizontally and the numbers along the y-axis tell us how to move vertically.

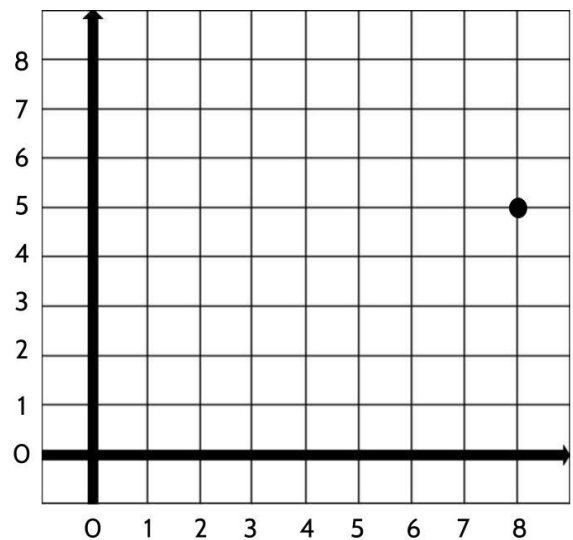
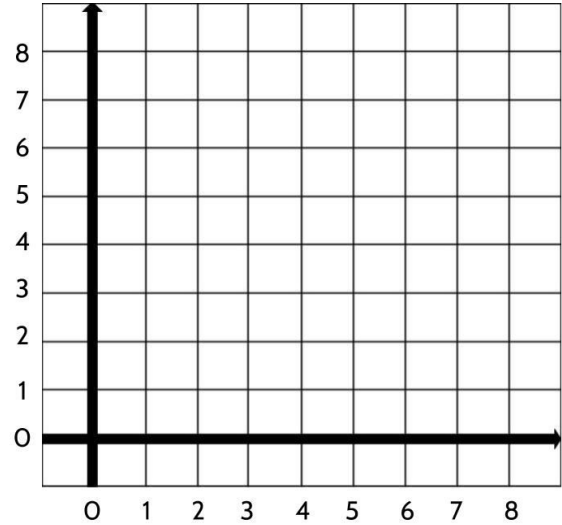
The second difference is that on the coordinate plane, we count where the grid lines meet (instead of counting squares the way we did on the chessboard).

On the coordinate plane, coordinates work like giving someone directions. You always start in the origin. That point is 0 on the x-axis and 0 on the y-axis. To describe an exact point on the coordinate grid, start at the origin, give the horizontal directions and then give vertical directions.

For example, if I wanted to tell you to place a point on the graph, I might tell you to *plot* it at

**(8,5)**

The 8 tells you to move 8 lines to the right from the origin. Then 5 tells you to move 5 lines up from the origin. (8, 5) is the point where the vertical line at 8 intersects with the horizontal line at 5.





## Rigid Transformations

The coordinates we use on the coordinate plane are sometimes called *ordered pairs* to help us remember that the order is always the same. The first number always tells us how far to move along the x-axis and the second number tells us how far to move along the y-axis. (8,5) is an example of an ordered pair.

- 1) Let's practice writing coordinates out as ordered pairs.

Write the ordered pairs for each point on this coordinate plane.

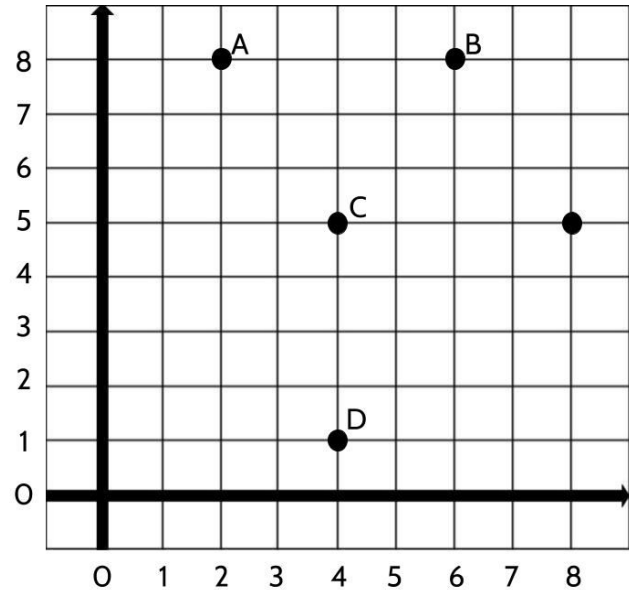
Point (8, 5) is included as an example.

**Point A:**

**Point B:**

**Point C:**

**Point D:**



- 2) Let's practice using ordered pairs to draw points.

Use these coordinates to draw the following ordered pairs on the coordinate plane. Make sure to label each point with the letter given.

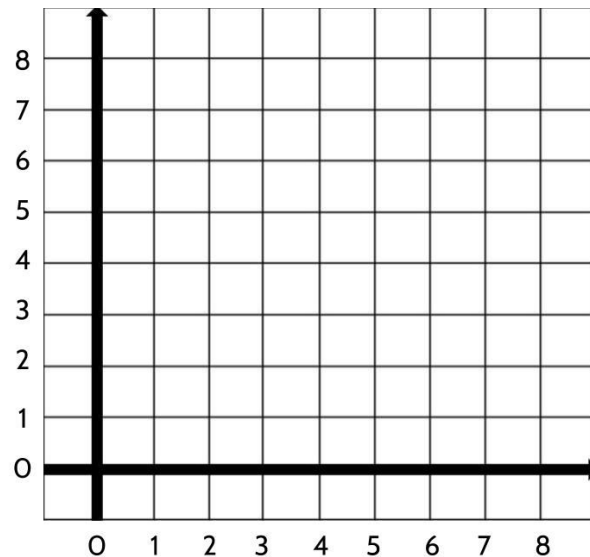
**H (2,1)**

**O (2,6)**

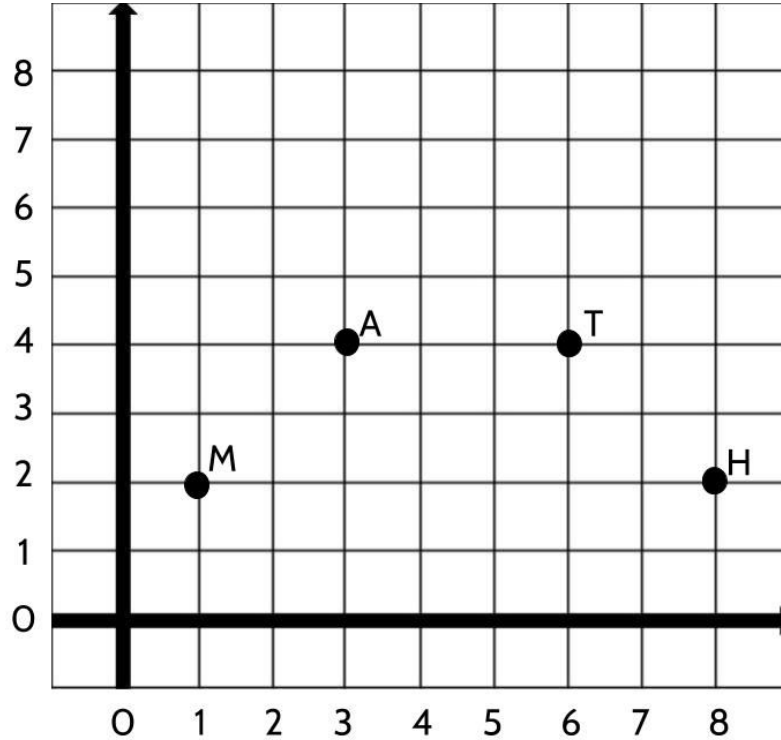
**S (6,6)**

**E (6,1)**

**U (4,8)**



3) Look at the points on this coordinate grid.



Which of the following statements are true? Underline the two true statements.

- A. Point T is 6 units to the right and 4 units up from the origin.
- B. The distance between Point M and Point H is 8 units.
- C. Point A and Point T have the same x coordinate.
- D. The ordered pair for Point M is (2, 1)
- E. To move from Point M to Point T, move 5 units to the right and two units up.

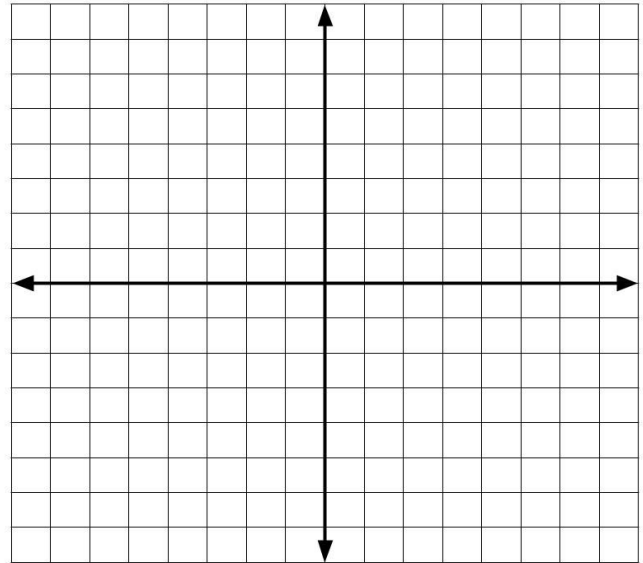
## Rigid Transformations

So far, we've been working in the top right section of the coordinate plane. Let's see what happens when we look at all four sections.

We know the coordinates of the origin are  $(0,0)$  because the origin represents the point of zero on the x-axis and zero on the y-axis.

As you move to the right along the x-axis from the origin, the numbers are getting bigger. As you move up along the y-axis from the origin, the numbers are also getting bigger.

But what happens to the numbers as you move to the left along the x-axis from the origin?



And what happens to the numbers as you move down along the y-axis from the origin?

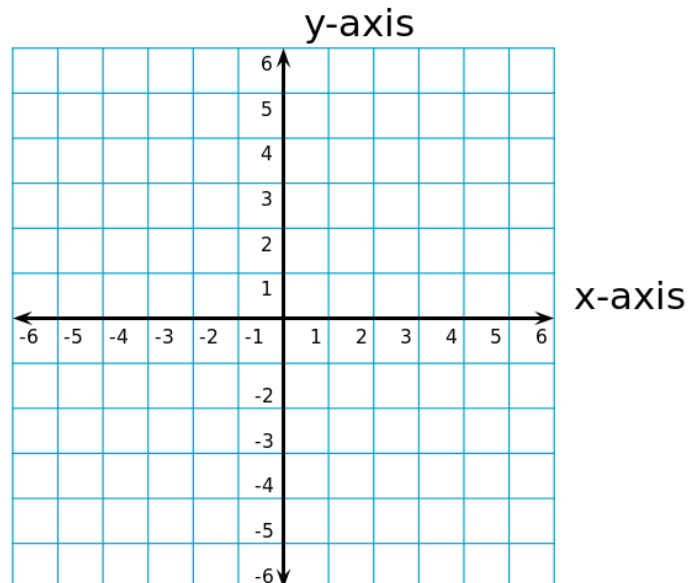
You can think of the x-axis and the y-axis as number lines. The same way number lines can continue below zero, so can the x-axis and y-axis, with negative numbers!

This is similar to what we saw on the fruit graph.

- The Tasty/Untasty axis was the dividing line between fruits that were difficult to eat and fruits that were easy to eat.

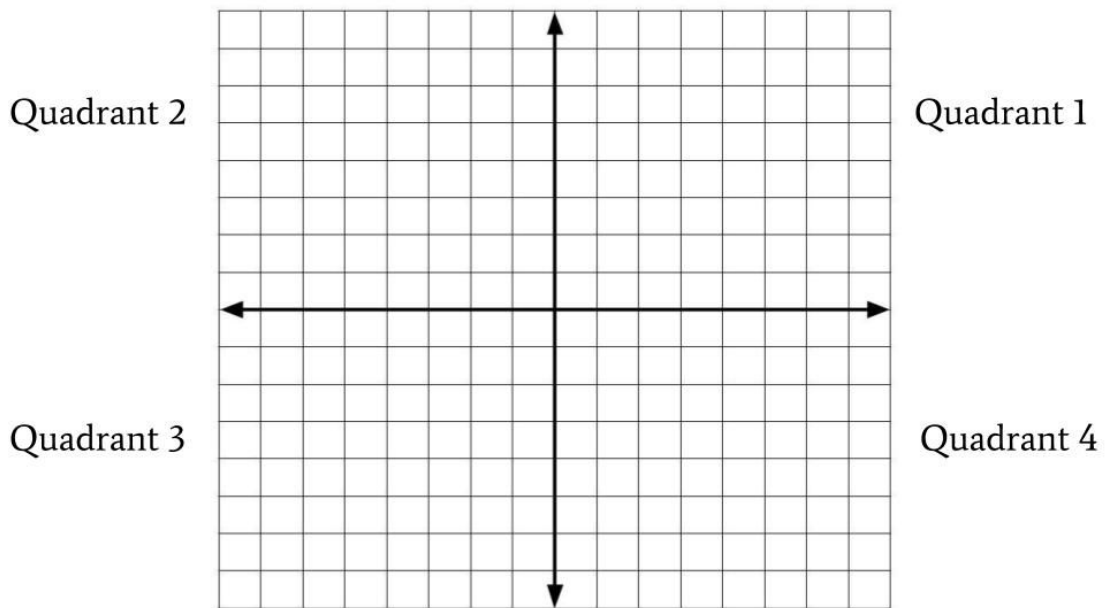
In the same way, the y-axis is the dividing line between positive numbers on the right and negative numbers on the left.

- The Difficult/Easy axis was the dividing line between tasty fruits above and untasty fruits below. In the same way, the x-axis is the dividing line between positive numbers above and negative numbers below.



## Rigid Transformations

As we've seen, the top right section of the coordinate plane is called Quadrant 1. The other Quadrants are numbered 2, 3, and 4, going in order, counterclockwise.



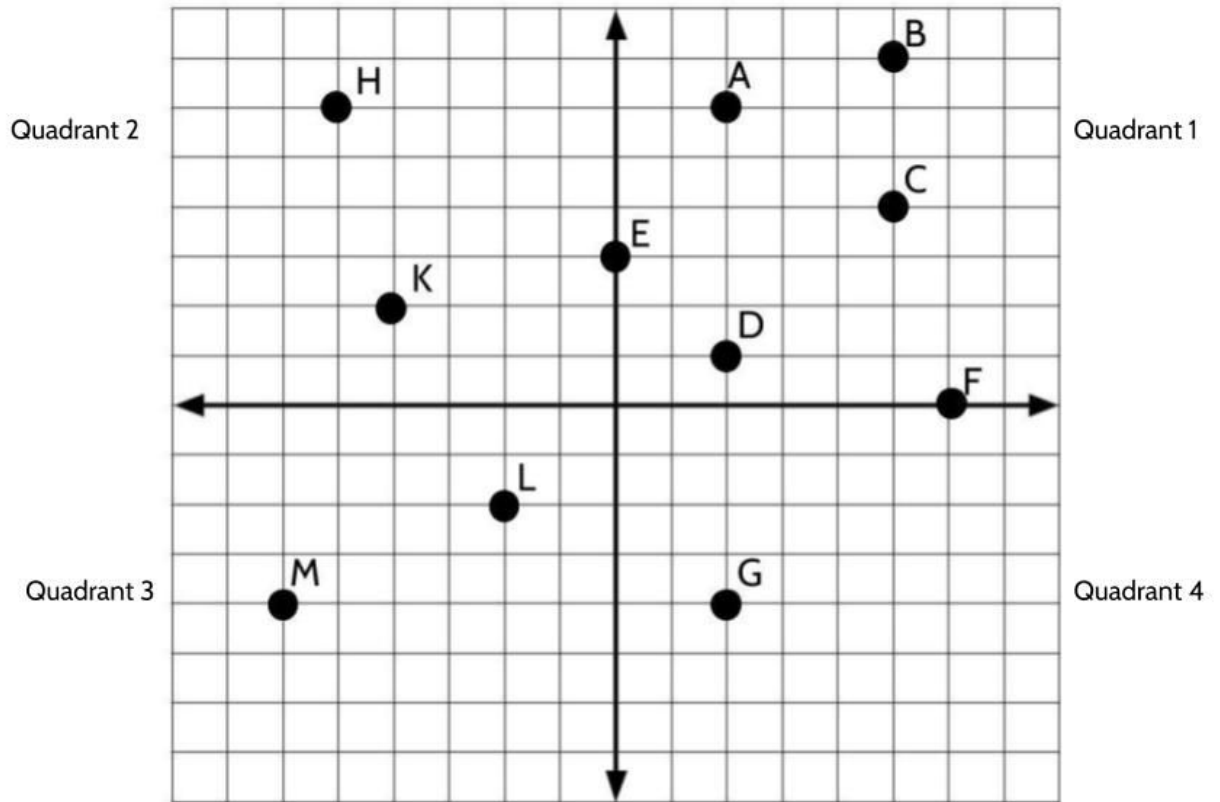
4) Complete the following statements:

Ex: Point (5, 4) is in Quadrant 1 because I start in the origin and move 5 units to the right / left and then I move 4 units up / down.

- Point (7, -3) would be in Quadrant \_\_\_\_ because I start in the origin and move \_\_\_\_ units to the right / left and then I move \_\_\_\_ units up / down.
- Point (-3, -6) would be in Quadrant \_\_\_\_ because I start in the origin and move \_\_\_\_ units to the right / left and then I move \_\_\_\_ units up / down.
- Point (16, 25) would be in Quadrant \_\_\_\_ because I start in the origin and move \_\_\_\_ units to the right / left and then I move \_\_\_\_ units up / down.
- Point (-10, 3) would be in Quadrant \_\_\_\_ because I start in the origin and move \_\_\_\_ units to the right / left and then I move \_\_\_\_ units up / down.

Rigid Transformations

5) Write the name of each point next to the ordered pairs provided below:



<i>Point D is at (2, 1)</i>		
	(-6, -4)	(6, 0)
	(-2, -2)	(0, 3)
	(-5, 6)	(5, 4)
	(-4, 2)	(2, 6)
	(2, -4)	(5, 7)

## The Coordinate Plane - Answer Key

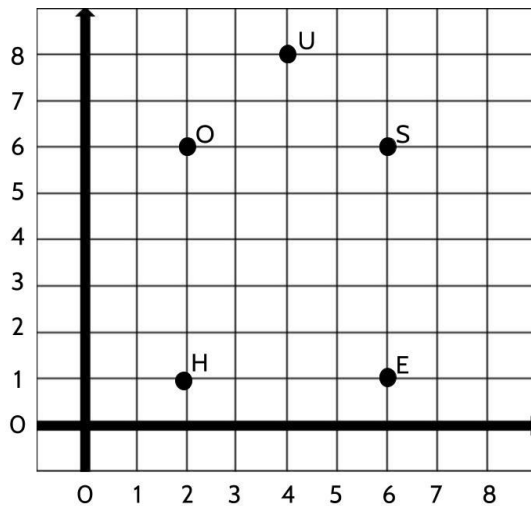
1) Point A: (2,8)

Point B: (6,8)

Point C: (4,5)

Point D: (4,1)

2)



3) Choices A and E are true.

Choice B is false because Point M and Point H are 7 units apart (not eight).

Choice C is false because Point A and Point T have the same y coordinate, but they have different x coordinates.

Choice D is false because the ordered pair for Point M is (1, 2)

4) Point (7, -3) is in Quadrant 4 because I start in the origin and move 7 units to the right and 3 units down.

Point (-3, -6) is in Quadrant 3 because I start in the origin and move 3 units to the left and 6 units down.

Point (16, 25) is in Quadrant 1 because I start in the origin and move 16 units to the right and 25 units up.

Point (-10, 3) is in Quadrant 2 because I start in the origin and move 10 units to the left and 3 units up.

## *Rigid Transformations*

5) Point M: (-6,-4)

Point L: (-2, -2)

Point H: (-5, 6)

Point K: (-4,2)

Point G: (2, -4)

Point F: (6,0)

Point E: (0,3)

Point C: (5,4)

Point A: (2,6)

Point B: (5,7)

Take a look at Point E and Point F. We can read the coordinate for Point E as moving 0 units to the right and 3 units up. Point F can be read as moving 6 units to the right and 0 units up. The value of x will be zero if the point is on the y-axis. Similarly, the y value in the ordered pair will be zero when the point is on the x-axis.

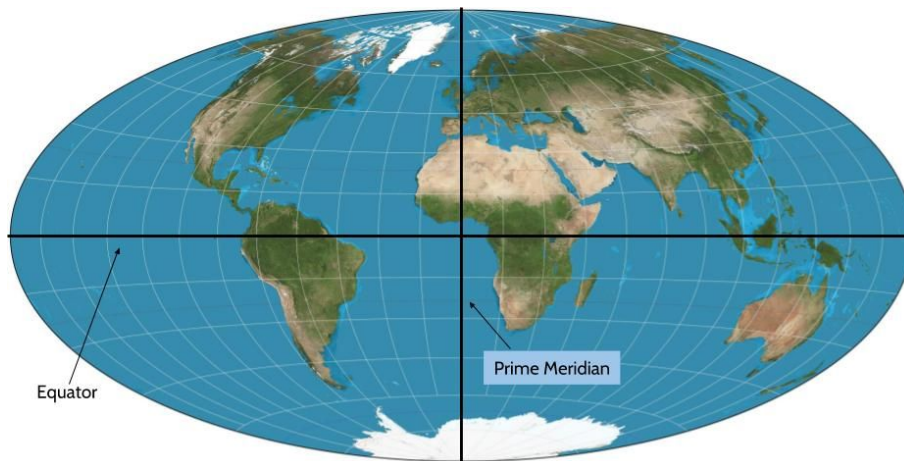
## Longitude Lines and Latitude Lines

There are many examples of coordinate systems where two pieces of information are used to pinpoint an exact location. We've already seen it at work in graphs and chessboards. You may have also seen a coordinate system when looking at maps.

There are two imaginary lines drawn on the Earth.

The equator is drawn around the middle of the Earth to divide it into the northern and southern hemispheres (halves). Latitude lines are horizontal lines used to describe how far to the north or south locations are from the equator. The Equator is the latitude line which is  $0^\circ$  North/South.

The Prime Meridian is drawn from the top of the Earth to the bottom, to divide the Earth into eastern and western hemispheres (halves). Longitude lines are vertical lines used to describe how far to the east and how far to the west locations are from the Prime Meridian. The Prime Meridian is  $0^\circ$  West/East



For example, in the center of New York State, the city of Utica is located at:

Latitude:  $43.100903^\circ$  and Longitude  $-75.232666^\circ$

That means Utica is about  $43^\circ$  to the north of the equator and about  $75^\circ$  to the west of the Prime Meridian.

Go to <https://www.latlong.net/> to find the exact coordinates (latitude, longitude) of your hometown.



## Shapes on the Coordinate Plane

Now that we've had some practice with naming and plotting (placing) points on the coordinate plane, let's use what we've learned to make a few shapes.

A rectangle is a four-sided shape with four right ( $90^\circ$ ) angles. In a rectangle, opposite sides are both equal and parallel.

Draw Point C to complete rectangle ABCD.

- 1) What are the coordinates of the ordered pair for Point C?

Point C: (   ,   )

- 2) A square is a special type of rectangle.

A square is a rectangle where all four sides are equal in length. On the coordinate grid, draw four points to create your own square.

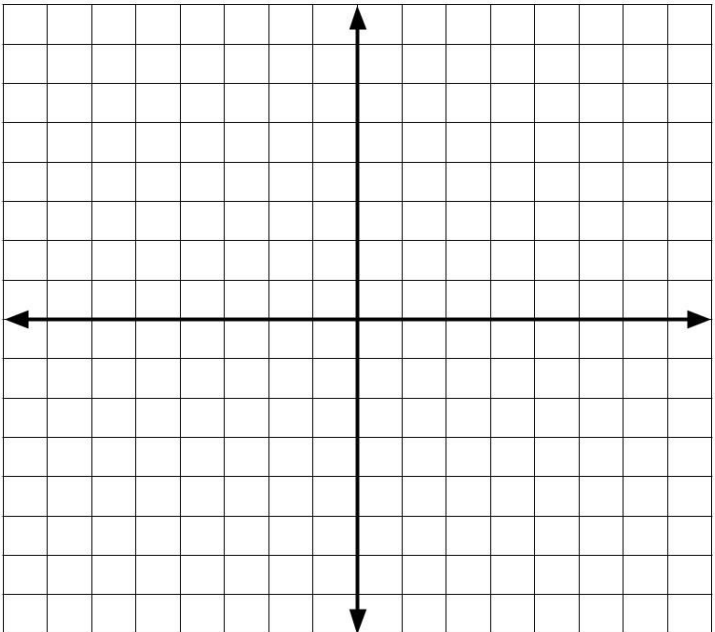
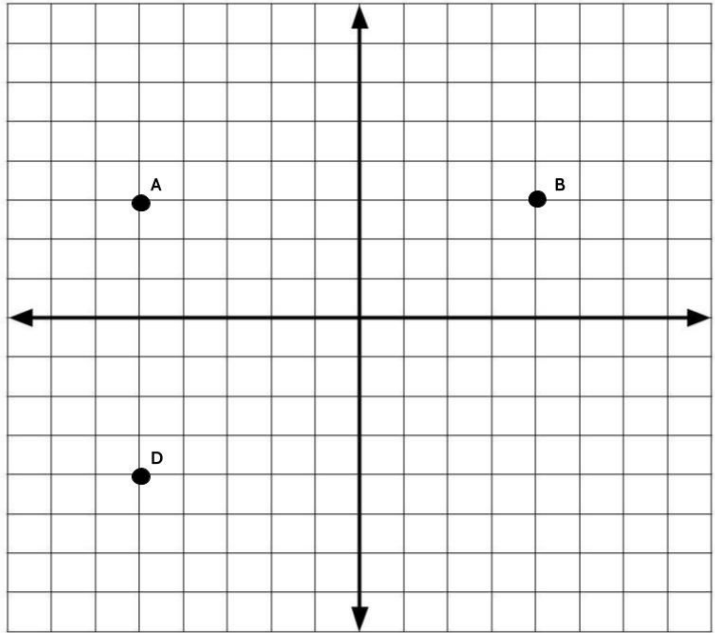
Label the four points E, F, G, and H. Write the coordinates for each point below.

Point E: (   ,   )

Point F: (   ,   )

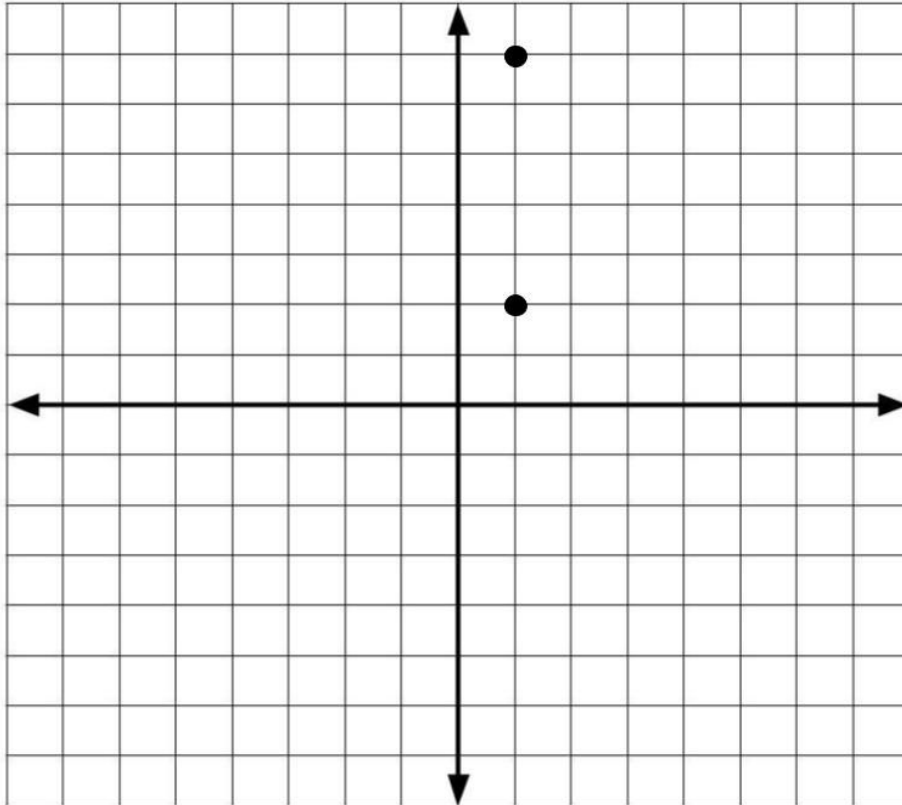
Point G: (   ,   )

Point H: (   ,   )



*Rigid Transformations*

3)



Which two sets of ordered pairs can be used to make a square with the two points on the coordinate grid?

- A.  $(7, -7)$  and  $(7, -2)$
- B.  $(6, 7)$  and  $(6, 2)$
- C.  $(-2, -2)$  and  $(-2, -7)$
- D.  $(-4, 2)$  and  $(-4, 7)$

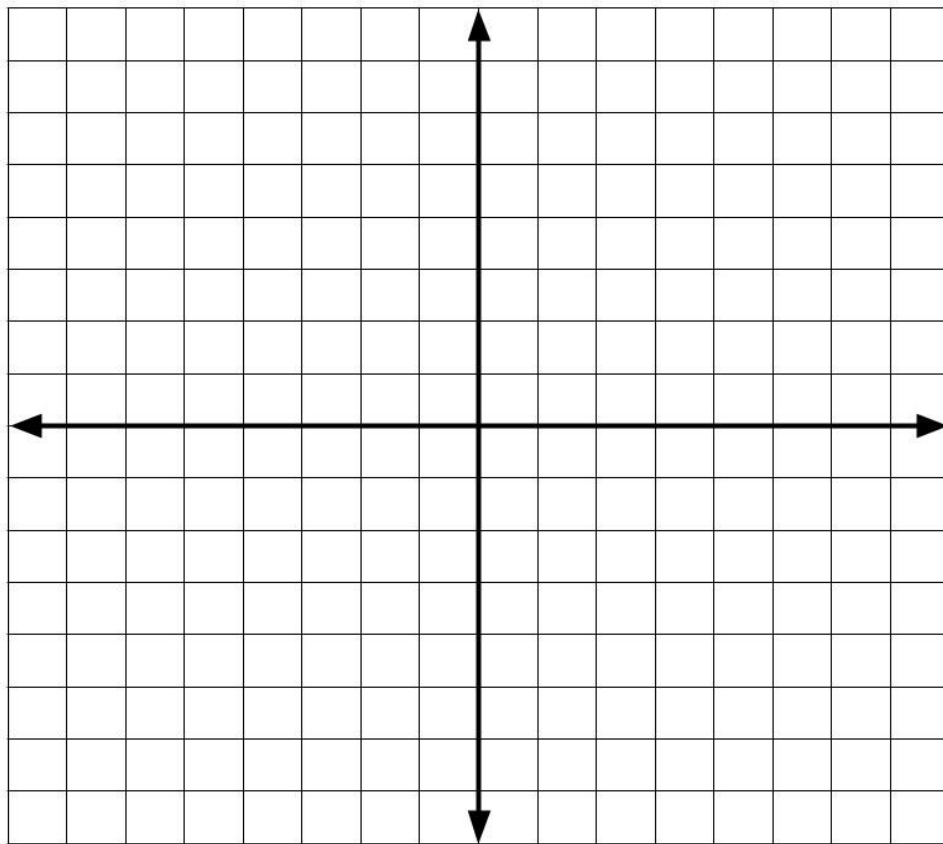
*Rigid Transformations*

4) Find the perimeter of the rectangle that is formed by these four points:

$(3, 2)$   $(5, 2)$   $(5, -5)$   $(3, -5)$

HINT: The perimeter of any shape can be found by adding up the length of each of its sides.

The perimeter of the rectangle is \_\_\_\_\_.



## Lines on the Coordinate Plane

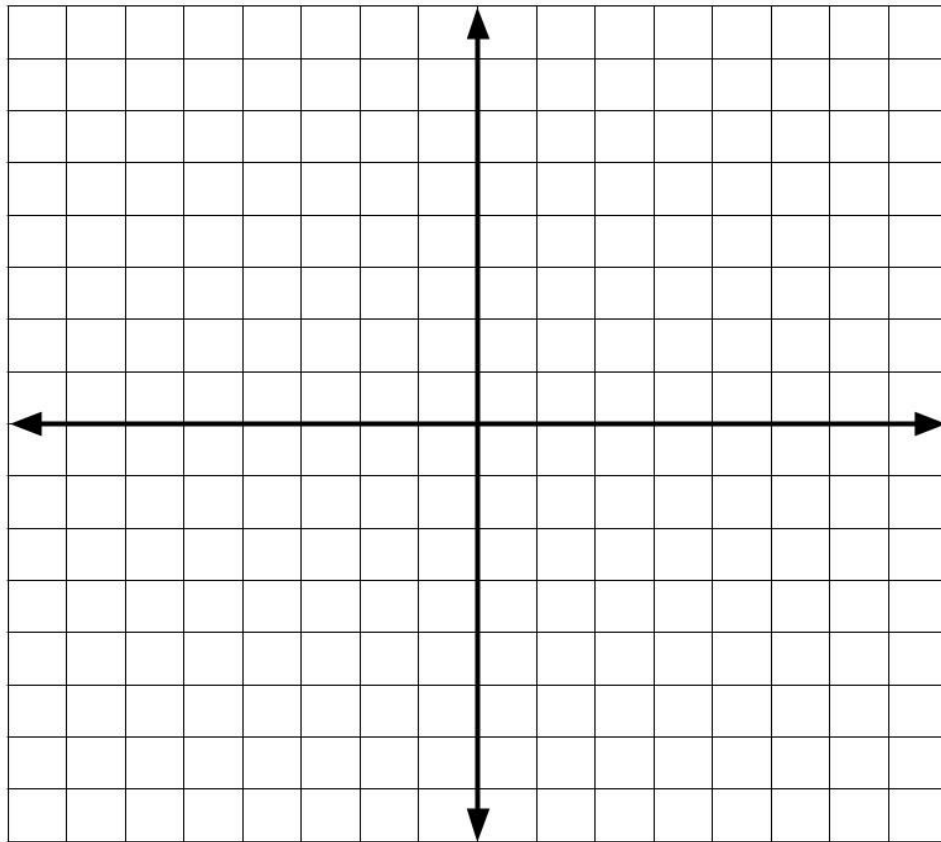
Something we will be working on in the next section is how to reflect points and shapes across different lines on the coordinate plane.

We've spent some time thinking about the two lines that define the coordinate plane - the x-axis and the y-axis. When working with rigid transformations you might be asked to reflect a shape about either.

Let's focus on a few other lines and begin with plotting some points.

5) Plot the following points on this coordinate plane:

$(-7,-7)$ ,  $(-5,-5)$ ,  $(-3,-3)$ ,  $(-1,-1)$ ,  $(0,0)$ ,  $(2,2)$ ,  $(4,4)$ ,  $(6,6)$ ,  $(8,8)$



What do you notice about these points?

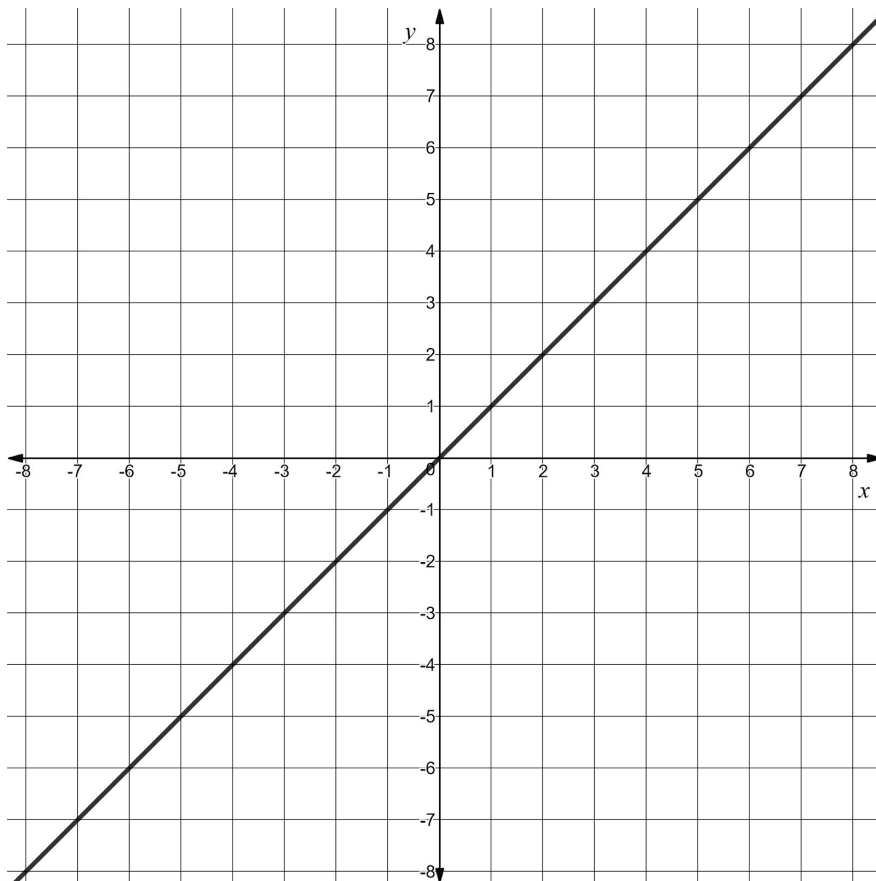
## Rigid Transformations

One thing you might have noticed is that the  $x$  and the  $y$  coordinates for each ordered pair are the same. You may also have noticed that the points form a line. In fact, if we continued plotting points, the line would continue.

Lines on the coordinate plane are named by an equation that can be used to construct the line. In this case, we call the line:

$$y = x$$

$y = x$  can be read as “ $y$  equals  $x$ ” and it refers to the line that cuts the coordinate plane in half, straight through the origin, where every ordered pair has the same  $x$  and  $y$  coordinates. The  $x$  value and the  $y$  value are the same at every point along the line.



- 6) Circle any of the following points that line  $y = x$  would pass through if we continued the line.

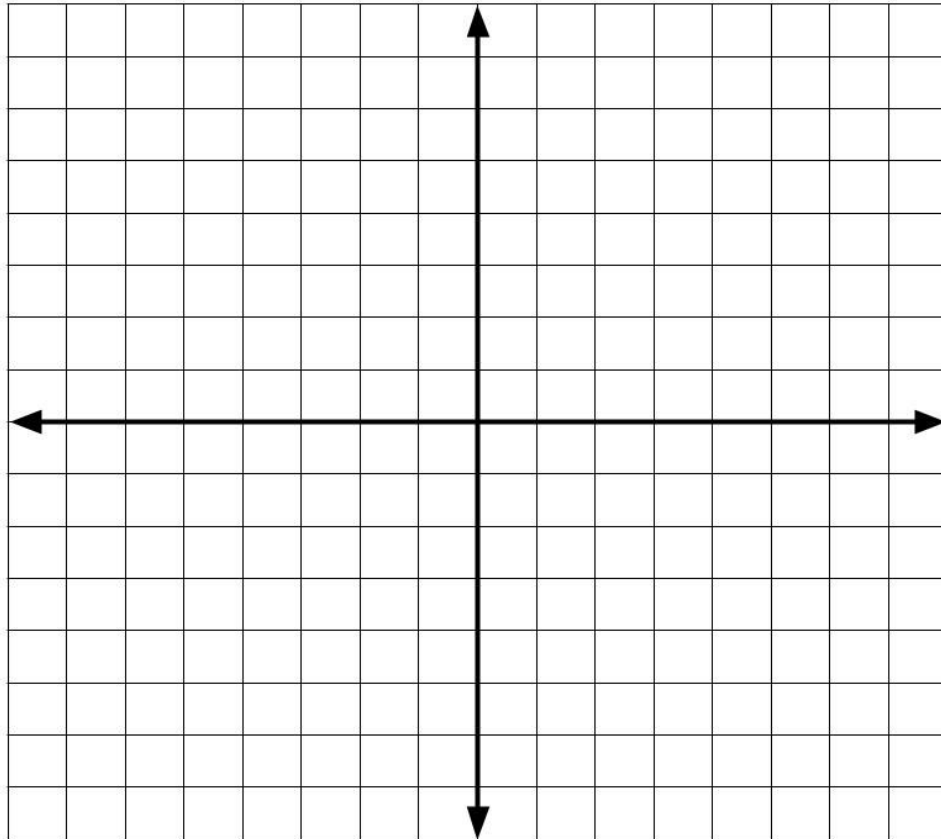
(-76, -76)    (16, -16)    (3, 3)    (24, 24)    (-9, -9)    (5, -5)

How do you know?

*Rigid Transformations*

7) Plot the following points on this coordinate plane:

$(3, 6)$ ,  $(-2, 6)$ ,  $(6, 6)$ ,  $(-5, 6)$ ,  $(7, 6)$ ,  $(-4, 6)$ ,  $(1, 6)$ ,  $(-7, 6)$



What do you notice about these points?

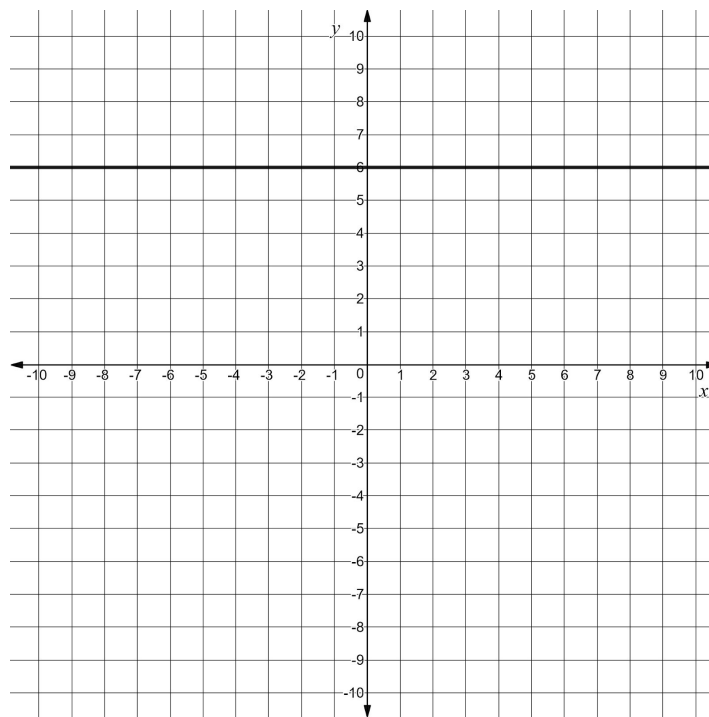
## Rigid Transformations

One thing you may have noticed is that the y coordinate in each ordered pair is always 6. You may have also noticed these points form a horizontal line.

The name for this line is:

$$y = 6$$

$y = 6$  can be read as “y equals 6”.  $y = 6$  refers to the line where every ordered pair has a 6 for the y coordinate. You may also have noticed that this line is parallel to the x-axis. It is parallel because the two lines (the x-axis and  $y=6$ ) are the same distance apart at any point along the line. They will always be 6 units apart.



8) Circle any points that line  $y = 6$  would pass through if we continued the line.

(32, 6)

(-19, 6)

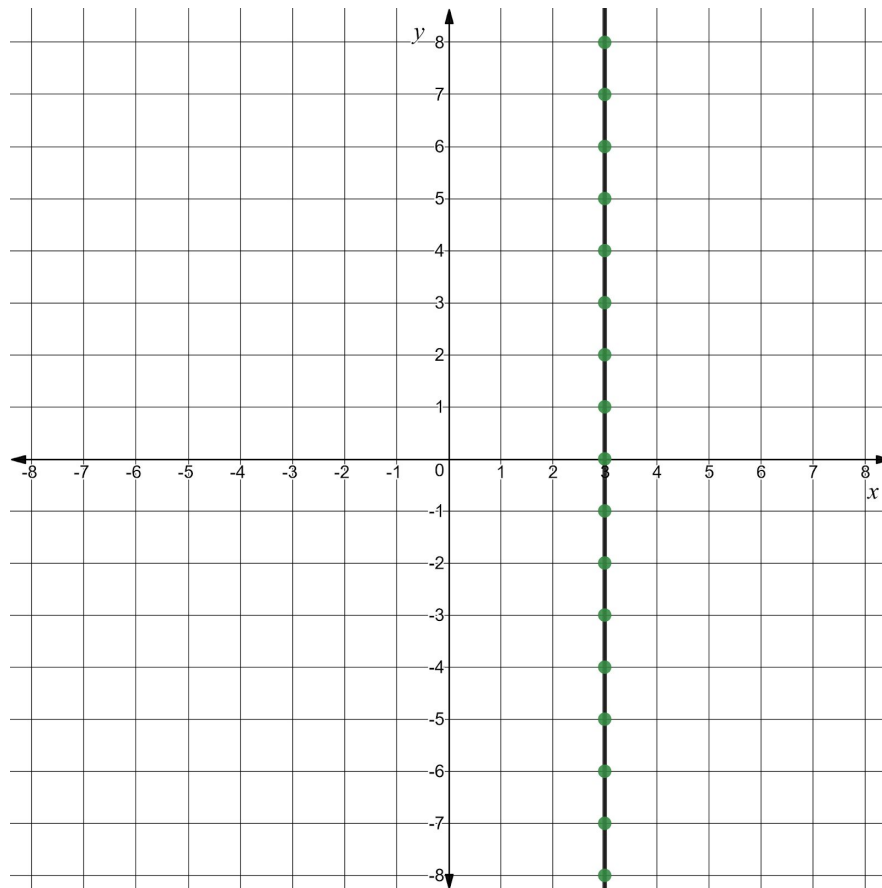
(6, 12)

(4, -6)

How do you know?

## Rigid Transformations

Use this coordinate plane to answer question #9.



9) Fill in the blanks.

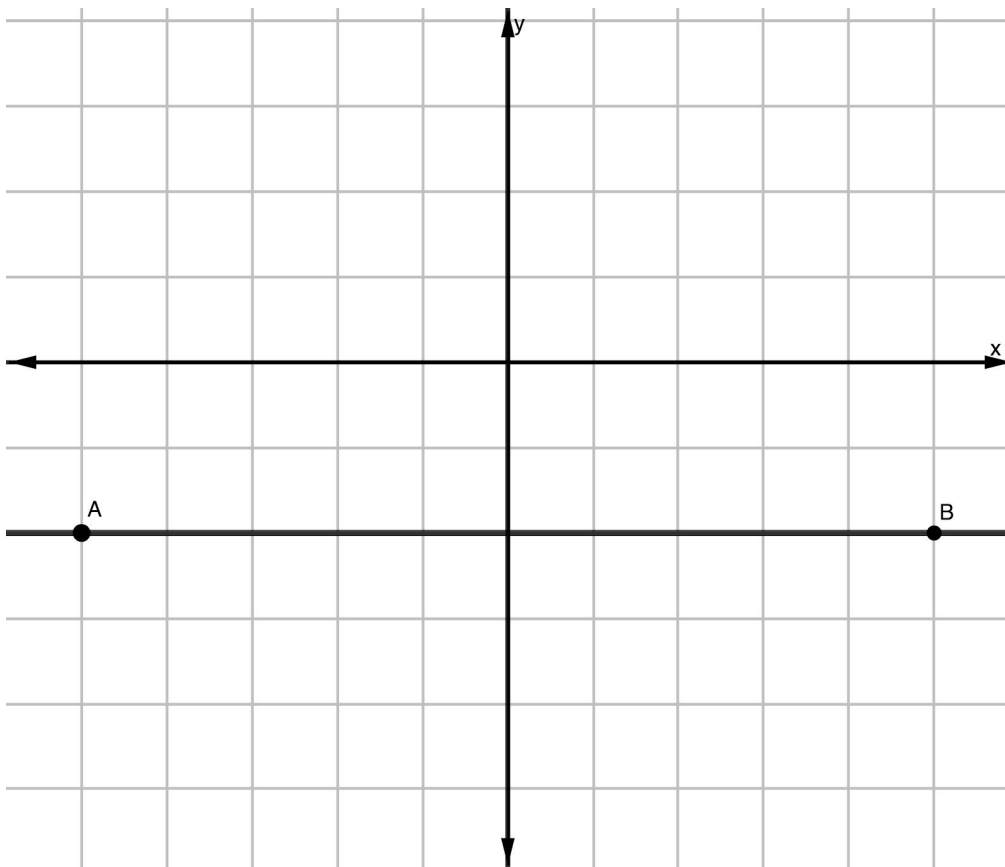
- $(\underline{\quad}, \underline{\quad})$ ,  $(\underline{\quad}, \underline{\quad})$ ,  $(\underline{\quad}, \underline{\quad})$ , and  $(\underline{\quad}, \underline{\quad})$  are all examples of points on this line.
- All points on the line have  $\underline{\quad}$  as the value of the  $\underline{\quad}$  coordinate.
- This line is parallel to the  $\underline{\quad}$ -axis, because each point is  $\underline{\quad}$  units away from that axis.
- I think the equation that names this line is  $\underline{\hspace{2cm}}$  because  $\underline{\hspace{2cm}}$

---

10) What is the name of the equation for line AB?

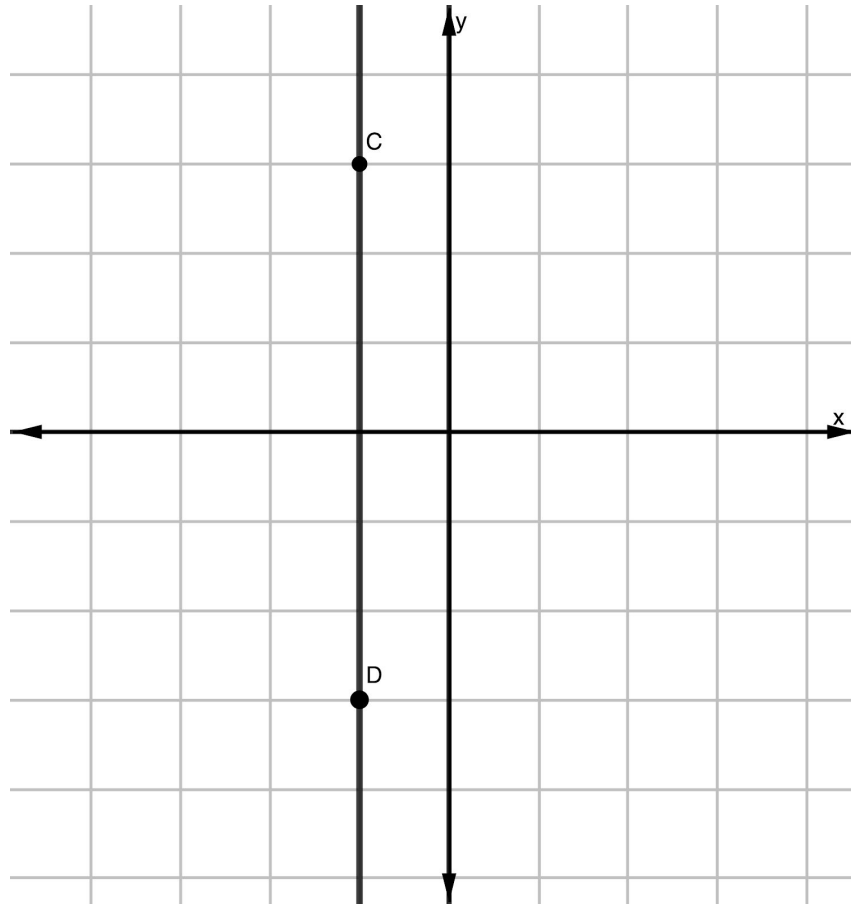


The equation for Line AB is \_\_\_\_\_



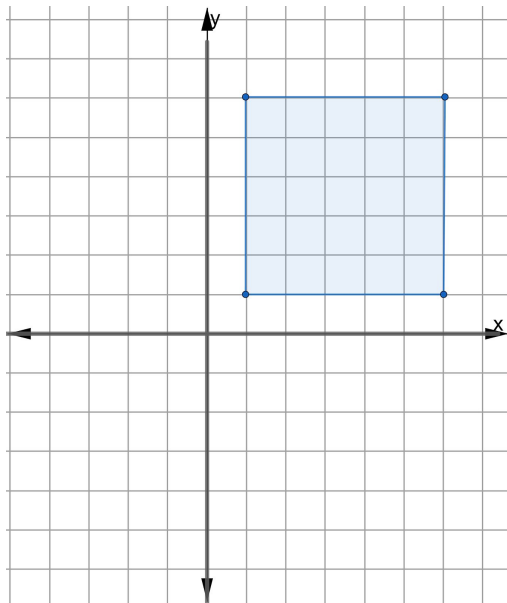
11) What is the name of the equation for the line CD?

The equation for Line CD is \_\_\_\_\_

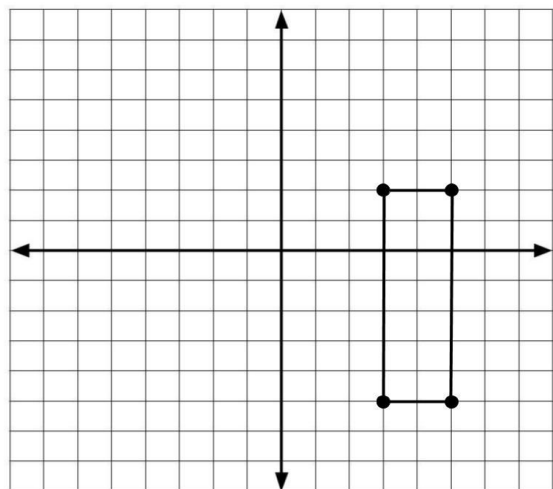


## Shapes and Lines on the Coordinate Plane - Answer Key

- 1) Point C is (4, -4)
- 2) There are many different ways to plot four points to form a square on the coordinate plane. There are two things to pay attention to:
  - 1 - The points should be in order, clockwise or counterclockwise, CDEF.
  - 2 - The distance of each side of the square should be the same.
- 3) The two given points are 5 units away from each other. Both choices B and D would form the remaining sides of a square.

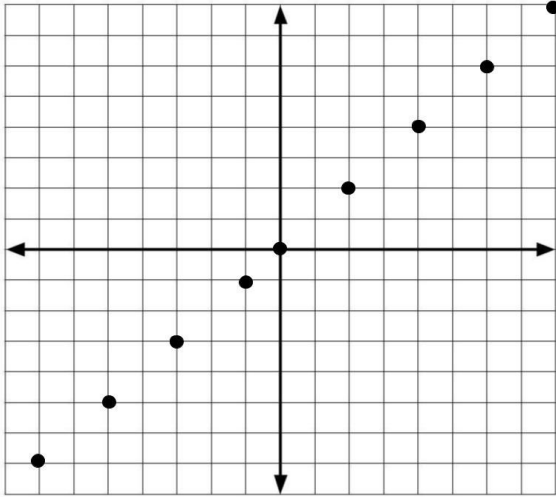


- 4) The perimeter of the rectangle would be 18 units.



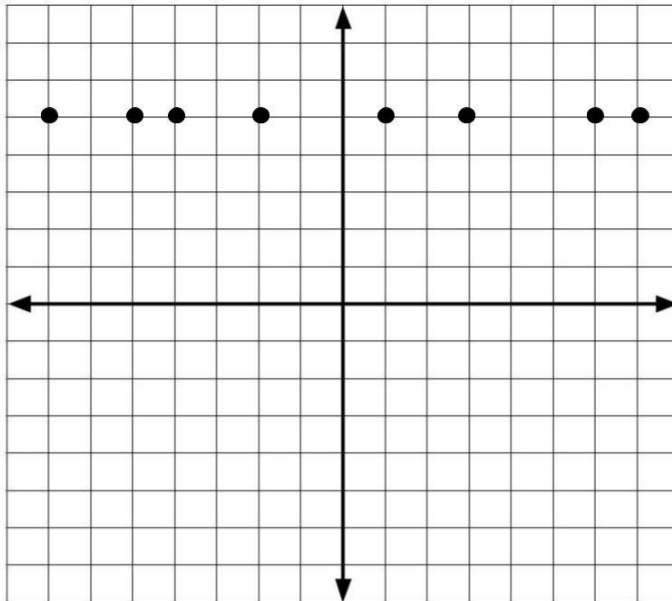
Rigid Transformations

5)



6)  $y = x$  would pass through four of the points:  $(-76, -76)$ ,  $(3, 3)$ ,  $(24, 24)$ ,  $(-9, -9)$ . We can tell because the  $x$  and  $y$  coordinate is the same. In the ordered pairs  $(16, -16)$  and  $(5, -5)$  the  $x$  and  $y$  values are not the same.  $(16, -16)$  and  $(5, -5)$  would be in Quadrant 4.  $y = x$  goes through Quadrants 1 and 3.

7)



8) Line  $y = 6$  would pass through  $(32, 6)$  and  $(-19, 6)$ . Both of those points have 6 as the  $y$  coordinate.

9)

- There are many correct answers. (3, 0), (3, 3), (3, -2), and (3, -4) are all examples of points on this line.
- All points on the line have 3 as the value of the x coordinate.
- This line is parallel to the y-axis, because each point is 3 units away from that axis.
- I think the equation that names this line is  $x = 3$  because it refers to a line where every ordered pair has the a 3 for the x coordinate.

10) The equation of the line is  $y = -2$ . The y coordinate for every ordered pair on the line is -2. The line is two units below the x-axis.

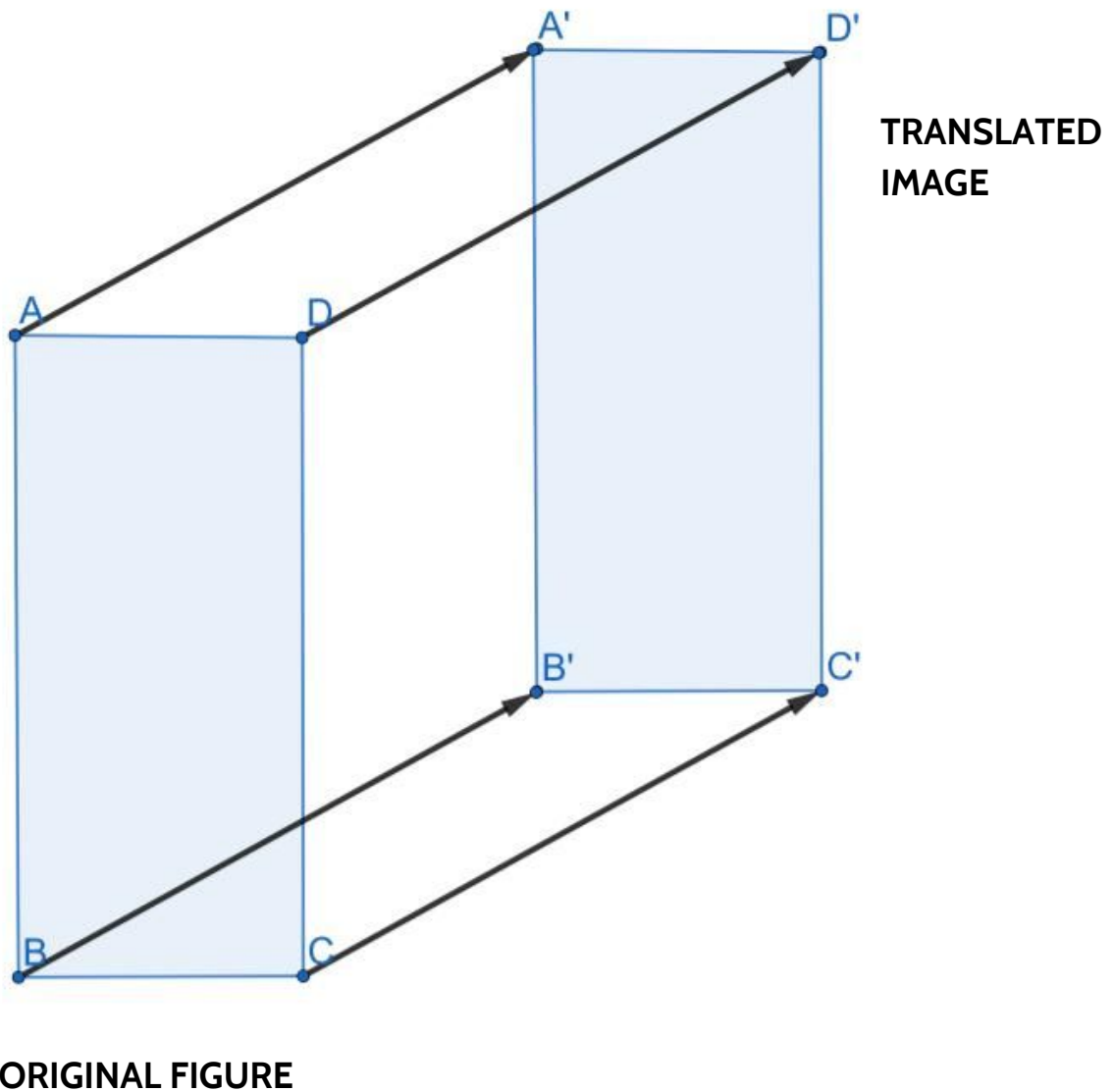
11) The equation of the line is  $x = -1$ . The x coordinate for every ordered pair on the line is -1. The line is 1 unit to the left of the y-axis.

## How Math is Written

In the next section you will be learning how to use rigid transformations on the coordinate plane.

One key idea in rigid transformations is that when we are sliding, turning, or flipping figures, we are sliding, turning, and flipping *every point* in that figure.

Consider the original figure on the left. We call this Rectangle ABCD.



## Rigid Transformations

We refer to shapes by the name of the shape (rectangle, triangle, square, etc) followed by the letter given to each point.

Notice how we slide each point the same distance in the same direction to create its image.

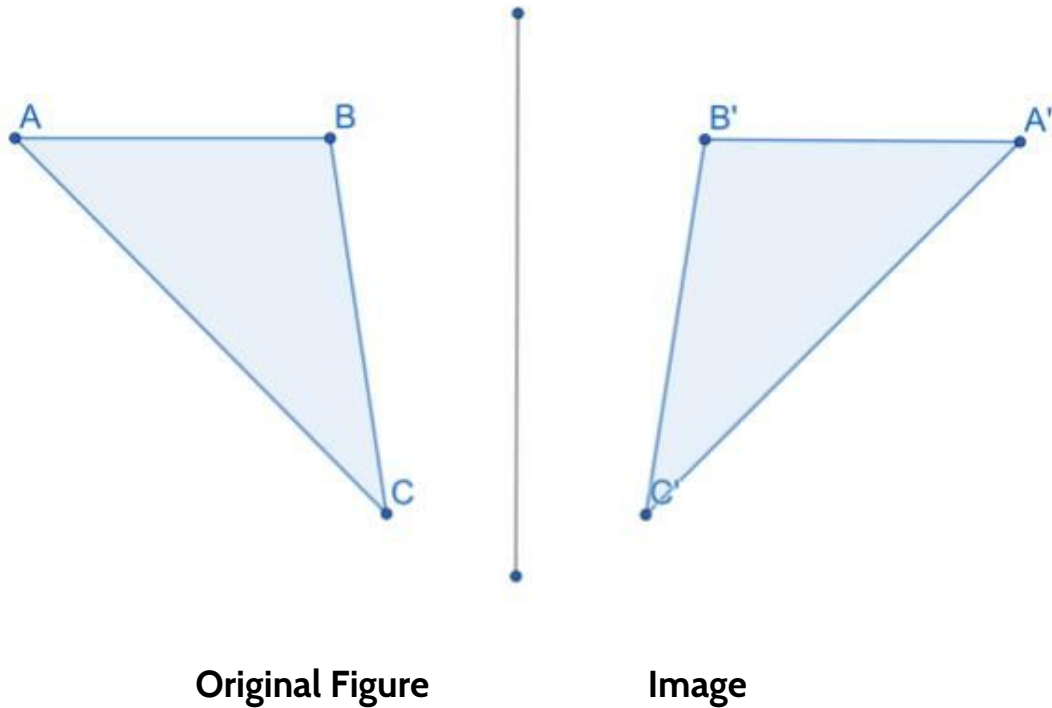
Because of this idea that each point on the original figure has a corresponding point in the image, we can say: **Rectangle ABCD maps onto its image Rectangle A'B'C'D'**.

Sometimes we name the corresponding points in the image produced by a rigid transformation by using a tick mark. This can help keep track of the corresponding points.

In the slide shown above:

- Point A maps to Point A'
- Point B maps to Point B'
- Point C maps to Point C'
- Point D maps to Point D'

One other important symbol is the symbol for congruence,  $\cong$ .  
In our example here, we can say Rectangle ABCD  $\cong$  Rectangle A'B'C'D'.



1) What kind of transformation is this? How do you know?

2) What is the name of the original figure?

3) Fill in the blanks

- Point A maps onto Point \_\_\_\_\_.
- Point B \_\_\_\_\_ onto Point B'.
- Point C \_\_\_\_\_ onto Point \_\_\_\_\_.
- $\triangle ABC$  is \_\_\_\_\_ to  $\triangle A'B'C'$  because \_\_\_\_\_

---



## How Math is Written - Answer Key

- 1) This is a reflection or a flip.
- 2) The original figure can be called Triangle ABC or  $\triangle ABC$ .
- 3) Point A maps onto Point A'.

Point B maps onto Point B'.

Point C maps onto Point C'.

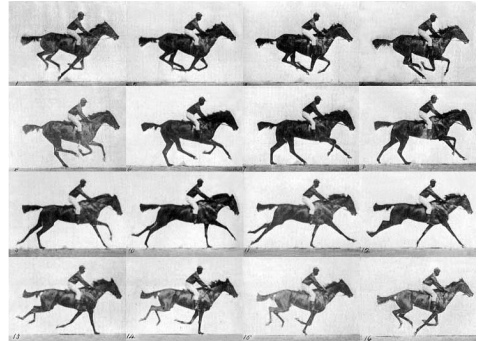
$\triangle ABC$  is congruent to  $\triangle A'B'C'$ .

There are different answers to explain why  $\triangle ABC$  is congruent to  $\triangle A'B'C'$ . Some possible answers are:

- $\triangle A'B'C'$  was produced by a rigid transformation and any image produced by a rigid transformation is congruent to the original shape.
- Each point on  $\triangle ABC$  maps to the points in  $\triangle A'B'C'$ .
- $\triangle ABC$  and  $\triangle A'B'C'$  meet our definition of congruent. They are the same size and shape. Also, if you moved them on top of one another other, one of them would exactly cover the other one.

## Working with Transformations on the Coordinate Plane

Movies create the illusion of movement by showing us 24 images per second. When the human eye processes 24 images per second it is interpreted in our brains as movement.



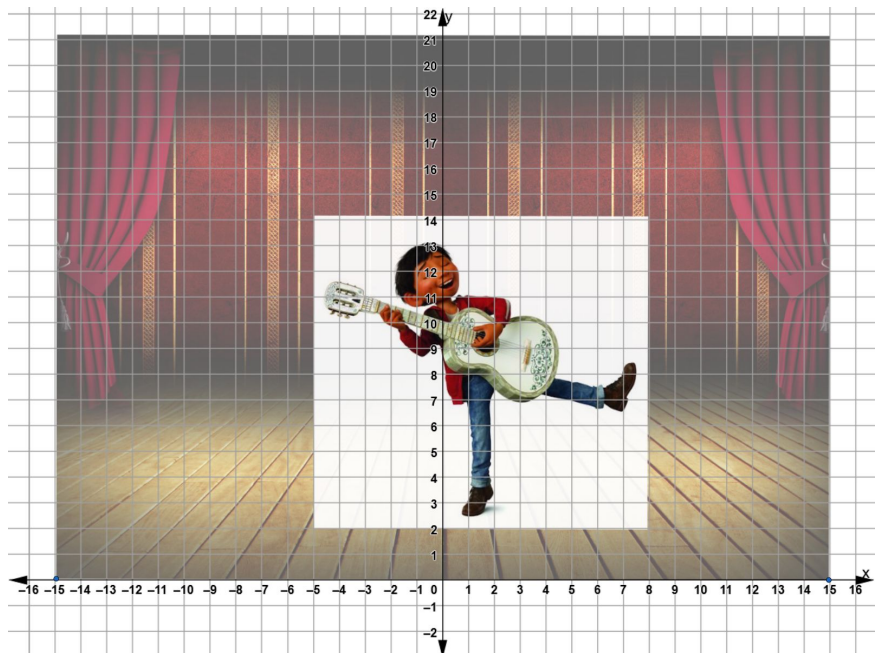
Computer animated films work the same way as other films in the sense that they are a series of single images played together quickly.

Question: Consider the recent film, *Coco* (2017) created by the computer animation studio, Pixar. That movie is 105 minutes long. If movies are created showing 24 images per second, how many images would you expect it took to make *Coco*?

In the introduction, we looked at some examples of rigid transformations in art that were done by hand. In this section, we are going to be looking at the skills needed to use computers to make rigid transformation. The coordinate grid allows human artists to communicate their ideas to computers. Rigid transformations on the coordinate grid are at the heart of computer animation and video game design.

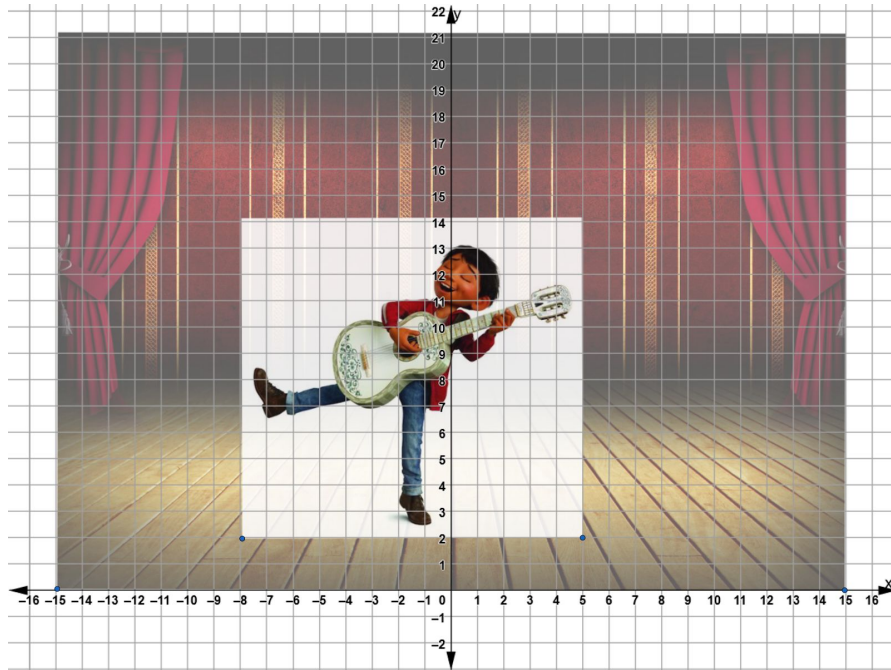
One of the main characters of the movie *Coco* is a young boy named Miguel who loves music and is always singing and practicing his guitar.

Consider this image of Miguel playing his guitar on stage.



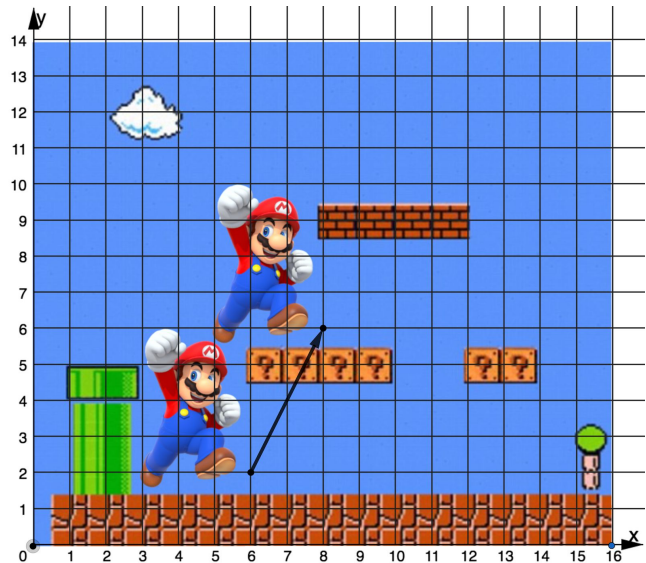
## Rigid Transformations

The art team at Pixar can use coordinates to create a reflection of that image across the  $y$ -axis, making it appear as though Miguel has moved from his right foot to his left foot.



Rigid transformations on the coordinate grid are also used in the design of video games. On this coordinate grid, the Nintendo character, Mario is being translated 2 units to the right and 4 units up.

In this section you'll be learning how to map translations, reflections, and rotations on the coordinate grid. You'll also have an opportunity to use rigid transformations on the coordinate grid to design and create patterns.



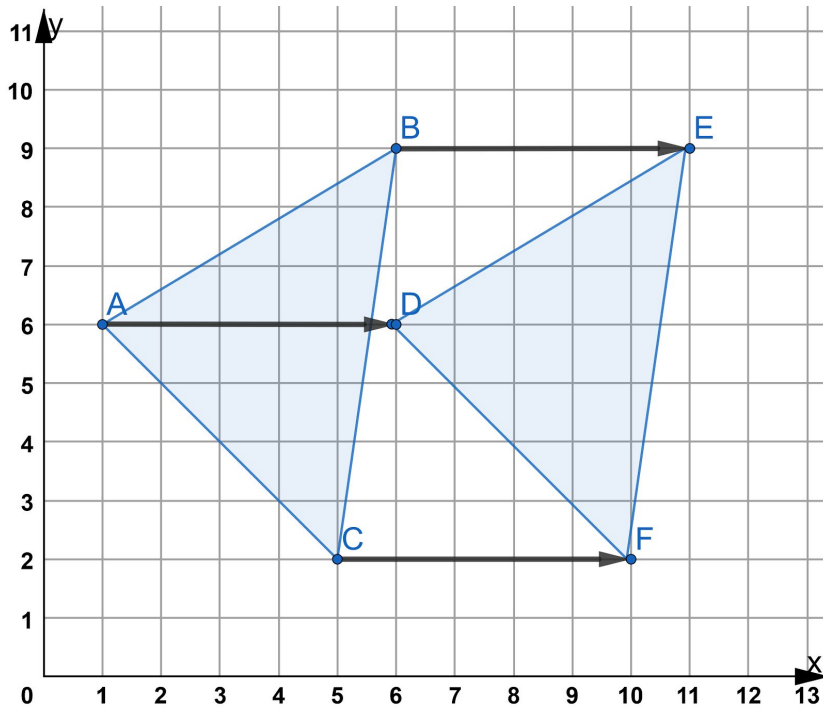
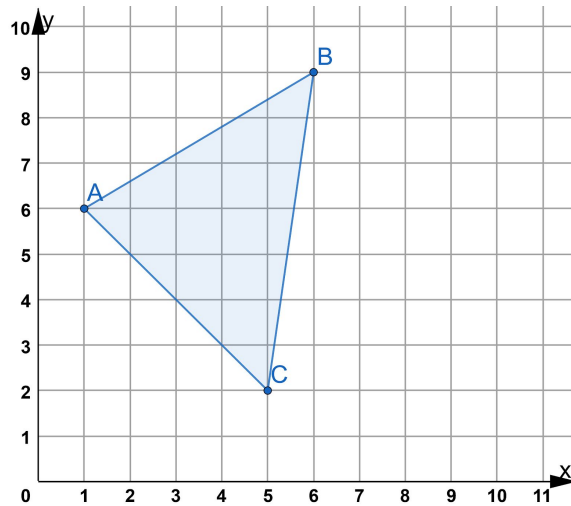
Answer: If *Coco* is 105 minutes long, we can multiply 105 minutes  $\times$  60 seconds for each minute to see that 105 minutes is 6,300 seconds. If film uses 24 frames per second, that's 24 images for each second of the film. 6,300 seconds  $\times$  24 images is 151,200. So Pixar had to create 151,200 images to make *Coco*.

## Translations on the Coordinate Plane

Translations involving sliding a point or shape to another location without turning it. As with all rigid transformations, any images created by translation are congruent to the original figure.

One thing you can do with translations on the coordinate grid is take a shape and translate it to make another congruent shape.

Let's take  $\triangle ABC$ .

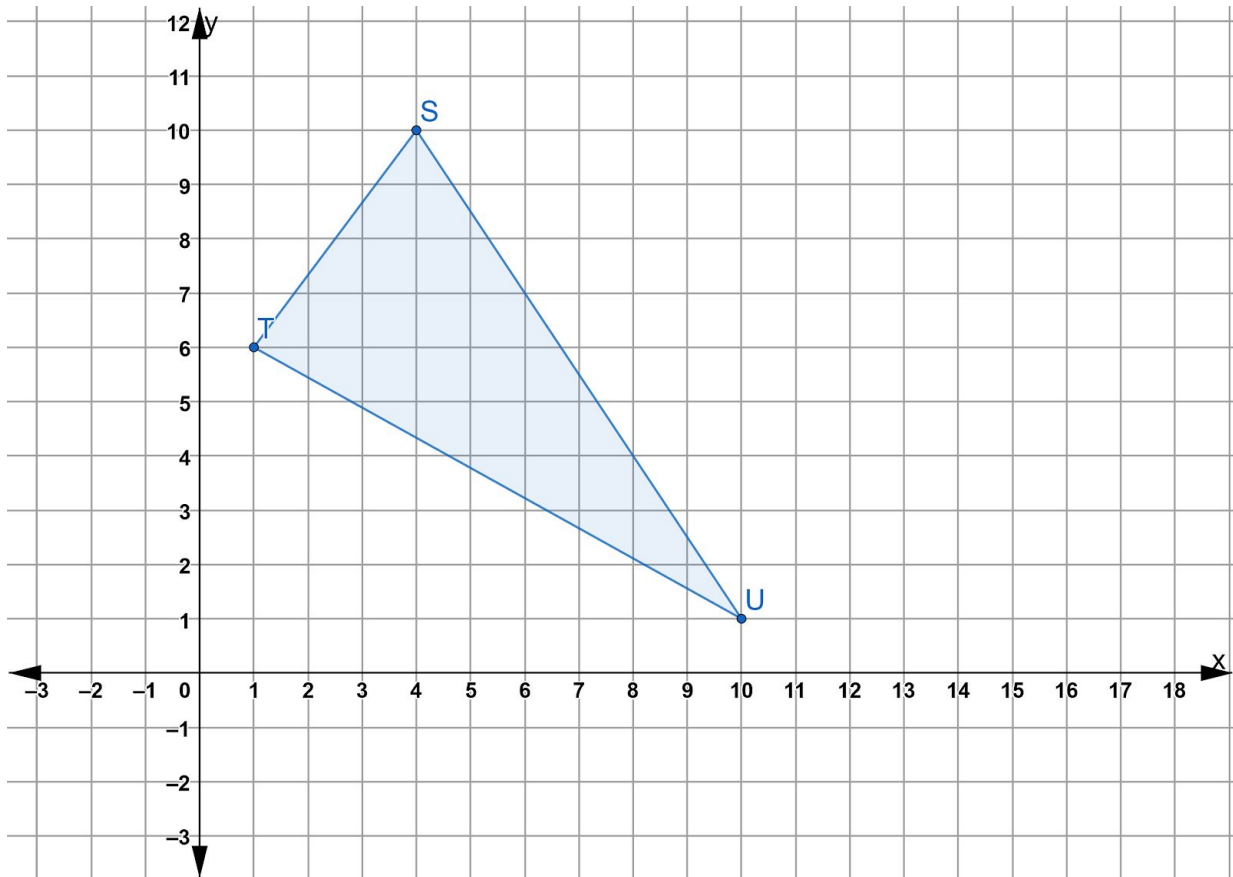


If we slide each point in  $\triangle ABC$  5 units to the right, we make  $\triangle DEF$ .  $\triangle DEF$  is congruent to  $\triangle ABC$ .

## Rigid Transformations

Now you try.

- 1) Translate  $\triangle STU$  6 units to the right to make  $\triangle S'T'U'$ .



What are the coordinates for:

Point S (     ,     )

Point T (     ,     )

Point U (     ,     )

What are the coordinates for:

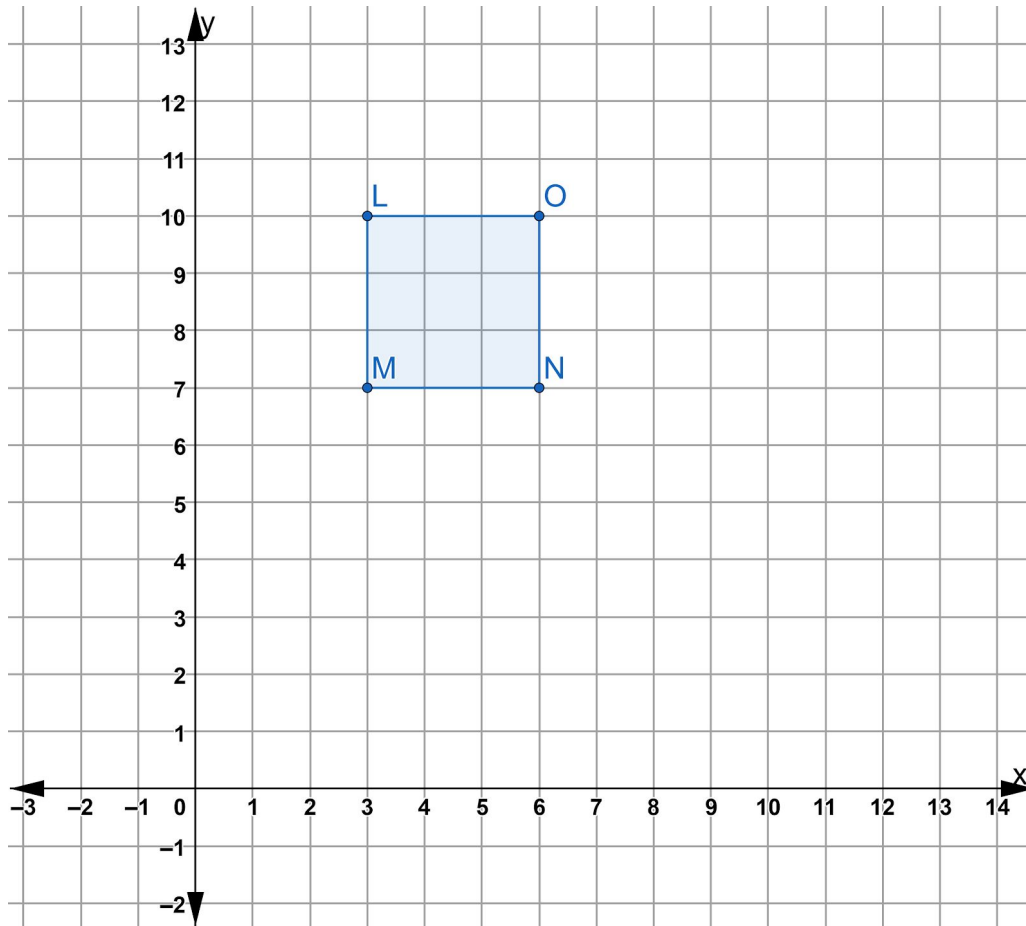
Point S' (     ,     )

Point T' (     ,     )

Point U' (     ,     )

*Rigid Transformations*

- 2) Translate Rectangle LMNO 4 units to the right and 4 units down. Label the points of the translated image L', M', N', O'.



What are the coordinates for:

Point L ( , )

Point M ( , )

Point N ( , )

Point O ( , )

What are the coordinates for:

Point L' ( , )

Point M' ( , )

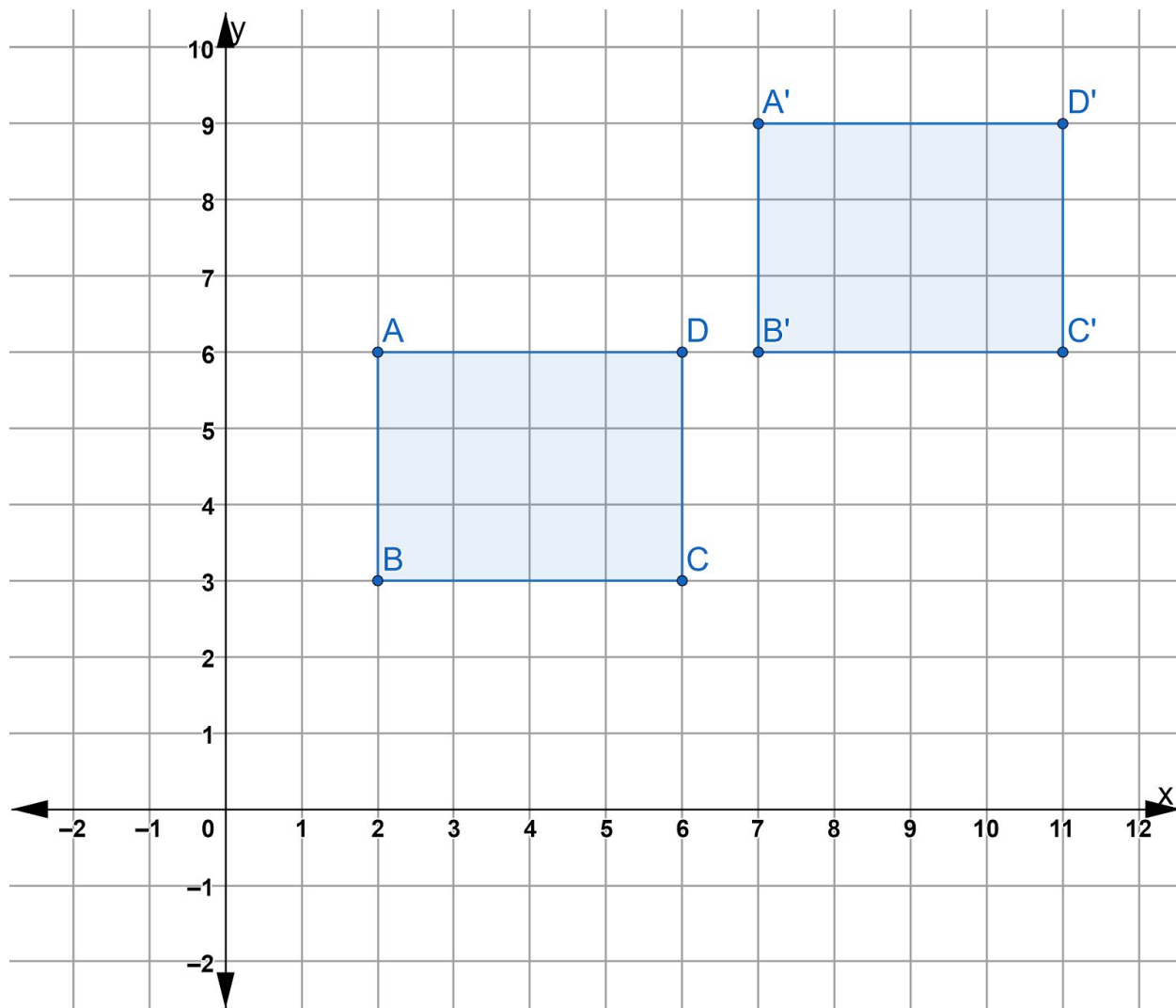
Point N' ( , )

Point O' ( , )

## Rigid Transformations

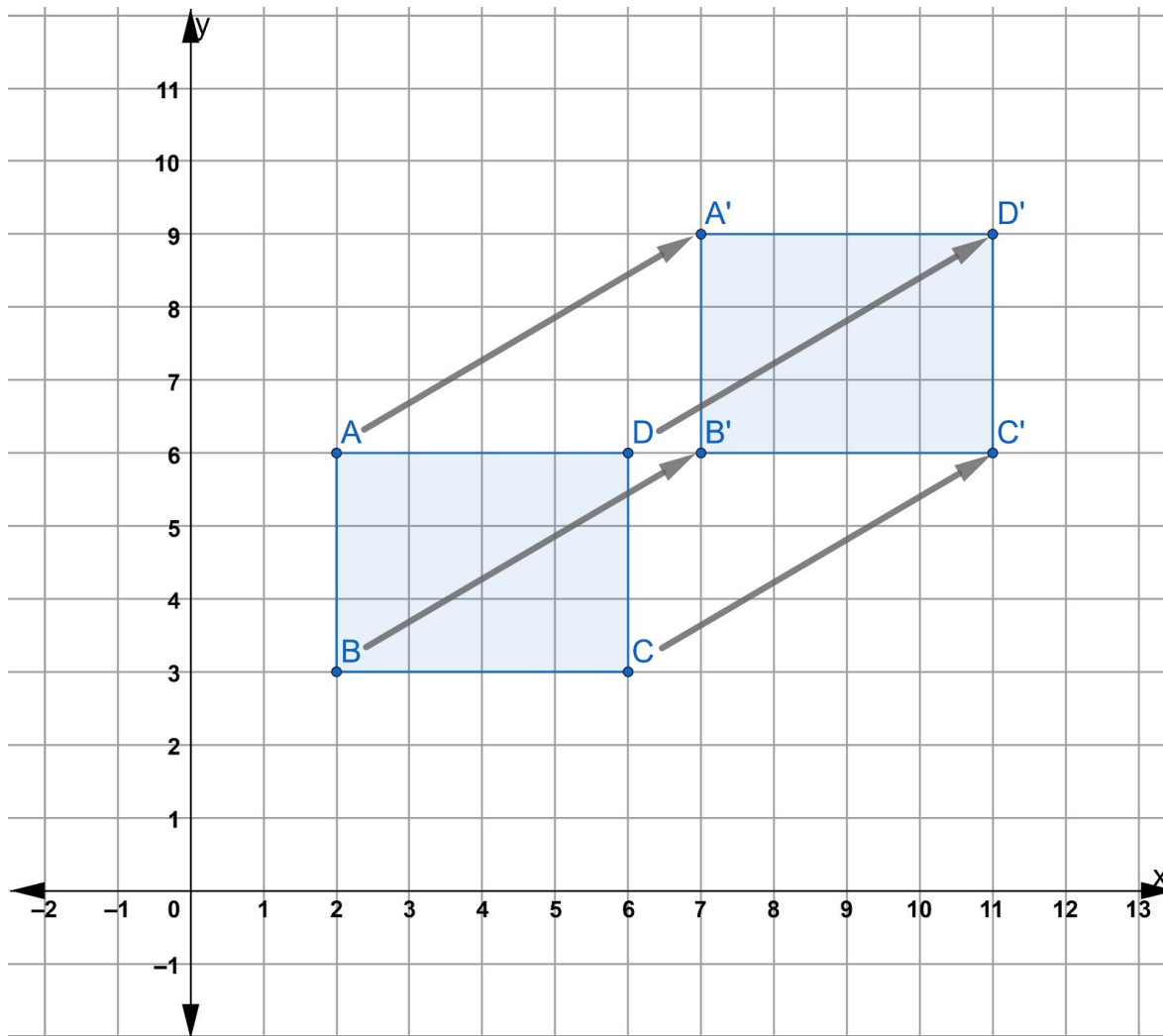
Once we understand how translations work, we can look at two congruent figures that have been created by a translation and figure out what the translation was.

Rectangle ABCD maps onto rectangle A'B'C'D'.



We can tell it is a translation because we can move the figures on top of each other with a slide. But how can we describe what the translation is? How do we get from our original rectangle to its translated image?

## Rigid Transformations



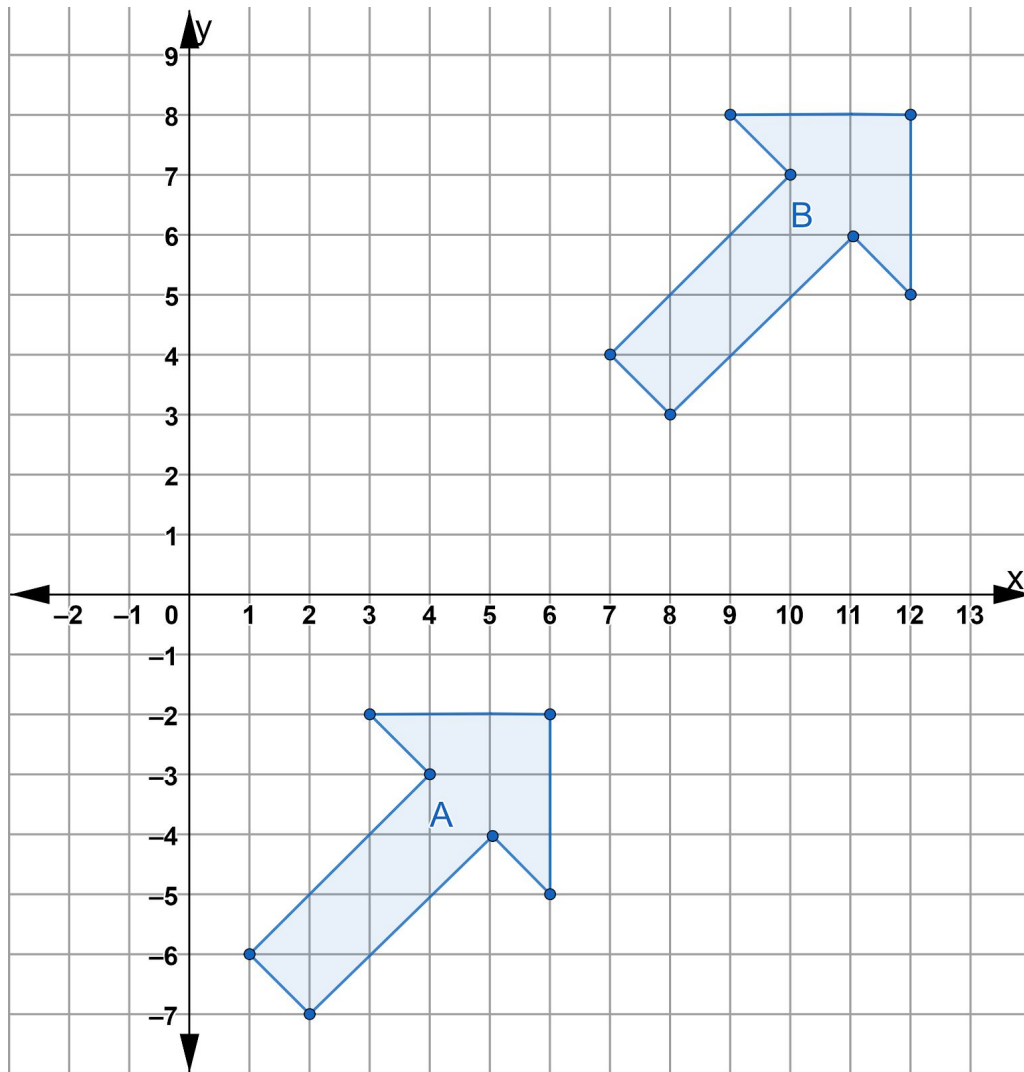
The first step when describing a translation is to be clear about which figure is the original figure. In this example, rectangle ABCD is the original and rectangle A'B'C'D' is its translated image.

The next step is to choose one point on the original figure and find the translated point on the image. For example, point C maps onto point C'. To get from point C to point C', we move 5 units to the right and three units up. If you count the units to move from point A to point A', you will see it is the same. In fact, you could use any point on the original figure to see how it moves onto its translated image. In the coordinate grid above, each point in rectangle ABCD has been translated 5 units to the right and 3 units up to make congruent rectangle A'B'C'D'. Note that when we write the directions for a translation, we give the horizontal (right/left) directions first and then the vertical (up/down).



## Rigid Transformations

Now you try.



- 3) Which translation can be used to map figure A to figure B?
- A) A translation 2 units to the right and 5 units up
  - B) A translation 2 units to the left and 5 units down
  - C) A translation 6 units to the left and 10 units down
  - D) A translation 6 units to the right and 10 units up

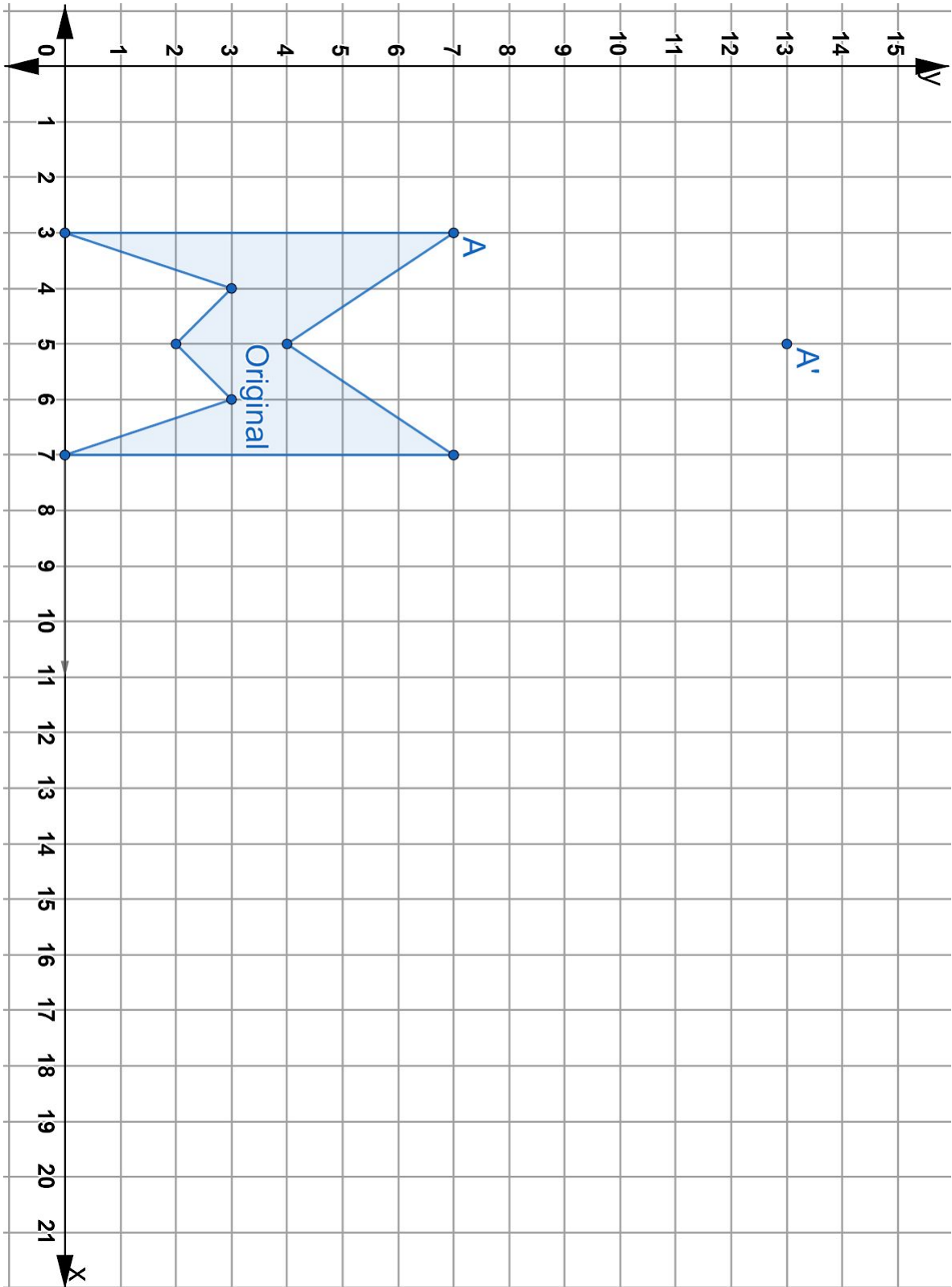
## Creating Designs with Translations on the Coordinate Plane

- 4) Textile designers can use translations can be used to create designs and patterns in clothing, fabric, tiles, etc.
  - a) On the coordinate plane on the next page, translate each point of the original figure 2 units to the right and 6 units up. One point has already been done.  
Label the translated image, "Figure B".
  - b) Now translate Figure B two units to the right and six units down.  
Label this image, Figure C.
  - c) Now translate Figure C two units to the right and six units up.  
Label this image Figure D.
  - d) Next translate Figure D four units to the right.  
Label this new image Figure E.
  - e) Next translate Figure E two units to the left and 6 units down.  
Label this new image Figure F.
  - f) Finally translate Figure F four units to the right.  
Label this new image Figure G.

You should end up with seven congruent figures.

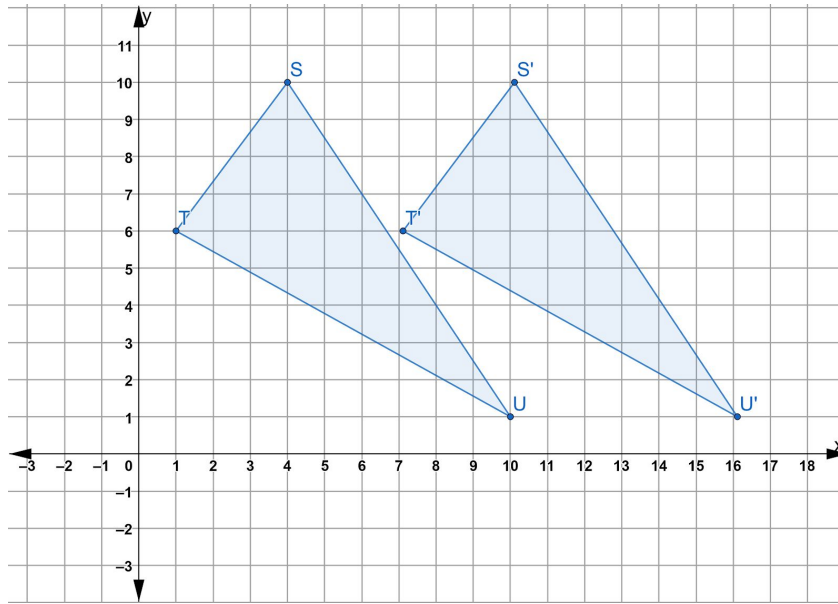
Feel free to color the figures in before moving on

Rigid Transformations



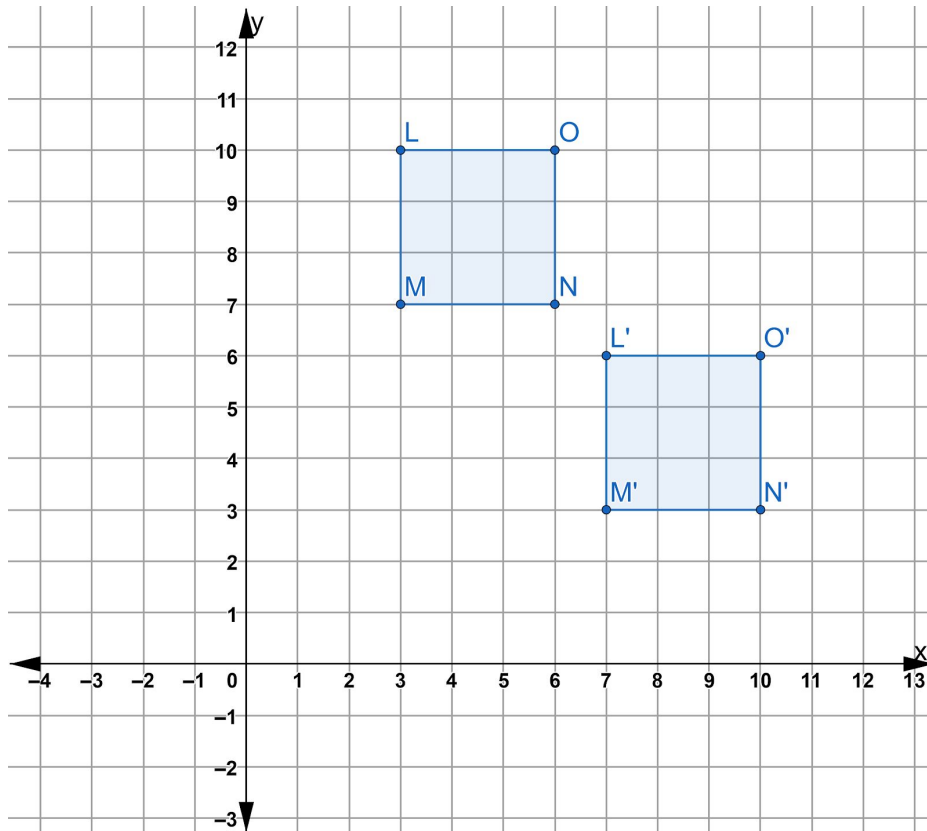
## Translations on the Coordinate Plane - Answer Key

1)



- Point S (4,10)
- Point T (1,6)
- Point U (10,1)
- Point S' (10,10)
- Point T' (7,6)
- Point U' (16,1)

2)

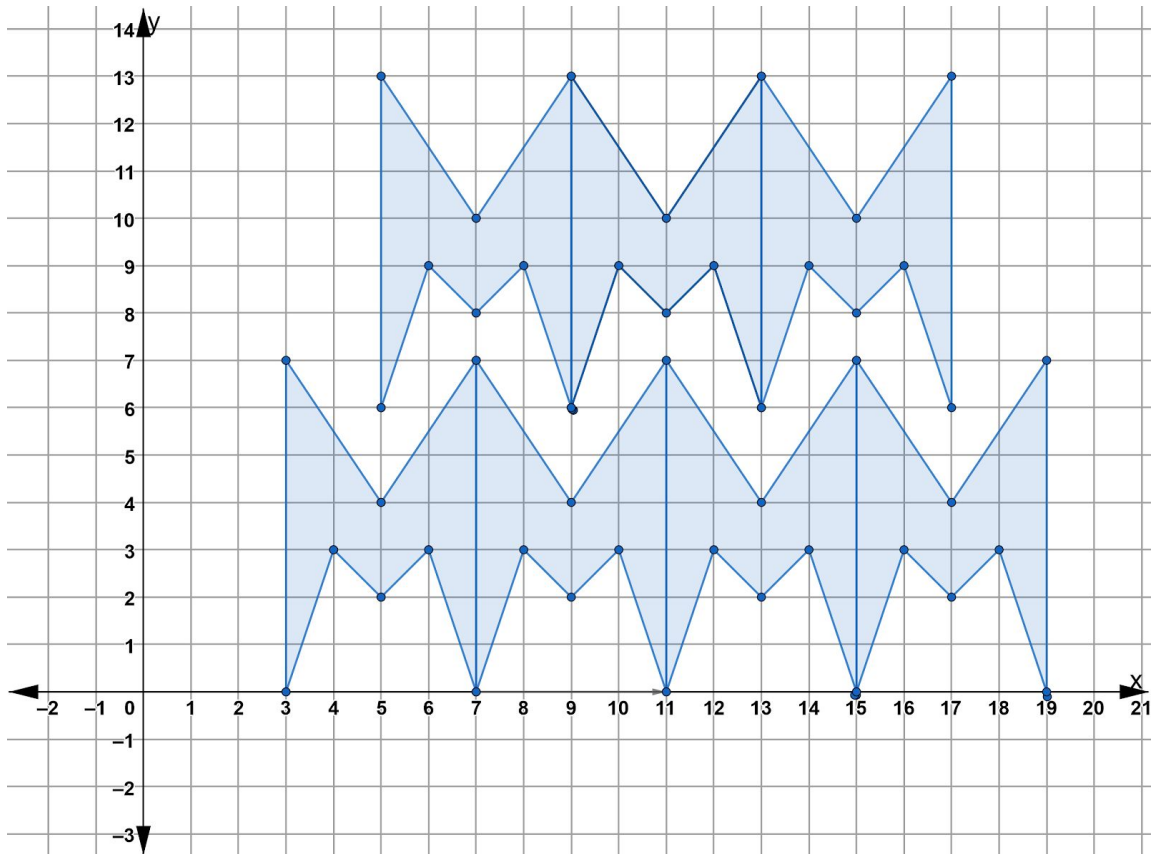


3) Choice D.

Choice A is a common mistake if you do not map the point on figure A to the same point on figure B. Choice A maps the point at the tip of the original arrow to the back of figure B.

Choice C is another common mistake if you confuse which is the original figure and which is the translated image. Choice C is the translation that maps Figure B onto Figure A. The question is asking about how we slide figure A onto Figure B.

4) The design built by those translations should look like this.

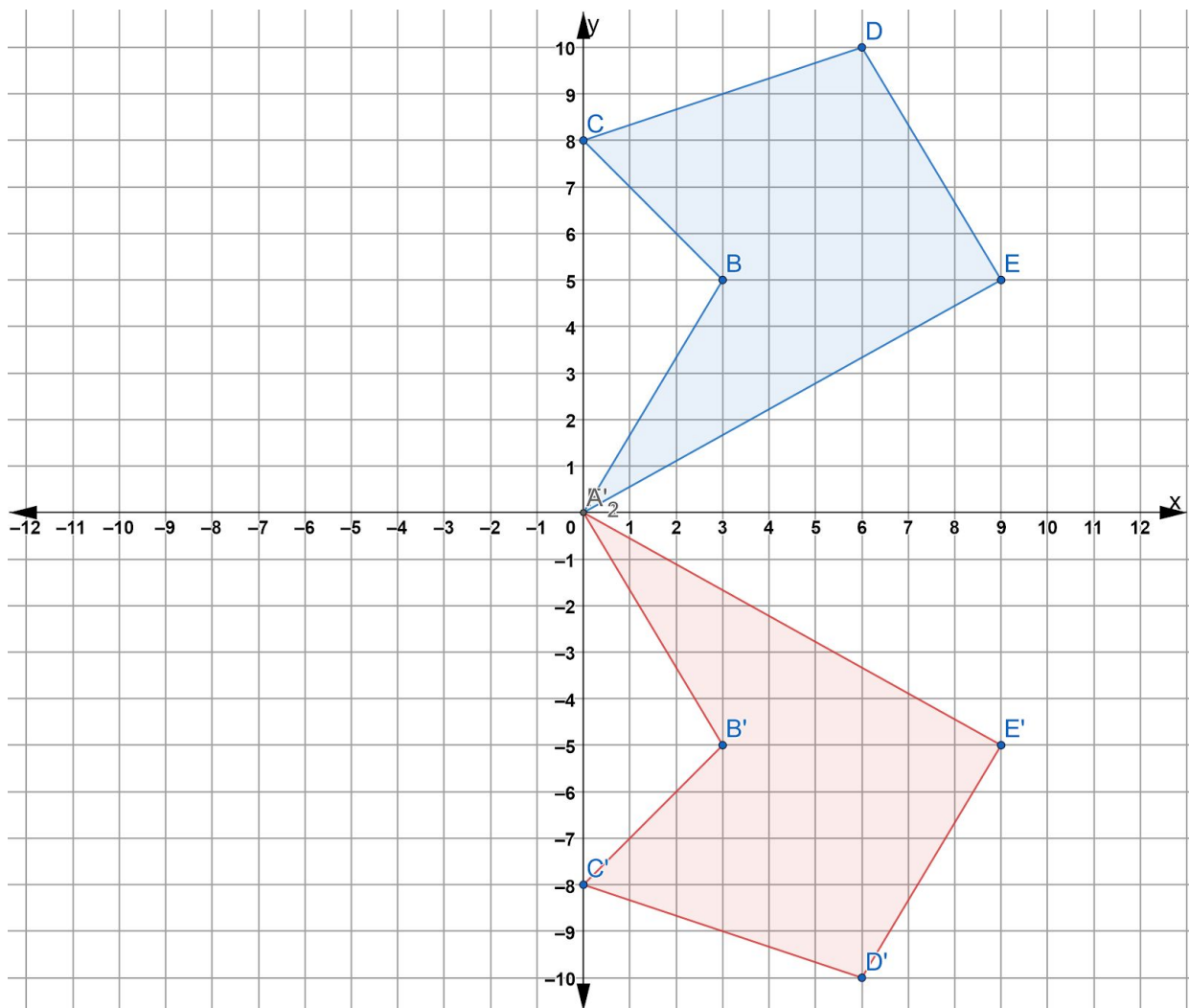


## Reflections on the Coordinate Plane

Remember that reflections involve flipping a point or a shape about a line of reflection. You will now learn to use the coordinate plane to do two things:

- (1) use a line of reflection to create an image of a shape, and
- (2) to use an original figure and an image to find the line of reflection.

The coordinate plane below shows original figure  $ABCDE$  being reflected about the  $x$ -axis to create image  $A'B'C'D'E'$ . If you folded this paper along the  $x$ -axis, the two shapes would exactly fit over one another. In this case, the  $x$ -axis is the line of reflection.



Write three things you notice on the coordinate grid.

- 1) Use the coordinate plane on the previous page to complete the following statements:
  - a) Point A and Point A' are both  $0$  units away from the line of reflection.
  - b) Point B and Point B' are both \_\_\_\_ units away from the line of reflection.
  - c) Point C and Point C' are both \_\_\_\_ units away from the line of reflection.
  - d) Point D and Point D' are both \_\_\_\_ units away from the line of reflection.
  - e) Point E and Point E' are both \_\_\_\_ units away from the line of reflection.

Note that each point in the original figure is the same distance from the line of reflection as the corresponding point in the image. For example, point B is 5 units above the line of reflection and point B' is 5 units below the line of reflection. Another way to look at this is to say that points B and B' are 10 units away from each other, and the line of reflection must be half way between them.

We can measure the distance between each point in the figure to the line of reflection. The image of each point will have to be the same distance away from the line of reflection.

- 2) If you reflected point B' across the y-axis, the reflected point B'' would appear in Quadrant 3. What are the coordinates of point B''? How do you know?

- 3) Reflect the remaining points of the shape in Quadrant 4 across the y-axis on the coordinate plane.

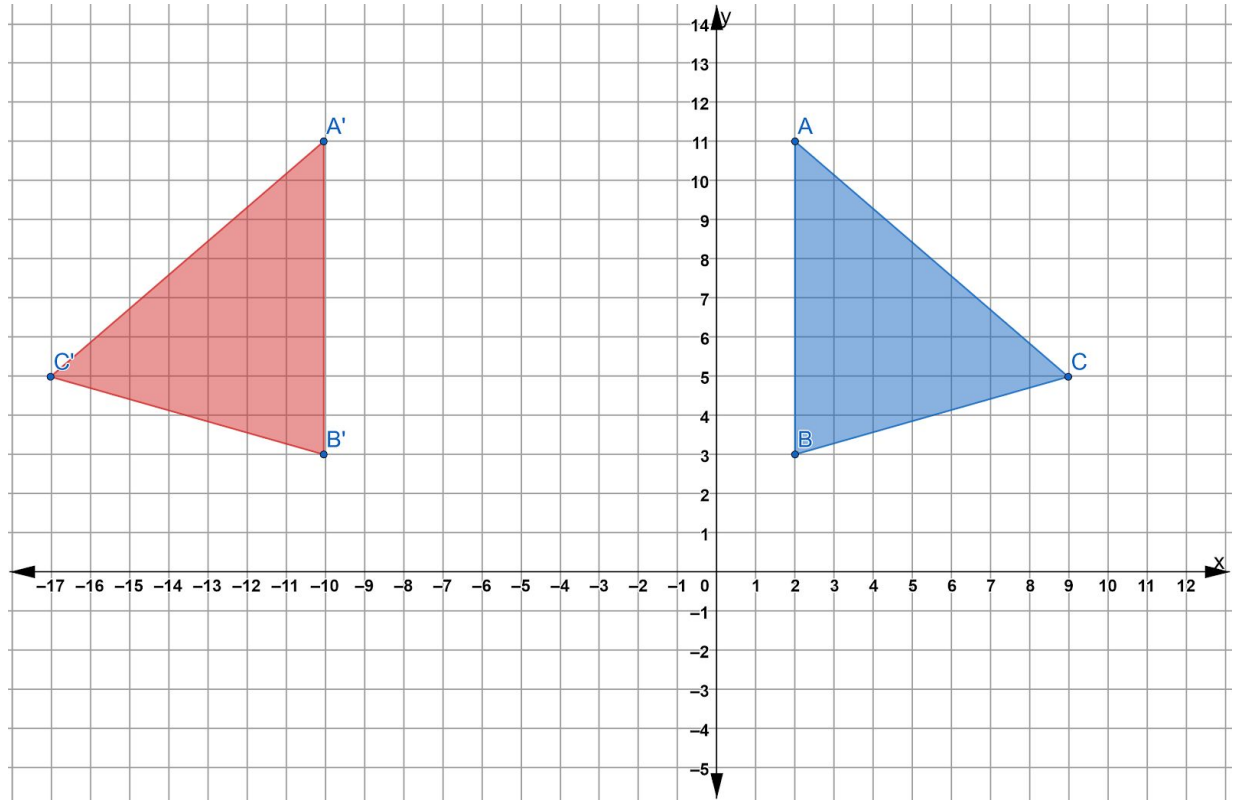
What are the coordinates of point E''?

- 4) Now reflect figure A''B''C''D''E'' over the x-axis. Your new image will be figure A'''B'''C'''D'''E''' and should appear in Quadrant 2.

What are the coordinates of point D'''?

- 5) Figure  $\triangle ABC$  below has been reflected about a line of reflection resulting in image  $\triangle A'B'C'$ . Draw the line of reflection and write the equation for the line on the coordinate plane below.

## Rigid Transformations



*HINT: You can start by finding the distances between the points in the original figure and the points on the image.*

- 6) What do you want to remember about reflecting figures and points across lines of reflection on the coordinate grid?

---

---

---

---

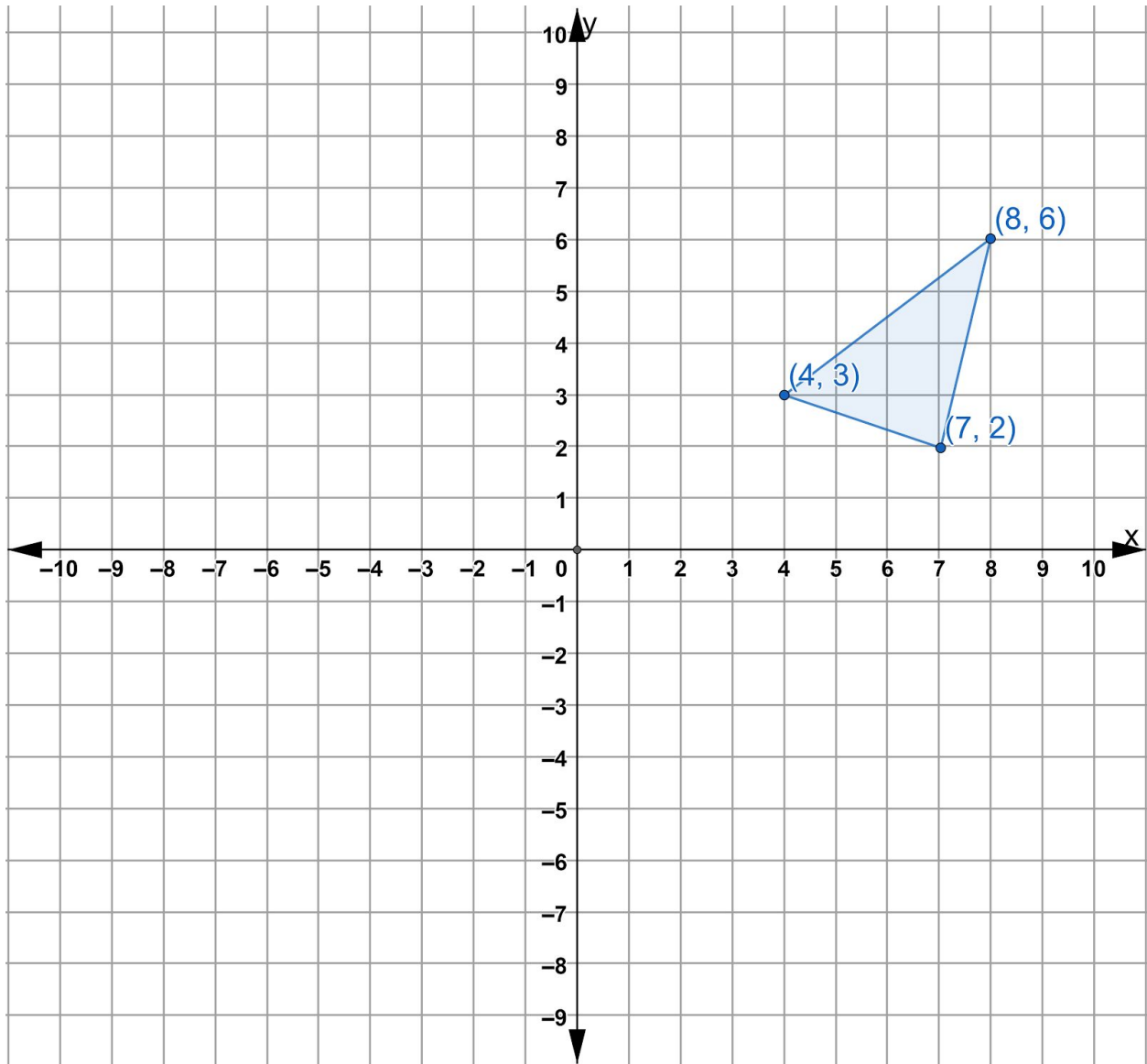
---

This next activity will help you see what happens to points and shapes when they are reflected across the x-axis, the y-axis, or the line  $y=x$ .

Consider the triangle on the coordinate grid below.



## Rigid Transformations

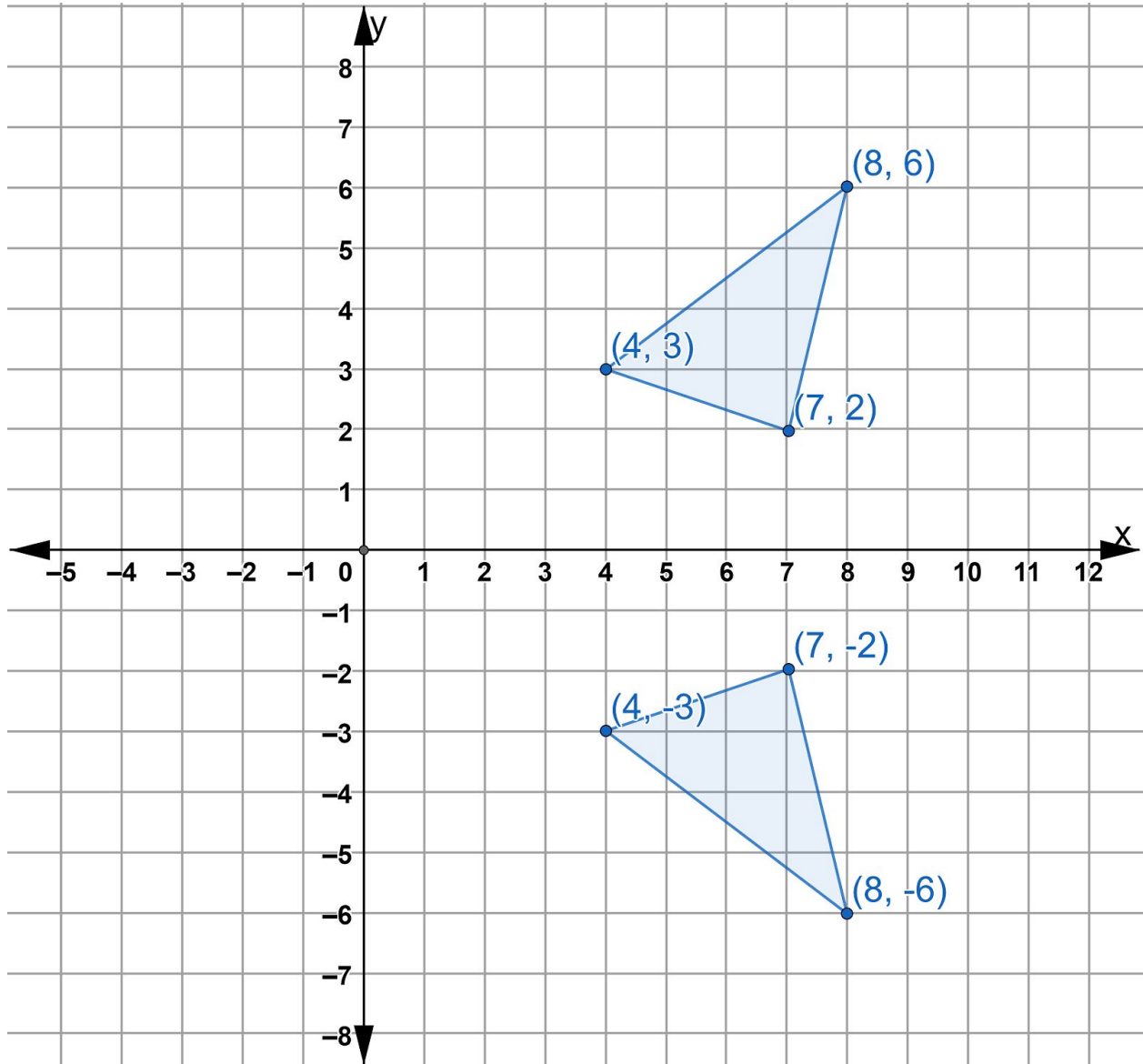


Reflect the triangle across the x-axis (from Quadrant 1 into Quadrant 4). Write the ordered pair next to each reflected point.

Then reflect the original triangle across the y-axis (from Quadrant 1 into Quadrant 2). Write the ordered pair next to each reflected point.

Let's focus on the reflection over the x-axis first.

## Rigid Transformations



Notice the coordinates of the reflected image.

Original Triangle	Corresponding Points in Reflected Image
(4,3)	(4,-3)
(8,6)	(8,-6)
(7,2)	(7,-2)

How are they similar to the original coordinates? \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

How are they different? \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

### The Relationship Between Points Reflected Over the X-axis

When the triangle is reflected over the x-axis, what is similar is the x coordinates and what is different are the y coordinates. The x coordinates remain the same and the y coordinates of the image become the opposite of the y coordinates in the original figure.

The original triangle here is in Quadrant 1. When we reflect it over the x-axis, it flips into Quadrant 4. Let's look at each specific point in the original triangle.

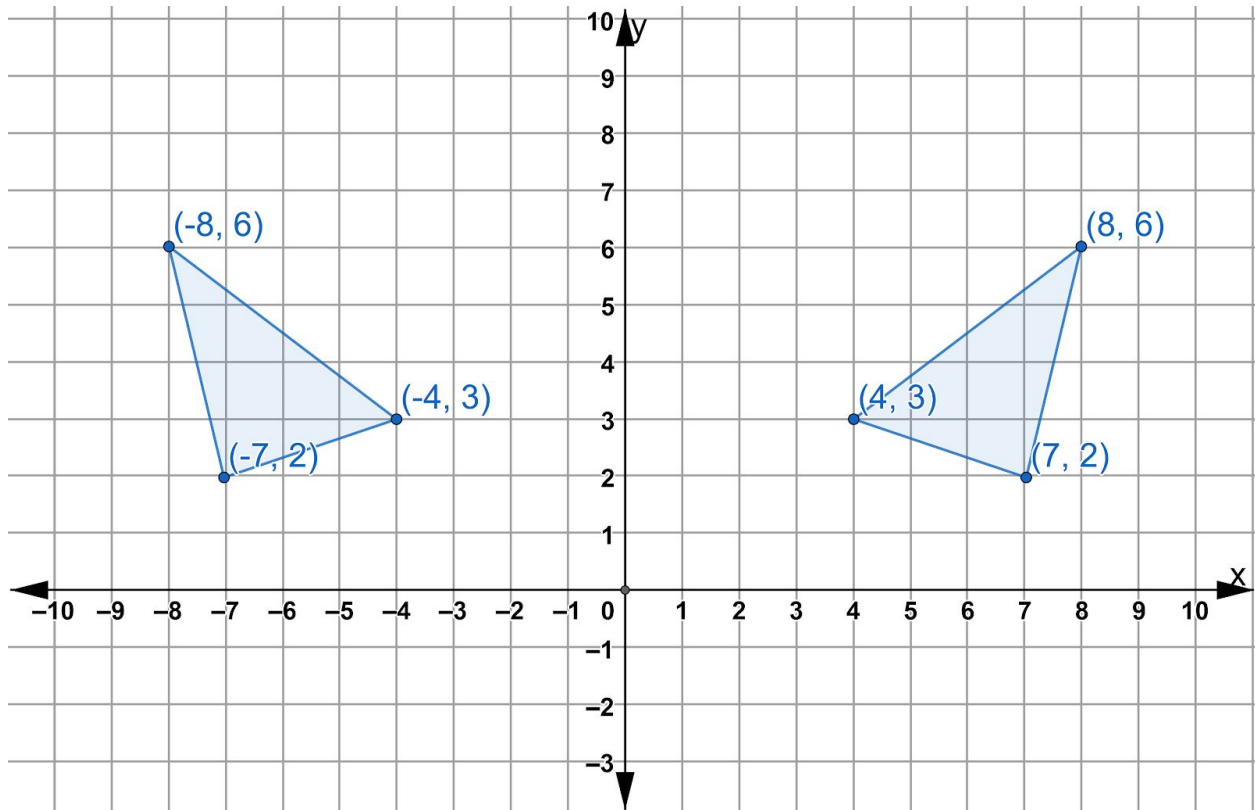
(7,2) is 7 units to the right of the origin and 2 units above the x-axis. If we reflect it across the x-axis, it will still be 7 units to the right of the origin. But since the original point is 2 units above the x-axis, the image point needs to be 2 units below the x-axis.

The same will be true for (8,6). Since (8,6) is 6 units above the x-axis, then its image needs to be 6 units below the x-axis. So (8,6) reflected over the x-axis becomes (8,-6).

(4,3) is 3 units above the x-axis, so its image reflected across the x-axis would be 3 units below the x-axis. (4,3) reflected over the x-axis becomes (4,-3).

## Rigid Transformations

Let's see what happens when we reflect the triangle over the y-axis.



Now the y coordinates stay the same and the x coordinates in the image become the negative of the x coordinates in the original.  $(4,3)$  becomes  $(-4,3)$ .  $(8,6)$  becomes  $(-8,6)$ .  $(7,2)$  becomes  $(-7,2)$

7) Describe what is happening to each point to explain why this happens.

---

---

---

---

---

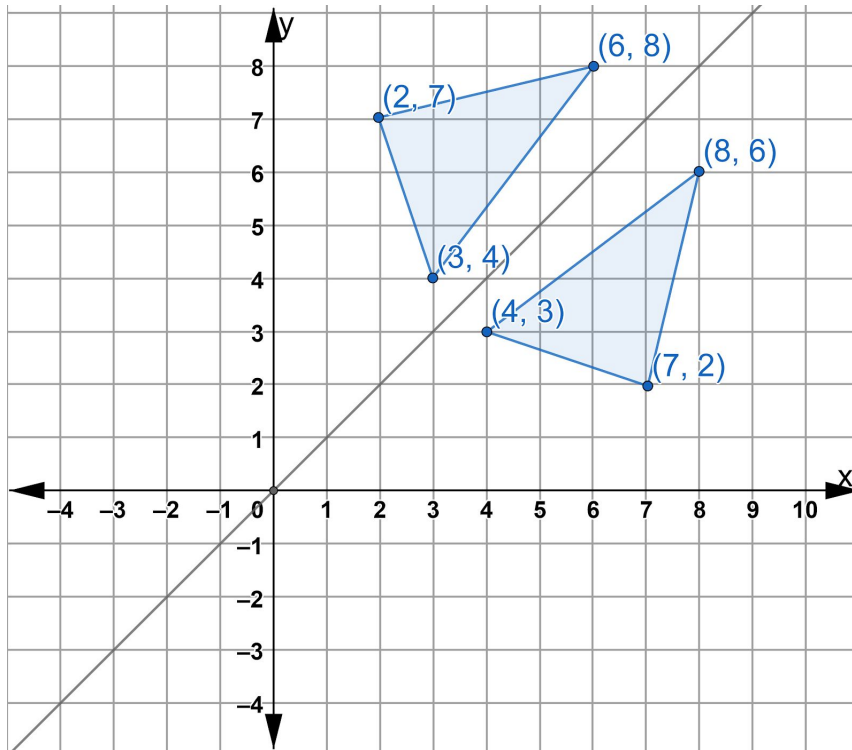
---

---

---

## Rigid Transformations

Let's look at one more reflection. This time let's see what happens when we reflect our original triangle over the line  $y=x$ .



What relationship do you see between the original points and their images reflected over  $y=x$ ? \_\_\_\_\_

---

---

---

---

---

---

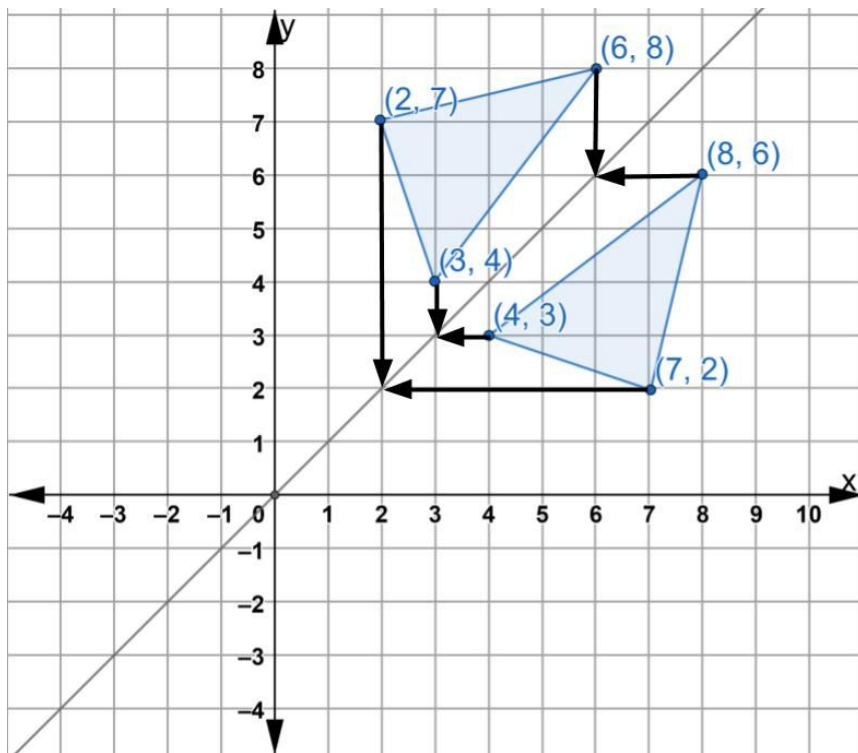
### The Relationship Between Points Reflected Over the Line $y=x$

When a point or a shape is reflected across the line  $y=x$  the coordinates of the  $x$  and  $y$  coordinates switch.

Point  $(4,3)$  becomes  $(3,4)$ , point  $(8,6)$  becomes  $(6,8)$  and point  $(7,2)$  becomes point  $(2,7)$ .

To help us see why, let's think about how we can figure out the distance between each point and the line of reflection  $y=x$ .

The line  $y=x$  is a diagonal line, so it is a little different when we want to find the distance between the points and the line of reflection. One strategy you can use is shown in the diagram below. To move from point  $(7,2)$  to the line of reflection, we move 5 units to the left. To move from the image point, we need to move 5 units down, forming an L shape with arms of equal length. You can do the same with the other points. To move from point  $(8,6)$  to the line of reflection, we move two units to the left and the two units up. So the  $x$  coordinate will go down by 2 and the  $y$  coordinate will go up by 2.

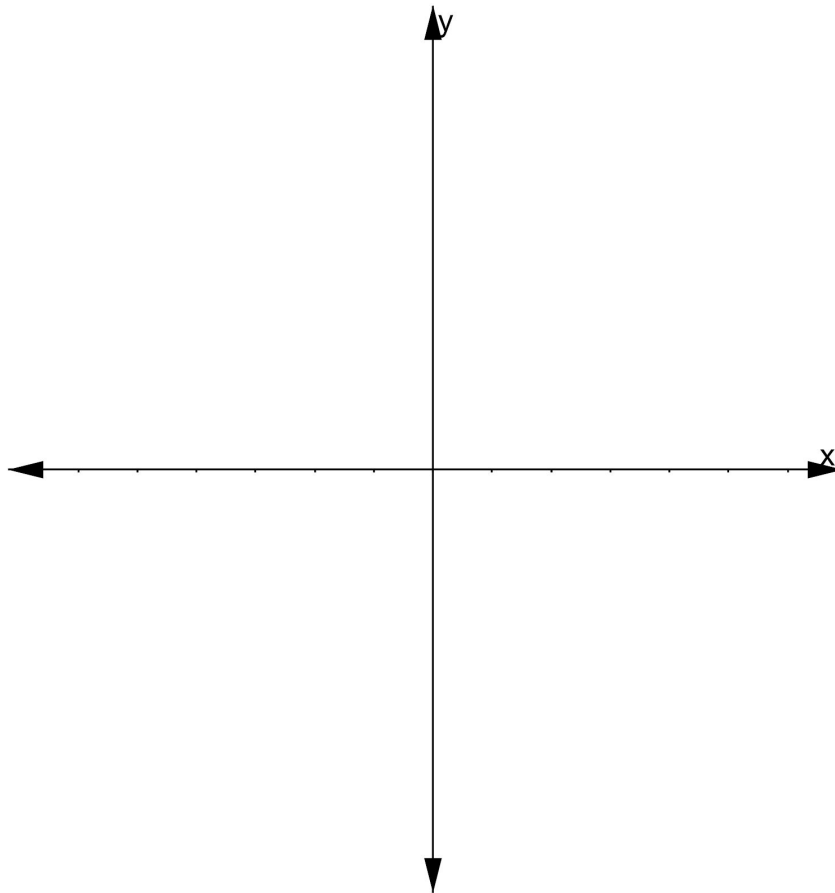


## Rigid Transformations

Let's practice what we've learned about reflecting shapes across the x-axis, the y-axis and the line  $y=x$ .

- 8) Use what you know about reflections to fill in the table below. A coordinate grid is provided to help you sketch out your ideas.

<b>Original Figure</b>	<b>(7,6)</b>	<b>(10,6)</b>	<b>(10,1)</b>	<b>(7,1)</b>
<b>Reflected over x-axis</b>				
<b>Reflected over y-axis</b>				
<b>Reflected over <math>y=x</math></b>				



## Rigid Transformations

One last thing we will look at in this section is what happens when a figure goes through a sequence of reflections. In other words, what can we notice about a figure that is reflected more than one time?

9) On the coordinate plane on the next page, plot figure ABCDE.

A (2,1)

B(2,7)

C(9,7)

D(5,1)

E(5, 3)

Imagine two different sequences of reflections:

**Sequence 1** - Figure ABCDE is reflected about the x-axis and the resulting image is reflected about the y-axis, or

**Sequence 2** - Figure ABCDE is reflected over the y-axis first and then the resulting image is reflected over the x-axis.

Would the final image in each sequence be different or the same?

What do you think will happen? (Circle the statement you agree with)

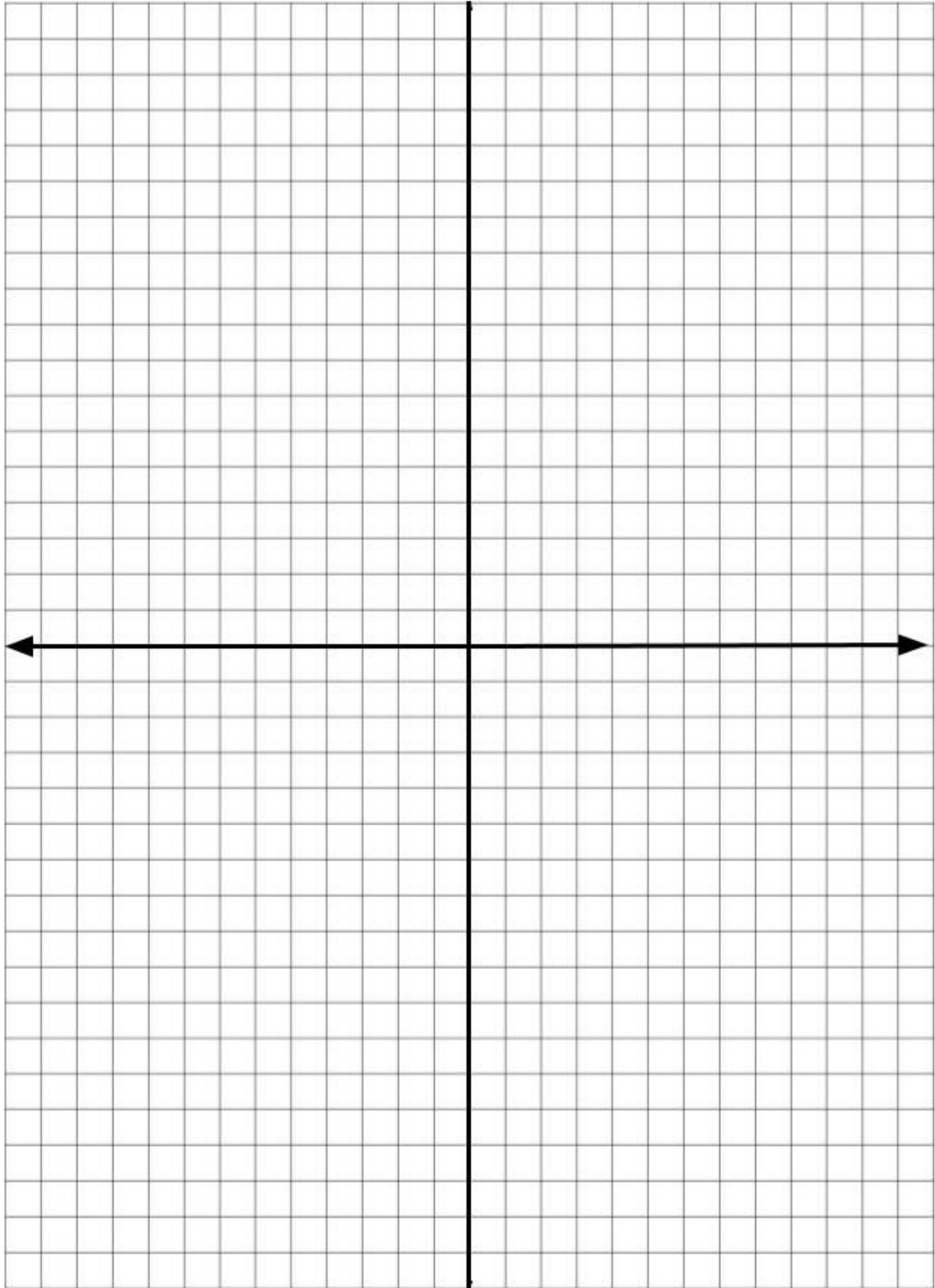
**I think the final figures will be the same.**

**I think the final figures will be different.**

Now use the coordinate plane on the next page to try out both sequences!

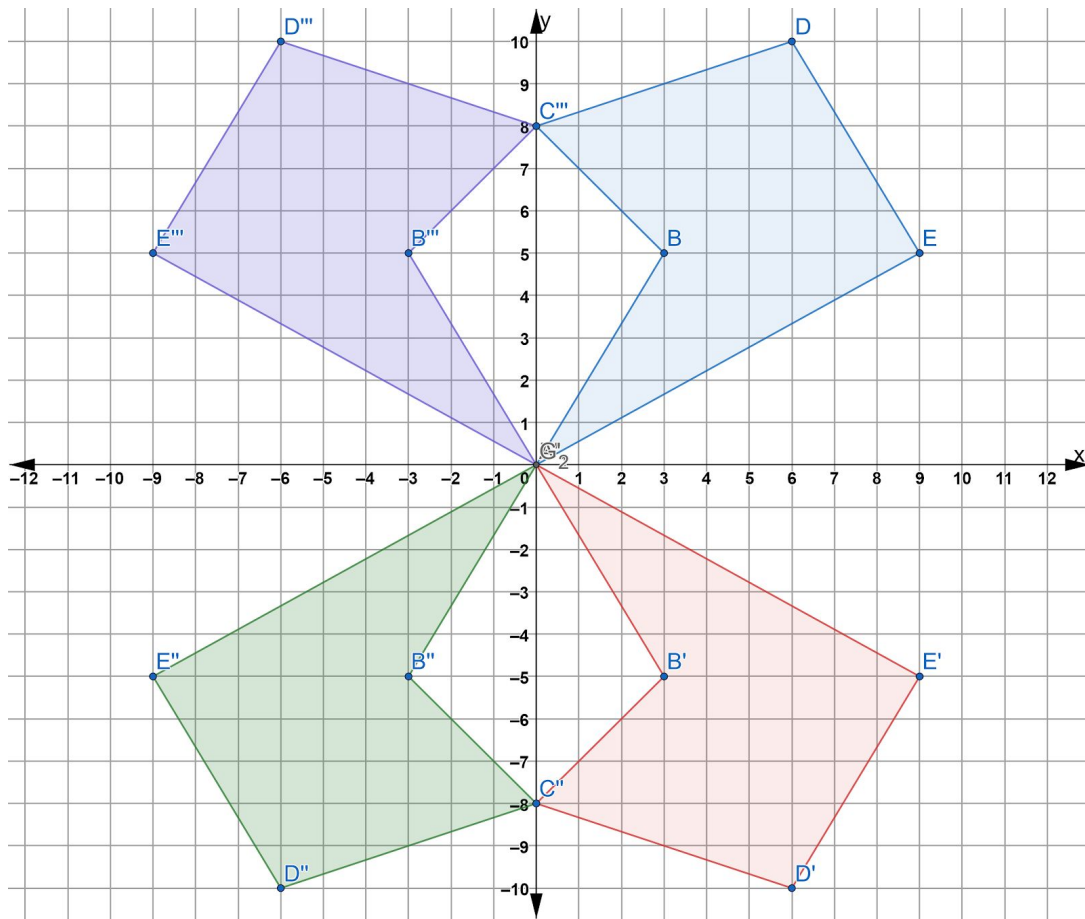


*Rigid Transformations*



## Reflections on the Coordinate Plane - Answer Key

- 1)
- Point A and Point A' are both 0 units away from the line of reflection.
  - Point B and Point B' are both 5 units away from the line of reflection.
  - Point C and Point C' are both 8 units away from the line of reflection.
  - Point D and Point D' are both 10 units away from the line of reflection.
  - Point E and Point E' are both 5 units away from the line of reflection.
- 2) Point B'' would be  $(-3, -5)$ . B' is at  $(3, -5)$ . From B', we have to move 3 units to get to the line of reflection. Since B'' has to be the same distance away from the line of reflection, we need to move another 3 units. That brings us to the point  $(-3, -5)$ .
- 3) The ordered pair for point E'' is  $(9, -5)$ .
- 4) The ordered pair for point D''' is  $(-6, 10)$ . This is what your coordinate grid should look like after you complete the reflections in questions 3 and 4.

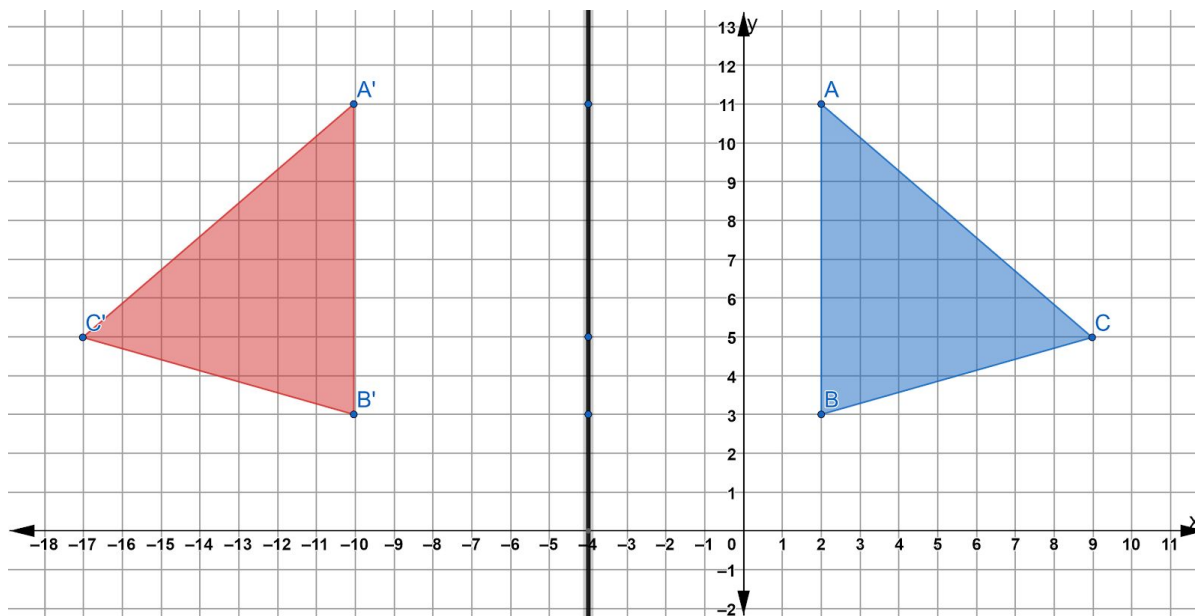


## Rigid Transformations

- 5) If we count the distance between point A and point A', we find that they are 12 units apart. The line of reflection needs to be halfway between that, which means the line of reflection needs to be 6 units from either point. The same is true of points B and B'. Points C and C' are 26 units away which means the line of reflection has to be 13 units away from each point.

Original Figure	Halfway Point	Image
A	(-4, 11)	A'
B	(-4, 3)	B'
C	(-4, 5)	C'

We can see that the three halfway points form a line. We can also see that they all have an x coordinate of -4. So the line of reflection is the line  $x = -4$



- 6) When we are reflecting a figure across a line of reflection, the line of reflection is like the mirror. Corresponding points are the same distance away from the line of reflection.
- 7) The original triangle here is in Quadrant 1. When we reflect it over the y-axis it flips into Quadrant 2. (7,2) is 7 units to the right of the y-axis. If we reflect it across the y-axis, it will need to be 7 units to the left of the y-axis.

## Rigid Transformations

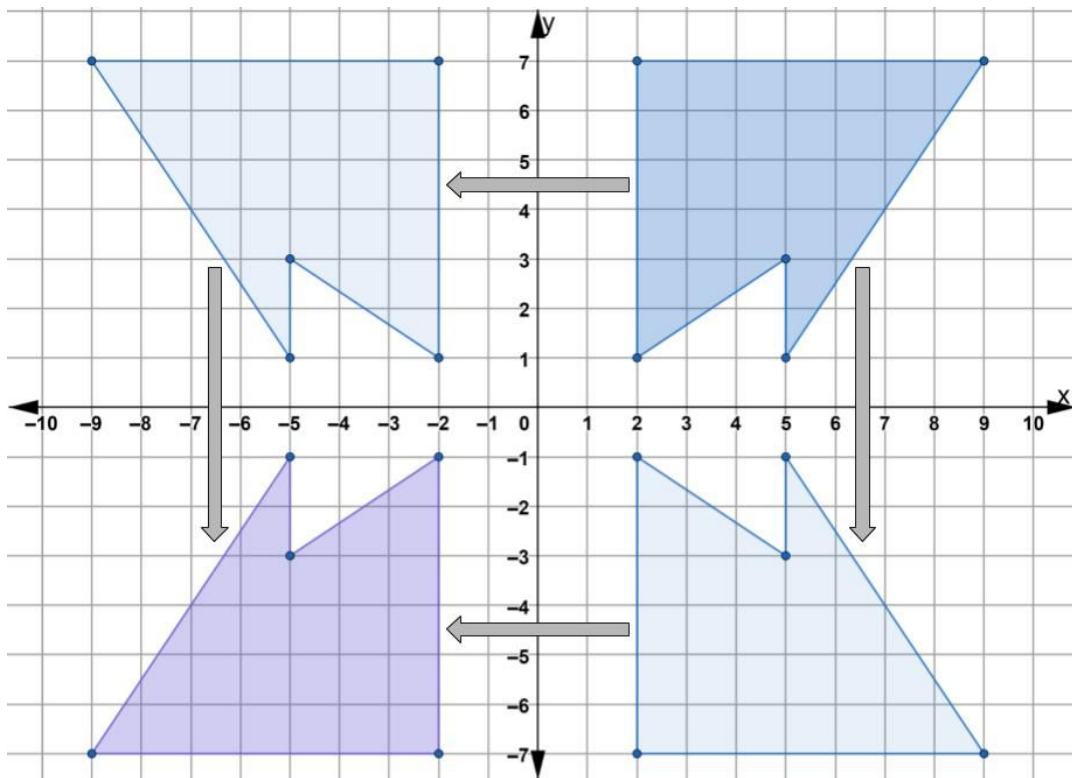
The same will be true for (8,6). Since (8,6) is 8 units to the right of the y-axis, then its image needs to be 8 units to the left of the y-axis. So (8,6) reflected over the y-axis becomes (-8,6).

(4,3) is 4 units to the right of the y-axis, so its image reflected across the y-axis would be 4 units to the left of the x-axis. (4,3) reflected over the y-axis becomes (-4,3).

8)

<b>Original Figure</b>	<b>(7,6)</b>	<b>(10,6)</b>	<b>(10,1)</b>	<b>(7,1)</b>
<b>Reflected over x-axis</b>	<b>(7,-6)</b>	<b>(10,-6)</b>	<b>(10,-1)</b>	<b>(7,-1)</b>
<b>Reflected over y-axis</b>	<b>(-7,6)</b>	<b>(-10,6)</b>	<b>(-10,1)</b>	<b>(-7,1)</b>
<b>Reflected over <math>y=x</math></b>	<b>(6,7)</b>	<b>(6,10)</b>	<b>(1,10)</b>	<b>(1,7)</b>

9) If you reflect ABCDE over the x-axis and then reflect the resulting figure over the y-axis you will end up with an image in the same place as you would if your reflected ABCE over the y-axis first and then over the x-axis.

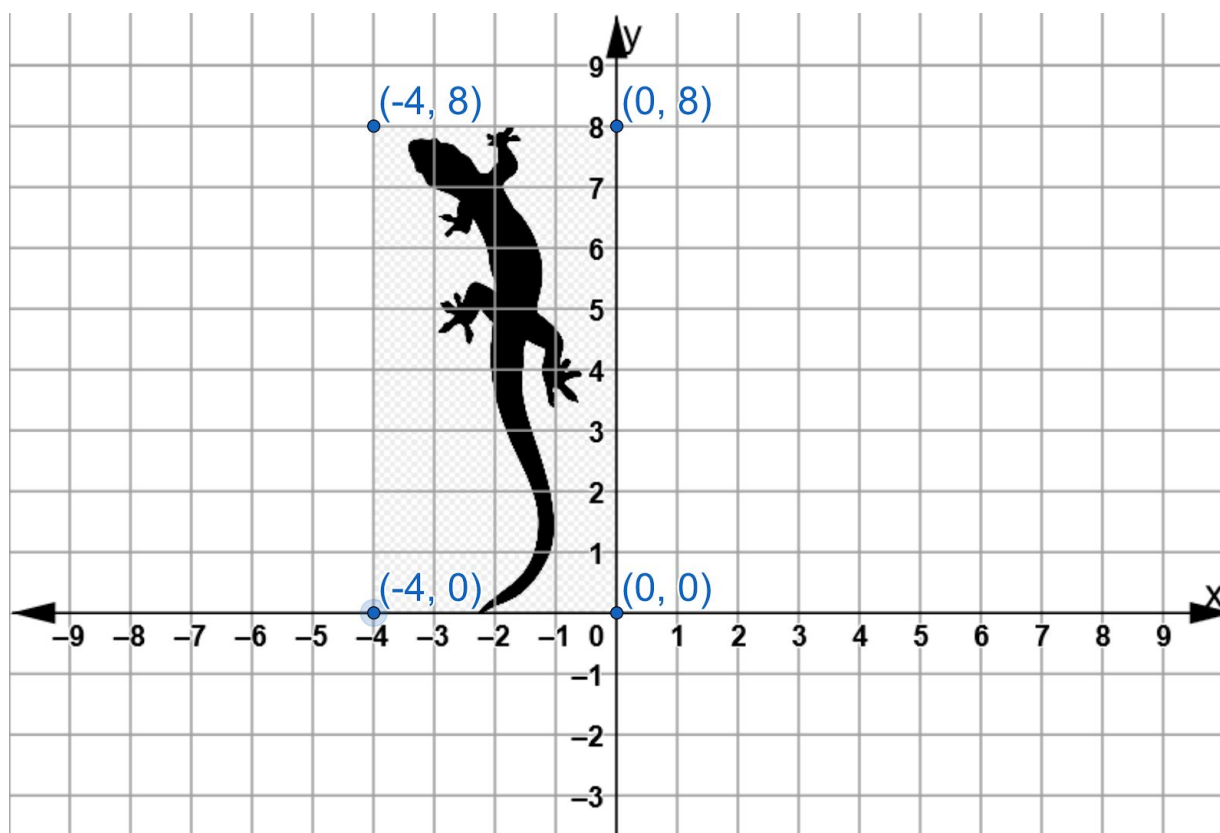


## Rotations on the Coordinate Plane

Rotations are the third type of rigid transformation. In a rotation, all of the points in the original figure rotate, or turn the same number of degrees around a fixed point. As with all rigid transformations, the rotated image is congruent to the original figure.

In this section, we will explore rotations on the coordinate plane, using the origin as the fixed point of rotation.

We have placed an image of a lizard in Quadrant 2 of this coordinate plane. We used a rectangle to make an outline around the lizard.

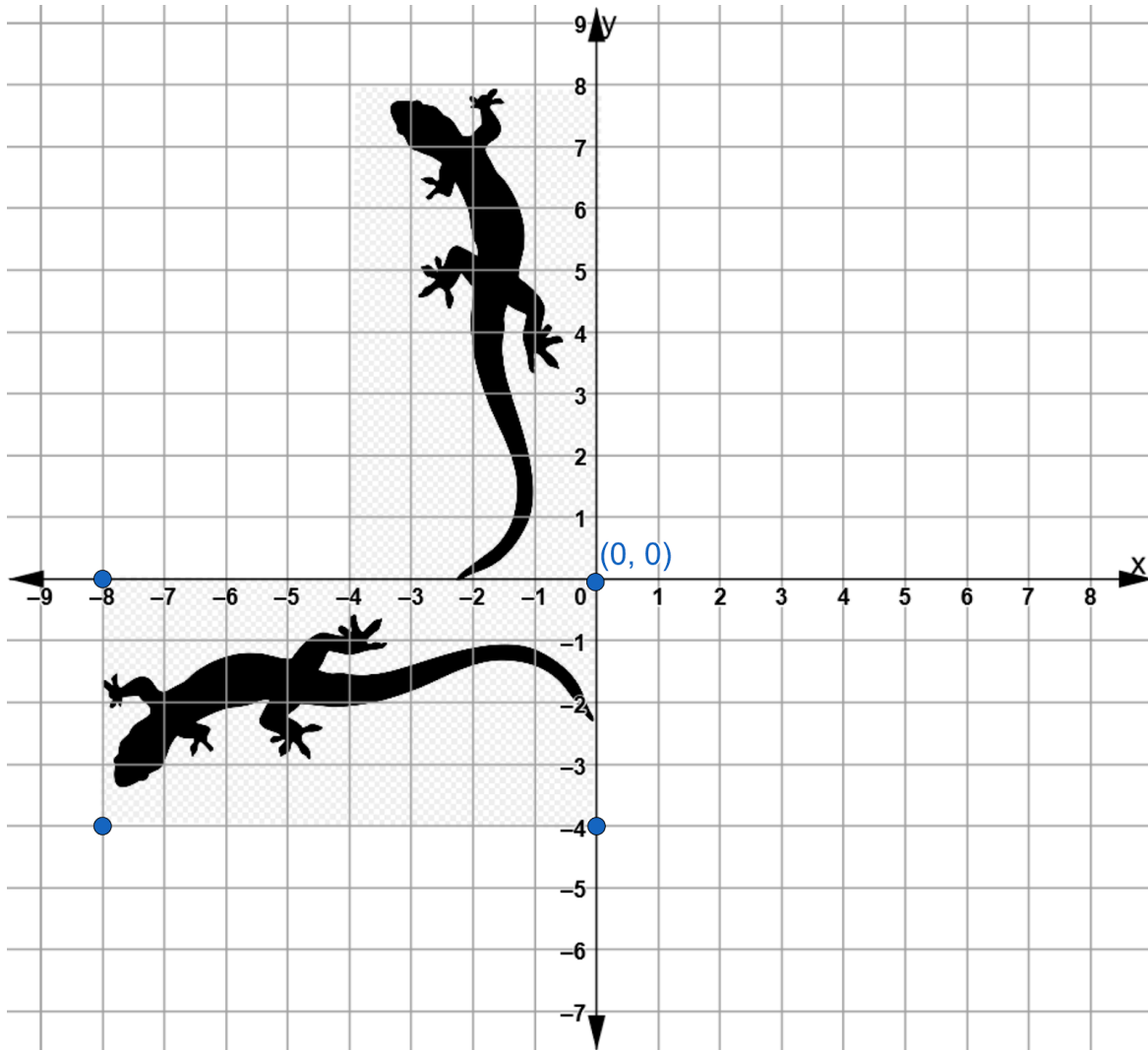
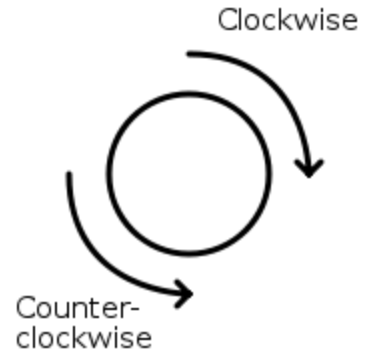


On the next few pages we will rotate this image counterclockwise around the origin.

## Rigid Transformations

Remember, a clockwise turn is a turn that moves in the same direction as the hands of a clock. A counterclockwise turn is a turn in the opposite direction.

The coordinate grid below shows the original lizard in Quadrant 2 and a congruent image in Quadrant 3. This new image was created by transforming the original lizard through a rotation of  $90^\circ$  counterclockwise around the origin.

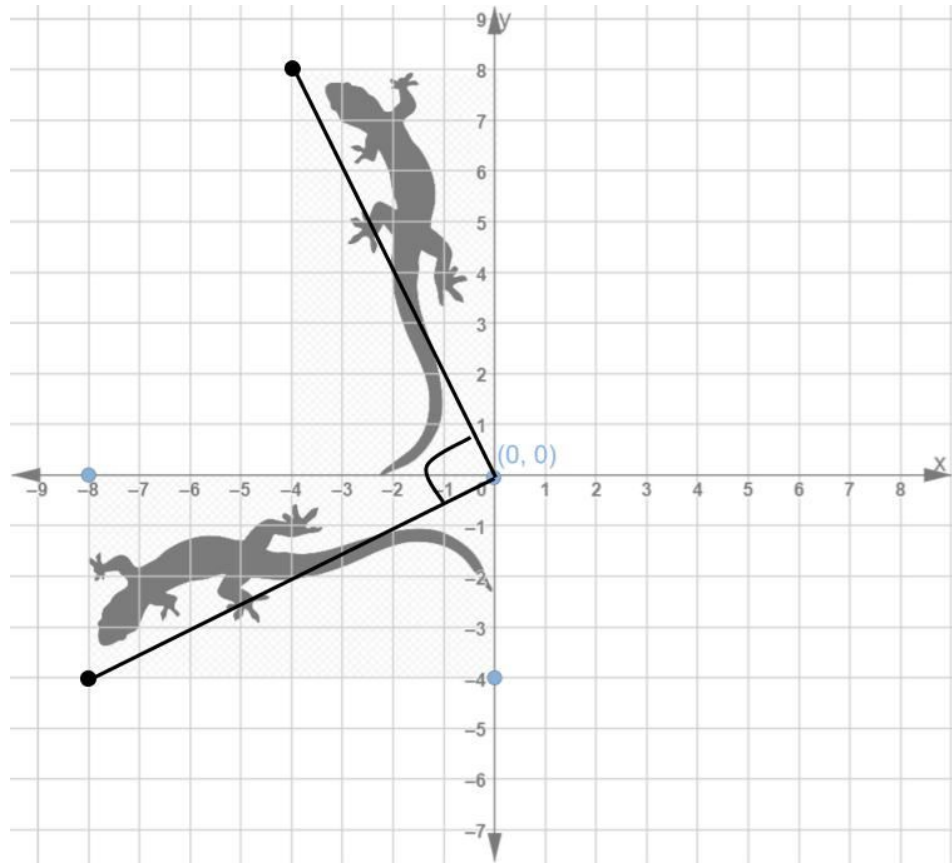


Can you see the  $90^\circ$  angle? It helps if you focus on two corresponding points -- one from the original lizard and one from the rotated image. Then draw a line from each point to the point of rotation, which in this case was the origin.

## Rigid Transformations

Because a rotation means we turn every point in the original figure the same number of degrees, this will work with any pair of corresponding points.

Let's focus on the corner of the rectangle that is closest to the lizard's mouth. When we draw a straight line from each corresponding point to the point of rotation, we can see the  $90^\circ$  angle.



Now you try.

- 1) Draw a straight line from any of the other corresponding points to the origin. The two lines should form a  $90^\circ$  angle.

If we return to our original lizard and rotate it  $180^\circ$  counterclockwise around the origin, where do you think the rotated image will appear?

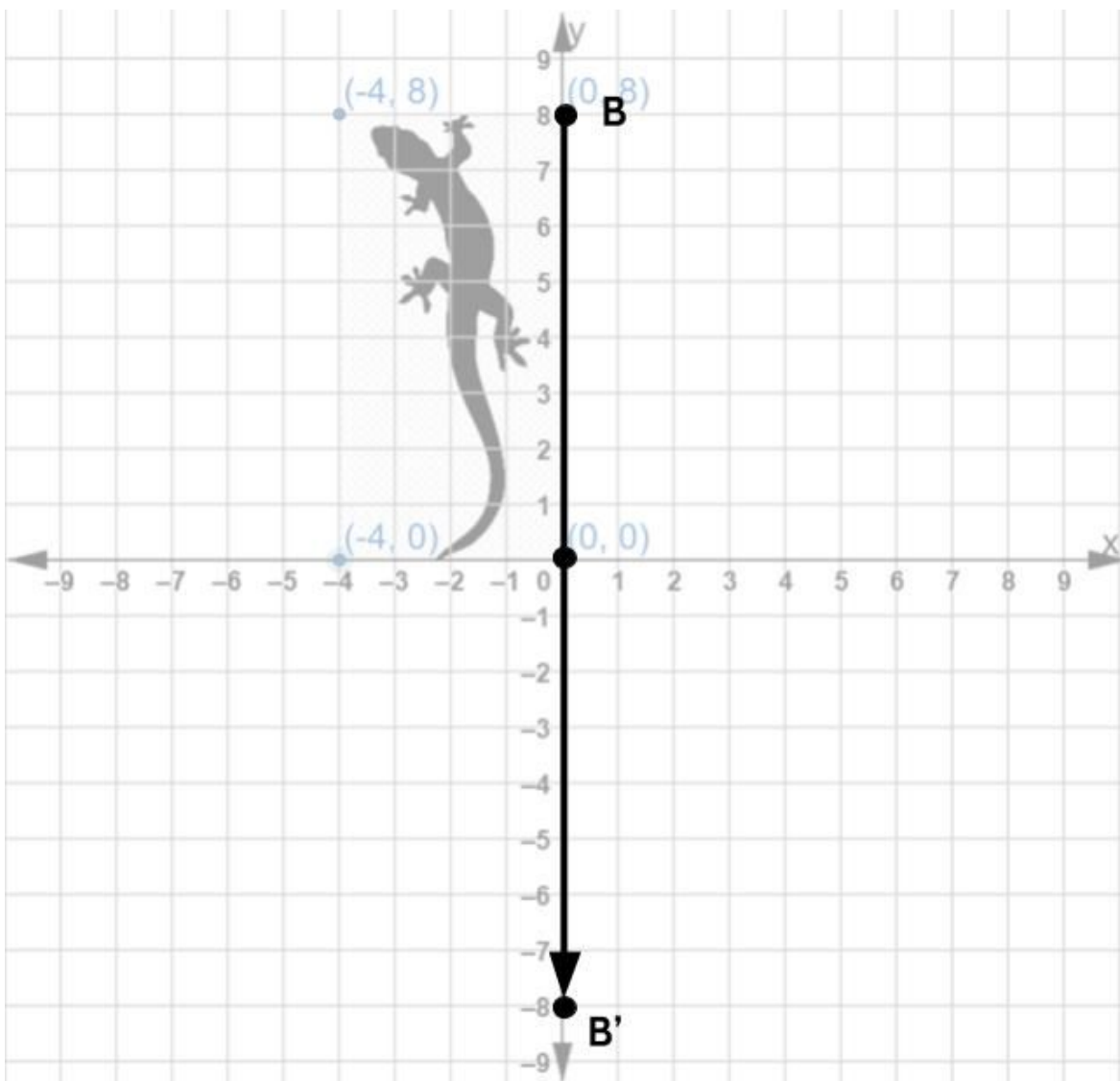
We can imagine it the  $180^\circ$  turn by making another  $90^\circ$  turn. Two  $90^\circ$  turns is the same as one  $180^\circ$  turn.

## Rigid Transformations

Here's another way to figure out each corresponding point on the image rotated  $180^\circ$ . Draw a straight line from a point on the rectangle to the origin. Then continue the line straight from the origin until your second line is the same length as your first line.

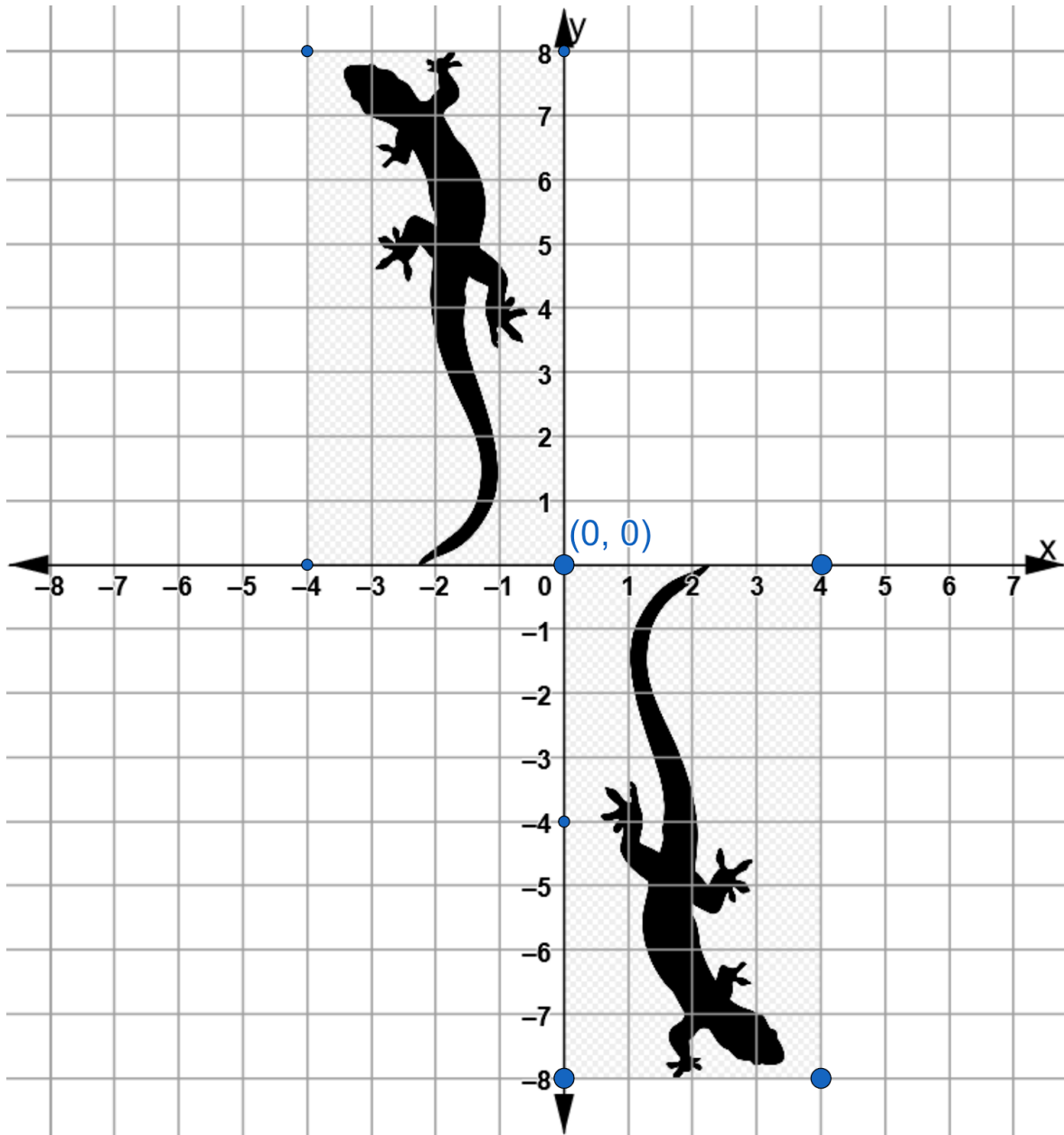
For example, start at Point B and draw a straight line to the point of rotation. Then continue to draw until you have doubled your first line. Since Point B is 8 units away from the origin, point B' must be another 8 units away, in a straight line.

- 2) Map the remaining two points  $(-4, 8)$  and  $(-4, 0)$  to their corresponding points on the image we would get if we rotated the original lizard  $180^\circ$  around the origin.





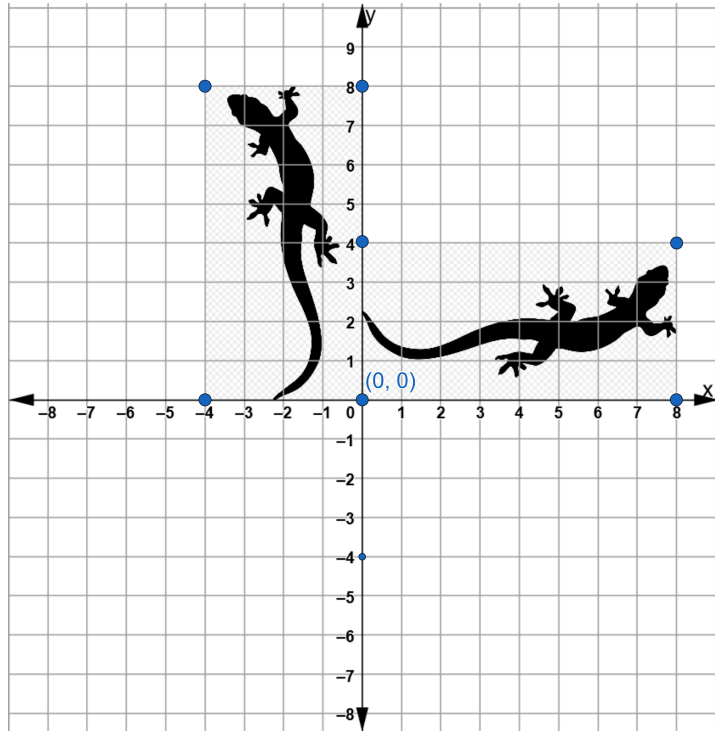
Rigid Transformations



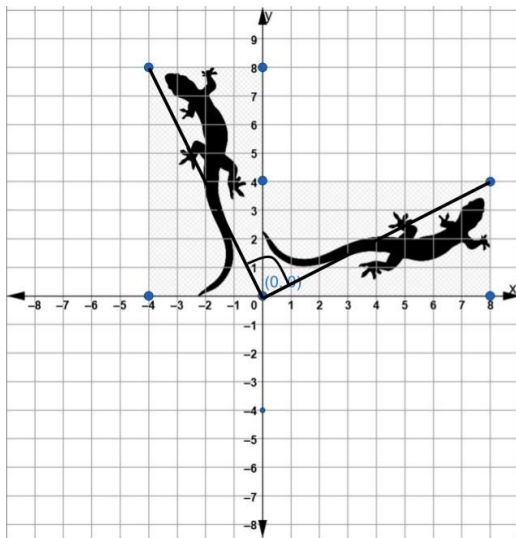
Write 3 things you notice when you compare the original lizard with its image rotated 180 degrees counterclockwise around the origin.

## Rigid Transformations

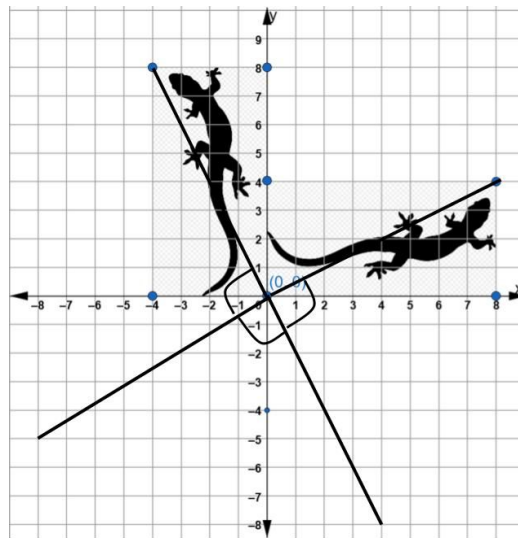
How would you describe the rotation of the original lizard around the origin on the coordinate grid to the right?



There are actually two ways to describe this rotation. .



A rotation of  $90^\circ$  *clockwise* around the origin.

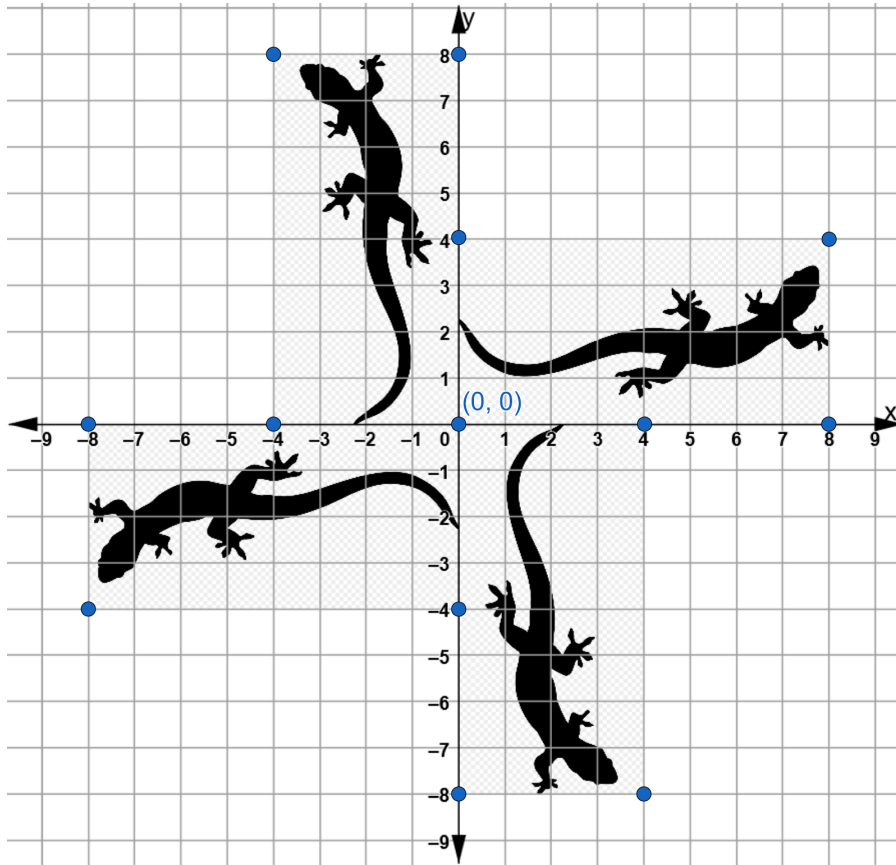


A rotation of  $270^\circ$  (or three  $90^\circ$  turns) *counterclockwise* around the origin.

Direction is important when describing rotation.

## Rigid Transformations

Here is the original lizard and all three counterclockwise rotations around the origin on one coordinate grid.

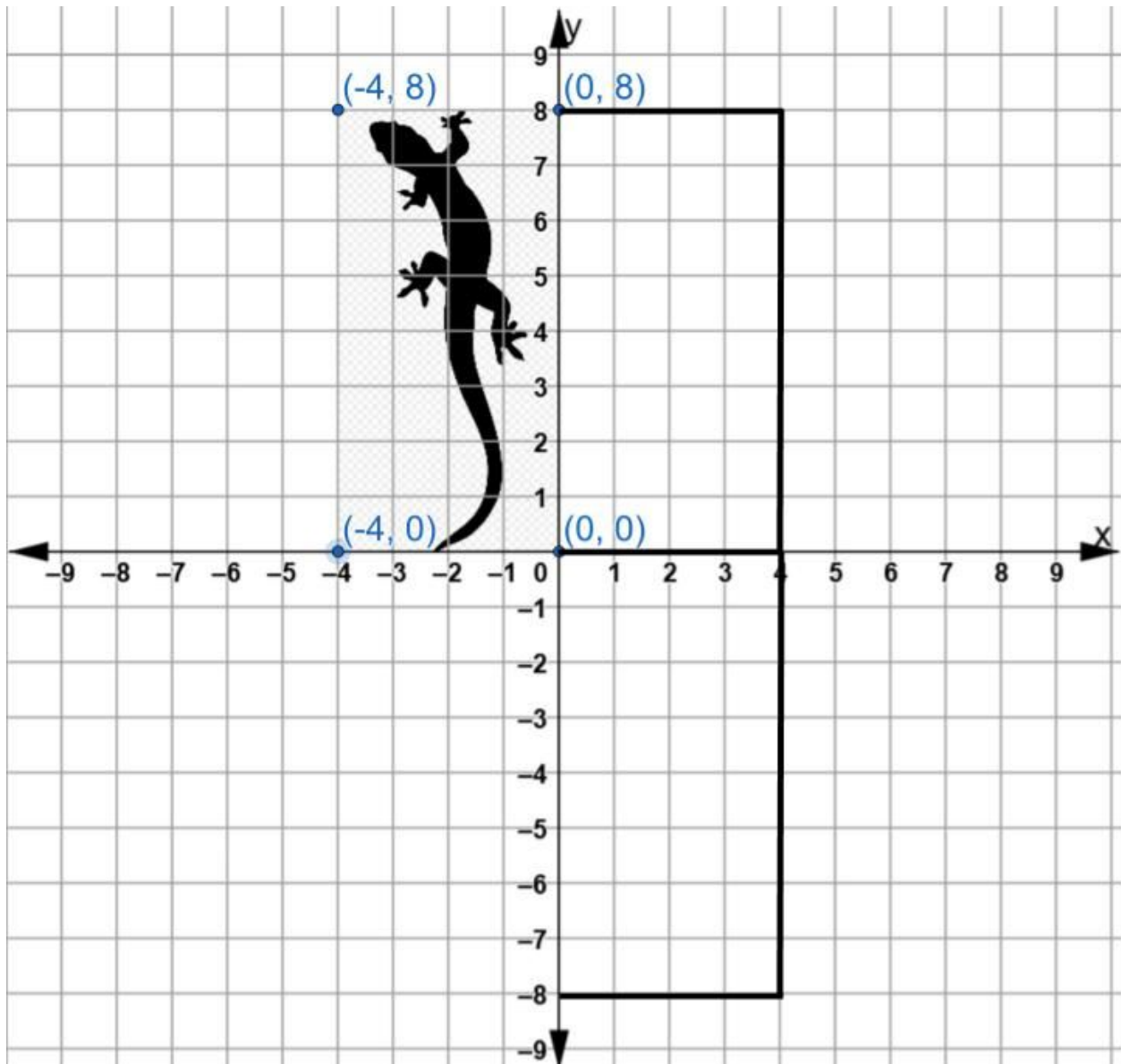


- 3) The coordinates for the rectangle around the lizard are recorded in the table below. Fill in the coordinates for the corresponding coordinate points for each of the 3 rotations.

Coordinate Points in Original Figure	Coordinate Points for Image rotated $90^\circ$ counterclockwise	Coordinate Points for Image rotated $180^\circ$ counterclockwise	Coordinate Points for Image rotated $270^\circ$ counterclockwise
(0,0)			
(0,8)			
(-4,8)			
(-4,0)			

## Rigid Transformations

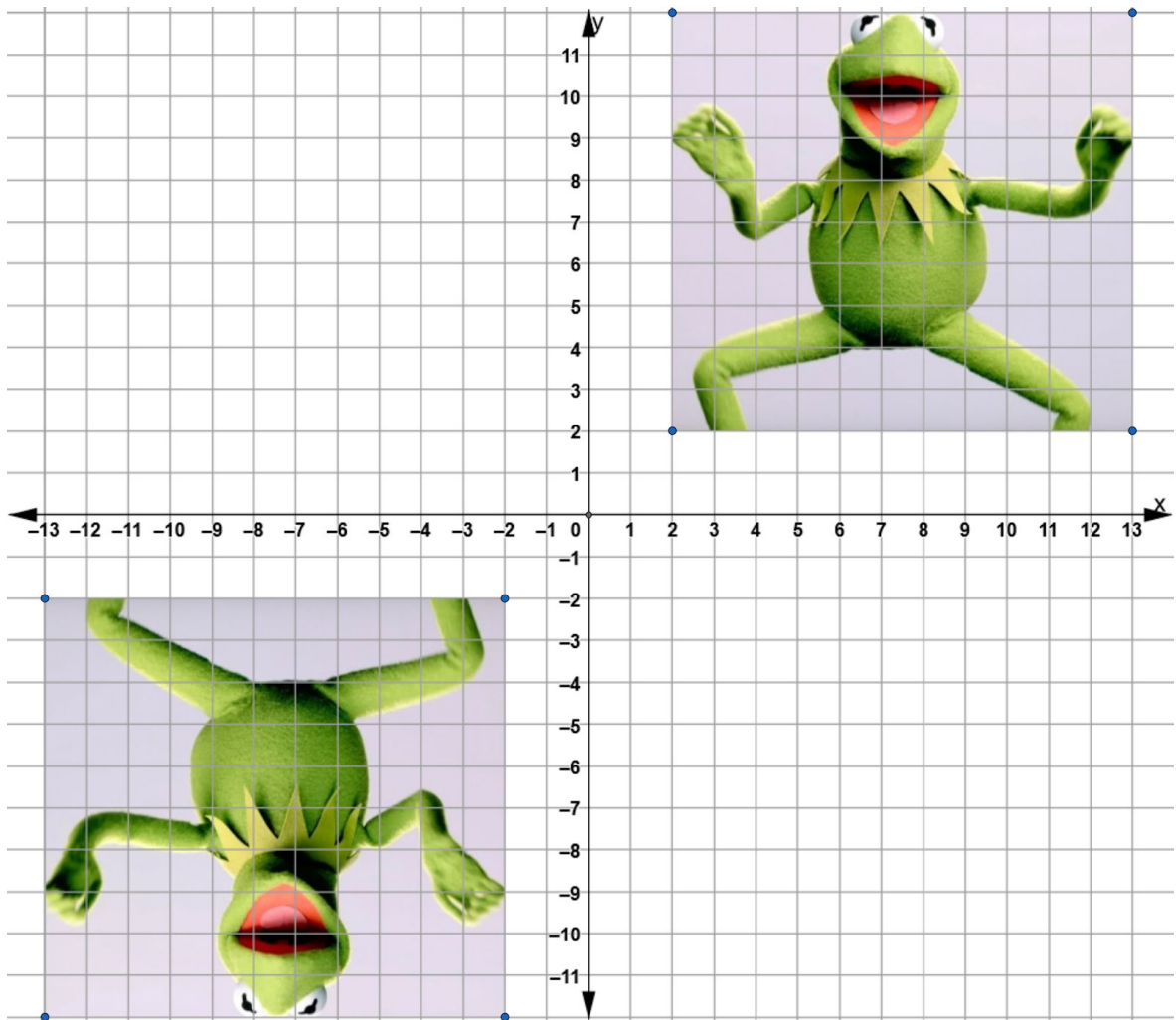
Let's do one final activity with our lizard. Draw an image of the lizard reflected over the y-axis. Then reflect that image about the x-axis.



What do you notice?

*Rigid Transformations*

- 4) The image of Kermit the Frog in Quadrant 1 of this coordinate grid maps onto the image in Quadrant 3.



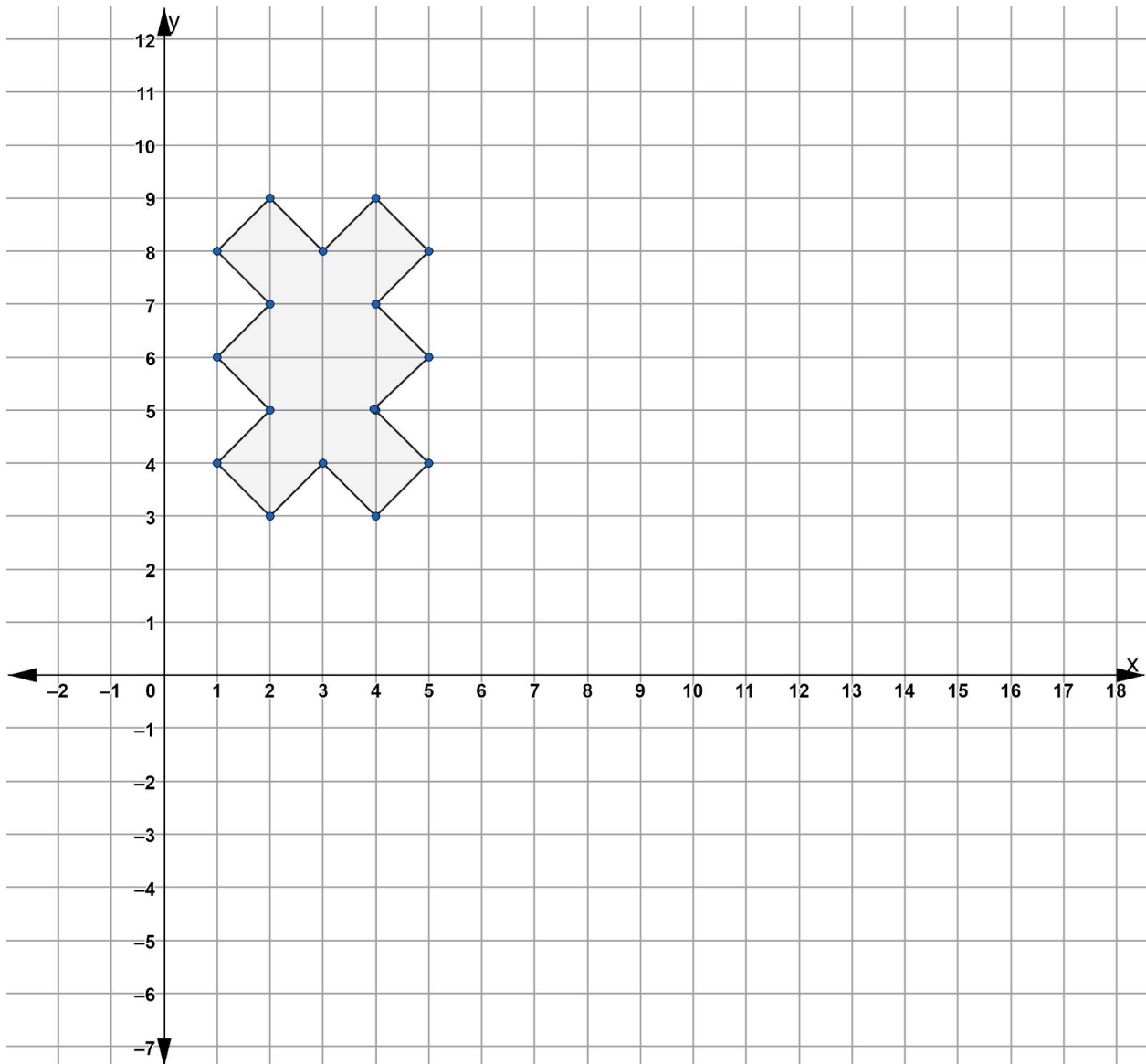
Which of the following statements are possible transformations that map the original Kermit onto the image? (*Note: There may be more than one possible answer*)

- A) A rotation of  $90^\circ$  clockwise about the origin.
- B) A rotation of  $180^\circ$  counterclockwise about the origin.
- C) Two rotations of  $90^\circ$  clockwise about the origin.
- D) A reflection over the y-axis followed by a reflection over the x-axis

## Rigid Transformations

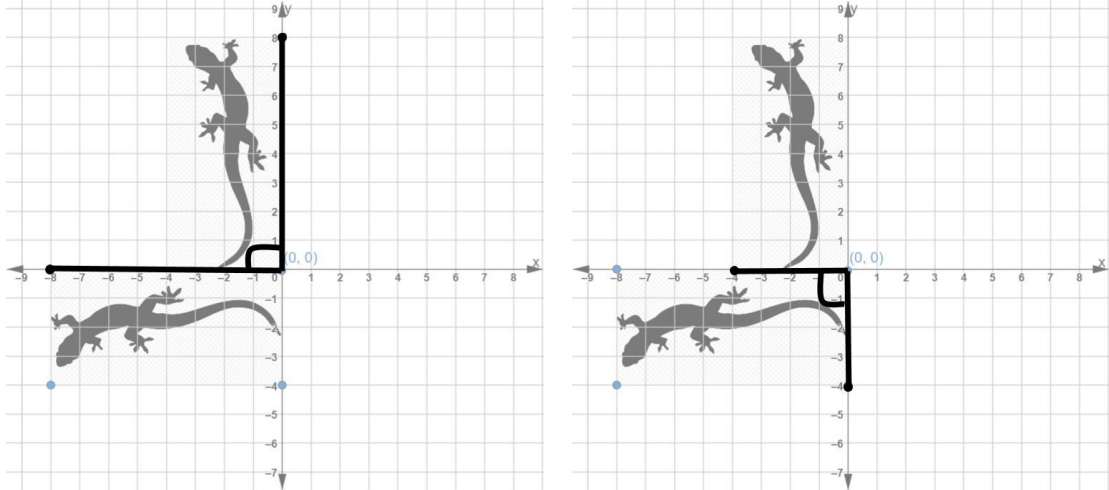
- 5) Remember a tessellation is an image that includes repeated congruent figures that cover a surface without gaps or overlapping. We can create tessellations by rotating figures on the coordinate grid.

Consider the figure below. Create three congruent figures by rotating the figure  $90^\circ$ ,  $180^\circ$  and then  $270^\circ$  counterclockwise. Use the point  $(5,4)$  as the point of rotation. Shade or color in each figure as you draw them.

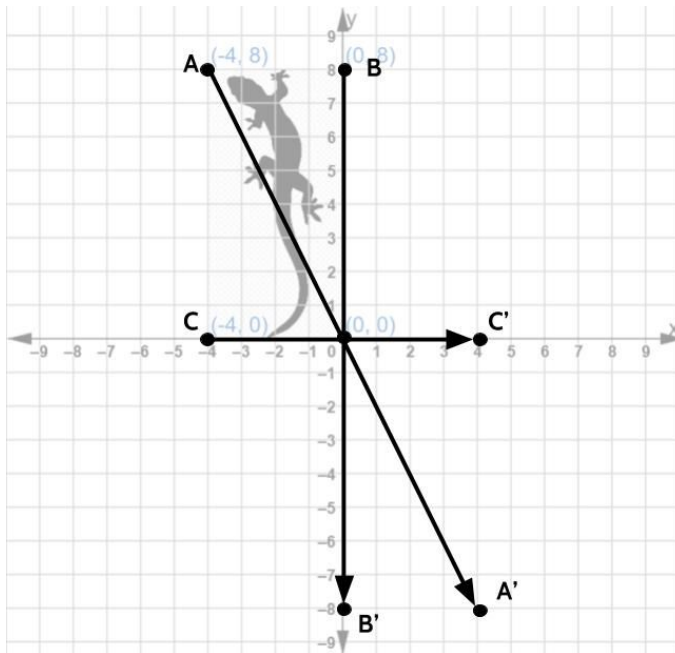


## Rotations on the Coordinate Grid - Answer Key

1)



2)

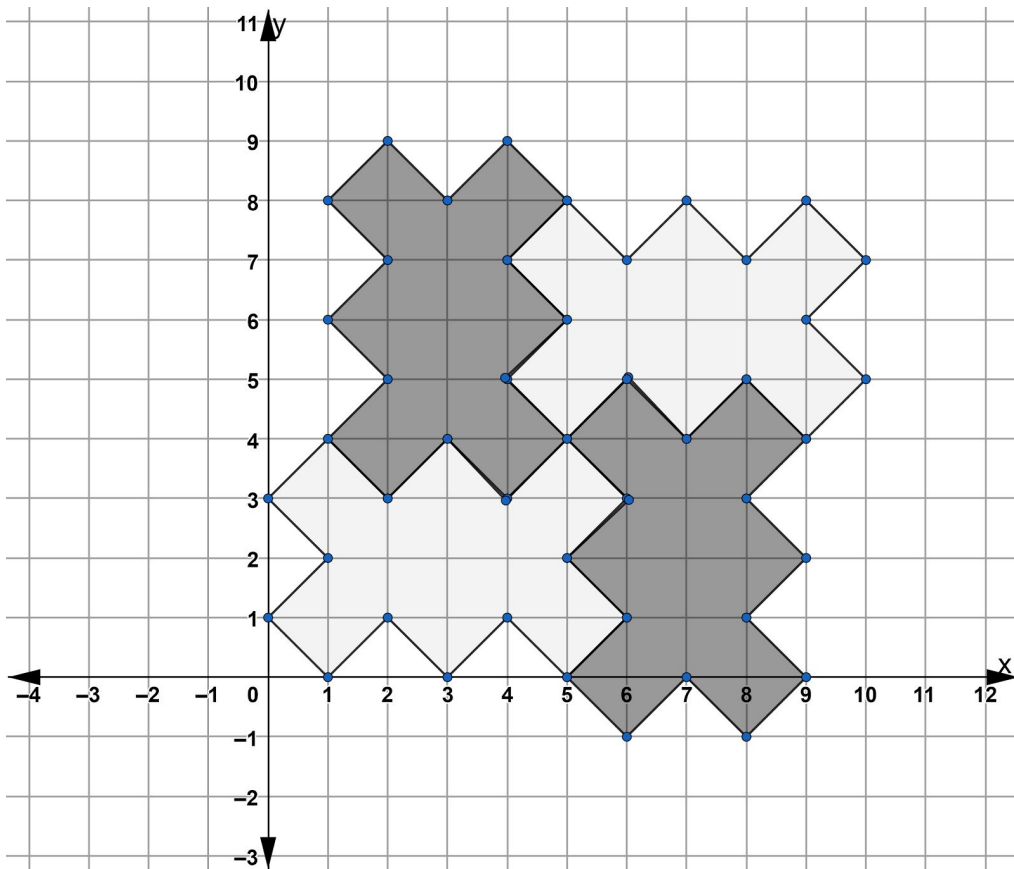


3)

Coordinate Points in Original Figure	Coordinate Points for Image rotated 90° counterclockwise	Coordinate Points for Image rotated 180° counterclockwise	Coordinate Points for Image rotated 270° counterclockwise
(0,0)	(0,0)	(0, 0)	(0, 0)
(0,8)	(-8, 0)	(0, -8)	(8,0)
(-4,8)	(-8, -4)	(4, -8)	(8, 4)
(-4,0)	(0, -4)	(4,0)	(0,4)

4) Choices B, C and D are all transformations that map the original Kermit to the image.

5) When you turn the original figure with rotations of 90, 180, and 270 counterclockwise about point (2,3) you will end up with a tessellation.

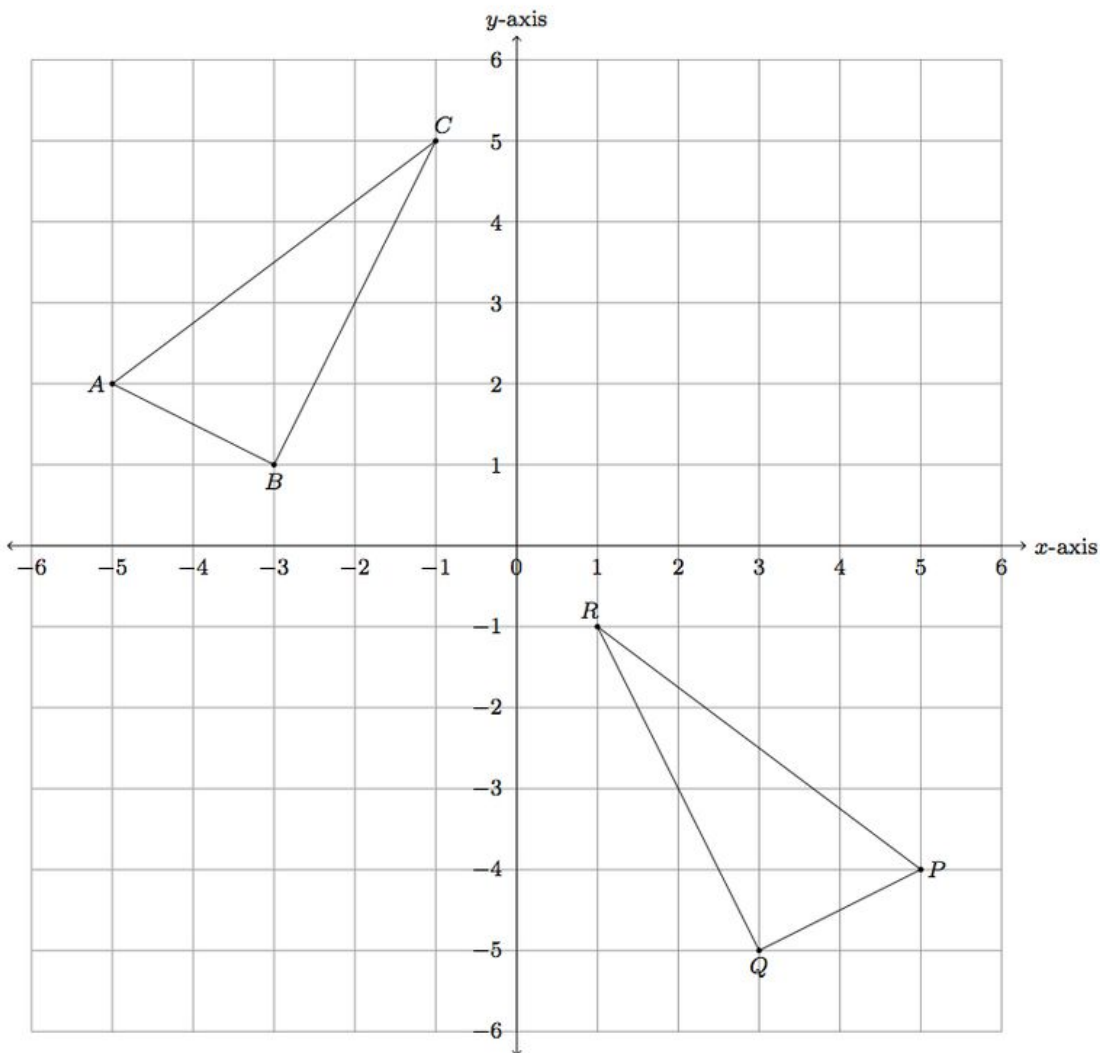




## Sequences of Transformations

So far, we have looked at what happens to when we reflect figures across a line of reflection, when we rotate figures around a point of rotation, and when we translate figures. In this section, we are going to use what you have learned to see what happens when we use combinations of rigid transformations.

$\triangle ABC$  and  $\triangle PQR$  are congruent.



There is no single translation that would map  $ABC$  to  $PQR$ . There is no single reflection or single rotation that would do it either. Triangle  $ABC$  can be mapped to Triangle  $PQR$  using a sequence of *two* rigid transformations.

## Rigid Transformations

One place to start when trying to figure out a sequence of transformations is look at which points in the original figure map to which points in the image.

By looking at the triangle you might be able to tell that point C maps to point R.

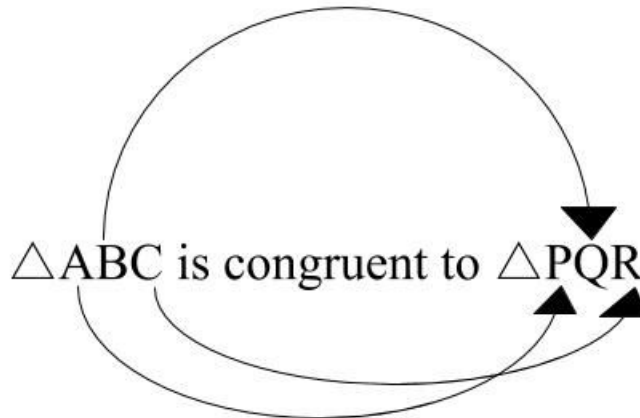
But which point in Triangle PQR corresponds to point A?

And which point in Triangle PQR corresponds to point B?

You might be able to see that point B maps to point Q and point A maps to point P also by looking at the figures, but there is another way.

$\triangle ABC$  and  $\triangle PQR$  are described as congruent. As you have already learned, that means that the two figures are the same size and shape. It means that you can map one to fit exactly over the other using rigid transformations. But there is also a clue in the way the names of the triangles (or any other shape) are written.

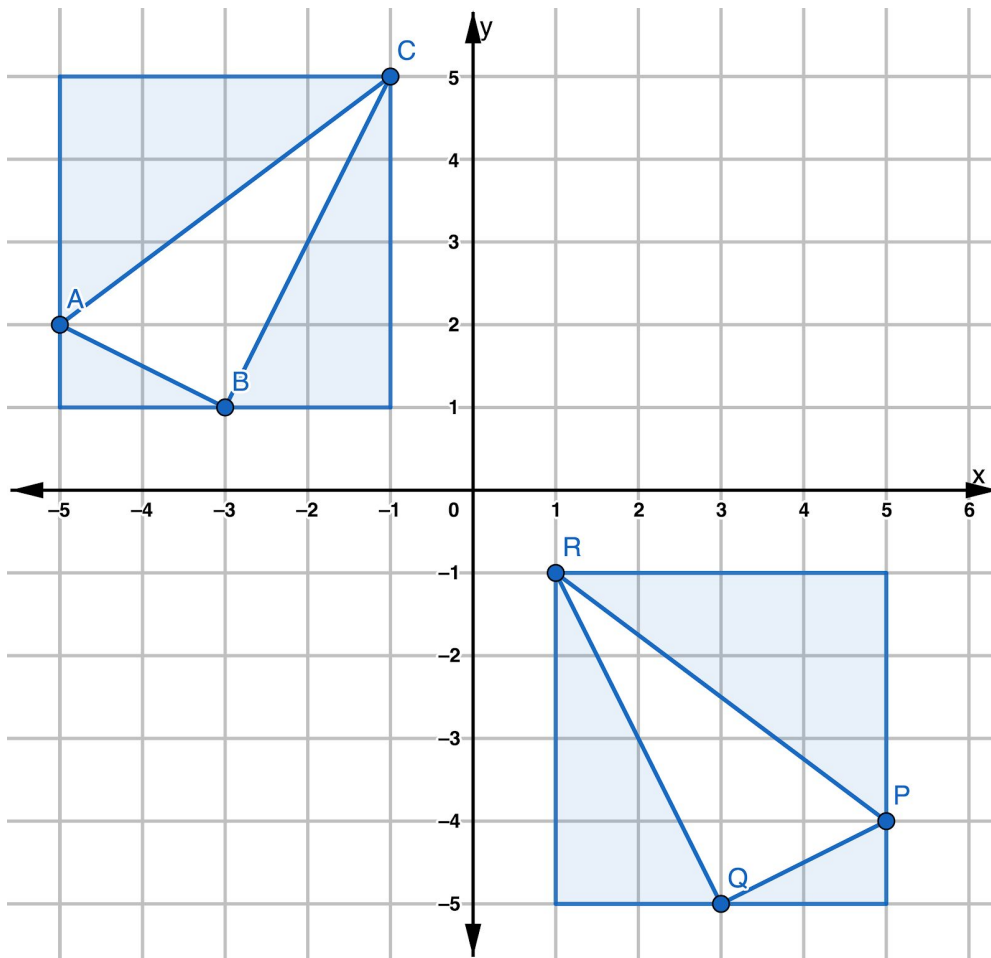
$\triangle ABC$  and  $\triangle PQR$  means that point A corresponds to point P, point B corresponds to point Q and point C corresponds to point R. When figures are described as congruent, the order of the points in the name of one figure must match the order of the corresponding points in the other figure.



Once you know which points in the original figure map to the points in the image, try different sequences to get from each point to its corresponding point.

## Rigid Transformations

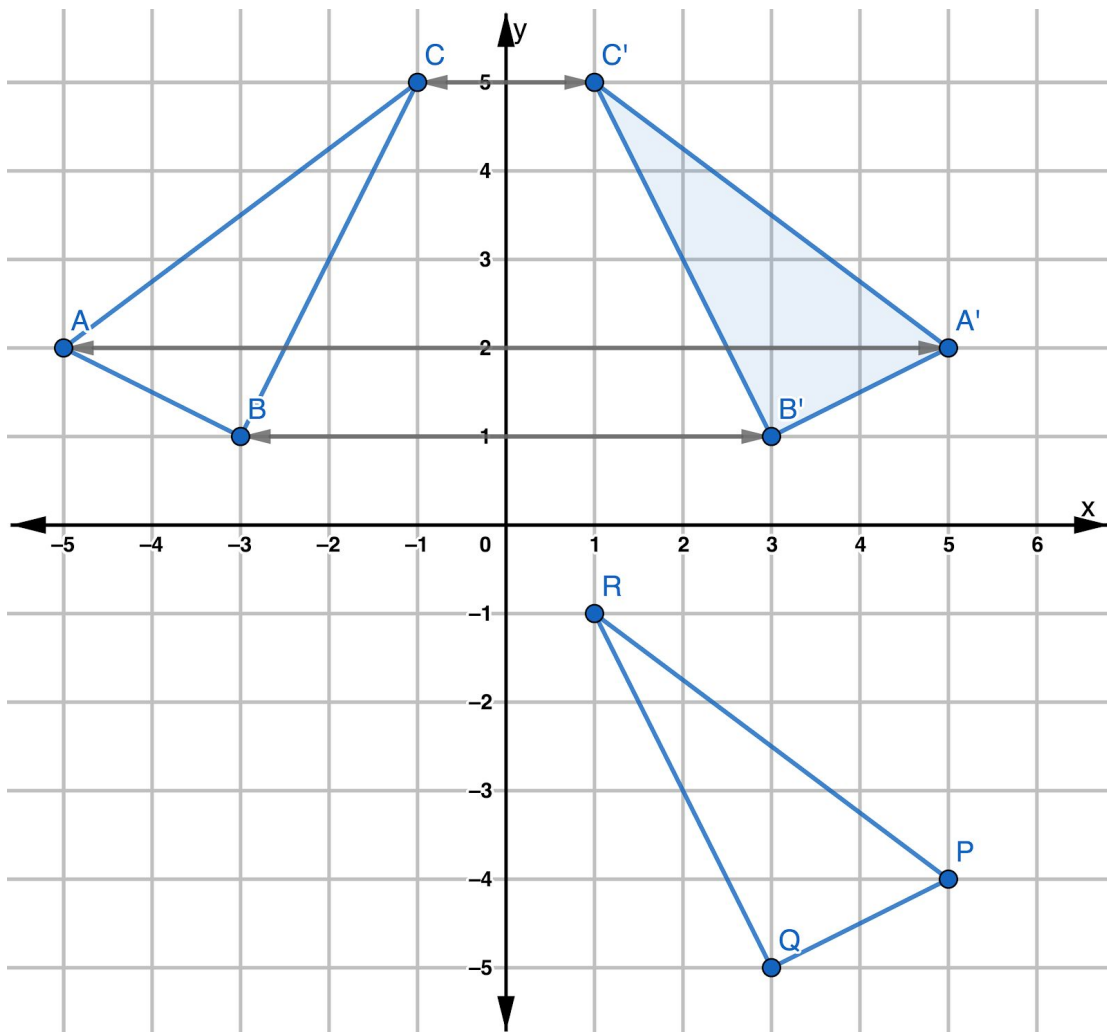
We shaded around each triangle to help you think about the transformations.



What do you notice?

## Rigid Transformations

If we reflect  $\triangle ABC$  across the  $y$ -axis, we end up with  $\triangle A'B'C'$ .



If we slide Points A', B', and C' down 6 units, the triangle will exactly fit over  $\triangle PQR$ .

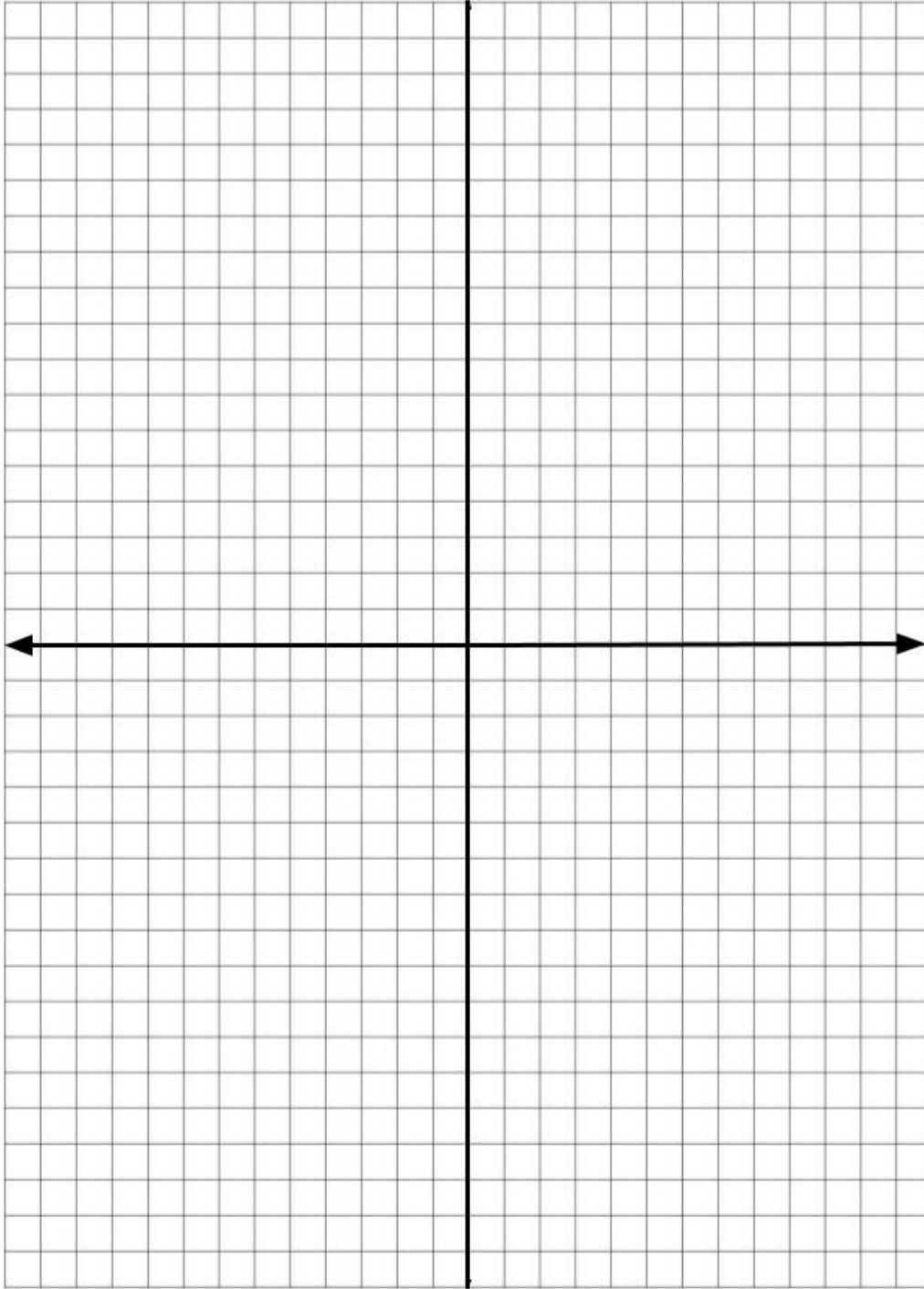
So  $\triangle ABC$  can be mapped to  $\triangle PQR$  using the following sequence of transformations:

- Step One: A reflection across the  $y$ -axis.
- Step Two: A translation of 6 units down.

You will have an opportunity to practice working with sequences of transformations in the Practice Questions section of this packet.

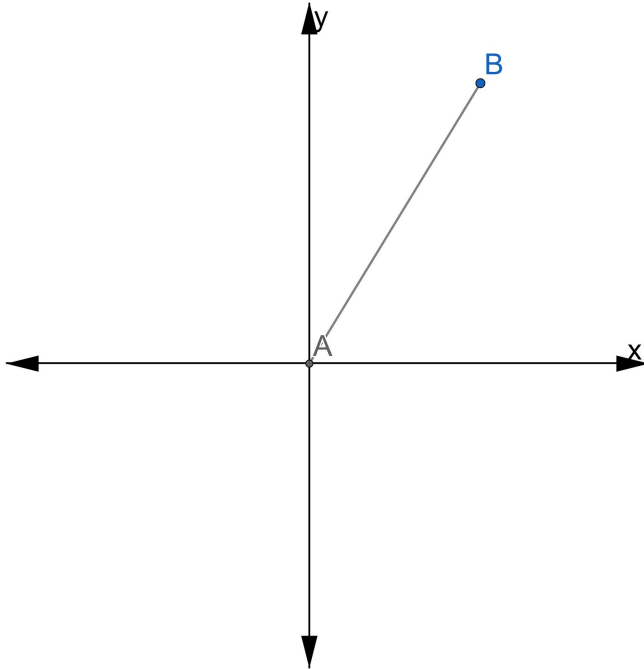
## Rigid Transformations

A good way to help practice a sequence of rigid transformations is to create your own. On the grid below, draw a shape that has between 5 to 8 sides with no curved edges. Label the shape Figure 1. Transform your shape through 3 rigid transformations of rotation, translation, and reflection — in any order. Label your 3 transformations Figure 2, Figure 3, and Figure 4.



## Practice Questions

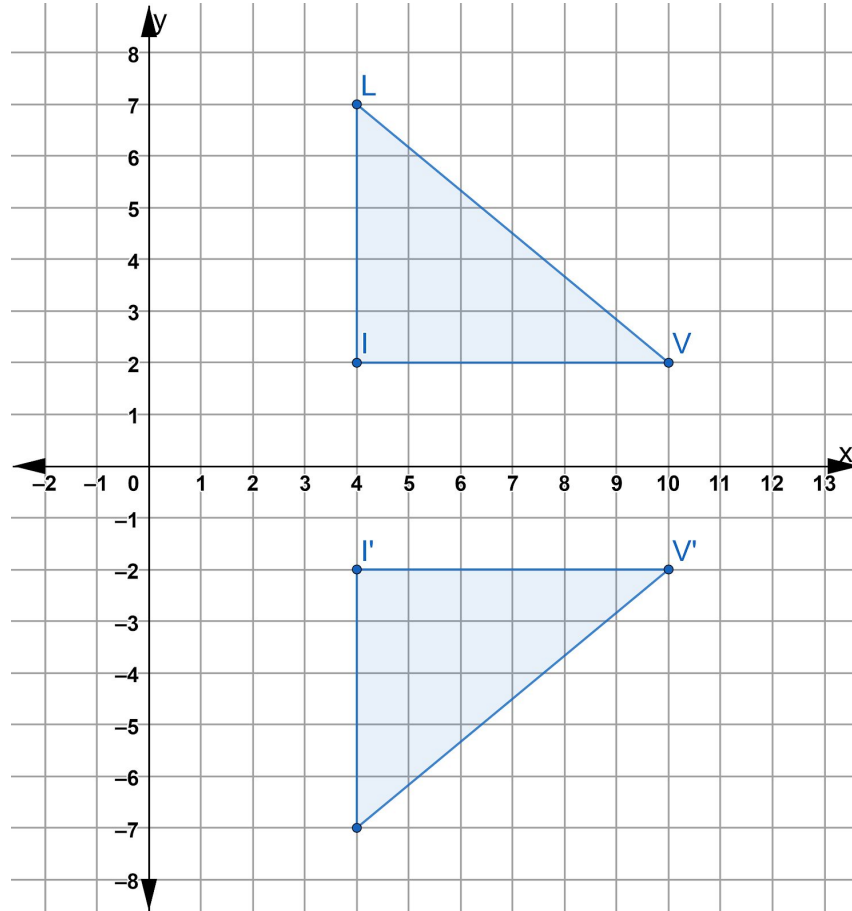
- 1) Which two transformations of line AB would result in an image parallel to line AB?



- A. translation of 2 units down
- B. reflection over the x-axis
- C. reflection over the y-axis
- D. clockwise rotation of  $90^\circ$  about the origin
- E. translation of 4 units to the right

Explain your answer:

2) Consider  $\triangle LIV$  and its image  $\triangle L'I'V'$  on the coordinate plane.



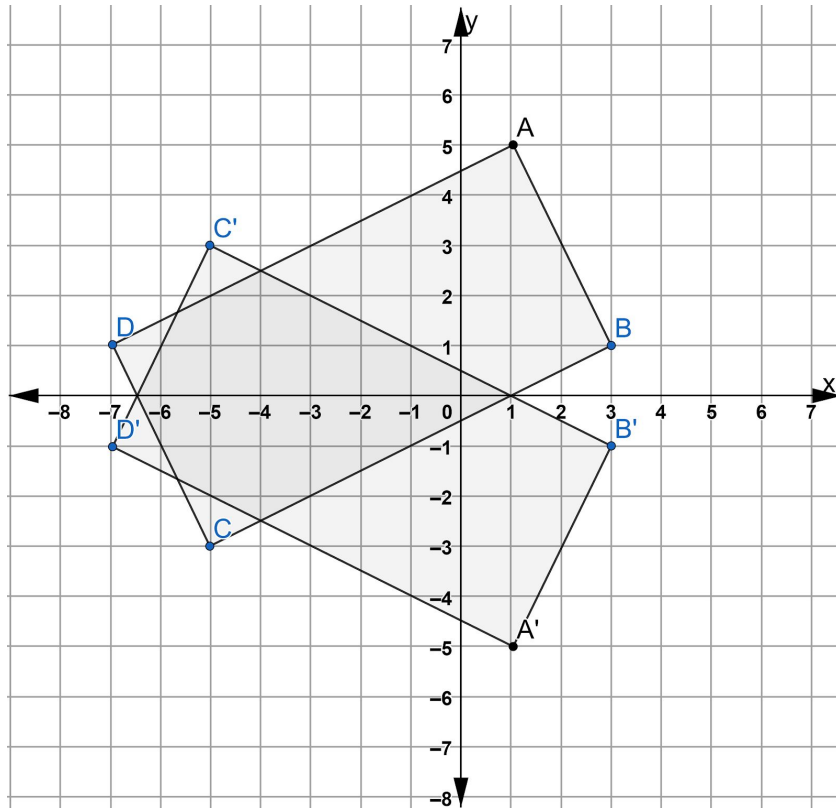
Which transformation maps  $\triangle LIV$  to  $\triangle L'I'V'$ ?

- A) translation 4 units down
- B) reflection about the x-axis
- C) translation 14 units down
- D) reflection about the line  $x=1$

Explain your answer:

Rigid Transformations

- 3) On the set of axes below, rectangle ABCD can be proven congruent to rectangle A'B'C'D' using which transformation?



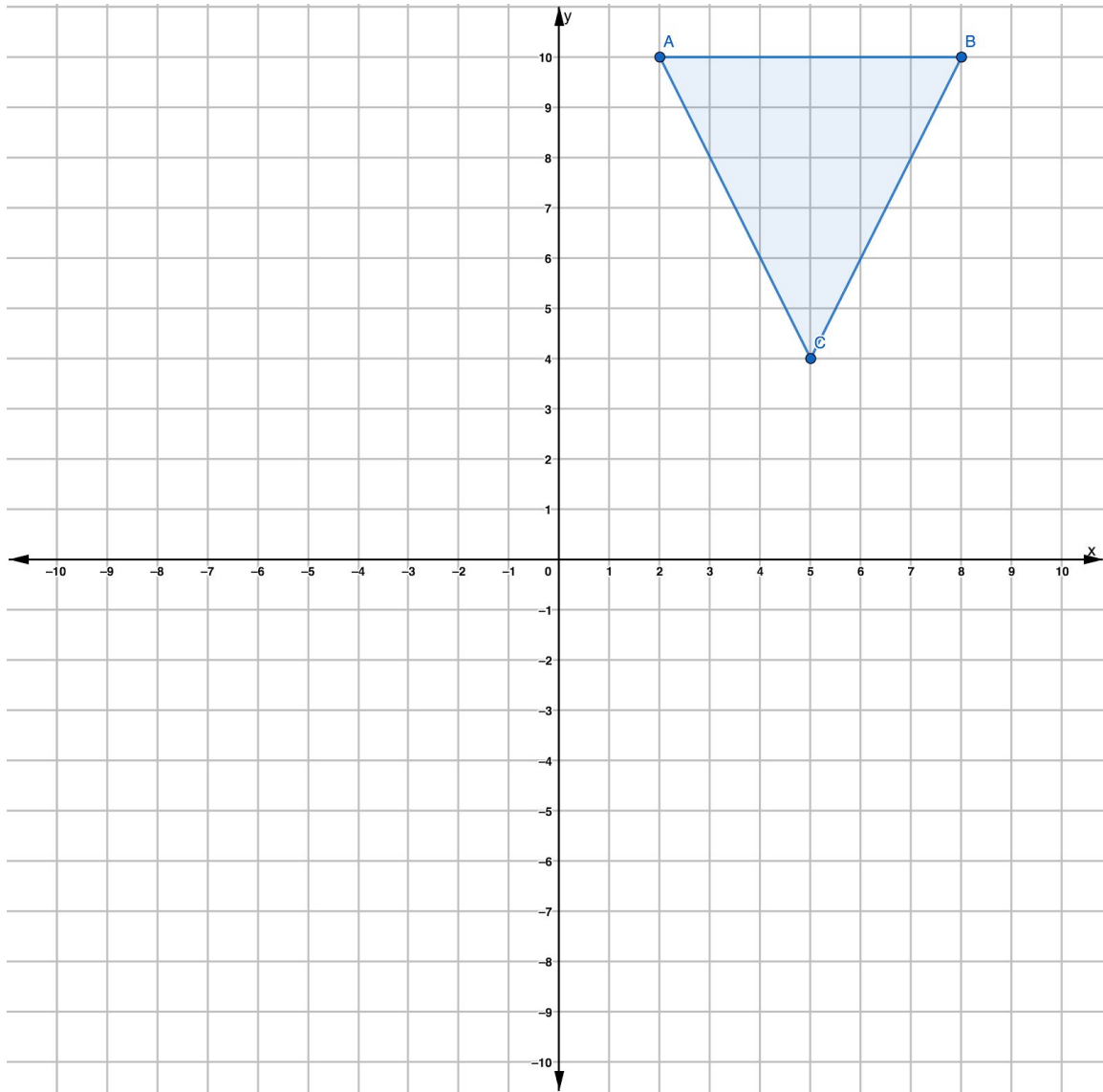
- A) reflection over the x-axis
- B) reflection over the y-axis
- C) rotation about the origin
- D) translation

Explain your answer:



Rigid Transformations

- 4) Which transformation on  $\triangle ABC$  would result in a triangle that is not congruent to  $\triangle ABC$ ?

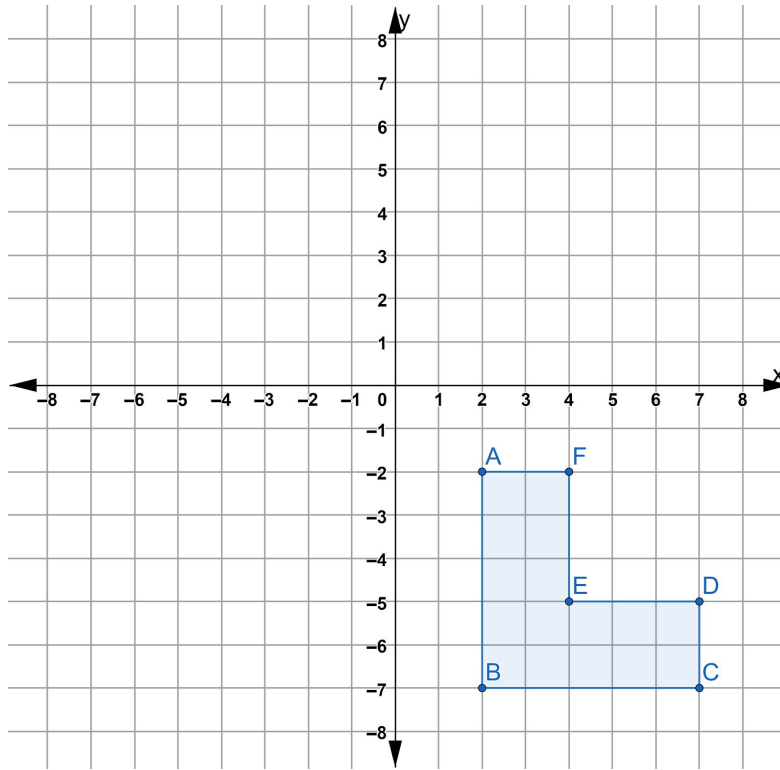


- A) reflection about the x-axis
- B) translation 4 units to the left and 6 units down
- C) dilation by a scale factor of 2
- D) rotation of  $180^\circ$  clockwise about the origin

Explain your answer:

*Rigid Transformations*

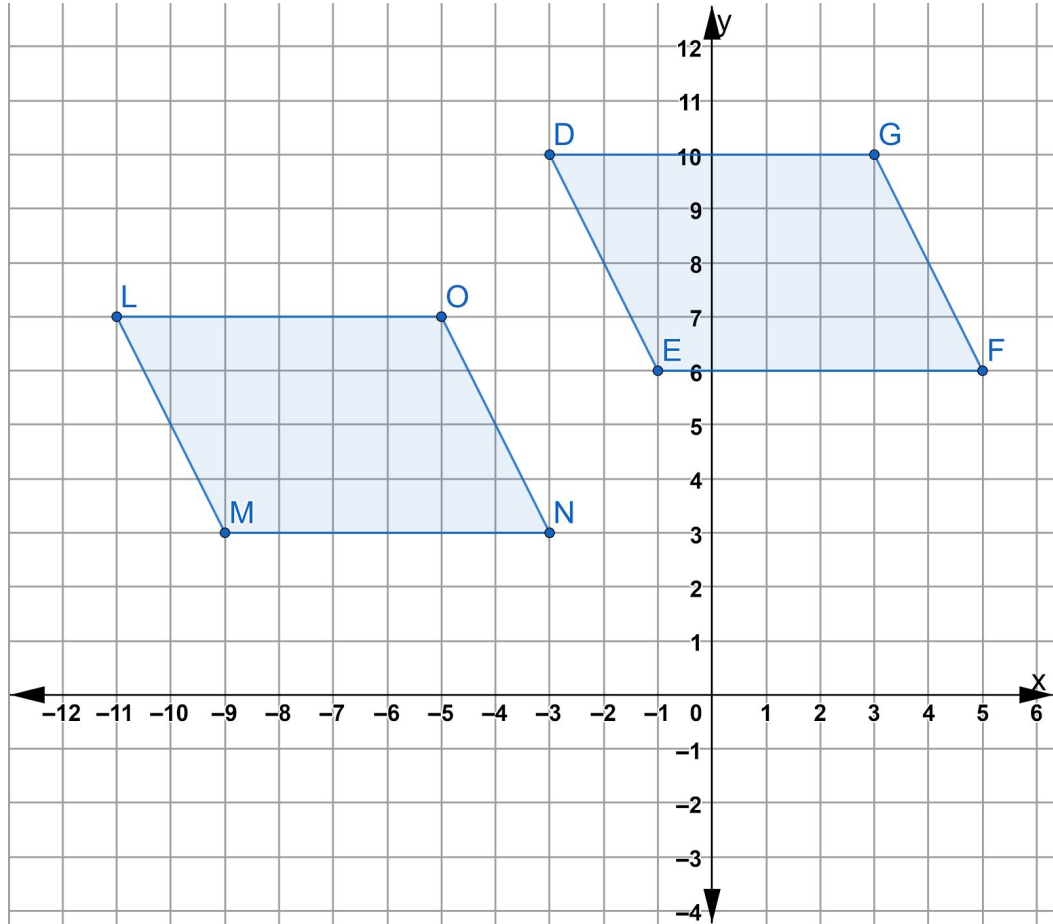
- 5) If the figure on the coordinate plane is translated 4 units to the right and 4 units down, what would the coordinates of point C' be?



- A) (7,-7)
- B) (7,-11)
- C) (11,-7)
- D) (11,-11)

Explain your answer:

6) Consider parallelogram LMNO on the coordinate plane.

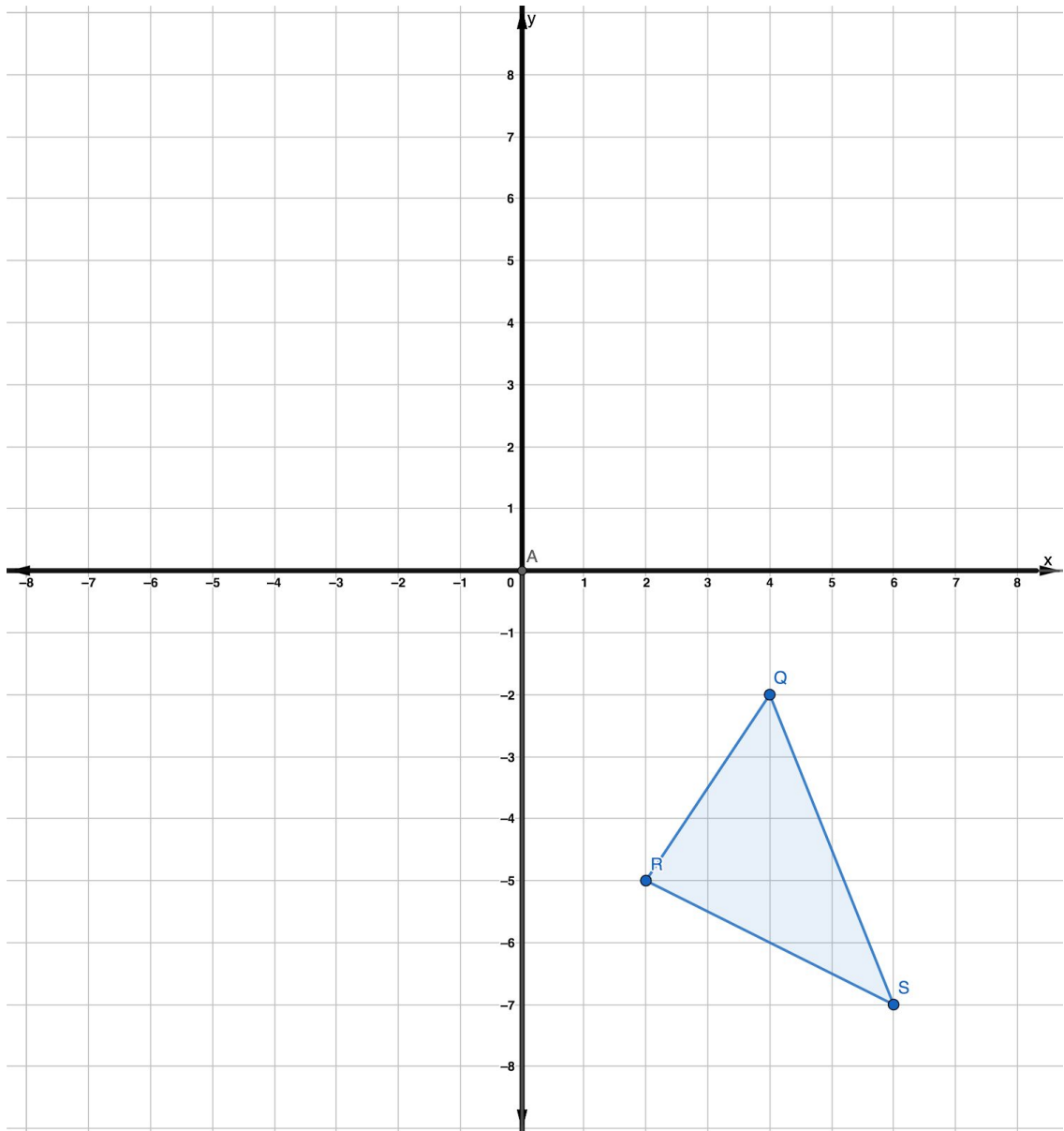


Which translation could have resulted in the congruent image, DEFG?

- A) translation of 2 units left and 3 units down
- B) translation of 2 units right and 3 units up
- C) translation 8 units to the right and 3 units up
- D) translation of 8 units left and 3 units down

Explain why answer B is incorrect:

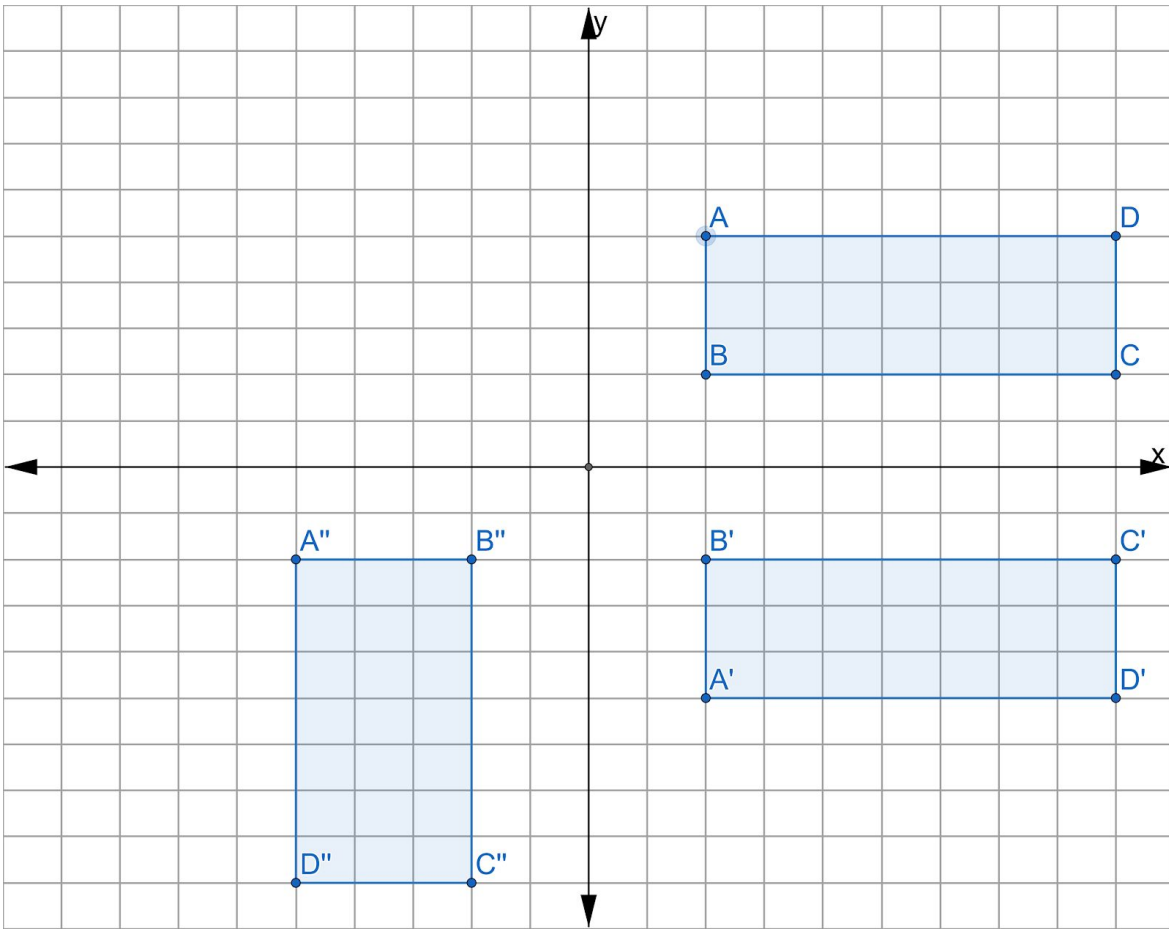
7) Triangle QRS (shown) maps on to triangle Q'R'S' (not shown) by a certain translation.



If two of the vertices of triangle Q' (1, 7) and S' (3, 2), which would be the coordinates of R'?

- A) (-1, 4)
- B) (-7, -2)
- C) (5, 4)
- D) (-2, 5)

- 8) A sequence of transformations maps rectangle  $ABCD$  onto rectangle  $A''B''C''D''$ , as shown in the diagram below.

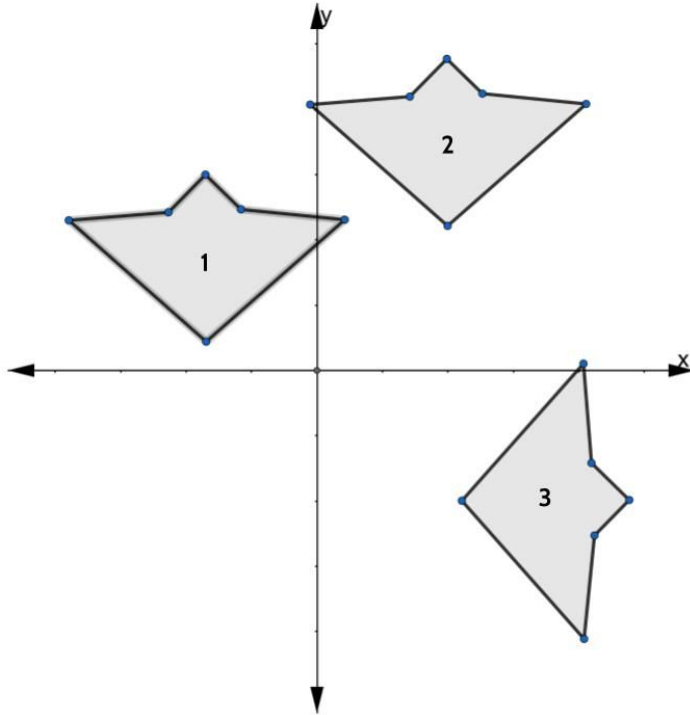


Which sequence of transformations maps  $ABCD$  onto  $A'B'C'D'$  and then maps  $A'B'C'D'$  onto  $A''B''C''D''$ ?

- A) a reflection followed by a rotation
- B) a reflection followed by a translation
- C) a translation followed by a rotation
- D) a translation followed by a reflection

Explain why answer C is incorrect:

9) In the diagram below, congruent figures 1, 2, and 3 are drawn.

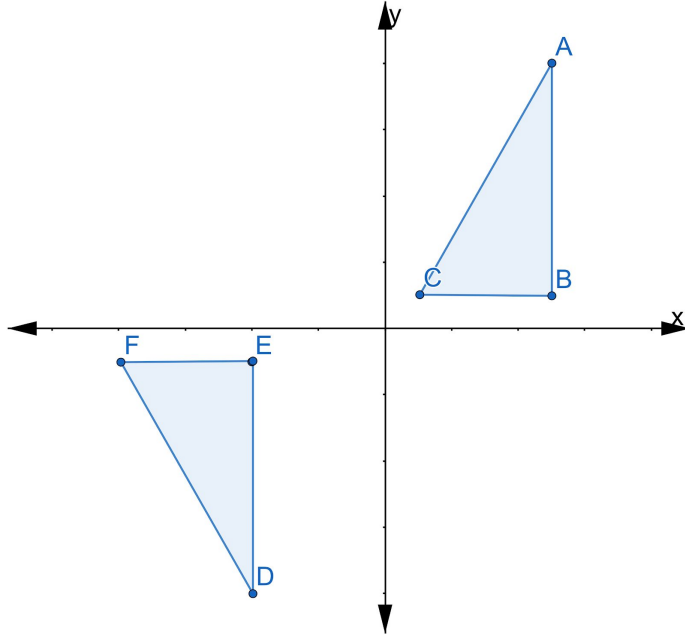


Which sequence of transformations maps figure 1 onto figure 2 and then figure 2 onto figure 3?

- A) a reflection followed by a translation
- B) a rotation followed by a translation
- C) a translation followed by a reflection
- D) a translation followed by a rotation

Explain your answer:

10) In the diagram below,  $\triangle ABC \cong \triangle DEF$



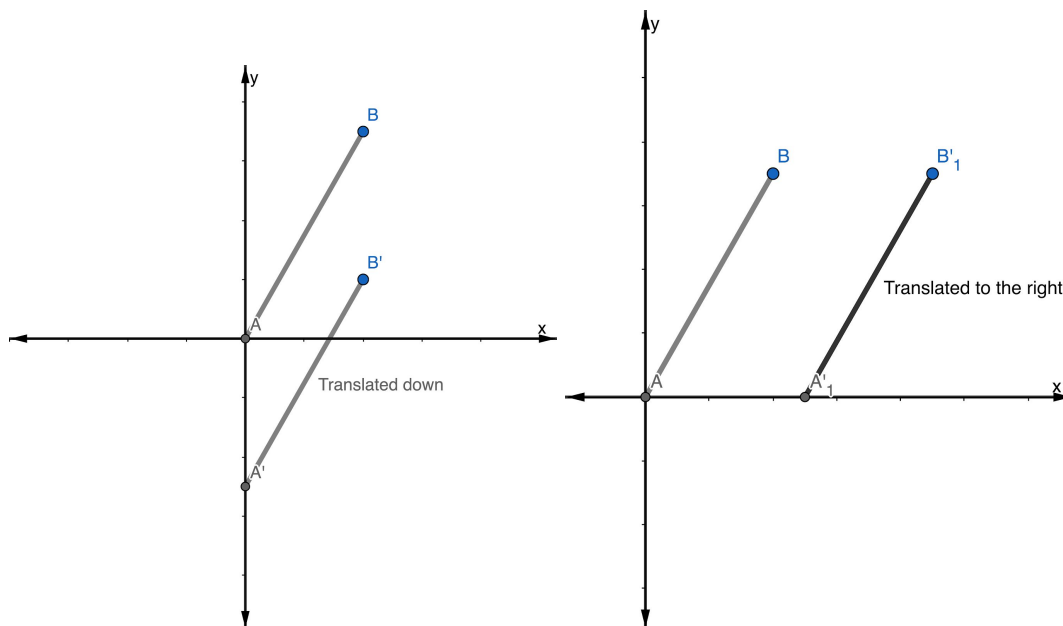
Which sequence of transformations maps ABC onto DEF?

- A. reflection over the y-axis followed by a translation
- B. reflection over the x-axis followed by a translation
- C. counterclockwise rotation of  $90^\circ$  about the origin followed by a translation
- D. counterclockwise rotation of  $180^\circ$  about the origin followed by a translation

Explain your answer:

## Practice Questions - Answer Key

- 1) Choice A and E. Parallel lines are lines that remain the same distance away from each other and never touch. If you slide every point on a line down or to the right, each point will be the same distance away from each other and the lines will be parallel.



- 2) Choice B.

We can see that  $\triangle LIV$  has been flipped, so we can tell that it is a reflection and not a translation. That narrows the options down to choice B or D. To identify the line of reflection, we can measure the distance between the points in the original figure to those in the image. For example, point V is 4 units away from point V'. We know the line of reflection needs to be halfway between them, which means the line of reflection in this transformation is the x-axis.

- 3) Choice A, a reflection over the x-axis.

The x-axis is halfway between each point in the original figure and in the reflected image. Because the x-axis goes through rectangle ABCD part of rectangle A'B'C'D' overlaps with the original rectangle.

- 4) Choice C.

Reflections, rotations and translations are all examples of rigid transformations. Rigid transformations are ways of moving figures without changing their shape or size.



Dilations change the size of a figure, which means dilations never produce congruent figures. A dilation of a scale factor of two would make  $\triangle ABC$  twice as big.

5) Choice D.

This problem is a little trickier because point  $C'$  is outside the coordinate plane that is provided. One strategy you might use to answer questions like this one is to sketch some lines to continue the coordinate plane. As a reminder, you are entitled to have graph paper during your HSE exam. If you are not given graph paper, you should request some from the person administering the exam. Another strategy is to write out the coordinates you are being asked to transform - in this case, point  $C(7, -7)$ . If you need to slide the figure 4 units to the right, then each  $x$  coordinate will increase by 4. If you need to slide the figure 4 units down, then the  $y$  coordinates will need to decrease by 4. So  $(7, -7)$  would slide to  $(11, -11)$ .

6) Choice C, a translation 8 units to the left and 3 units down.

We can tell Choice B is incorrect by looking at the corresponding points. Point  $N$  on the original figure maps to point  $F$  on the image, but moving 2 units to the right and 3 units up would map point  $N$  to point  $E$ .

Choice D is the translation that would take us from image  $LMNO$  to original figure  $DEFG$ . Choice D is a common mistake if you confuse which is the original figure and which is the translated image.

7) Choice A,  $(-1, 4)$ .

If point  $Q(4, -2)$  maps to point  $Q'(1, 7)$  then triangle  $QRS$  maps to triangle  $Q'R'S'$  by a translation of 3 units left and 9 units up.

8) Choice A, a reflection followed by a rotation.

Choice C is a very compelling answer. It appears like rectangle  $ABCD$  slides down and then rotates. Only by looking at the names of the points can you see that  $A'B'C'D'$  is made by reflecting rectangle  $ABCD$  over the  $x$ -axis.

9) Choice D, a translation followed by a rotation.

10) Choice B, reflection over the  $x$ -axis followed by a translation.

## Final Project

On the front cover of this packet is a detail of tile work from the Alhambra Palace in Granada, Andalusia, Spain.

You can recreate this image using everything you have learned about rigid transformation.

1. Plot the following points on the coordinate grid on the next page and shade in the figure. This is your original figure.

(0,0)      (-4,2)      (-3,3)      (-2,2)      (0,6)      (2,2)      (3,3)      (4,2)

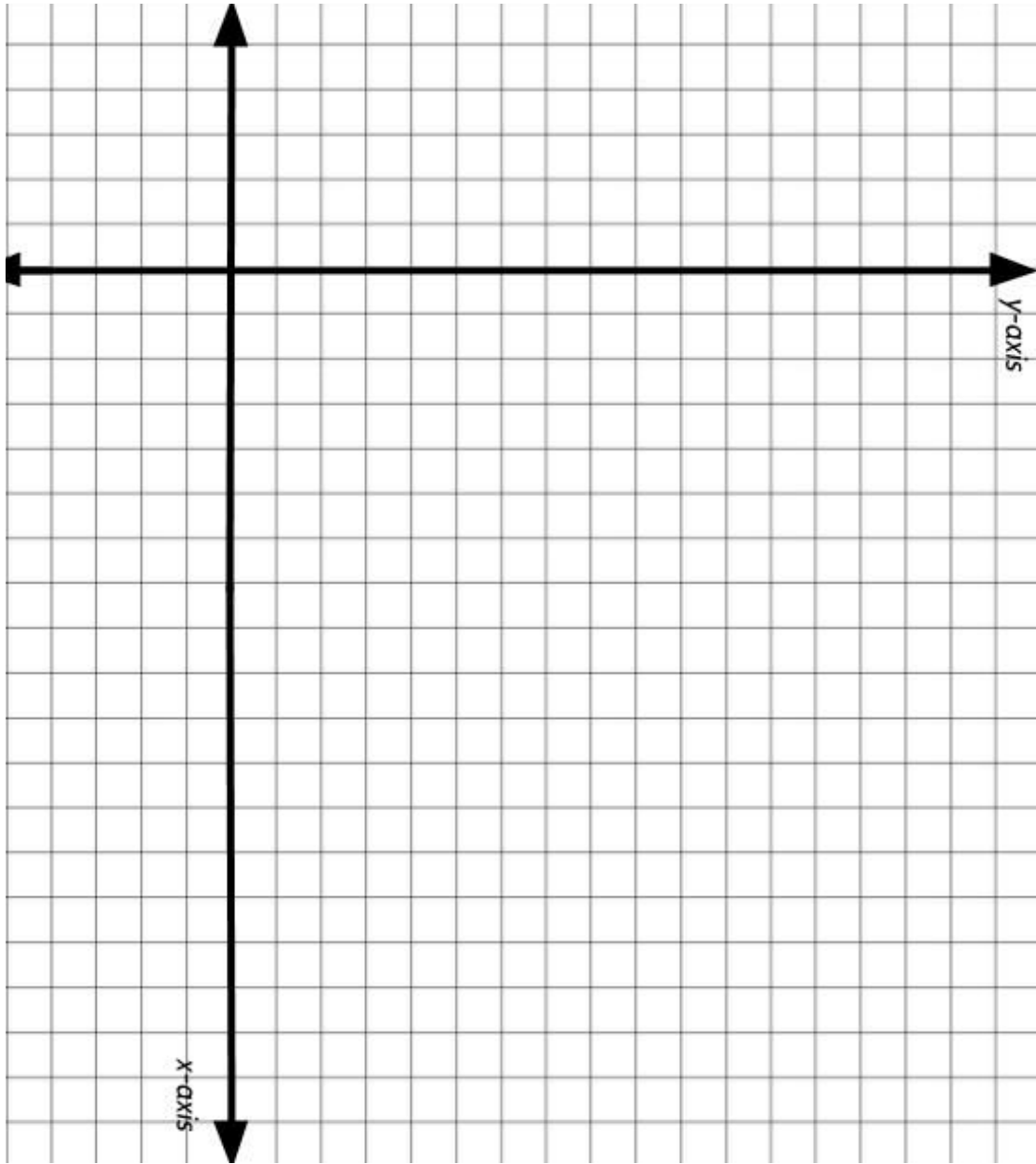
2. Reflect the original figure about the line  $y = 6$ . Call this figure 2.
3. Reflect the original figure about the  $x$ -axis. Call this figure 3.
4. Rotate the original figure  $180^\circ$  clockwise around point (3,3). Call this figure 4.
5. Reflect the image you create in step 4 about the line  $y = 6$ . Call this figure 5.
6. Reflect the image you created in step 4 over the  $x$ -axis. Call this figure 6.
7. Rotate figure 5  $180^\circ$  clockwise around the point (9,9). Call this figure 7.
8. Reflect figure 7 about the line  $y = 6$ . Call this figure 8.
9. Rotate figure 6  $180^\circ$  clockwise about point (9,-3). Call this figure 9.
10. Shade in figures 1 through 9.

Can you continue the rigid transformations until the grid is covered?

If you'd like to see what steps 1-9 look like on the coordinate plane,

go to [http://bit.ly/RigidTransformations\\_FinalProject](http://bit.ly/RigidTransformations_FinalProject)

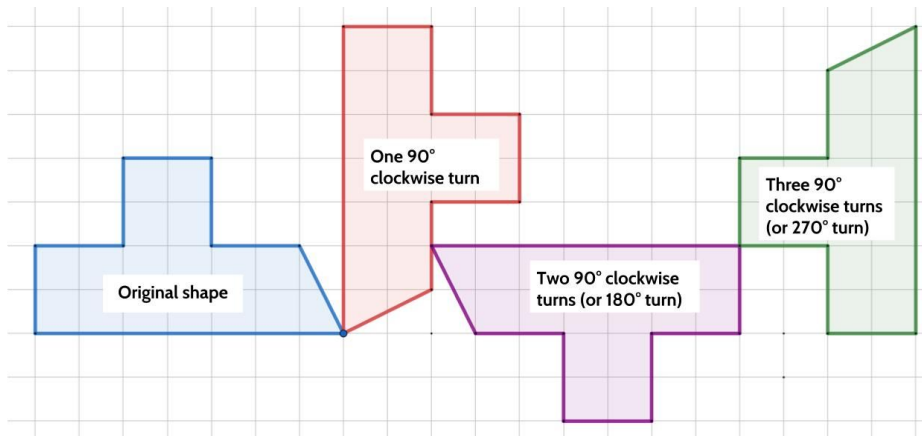
*Rigid Transformations*



## The Language of Rigid Transformations

### Vocabulary Review

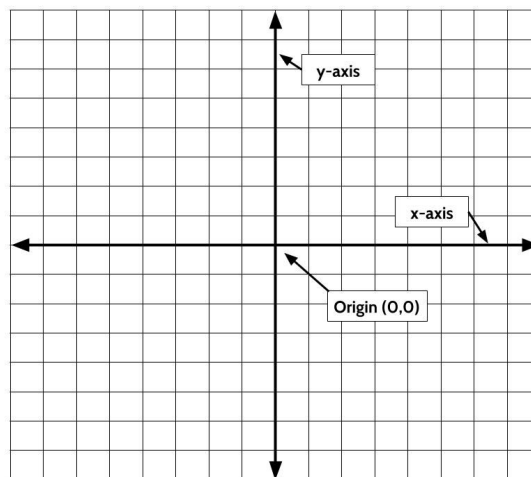
**90 degree turn:** A  $90^\circ$  turn is a quarter turn. A figure that is turned  $360^\circ$  turns in a complete circle and returns to its original position. In the diagram to the right, the original shape is being turned clockwise  $90^\circ$ . (See *ROTATION, ROTATIONAL SYMMETRY*)



**clockwise / counterclockwise (adjective):** clockwise means turning something in the same direction in which the hands of a clock move. Counterclockwise means turning something in the opposite direction in which the hands of a clock move. Turning a screw clockwise will tighten the screw, turning a screw counterclockwise will loosen the screw. (See *ROTATION, ROTATIONAL SYMMETRY*)

**congruent (adjective):** two shapes are congruent to each other if you can slide, flip or turn one figure so it fits exactly on top of the other figure. Any image that is the result of a rigid transformation is congruent to the original figure.

**coordinate plane (noun):** a completely flat surface formed when two straight number lines go across each other at right angles. The point where the lines intersect is called the origin and represents the zero on each axis. An exact position on the grid can be described using coordinates. The coordinate plane is also called the coordinate grid.

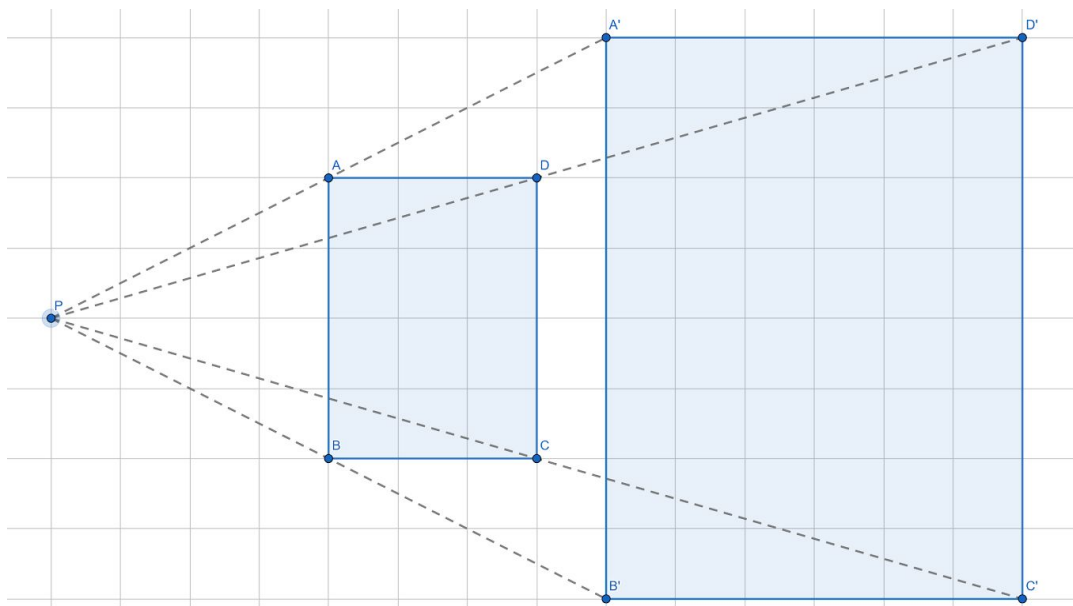


**corresponding (adjective):** If a part of the original figure matches up with a part of the image, we call them corresponding parts. The

## Rigid Transformations

part could be an angle, point, or side, and you can have corresponding angles, corresponding points, or corresponding sides.

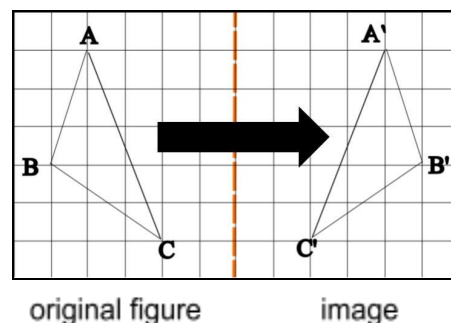
**dilation (noun):** a dilation is a *non-rigid* transformation that makes a figure larger or smaller without changing its shape. A figure that is dilated is not congruent to the original figure because you cannot slide, flip, or turn the image to fit exactly on top of the original figure. Dilations can be described by the term *scale factor*. In this diagram, rectangle ABCD is dilated by a scale factor of 2. That means the image is 2 times larger than the original figure. A scale factor of  $\frac{1}{2}$  means that the dilated image is half the size of the original figure.



**figure (noun):** a shape. *A triangle is a three-sided figure.*

**image (noun):** in everyday English, an image is a picture. In the language of rigid transformations, an image specifically refers to a figure that has gone through a rigid transformation. An image is congruent to the original figure. (See *ORIGINAL*)

**map (verb):** in everyday English, map means to make a drawing of an area showing roads, rivers, cities, etc. In the language of rigid transformations, to map a figure is to move all of the points of an original figure onto an image. For example, in the diagram to the right, we use a reflection to map  $\triangle ABC$  to  $\triangle A'B'C'$ .



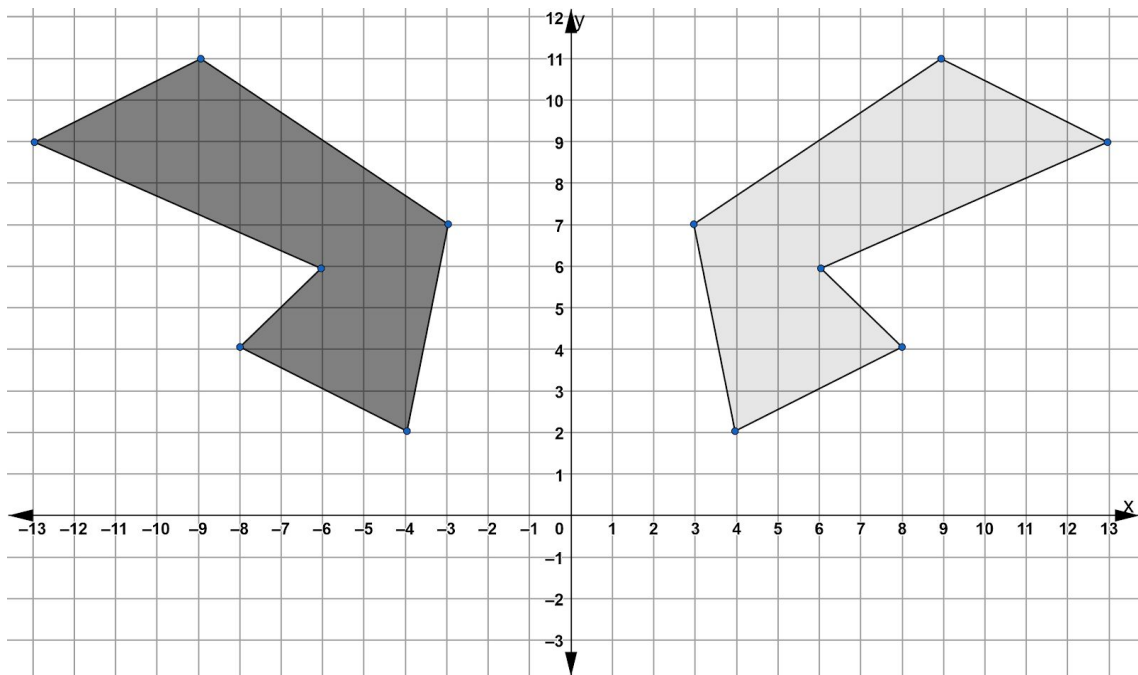
## Rigid Transformations

**non-rigid transformation:** a transformation that does not keep the size and shape of the original figure. (See *DILATION*)

**origin:** the point where the x-axis and the y-axis intersect on the coordinate grid. The origin has the coordinates (0,0).

**original:** In everyday English, original means the first or earliest. In the language of transformations, original specifically refers to the figure you start with. <See *IMAGE*>

**reflection:** A transformation that maps each point of a figure to a point in its mirror image. In the diagram below, the darker shape is reflected about the y-axis into the lighter image. The line of reflection acts as the mirror and is the halfway point between each point and its image. In this case, the y-axis is the line of reflection. So each on the darker shape is the same distance away from the y-axis as its reflected point in the lighter figure.



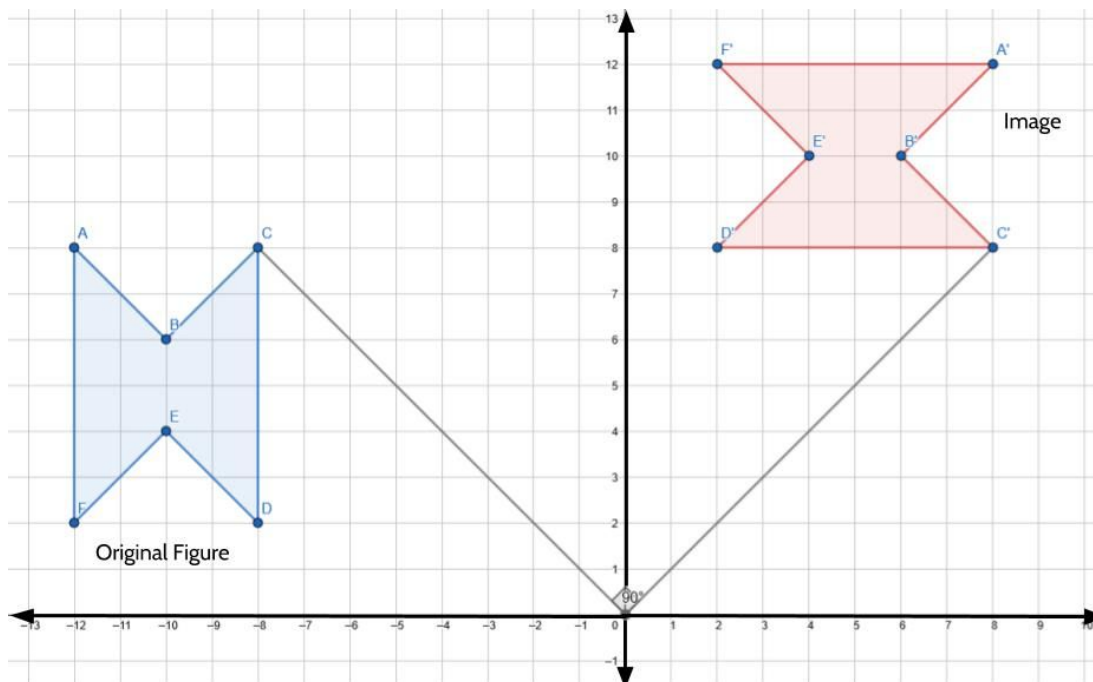
**reflectional symmetry:** a figure or design has reflectional symmetry if you can draw a line that divides the figure (or design) into mirror images. We call the line that cuts the figure (or design) into halves, the line of symmetry or the line of reflection.



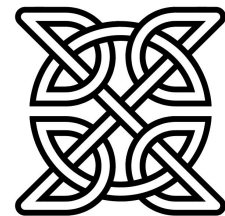
## Rigid Transformations

**rigid transformation:** When you move all the points of a figure in such a way that produces an image that is identical in size and shape to the original figure. With rigid transformations, the image and original figure should fit exactly on top of each other. If two figures are congruent, that means there is a sequence of rigid transformations we could describe that would make one of the figures look like the other. (See *REFLECTION*, *ROTATION*, *TRANSLATION*)

**rotation:** a transformation that rotates all of the points in a figure about a point. When we describe rotations, we describe the number of degrees it is turned, and whether it is turned clockwise or counterclockwise. For example, in the diagram below, the original figure was rotated  $90^\circ$  clockwise about the origin. Since the original figure and the image are the same size and shape, we can say they are congruent.



**rotational symmetry:** a figure or design has rotational symmetry if it can be rotated less than  $360$  degrees about a point to a position where it looks the same as it did in the original position. The Celtic knot to the right has rotational symmetry because it can be rotated  $180$  degrees and look identical to the way it did in its original position.



**sequence (noun):** a series of related events that happen in a particular order. A *sequence* of transformations would mean a figure going through multiple transformations.

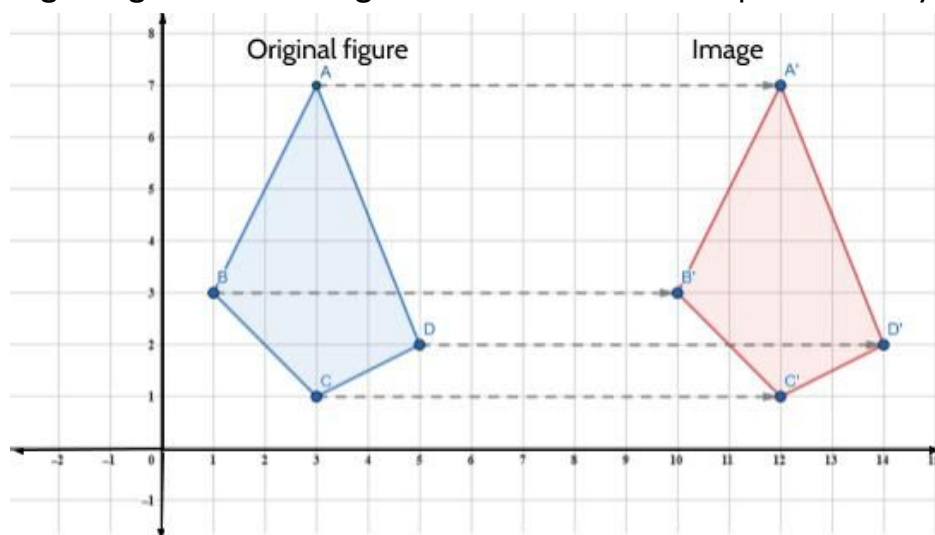
## Rigid Transformations

**step (noun):** one in a series of things that are done to achieve something. If you wanted to plot a point on the coordinate grid, your first *step* would be to start in the origin. Your next *step* would be to count along the x-axis. Your third *step* would be to count along the y-axis.

**symmetry:** Something has symmetry if it has two or more sides that are identical after a flip, a slide, or a turn. (See *ROTATIONAL SYMMETRY, REFLECTIONAL SYMMETRY*)

**tessellation:** a image that includes repeated congruent figures that cover a surface without gaps or overlapping. The art of MC Escher often uses tessellations. Tiles in a bathroom is another example.

**translation:** In everyday English, translation means to change speech or writing from one language to another. It has a different definition in the language of rigid transformations. A translation means sliding each point of a figure the same distance, in the same direction. In the diagram below, each point of the original figure is translated 9 units to the right. Since the original figure and the image are the same size and shape, we can say they are congruent.



**x-axis:** the line that goes from left to right of a graph. The horizontal axis on the coordinate grid.

**y-axis:** the line that goes from top to bottom of a graph. The vertical axis on the coordinate grid.

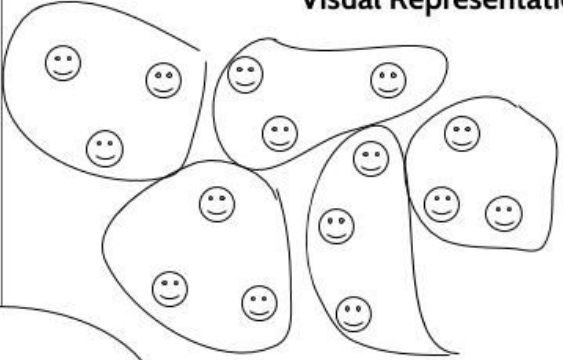


## Using Graphic Organizers to Learn Vocabulary

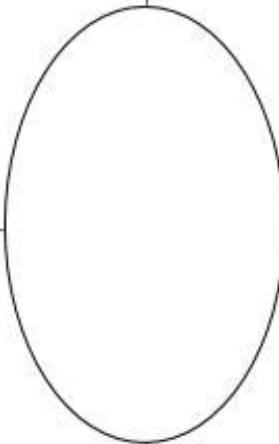
In order to learn math vocabulary, we need practice using it in different ways. In this activity, you will choose a few words from this packet that you want to practice, then you will complete a graphic organizer for each word. Look at the sample for the word *quotient* below.

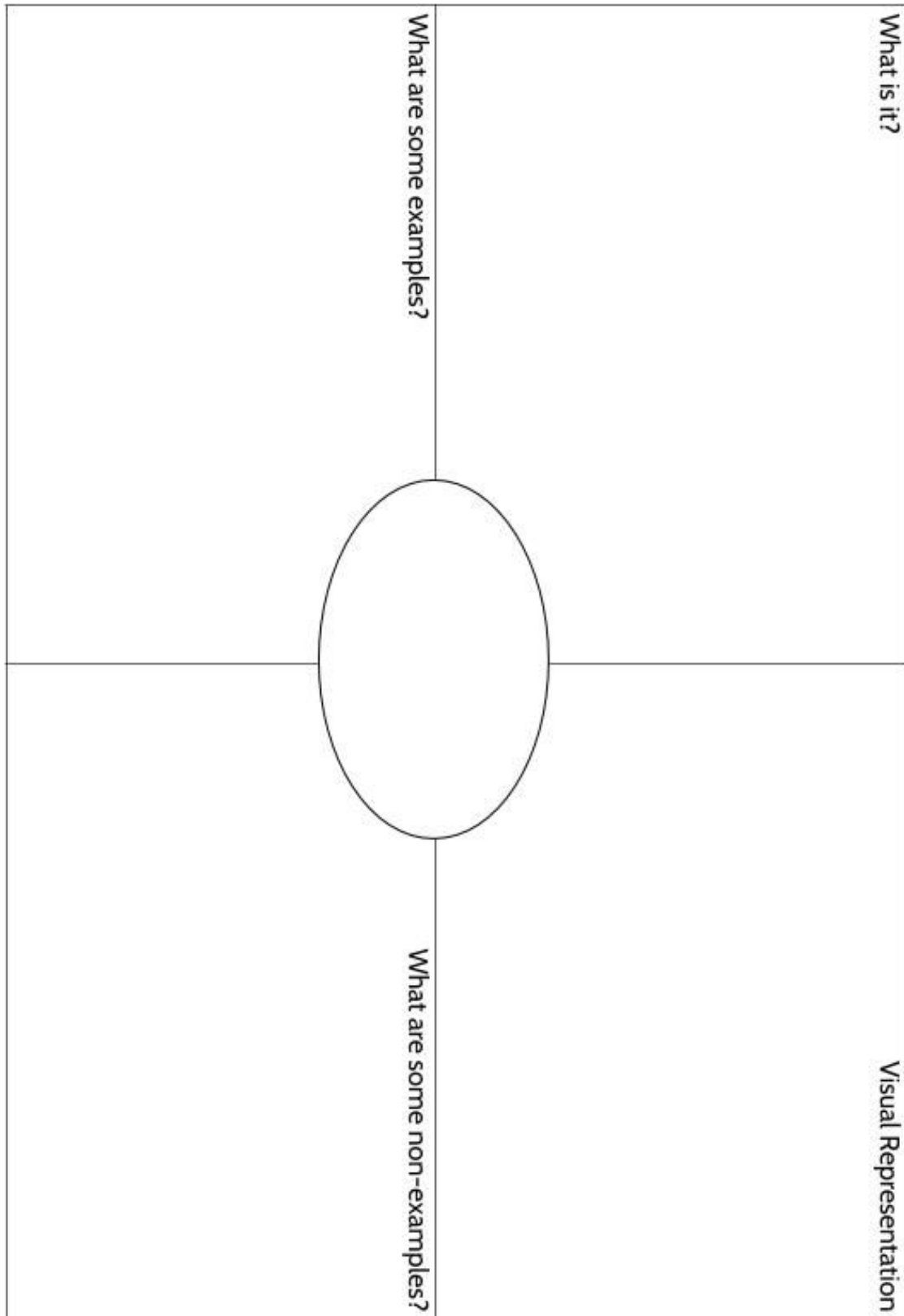
To complete the graphic organizer, you will choose a word and then answer four questions:

1. What is the definition of the word. You can look at the vocabulary review on page 120 for help. Try to write the definition in your own words to help remember what it means.
2. Make a Visual Representation. You can make a drawing or diagram that will help you remember what the word means.
3. What are some examples of the word you're studying? Below you can see that there are examples of *quotients*, which are the answers to division problems.
4. What are some non-examples of this word? These are things that are **not** the word you're studying. For example, 24 is **not** the quotient of 4 divided by 6.

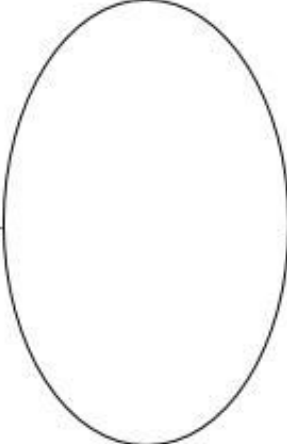
<p><b>What is it?</b></p> <p>A quotient is the result of dividing one number by another. It is the answer to a division question.</p>	<p><b>Visual Representation</b></p> 
<p><b>What are some examples?</b></p> <p>15 divided by 3 equals 5</p> <p><math>66 \div 6 = 11</math></p> <p><math>63/18 = 3.5</math></p> <p>5, 11 and 3.5 are quotients in these calculations.</p> <p>population <math>\div</math> area = <b>density</b></p>	<p><b>What are some non-examples?</b></p> <p>4 times 6 equals 24</p> <p><math>18 + 5 = 23</math></p> <p><math>17 - 2.5 = 14.5</math></p> <p><math>3.5 \times 18 = 63</math></p>

<p>What are some examples?</p>	<p>What is it?</p>
<p>What are some non-examples?</p>	<p>Visual Representation</p>

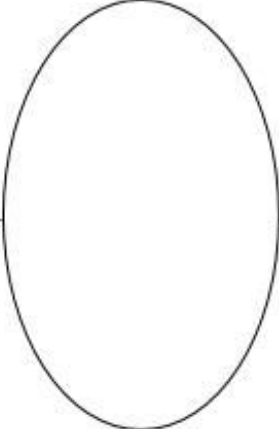




<p>What is it?</p>	<p>Visual Representation</p>
<p>What are some examples?</p>	<p>What are some non-examples?</p>



<p>What are some examples?</p>	<p>What is it?</p>
<p>What are some non-examples?</p>	<p>Visual Representation</p>

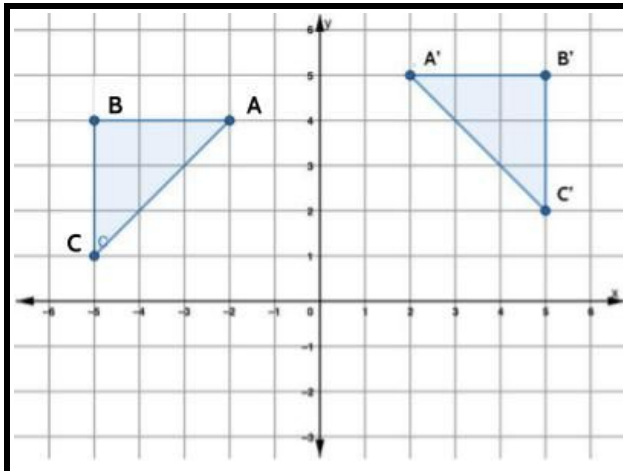


## Matching Pictures and Descriptions

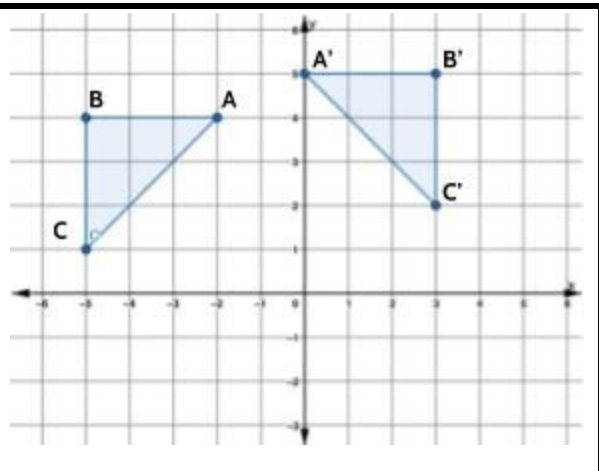
Match the sentence with the picture it describes. Write the letter of that sentence in the blank box below each picture. You will have one sentence left. Write that letter in the box for #4 and complete the picture to match that description.

Sentences

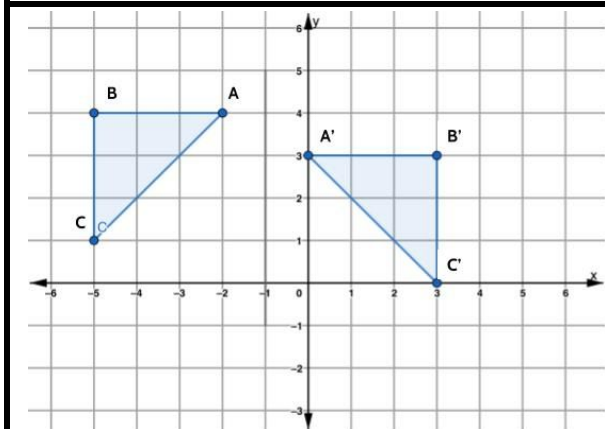
- A) a reflection about the  $y$ -axis and a translation 1 unit down
- B) a reflection about the line  $x = -1$  and a translation 1 unit down
- C) a reflection about the line  $x = -1$  and a translation 1 unit up
- D) a reflection about the  $y$ -axis and a translation 1 unit up



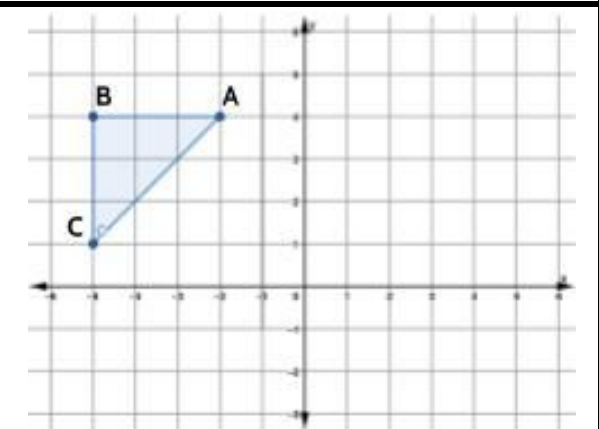
1) The transformation above matches with sentence \_\_\_\_\_



2) The transformation above matches with sentence \_\_\_\_\_



3) The transformation above matches with sentence \_\_\_\_\_



4) The transformation above matches with sentence \_\_\_\_\_

### Concept Circle

5) Explain the four words in each circle and the connections you see between them.

---

---

---

---

---

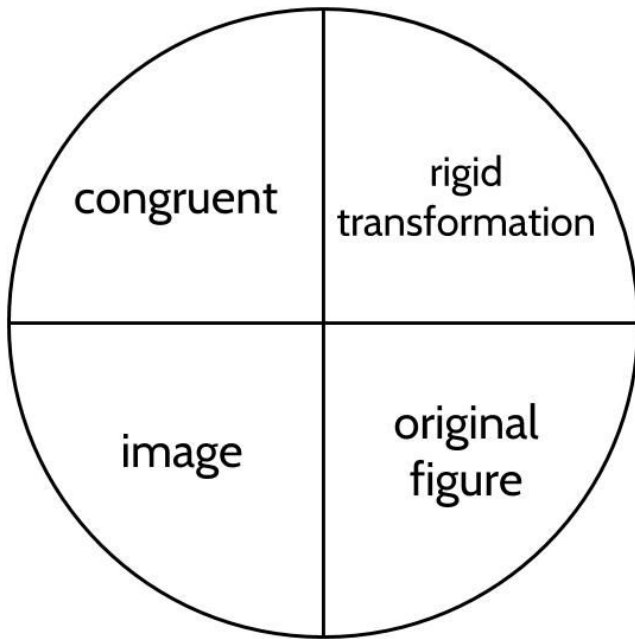
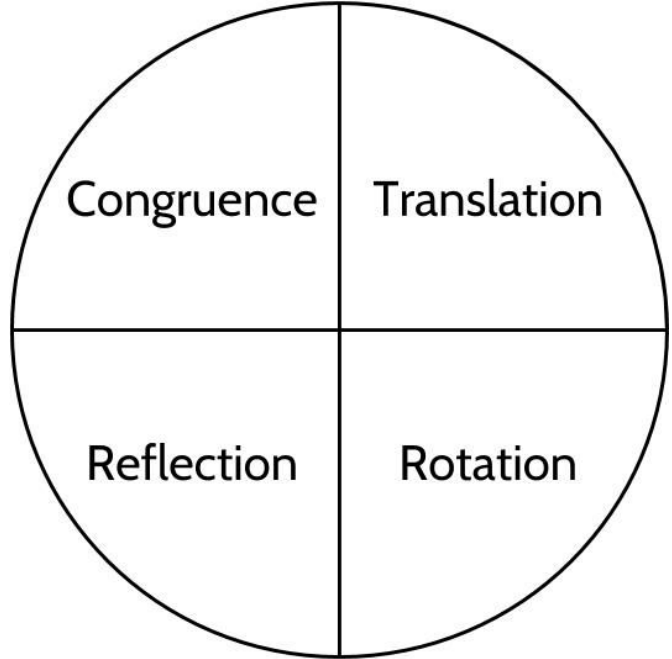
---

---

---

---

---



---

---

---

---

---

---

---

---

---

---

---

---

---

---

---

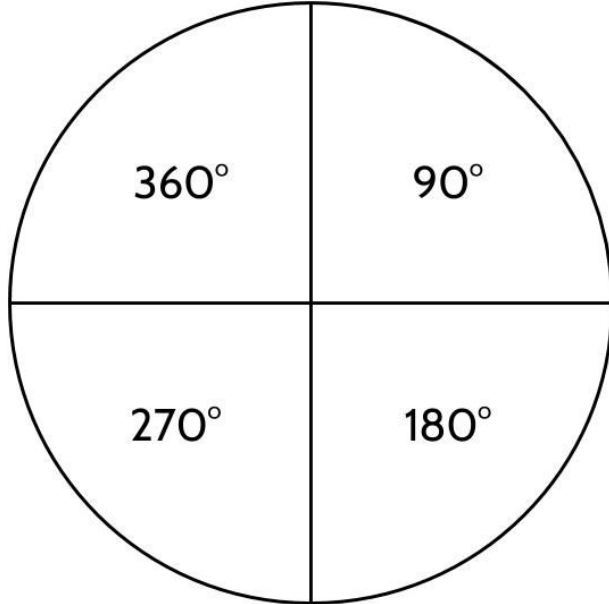
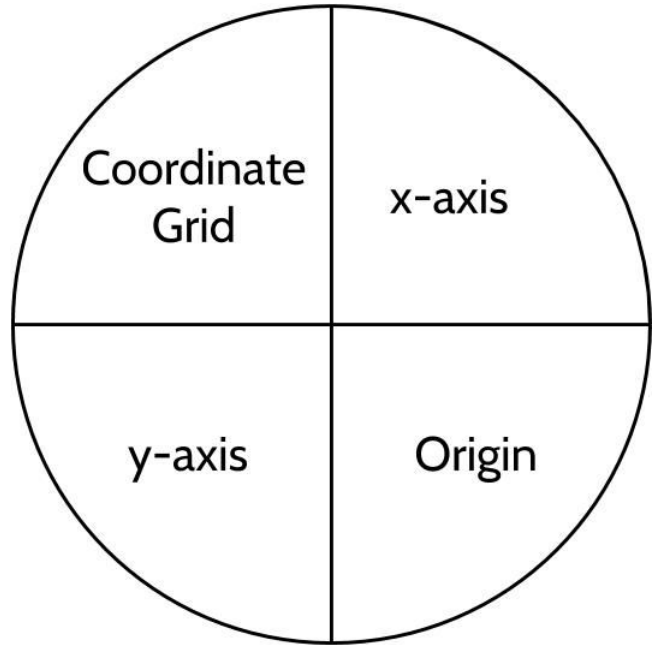
---

---

---

---

---



---

---

---

---

---

---

---

---



## Rigid Transformations and You

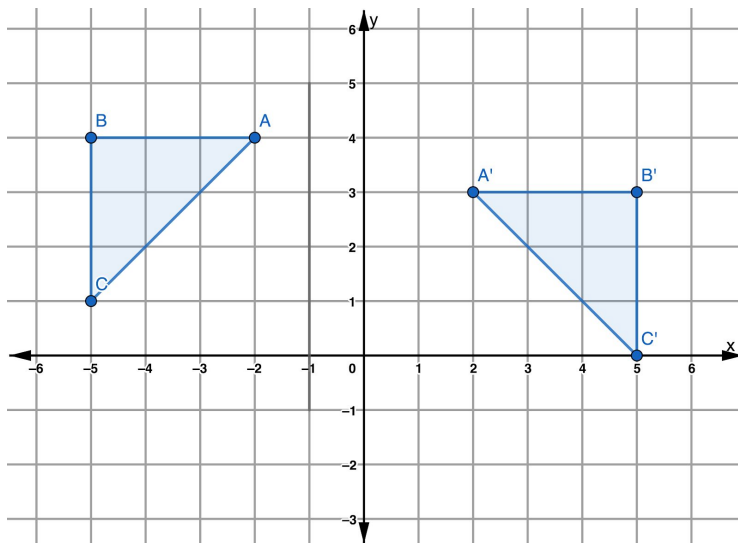
6) Draw at least one example of each of the following from your life:

- An image or pattern created by translation
- An image or pattern created by reflection
- An image or pattern created by rotation.

Label your pictures with as many vocabulary words as you can.

## The Language of Rigid Transformations - Answer Key

- 1) The transformation above can be described as a reflection about the y-axis and a translation 1 unit up. (Sentence D)
- 2) The transformation above can be described as a reflection about the line  $x = -1$  and a translation 1 unit up. (Sentence C)
- 3) The transformation above can be described as a reflection about the line  $x = -1$  and a translation 1 unit down. (Sentence B)
- 4) Sentence A is the one that does not have a corresponding picture. A reflection of the original about the y-axis and a translation 1 unit down would look like this:



- 5) Each paragraph should use the 4 vocabulary words in the circle. Be creative. There is no wrong way to do this activity!
- 6) Take your time with this activity. Look around and describe what you see. Explain what rigid transformations are to someone else and have them help you find examples. This is an opportunity to practice all the vocabulary and math skills you have learned.