Essential Geometry Practice for Students of TASC-math

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Essential Geometry Practice for Students of TASC-math

Approximately 23% of the questions on the TASC math subtest focus on geometry. That is nearly 1 in 4 questions. It's clear that you need to be strong in this area of math to excel on this test.

Some, but probably not most, of the questions may involve a formula. Some of the formulas you must memorize and some of the formulas will be given to you on the TASC Math Reference Sheet.

Geometry Information Provided on the TASC Math Reference Sheet

Volume

Cylinder: $V = \pi r^2 h$

Pyramid: $V = \frac{1}{3}Bh$

Cone: $V = \frac{1}{3}\pi r^2 h$

Sphere: $V = \frac{4}{3}\pi r^3$

V = volume

r = radius

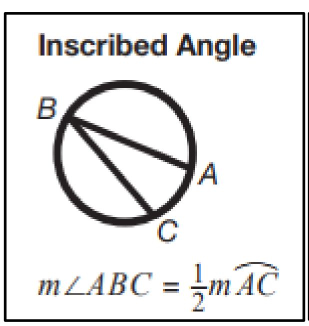
h = height

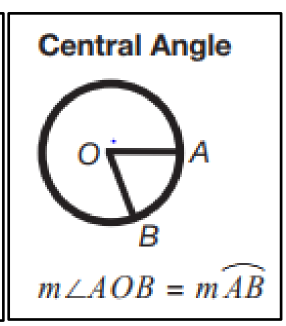
B = area of base

Pythagorean Theorem

$a^2 + b^2 = c^2$

Inscribed and Central Angles





Geometry Formulas You Should Memorize

Area

Rectangle: $A = lw$

Triangle: $A = \frac{1}{2}bh$

Circle: $A = \pi r^2$

A = area

l = length

w = width

b = base

h = height

r = radius

Circumference

$C = \pi D$

C = circumference

D = diameter

Density

$D = \frac{m}{V}$

D = density

m = mass

V = volume

Essential Geometry Practice Questions

Below you will find practice geometry questions like those you will see on the TASC. There are many more types of questions you will see on the exam, but these represent some of the most common topics on the test.

I. Precise Definitions of Geometric Figures

On the TASC, you need to know the definition for common geometric figures such as angle, circle, perpendicular line, parallel line, and line segment. You may know what a circle is, but can you define it?

Example 1

Two rays that share a common endpoint form A. a line segment. B. parallel lines. C. a circle. D. an angle.

Solving Example 1

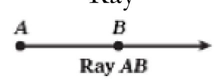
For the purposes of the TASC math, you need to be familiar with defining geometric figures using the "undefined terms" *point* and *line*.

Point: a position in space with no size.

Line: a set of points extending in either direction infinitely. A line has no thickness.

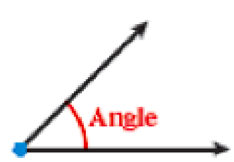
Since we cannot measure the size of a point, or the thickness of a line, we call these terms *undefined*.

To answer Example 1, let's look at some diagrams to help us visualize some terms.

Point	Line	Ray	Line Segment
A • Written Point A	 Lines have no endpoints and extend infinitely in either direction. Written Line AB or $\overleftrightarrow{AB}$	 Rays have one endpoint and extend infinitely in one direction. Written Ray AB or $\overrightarrow{AB}$	 Line segments have two endpoints. Written Line Segment AB or $\overline{AB}$

Two rays that share a common endpoint form an angle.

The correct answer is Choice D.



You Try – Precise Definitions of Geometric Figures

1. Which defines a line segment?

   A. A set of infinite points found between two endpoints.

   B. A set of infinite points extending from one endpoint in a single direction.

   C. A set of infinite points extending in either direction.

   D. A set of infinite points extending in every direction.

2. Perpendicular lines form what type of angle?

   A. Right

   B. Straight

   C. Acute

   D. Obtuse

3. What is the definition of a circle?

   A. The set of all lines that are an equal distance from a line

   B. The set of all lines that are an equal distance from a point

   C. The set of all points that are an equal distance from a line

   D. The set of all points that are an equal distance from a point

4. One-fourth of the distance around the circumference of a circle is

   A. the radius

   B. the diameter

   C. an arc measuring 45°

   D. an arc measuring 90°

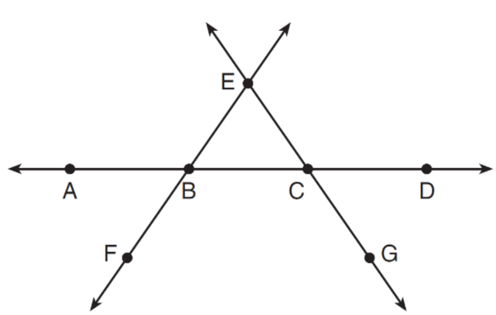
5. Which two angles in the diagram below must be congruent?

   A. $\angle EBC \cong \angle BEC$

   B. $\angle ABE \cong \angle ABF$

   C. $\angle ACG \cong \angle BCG$

   D. $\angle EBC \cong \angle EAC$



II. Volume of Prisms (including Cubes)

Example 2

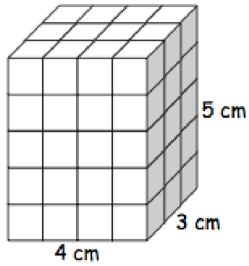
Find the volume of the rectangular prism in cubic centimeters.

A. 12 cm^3

B. 35 cm^3

C. 47 cm^3

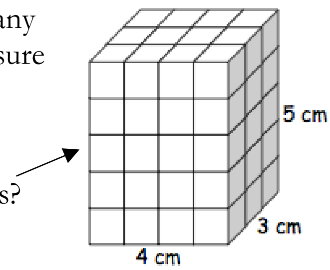
D. 60 cm^3



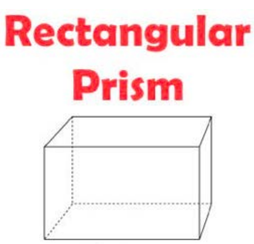
Solving Example 2

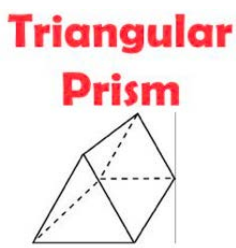
Volume is used to measure the space inside of three dimensional (3-D) figures. On the TASC math subtest, you will need to be able to work with volume measurements for many figures including prisms, pyramids, cylinders, cones, and spheres. We always measure volume using a cubic unit. In Example 2, we are using cubic centimeters.

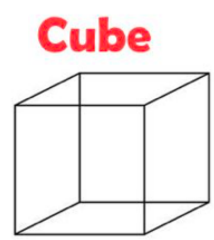
Essentially, you are being asked how many of these  does it take to make this?



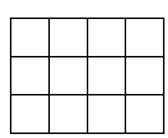
A *prism*, loosely, is a solid object with two identical bases, and flat sides. The shape of the base gives the prism its name. Here are some different types of prisms:







To determine the volume of the rectangular prism in Example 2, we will use the formula $V = Bh$. Here, V stands for volume, B stands for the area of the base, and h stands for height. Let's examine what is meant by the area of the base. The base is a 4 cm by 3 cm rectangle that looks like this:



The area of the base is 12 cm^2. We can find this by multiplying $4\ cm \times 3\ cm = 12\ cm^2$. Since the height is 5 cm, we can find the volume like this:

$$V = Bh$$
$$V = (3 \times 4) \times 5$$
$$V = (12) \times 5$$
$$V = 60\ cm^3$$

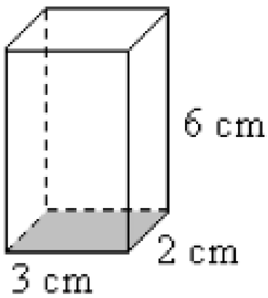
The correct answer is Choice D.

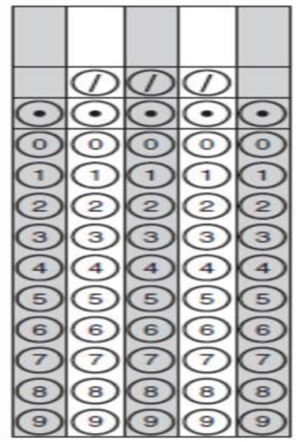
Testing Tips

- You may be more familiar with the formula $volume = length \times width \times height$. This certainly works, but only for rectangular prisms. The formula $V = Bh$ will work for various prisms as well as cylinders. And, we will see this use of B = area of the base later in other formulas. While other volume formulas will be given to you, **you need to memorize how to find the volume of prisms**.
- Remember, cubic units, such as in^3, take up the space of a $1 \times 1 \times 1$ cube (inches in this case). But, 4 in^3 does not indicate the third power of 4 itself, only the inches.

You Try – Volume of Prisms (including Cubes)

6. Find the volume of the rectangular prism in cubic centimeters.





7. A fish tank with a rectangular base has a volume of 5,040 cubic inches. The length and width of the tank are 20 inches and 14 inches, respectively. Find the height, in inches, of the tank.

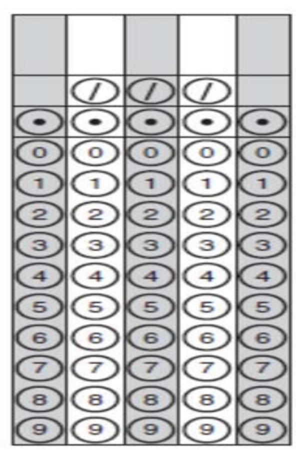
   A. 16

   B. 18

   C. 34

   D. 280

8. A box in the shape of a cube has a volume of 64 cubic inches. What is the length of a side of the box in inches?



9. Brooke needs to buy an exhaust fan for her bathroom. The bathroom has a width of 8 feet, a length of 10 feet, and a height of 8 feet, and the duct for the fan is 20 feet long. Using the chart below, what size bathroom fan should she purchase?

A. ≥ 60 cfm

B. ≥ 70 cfm

C. ≥ 90 cfm

D. ≥ 110 cfm

Bathroom Fan Sizing Chart				
Bathroom Size	Duct Length			
	10 ft.	20 ft.	30 ft.	40 ft.
400 ft^3	60 cfm	60 cfm	60 cfm	60 cfm
480 ft^3	60 cfm	60 cfm	60 cfm	60 cfm
560 ft^3	70 cfm	70 cfm	90 cfm	90 cfm
640 ft^3	90 cfm	90 cfm	90 cfm	90 cfm
730 ft^3	90 cfm	90 cfm	110 cfm	110 cfm

III. Volume of Cylinders

Example 3

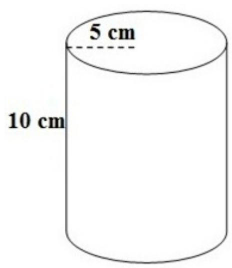
Find the volume of the cylinder to the nearest cubic centimeter.

A. 15 cm^3

B. 50 cm^3

C. 250 cm^3

D. 785 cm^3



Solving Example 3

The formula for finding the volume of a cylinder is $V = \pi r^2 h$ (given to you on the Reference Sheet). To use the formula, use the following steps.

Step 1: Identify *r* and *h*.

r = radius = 5 cm
h = height = 10 cm

Step 2: Substitute the values for *r* and *h* into the formula $V = \pi r^2 h$.

$V = \pi(5^2)(10)$

Step 3: Change π to 3.14, and then evaluate using the order of operations.

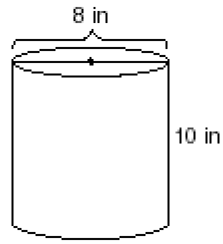
$V = (3.14)(5^2)(10)$
$V = (3.14)(25)(10)$
$V = 785\ cm^3$

Testing Tip

The volume of a cylinder formula, $V=\pi r^2 h$ is essentially the same as the formula for a prism $V = Bh$, only instead of B we have πr^2. This makes sense because **πr^2 is used to find the area of a circle** and the base of a cylinder is a circle. Look to $V=\pi r^2 h$ on the formula sheet if you forget the area of a circle formula $A = \pi r^2$.

You Try – Volume of Cylinders

10. A storage container in the shape of a right circular cylinder is shown in the accompanying diagram. What is the volume of this container, to the *nearest tenth* of a cubic inch?



A. 125.6 in^3

B. 251.2 in^3

C. 502.4 in^3

D. 2009.6 in^3

11. Determine the volume of a cylinder with a radius of 5 inches and a height of 7 inches.

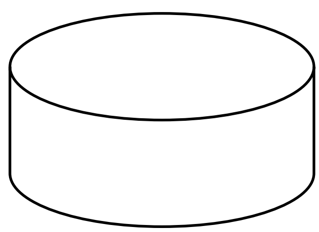
A. 12π in^3

B. 35π in^3

C. 175π in^3

D. $1{,}225\pi$ in^3

12. The cylinder below has a diameter of 5 meters and a height of 2 meters. Choose the closest approximate volume of the cylinder below.



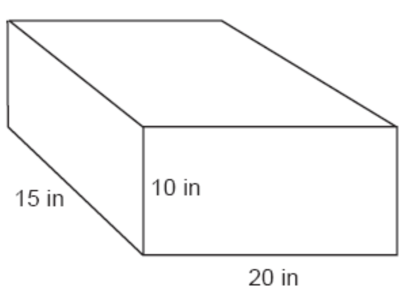
A. 39.25 m^3

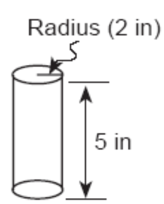
B. 78.5 m^3

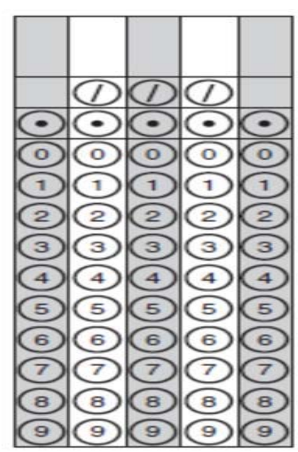
C. 157 m^3

D. 314 m^3

13. In the accompanying diagram, a rectangular container with the dimensions 10 inches by 15 inches by 20 inches is to be filled with water, using a cylindrical cup whose radius is 2 inches and whose height is 5 inches. What is the maximum number of full cups of water that can be placed into the container without the water overflowing the container?



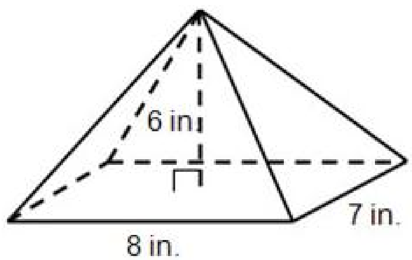


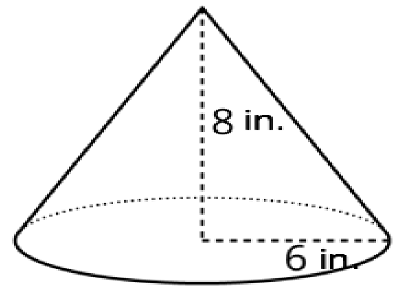


IV. Volume of Pyramids and Cones

Example 4

Choose the statement that most accurately compares the volumes of the figures below (figures not drawn to scale).





A. The cone is approximately 200 in^3 larger than the pyramid.

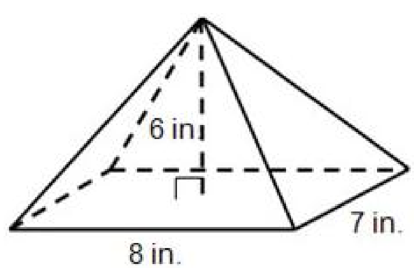
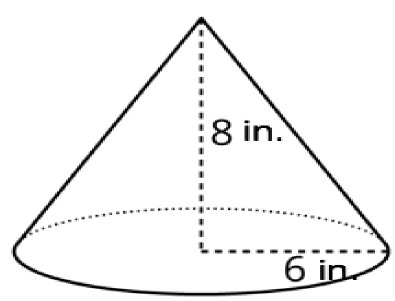
B. The pyramid is approximately 290 in^3 larger than the cone.

C. The pyramid is 7 in^3 larger than the cone.

D. The cone is approximately 2 in^3 larger than the pyramid.

Solving Example 4

To answer this question, we must calculate the volume of both the pyramid and the cone in the diagram. We know that the images are not drawn to scale, so it is not reliable to go by what the two figures look like. Let's first find the volume of the pyramid, and then find the volume of the cone.

Finding the Volume of a Pyramid	**Finding the Volume of a Cone**
To find the volume of a pyramid, we use the formula $V = \frac{1}{3}Bh$. As before with finding the volume of a prism, the letter *B* stands for the area of the base, and the letter *h* stands for the height of the pyramid. The base of this pyramid is a 8 in. by 7 in. rectangle.  Step 1: Identify *B* and *h* $B = 8\ in \times 7\ in = 56\ in^2$ *h* = 6 in	To find the volume of a cone, we use the formula $V = \frac{1}{3}\pi r^2 h$. As before with finding the volume of a cylinder, *r* stands for radius and *h* stands for the height of the cone.  Step 1: Identify *r* and *h* *r* = 6 in *h* = 8 in

Step 2: Substitute the values for *B* and *h*. $V = \frac{1}{3}Bh$ $V = \frac{1}{3}(8 \times 7)(6)$ $V = \frac{1}{3}(56)(6)$ Step 3: Evaluate using the order of operations. $V = \frac{1}{3}(56)(6)$ $V = \frac{1}{3}(336)$ $V = 112\ in^3$	Step 2: Substitute the values for *r* and *h*. $V = \frac{1}{3}\pi r^2 h$ $V = \frac{1}{3}\pi(6^2)(8)$ Step 3: Change π to 3.14, and then evaluate using the order of operations. $V = \frac{1}{3}(3.14)(6^2)(8)$ $V = \frac{1}{3}(3.14)(36)(8)$ $V = \frac{1}{3}(3.14)(36)(8)$ $V = \frac{1}{3}(904.32)$ $V = 301.44\ in^3$

Now that we know the volume of both the cone and the pyramid, it is clear that the cone is much larger. To find out how much, we can subtract the volume of the pyramid from the volume of the cone using:

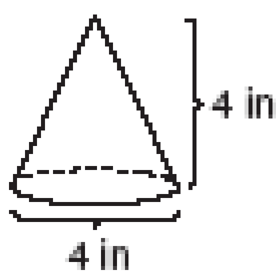
$301.44\ in^3 - 112\ in^3 = 189.44\ in^3$

The correct answer is Choice A, since the cone is almost, or approximately, 200 in^3 larger than the pyramid.

You Try – Volume of Cones and Pyramids

14. What is the volume of the container below, to the nearest hundredth of an in^3?

   A. 16 in^3
   B. 16.75 in^3
   C. 33.49 in^3
   D. 66.99 in^3

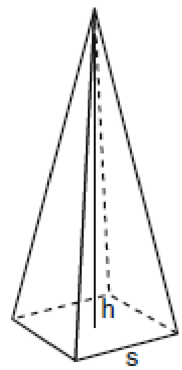


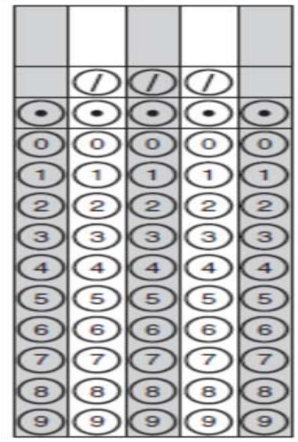
15. How is the volume of a cone affected by doubling the length of its radius?

   A. The volume will be unchanged
   B. The volume will be half as big
   C. The volume will be twice as big
   D. The volume will be four times greater

16. A regular pyramid with a square base is shown in the diagram below.

A side, *s*, of the base of the pyramid is 12 meters, and the height, *h*, is 42 meters. What is the volume of the pyramid in cubic meters?





17. A regular pyramid has a height of 12 centimeters and a square base. If the volume of the pyramid is 256 cubic centimeters, how many centimeters are in the length of one side of its base?

A. 8

B. 16

C. 32

D. 64

V. Volume of Spheres

Example 5

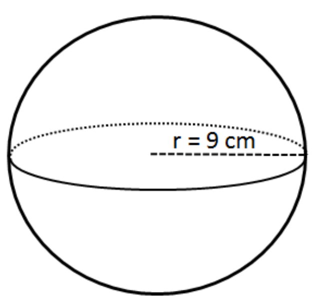
Find the volume of the sphere below to the nearest cubic centimeter.

A. 113 cm^3

B. 2916 cm^3

C. 3,052 cm^3

D. 30,092 cm^3



Solving Example 5

Use the volume of a sphere formula $V = \frac{4}{3}\pi r^3$ by following these steps.

Step 1: Identify *r*

$r = 9$ cm

Step 2: Plug *r* into the formula.

$$V = \frac{4}{3}\pi r^3$$

$$V = \frac{4}{3}\pi(9)^3$$

Step 3: Change π to 3.14, and then evaluate using the order of operations.

$$V = \frac{4}{3}(3.14)(9)^3$$
$$V = \frac{4}{3}(3.14)(729)$$
$$V = \frac{4}{3}(2{,}289.06)$$
$$V = 3{,}052.08 \text{ cm}^3$$

To the nearest cm^3, the answer is 3,052 cm^3, Choice C.

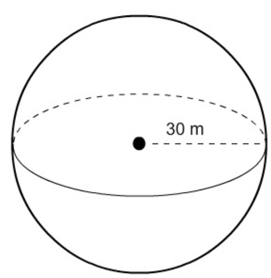
Testing Tip

There are several ways to find $\frac{4}{3}$ of a number. For example, if we want to find $\frac{4}{3}(9)$ we can:

- Change 9 to a fraction by placing it over 1. Then multiply across, and simplify.
  $\frac{4}{3}\left(\frac{9}{1}\right) = \frac{4\times9}{3\times1} = \frac{36}{3} = 12$
- Change 9 to a fraction by placing it over 1. Then divide 9 by 3. Then multiply across, and simplify.
  $\frac{4}{3}\left(\frac{9}{1}\right) = \frac{4}{\cancel{3}_{1}}\left(\frac{\cancel{9}^{3}}{1}\right) = \frac{4\times3}{1\times1} = \frac{12}{1} = 12$
- Using the official TASC TI-30XS calculator, type $\boxed{4}$ $\boxed{\div}$ $\boxed{3}$ $\boxed{\times}$ $\boxed{9}$ $\boxed{ENTER}$ and the result is 12.

You Try – Volume of a Sphere

18. What is the volume of the container below, to the nearest hundredth of an m^3?



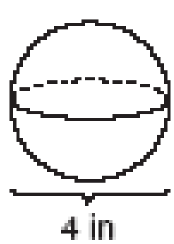
A. 376.8 m^3

B. 63,585 m^3

C. 113,040 m^3

D. 1,114,529.18 m^3

19. What is the volume of the container below, in terms of π in^3?



A. 8π in^3

B. 10.67π in^3

C. 16π in^3

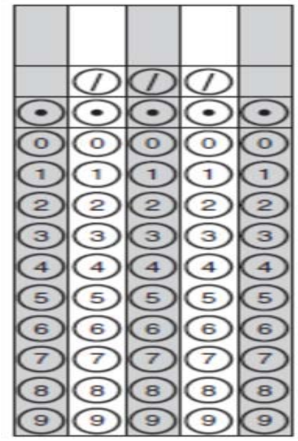
D. 85.33π in^3

VI. Density

Example 6

A tank in the shape of a cube contains 64 cubic feet of ammonium nitrate. The weight of the ammonium nitrate in the tank is 3,904 pounds.

What is the density of the ammonium nitrate in the tank in pounds per cubic foot?



Solving Example 6

We can loosely say that density is a comparison of how much stuff we have to the amount of space the stuff is found in. The same amount of stuff crammed into a smaller space is said to be more dense. For example, we can take a nice, fluffy blanket and cram into into a small sack so that it forms a small hard ball. We still have the same blanket, only now it is squeezed into a tiny space. We have the same amount of stuff, the blanket, but now that it is crammed into a smaller space, we say that it is *denser*.

More scientifically, we refer to stuff as *mass* and 3-D space as *volume*. The formula for density then is:

$$Density = \frac{mass}{Volume} \quad \text{or} \quad D = \frac{m}{V}$$

To solve Example 6, we can follow these steps.

Step 1: Identify the mass and the volume.

Remember, mass is generally measured in units such as pounds (lb), ounces (oz), grams (g) or kilograms (kg). And volume is generally measured in units such as cubic feet (ft^3), cubic inches (in^3), cubic meters (m^3), liters (l), or gallons (gal).

mass = m = 3,904 pounds

Volume = V = 64 cubic feet

Step 2: Substitute the values for m and and V into the formula.

$$D = \frac{3{,}904}{64}$$

Step 3: Divide

$$D = \frac{3{,}904}{64} = 3{,}904 \div 64 = 61\ lb/ft^3$$

You Try – Density

20. A tank in the shape of a right cylinder contains 8 m^3 of gasoline. The mass of the gasoline in the tank is 5,757 kilograms (kg). What is the density of the gasoline in the tank in kilograms per cubic meter?

    A. 0.00139 kg/m^3

    B. 11.244 kg/m^3

    C. 719.625 kg/m^3

    D. 5765 kg/m^3

21. A wooden cube has an edge length of 6 centimeters and a mass of 146 grams. Determine which type of wood the cube is made of given its density below in g/cm^3.

    A. Hemlock: density 0.431 g/cm^3

    B. Elm: density 0.554 g/cm^3

    C. Birch: density 0.601 g/cm^3

    D. Maple: density 0.676 g/cm^3

22. What is the mass of a 250 cm^3 block of plastic if the density of the plastic is 0.94 g/cm^3?

    A. 0.00376 grams

    B. 235 grams

    C. 266 grams

    D. 376 grams

VII. Population Density

Example 7

The city of Tulsa measures 186.8 mi^2 and the population is 403,090. What is the population density of Tulsa measured in people per square mile?

A. 11.55 $people/mi^2$

B. 2157.9 $people/mi^2$

C. 4634 $people/mi^2$

D. 8656 $people/mi^2$

Solving Example 7

Population density compares a number of people to the size of the space that the people live in using the proportion $Population\ Density = \frac{population}{land\ area}$. This is very similar to our other density problems, only the "stuff" here is people, and the space is an area, not a volume.

To solve example 7, we can follow these steps:

Step 1: Identify the population and land area.

Population = 403,090 people
Land area = 186.8 mi^2

Step 2: Substitute the population and land area into the formula

$$Population\ Density = \frac{403{,}090\ people}{186.8\ mi^2}$$

Step 3: Divide

$403{,}090 \div 186.8 = 2157.9$ people/mi^2 (**Choice B).**

You Try – Population Density

23. A 300 square mile city has a population of 2.5 million. What is the population density per square mile of the city?

A. 12 people per square mile

B. 83 people per square mile

C. 120 people per square mile

D. 8,333 people per square mile

24. The population and area of four cities are shown in the table below.

City	**Population**	**Area (km^2)**
Chennai, India	4,681,087	181.06
Guttenberg, New Jersey	11,481	0.507
Colombo, Sri Lanka	323,257	37
New York, New York	8,175,133	783.73

Based on the table, which choice below accurately lists the cities in order from least population density to greatest population density?

A. Colombo, New York, Guttenberg, Chennai

B. Guttenberg, Colombo, Chennai, New York

C. Guttenberg, Chennai, Colombo, New York

D. Chennai, Guttenberg, New York, Colombo

25. According to the 2010 Census, New York State had a population density of 411.2 people/mi^2 and a population of 19,378,102 million people. Based on this information, what is the area of New York State to the nearest square mile?

A. 2,122 mi^2

B. 9,640 mi^2

C. 47,126 mi^2

D. 826,110 mi^2

VIII. Transformations

Example 8

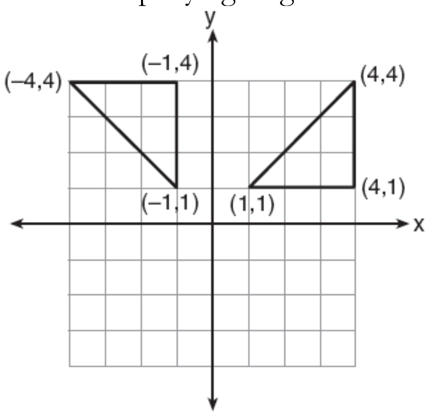
Which type of transformation is illustrated in the accompanying diagram?

A. Reflection across the *y*-axis

B. Dilation by a scale factor of 2

C. Rotation 90° about the origin

D. Translation 2 units right



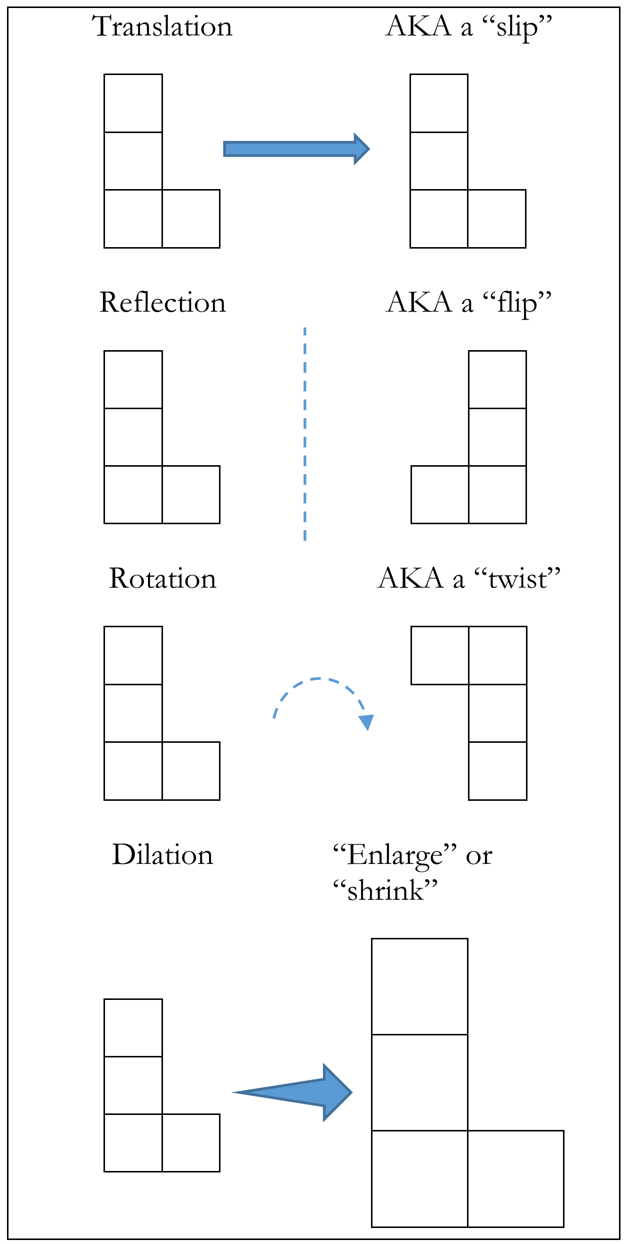
Solving Example 8

Let's first define *transformation*.

Transformation: a change in the position, orientation, or size of a figure (bot not shape).

On the TASC exam, there are four types of transformations you may see: translation, reflection, rotation, and dilation.

In Example 8, we can't tell which figure is the original. Let's say we start with the triangle in the upper left had corner and then translate, or move, it to become the triangle in the upper right hand corner. We can see that the triangle was "twisted", or *rotated*, because the right angle for the triangle went from the upper right hand corner to the lower right hand corner. We can also see that the rotation was 90° about the origin because the rotation was a quarter turn. To see this, turn your paper one quarter turn to the right. Each triangle twists to be in the next quadrant, or quarter, of the coordinate plane. Therefore, the correct answer is **Choice C.**



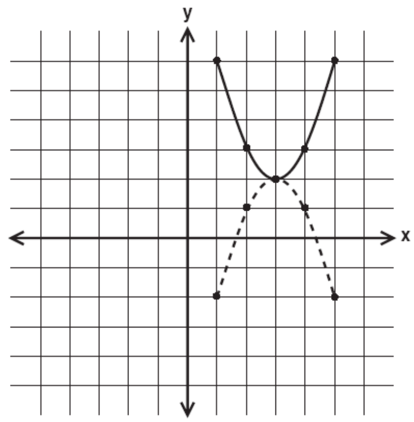
Testing Tip

According to the DRC, the makers of TASC, all testers are entitled to graph paper and scrap paper when they take the TASC or the Readiness Test. Graph paper can be incredibly helpful on transformation problems. Ask your testing coordinator for a piece of graph paper if one is not given to you.

You Try – Transformations

26. Which type of transformation is illustrated in the accompanying diagram?

    A. Translation
    B. Line reflection, only
    C. Rotation, only
    D. Line reflection or rotation



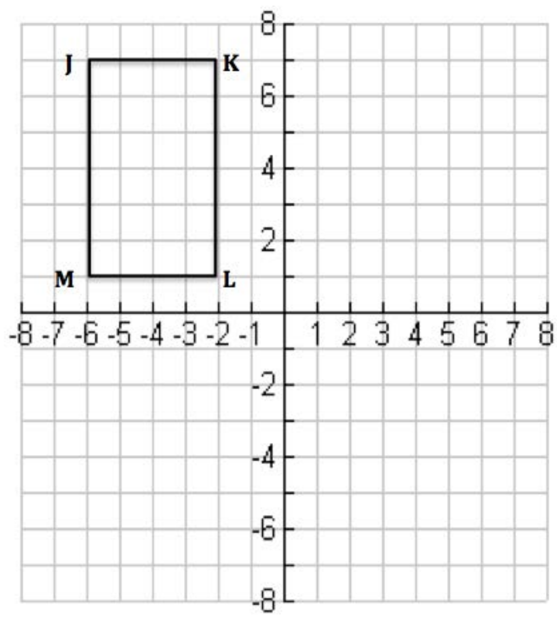
27. Which transformation does *not* always produce an image that is congruent to the original figure?

    A. Translation
    B. Dilation
    C. Rotation
    D. Line Reflection

28. Suppose Rectangle JKLM, shown at right, is translated to another location to form J'K'L'M'.

    If three vertices of J'K'L'M' are J'(-3, 5), K'(1, 5), and L'(1, -1), which could be the coordinates of vertex M'?

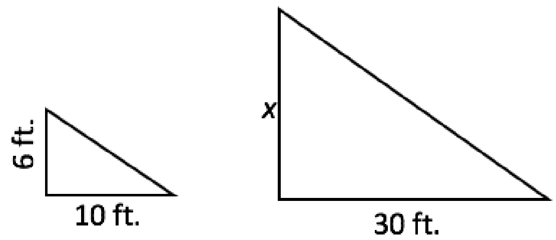
    A. (-6, 1)
    B. (5, -3)
    C. (1, 1)
    D. (-3, -1)

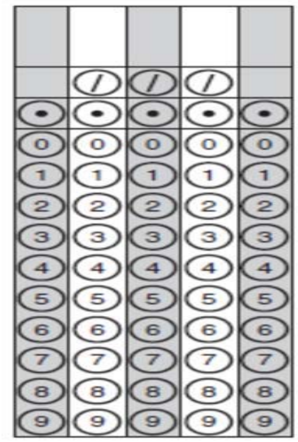


IX. Similar Triangles

Example 9

On a sunny day, the village inspector used similar triangles to find the height of a flagpole without climbing it. She found that her 6-foot tall coworker cast a 10-foot shadow at the same time the flagpole cast a 30-foot shadow. How tall is the flagpole?





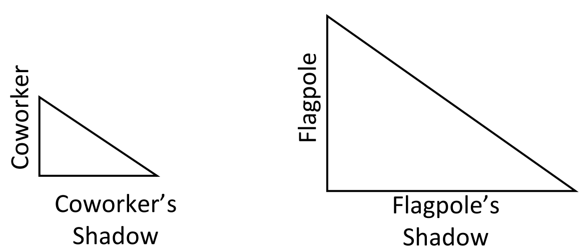
Solving Example 9

The key to Example 9 is that these triangles are *similar.*

Similar figures: two figures that are the same shape, but a different size.

When you think of similar figures think of a photograph of you. The picture of you is exactly like you in shape, but not in size.

A defining factor of similar figures is that their *corresponding sides* are *proportional.* Corresponding sides are sides that are in the same position. Let’s look at our example first without numbers and set up a proportion using corresponding sides. Tthe coworker corresponds with the flagpole, and the coworker’s shadow corresponds with the flagpole’s shadow.



$$\frac{\text{coworker}}{\text{coworker's shadow}} = \frac{\text{flagpole}}{\text{flagpole's shadow}}$$

Replacing our labels with the values we have, we get:

$$\frac{6}{10} = \frac{x}{30}$$

To find x, we can cross-multiply, and then divide.

$10x = (6)(30)$

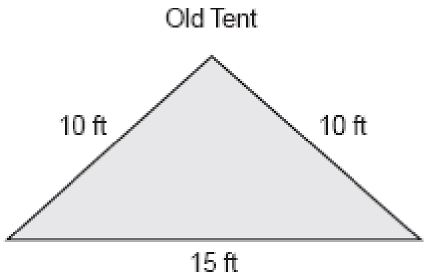
$10x = 180$

$\frac{10x}{10} = \frac{180}{10}$

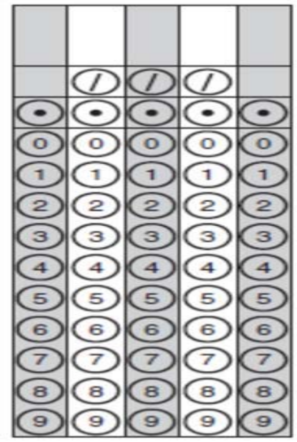
$x = 18$ ft.

You Try – Similar Figures

29. The Castro family bought a new tent for camping. Their old tent had equal sides of 10 feet and a floor width of 15 feet, as shown in the accompanying diagram.



If the new tent is similar in shape to the old tent and has equal sides of 16 feet, how wide is the floor of the new tent?



30. The base of an isosceles triangle is 5 in and its perimeter is 11 in. The base of a similar isosceles triangle is 10 in. What is the perimeter of the larger triangle?

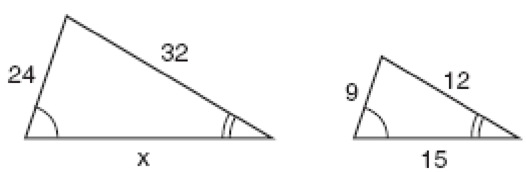
A. 15 in

B. 22 in

C. 21 in

D. 110 in

31. The triangles at right are similar.



Which proportion could be used to solve for *x*?

A. $\frac{x}{24} = \frac{9}{x}$

B. $\frac{24}{9} = \frac{15}{x}$

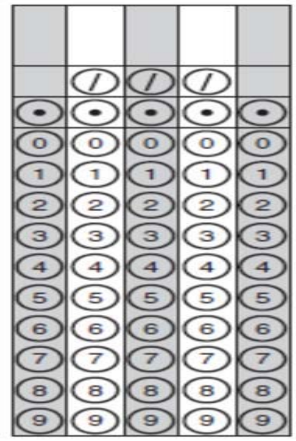
C. $\frac{32}{x} = \frac{12}{15}$

D. $\frac{32}{12} = \frac{15}{x}$

X. Pythagorean Theorem

Example 10

A painter leans the top of a ladder against a windowsill that sits 12 feet from the ground. The base of the ladder rests 5 feet from the house. How long is the ladder?

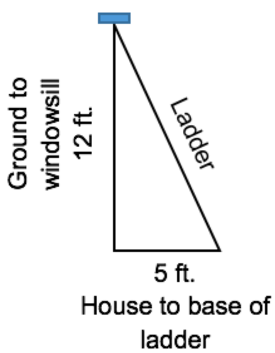


Solving Example 10

Many people recall that the Pythagorean Theorem has something to do with $a^2 + b^2 = c^2$. But, what does that have to do with ladders, and how do we know when we have a Pythagorean Theorem problem?

Pythagorean Theorem: used to find the third side of a right triangle when two sides are known.

Let's <u>make a diagram</u> to see how this question relates to a right triangle.



When we draw the diagram, we see that we are dealing with a right triangle with legs that measure 5 ft. and 12. We need to find the length of the third side, the ladder, or *hypotenuse*. To do so we can use $a^2 + b^2 = c^2$, where a and b stand for the lengths of the legs, and c stands for the length of the hypotenuse. Remember, the hypotenuse is always the longest side of a right triangle. It is found opposite the right angle.

Step 1: Identify the known sides.

$a = 5$ ft.
$b = 12$ ft.

Step 2: Substitute the known sides into $a^2 + b^2 = c^2$.

$5^2 + 12^2 = c^2$

Step 3: Evaluate and solve for the missing side.

$25 + 144 = c^2$
$169 = c^2$
$c = \sqrt{169}$
$c = 13$ ft. The ladder is 13 ft. long.

Testing Tip

The most common mistake when using the Pythagorean Theorem is to forget to take the square root. This happens because students often work across instead of down and lose their variable in the process. **Incorrect** $3^2 + 4^2 = c^2$ $9 + 16 = 25$ Answer: 25 **Correct** $3^2 + 4^2 = c^2$ $9 + 16 = c^2$ $25 = c^2$ $c^2 = 25$ $c = \sqrt{25} = 5$ Answer: 5

You Try – Pythagorean Theorem

32. If the length of the legs of a right triangle are 5 meters and 7 meters, what is the approximate length of the hypotenuse in meters?

A. 6 meters

B. 8.6 meters

C. 12 meters

D. 74 meters

33. A painter leans a 10 foot ladder against a windowsill. The base of the ladder rests 6 feet from the house. How high is the windowsill from the ground?

A. 8 feet

B. 10 feet

C. 16 feet

D. 64 feet

34. Two college roommates, Henry and Harry, leave college at the same time. Henry travels south at 25 miles per hour and Harry travels west at 45 miles per hour. To the nearest tenth of a mile, how far apart are they at the end of two hours?

A. 51.5 miles

B. 70 miles

C. 103 miles

D. 10,600 miles

Essential Geometry Practice for Students of TASC-math
Answer Key

Solutions to: You Try – Precise Definitions of Geometric Figures

#	√	Explanation
1	A	A line segment has two endpoints. Between the two endpoints we find an infinite number of points since points do not have size.
2	A	Perpendicular lines are lines that intersect to form right angles, which measure 90°.
3	D	Every point on a circle is the same distance from the center of the circle. If we take any point, and extend exactly *r* distance, for example 2 inches, from that point in every direction, we would form a circle. Our example circle would have a radius of 2 inches and a diameter of 4 inches.
4	D	Circumference is the distance around a circle. The entire circumference of a circle, measured in degrees, would measure 360°. An arc is known as a portion of a circumference. One-fourth of 360° can be found using $360° \div 4 = 90°$.
5	C	To name an angle we often use three capital letters where each of the letters name a point on the angle. The middle letter always names the vertex, and the other two letters name points on the two legs of the angle. Choice C is correct because $\angle ACG$ and $\angle BCG$ actually name the same angle. Since they are the same angle with a different name, they are congruent, or the same.

Solutions to: You Try – Volume of Prisms (Including Cubes)

6	36	The formula for the volume of a rectangular prism is $V = Bh$ or $V = lwh$. We know: l = length = 3 cm. w = width = 2 cm. h = height = 6 cm $V = (3 \times 2)(6) = (6)(6) = 36$ cm^3
7	B	The formula for the volume of a rectangular prism (a fish tank) is $V = Bh$ or $V = lwh$. We know: V= Volume = 5,040 in^3 l = length = 20 in. w = width = 14 in. We can then substitute those values into the formula: $5,040 = (20)(14)h$ Then, evaluate. $5,040 = 280h$ And then divide by 280 to solve for h. $\frac{5,040}{280} = \frac{240}{280}h$ h = 18 in.
8	4	The volume of a cube can be found using the formula $V = lwh$. But, since we know that all of the edges of a cube have the same measurement, we know that $l = w = h$. We can then say that $V = l^3$ or $V = s^3$, where s = side length. We know the volume of this cube measures 64 in^3, so we can then write $64in^3 = s^3$. To find s, we need to find a number whose third power is 64, or $s \times s \times s = 64in^3$. We can take the cube root of 64 by doing $s = \sqrt[3]{64in^3} = 4in$
9	C	Before we can use the chart to answer the question, we must first determine the volume of the room using $V = (8)(10)(8) = 640$ ft^3. Now that we know we have a 640 ft^3 room, with a 20 ft. duct length, we can look on the chart and determine that Brooke should buy at least a 90 cfm bathroom fan.

Solutions to: You Try – Volume of Cylinders

10	C	Use the volume of a cylinder formula $V = \pi r^2 h$. Step 1: Identify *r* and *h*. $r = 4$ in (find the radius by taking half of the diameter, 8 in.) $h = 10$ in Step 2: Substitute the values for *r* and *h* into the formula. $V = \pi(4^2)(10)$ Step 3: Change π to 3.14, and then evaluate using the order of operations. $V = (3.14)(16)(10)$ $V = 502.4\ in^3$
11	C	Use the volume of a cylinder formula $V = \pi r^2 h$. Step 1: Identify *r* and *h*. $r = 5$ in $h = 7$ in Step 2: Substitute the values for *r* and *h* into the formula. $V = \pi(5^2)(7)$ Step 3: Evaluate using the order of operations. Keep the π symbol until the end because all of the answer choices are in terms of pi. $V = \pi(25)(7)$ $V = 175\pi\ in^3$
12	A	Use the volume of a cylinder formula $V = \pi r^2 h$. Step 1: Identify *r* and *h*. $r = 2.5$ m (find the radius by taking half of the diameter, 5 m.) $h = 2$ m Step 2: Substitute the values for *r* and *h* into the formula. $V = \pi(2.5^2)(2)$ Step 3: Change π to 3.14, and then evaluate using the order of operations. $V = (3.14)(6.25)(2)$ $V = 39.25\ m^3$
13	47	First, find the volume of the rectangular container using $V = Bh$ or $V = lwh$. $V = (15 \text{ in} \times 20 \text{ in})(10 \text{ in}) = 3000 \text{ in}^3$ Next, find the volume of the cylindrical cup using the formula $V = \pi r^2 h$. Step 1: Identify *r* and *h*. $r = 2$ in (find the radius by taking half of the diameter, 5 m.) $h = 5$ in Step 2: Substitute the values for *r* and *h* into the formula. $V = \pi(2^2)(5)$ Step 3: Change π to 3.14, and then evaluate using the order of operations. $V = (3.14)(4)(5)$ $V = 62.8\ in^3$ By dividing 3000 in^3, the size of the rectangular container, by 62.8 in^3, the size of the cup, we can find out how many full cups of water can be placed <u>into</u> the container. $3000 \div 62.8 = 47.77$ cups. We need to figure out how many full cups of water fit. If we tried to pour 48 full cups of water in the container would overflow since it only holds 47.77 cups. Therefore, the answer is 47 full cups of water.

Solutions to: You Try – Volume of Pyramids and Cones

14	B	To find the volume of a cone, we use the formula $V = \frac{1}{3}\pi r^2 h$. Step 1: Identify *r* and *h*, $r = 2$ in, $h = 4$ in.

		Step 2: Substitute the values for *r* and *h* into the formula. $V = \frac{1}{3}\pi(2^2)(4)$ Step 3: Change π to 3.14, and then evaluate using the order of operations. $V = \frac{1}{3}(3.14)(4)(4)$ $V = \frac{1}{3}(50.24)$ $V = 16.75\ in^3$
15	D	To see how the volume of a cone is affected by doubling the length of its radius we can take a cone, find its volume, and then double its radius, and find the new volume. Remember, when we use 3.14 as our approximate value for π, our calculations are not exact. Here, we will leave π to make comparing easier. Let's say, for example, we have a cone with a radius of 2 m, and a height of 3 m. To find the volume of this cone we use the formula $V = \frac{1}{3}\pi r^2 h$. Substituting the values for *r* and *h*, we find $V = \frac{1}{3}(\pi)(2^2)(3) = 4\pi\ m^3$. Then, we can double the radius of our original cone from 2 m to 4 m, and keep the same height of 3 m. The volume of our new cone is $V = \frac{1}{3}(\pi)(4^2)(3) = 16\pi\ m^3$. When we compare the original volume of $4\pi\ m^3$, to the new volume of $16\pi\ m^3$ we see that the volume became 4 times greater as a result of doubling the radius.
16	2016	To find the volume of a pyramid, use the formula $V = \frac{1}{3}Bh$. Because we are dealing with a square pyramid, we know that the sides of the base each measure 12 m. To find *B*, the area of the base, we use $B = 12 \times 12 = 144\ m^2$. To find the volume of the pyramid we substitute 16 m^2 for *B*, and 42 for *h*, the height. $V = \frac{1}{3}(144)(42) = \frac{1}{3}(6048) = 2016\ m^3$.
17	A	Here we know the volume of the pyramid, 256 cm^3, and the height is 12 cm. Substituting 256 for *V* and 12 for *h,* we have $256 = \frac{1}{3}B(12)$. To find *B*, we evaluate and then divide. $256 = \frac{12B}{3}$ 256 = 4*B* $\frac{4B}{4} = \frac{256}{4}$ $B = 64\ cm^2$ We know that the base is a square with an area of 64 cm^2. We can determine the length of a side of the square base, *s*, by taking the square root of 64 cm^2. $s = \sqrt{64} = 8\ cm$.

Solutions to: You Try – Volume of Spheres

18	C	To find the volume of a sphere use $V = \frac{4}{3}\pi r^3$. Here *r* = 30 *m*. $V = \frac{4}{3}\pi(30^3)$ $V = \frac{4}{3}\pi(27{,}000)$ $V = 36{,}000\pi = 36{,}000(3.14) = 113{,}040\ m^3$
19	B	To find the volume of a sphere use $V = \frac{4}{3}\pi r^3$. Here *r* = 2 *in*. $V = \frac{4}{3}\pi(2^3)$ $V = \frac{4}{3}(8\pi)$ $V = 10.67\ in^3$

Solutions to: You Try – Density

20	C	To find Density, use $D = \frac{m}{V}$, where *m* = mass, and *V* = volume. First, identify *m* and *V*. *m* = 5,757 kg, *V* = 8 m^3. Then substitute the values into the formula.

		$D = \frac{5{,}757}{8}$, and then divide. D = 719.625 kg/m^3
21	D	Here we are given the mass, m, of 146 g, and only the side length of the wooden cube of 6 cm. To find the volume of the cube we find V = (6)(6)(6) = 6^3 = 216 cm^3. To find the density of the cube use $D = \frac{m}{V}$. $D = \frac{146\ g}{216\ cm^3} = 0.676$ g/cm^3. The wooden cube is made of maple.
22	B	Here we're given the volume, V = 250 cm^3, and the density, D = 0.94 g/cm^3, of the sample. To find the mass, or m, we need to first substitute our values into the formula. $0.94 = \frac{m}{250}$. To solve for m we can multiply both sides of the equation by 250. $250 \times 0.94 = \frac{m}{250} \times 250$ m = 235 grams

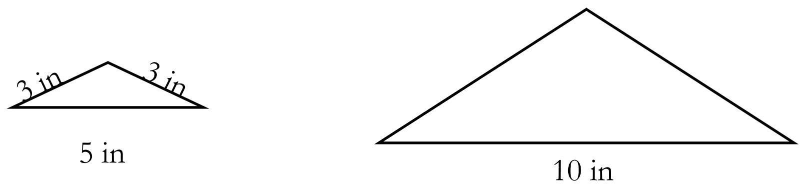
Solutions to: You Try – Population Density

<table>
<tr><td>23</td><td>D</td><td>To find population density we use $population\ density = \frac{population}{area}$.
Population density = $\frac{2{,}500{,}000\ people}{300\ mi^2} = 8333.\overline{3}$ people/mi^2.</td></tr>
<tr><td>24</td><td>A</td><td>To compare the population densities of the cities on the table, we will first calculate the population densities of each. We added a column for population density.
<table>
<tr><th>City</th><th>Population</th><th>Area (km^2)</th><th>Population Density</th></tr>
<tr><td>Chennai, India</td><td>4,681,087</td><td>181.06</td><td>$\frac{4{,}681{,}087}{181.06}$ = 25,854 people/km^2</td></tr>
<tr><td>Guttenberg, New Jersey</td><td>11,481</td><td>0.507</td><td>$\frac{11{,}481}{0.507}$ = 22,645 people/km^2</td></tr>
<tr><td>Colombo, Sri Lanka</td><td>323,257</td><td>37</td><td>$\frac{323{,}257}{37}$ = 8,737 people/km^2</td></tr>
<tr><td>New York, New York</td><td>8,175,133</td><td>783.73</td><td>$\frac{8{,}175{,}133}{783.73}$ =10,431 people/km^2</td></tr>
</table>
We can now see that put in order from least population density to greatest we have: Colombo, New York, Guttenberg, Chennai.</td></tr>
<tr><td>25</td><td>C</td><td>Here we know population density, 411.2 people/mi^2, and population, 19,378,102 people, of New York State, but need to find its area using this information. Using $population\ density = \frac{population}{area}$, we can establish $411.2\ people/mi^2 = \frac{19{,}378{,}102\ people}{A}$, where A stands for area. To solve for A, we can establish a proportion, and then cross-multiply and then divide.
$\frac{411.2\ people/mi^2}{1} = \frac{19{,}378{,}102\ people}{A}$
(411.2 people/mi^2)(A) = 19,378,102 people
$A = \frac{19{,}378{,}102\ people}{411.2\ people/mi^2}$
$A = 47{,}125.734\ mi^2$</td></tr>
</table>

Solutions to: You Try – Transformations

26	D	The pre-image, shown with the solid line, could move to the position of the image, shown with the dashed line, by either line reflection, "flipping," or by rotation, "twisting."
27	B	Translation, rotation, and line reflection change the position of a figure, but not it's shape or size. Dilation however, involves a change in size of a figure. Shapes that are the same shape, but a different size are called *similar*, but not *congruent*.
28	D	Under a translation, all points of Rectangle JKLM will move in the same manner to make J'K'L'M'. We can put our pencil at J, and see that it was moved two units down, and three units right to become J'. We can then take our pencil, and move M two units down, and three units right and find that M' will be located at (-3, -1).

Solutions to: You Try – Similar Triangles

29	24	We can set up a proportion to compare the sides of the old tent, to the sides of the new tent $\frac{10\,ft}{15\,ft} = \frac{16\,ft}{x}$, where x stands for the width of the floor of the new tent. We can then cross-multiply and divide to solve for x. $10x = 240$ $\frac{10x}{10} = \frac{240}{10}$ $x = 24$ ft
30	B	It is best here to draw a diagram. An isosceles triangle has two legs that are the same length, and a base that is a different length than the legs. Since we know the perimeter of the original triangle is 11 in, that means the legs must be 3 in since 3 + 3 + 5 = 11.  We can see that the similar triangle is twice as big as the original triangle since the 5 in side corresponds with the 10 in side. That means that the other two sides must measure 6 in each. We can find the new perimeter by adding the lengths of the three sides of the new, similar triangle. Perimeter = 6 in + 6 in + 10 in = 22 in.
31	C	We can use several proportions to solve the same similar triangle problem. Whichever proportion we use it must be one using corresponding sides in the same ratio on both sides of the equation. Only Choice C, $\frac{32}{x} = \frac{12}{15}$, does that here.

Solutions to: You Try – Pythagorean Theorem

32	B	Knowing the lengths of two sides of the right triangle, 5 m and 7 m, we can find the length of the hypotenuse using $a^2 + b^2 = c^2$. $5^2 + 7^2 = c^2$ $25 + 49 = c^2$ $c^2 = 74$ $c = \sqrt{74} = 8.6$ m
33	A	A ladder leaning against a wall forms a right triangle with the ladder making up the hypotenuse. Using 6 ft as a leg, and 10 ft as the hypotenuse of our right triangle, we have: $6^2 + b^2 = 10^2$ $36 + b^2 = 100$ $b^2 = 100 - 36$ $b^2 = 64$ $b = \sqrt{64} = 8$ ft
34	C	Travelling south for two hours at 25 mph Henry travels a total of 50 mi south. Harry travels west for two hours at 45 mph. He travels a total of 90 mi west. If we draw a diagram, we will see that their paths form a right angle. The distance between the two of them then forms the hypotenuse of a right triangle. We can then use the Pythagorean Theorem to calculate the distance between them. $50^2 + 90^2 = c^2$ $2{,}500 + 8{,}100 = c^2$ $10{,}600 = c^2$ $c = \sqrt{10{,}600}$ $c = 102.956$ miles, to the nearest tenth of a mile is 103 miles.