When Does Common Core Math = Fun?

At a Math Fair!

Patricia Helmuth

A How-To Guide

In the Spring of 2015, the Adult Program of Sullivan County BOCES, NY hosted a full day *Student Numeracy Adventures Day*. This is the story of what happened that day and how you can plan a math fair for your program.

A Math Fair How-To Guide

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A digital copy of this How-To Guide, PowerPoint Presentation for Professional Development, videos, and more available at: www.hudsonvalleyraen.org.

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INTRODUCTION

Let’s face it. Adult students don’t usually equate math with having fun. On occasion we may have an adult student enter our class who actually likes math, because she was good at making application of rules and procedures when last she was in a math class. When asked, that student will tell you that she just needs to “brush up” on her math skills. Meaning, she’s forgotten those rules and procedures that allowed her to get by in math class before.

For the student who breaks out into a cold sweat at the thought of math, which I’ve actually seen happen, he thinks that his prior negative experiences in math class were his fault. He could never remember the times tables or the rules for manipulating fractions, decimals, or percentages. Algebra was a foreign language altogether, spoken only by math people.

Interestingly, what both of these students have in common is a need to develop number sense, algebraic thinking, and flexibility with numbers. Instead of focusing on memorizing rules and disconnected facts, they are better served when they learn connections within and between mathematical concepts and content.

So, what if, instead of showing our students how to solve a math problem, we provide them with opportunities to develop problem solving strategies? What if we give them problems that they can solve even though they don’t know, or can’t remember, rules and procedures? What if they have fun doing it, together, in a paired or group activity?

Jason exemplifies the results of implementing the “what ifs”. When I met him he had just entered a recovery program for substance abuse. He dropped out of school in the eighth grade and when he enrolled in my class, it was, according to him, “for the wrong reasons. I didn’t have high expectations for myself.” However, his attitude changed. “After a few weeks in class,” he continues, “I excelled in things I never thought I would and was focused on getting my High School Equivalency Diploma.”

While he was attending my class we did not cover every topic that I thought he might see on the TASC exam. Rather, we focused on problem solving in group settings. When he took the TASC exam he passed! He exclaimed, “To my surprise I passed with a higher score than I imagined! I am now excited about a future I never thought I would have. My diploma hangs on my wall and when I look at it, it inspires me.” He has plans to enroll in a community college in September.

Jason was at the Student Numeracy Adventures Day last year, and in my final thoughts, I’ll talk about his reflections of that day. One student story is worth a thousand words.

STUDENT NUMERACY ADVENTURES DAY
MAY 14, 2015 – A REVIEW

SULLIVAN COUNTY BOCES HOSTS "STUDENT NUMERACY ADVENTURES DAY"

Andrew Carnright, Director
Hudson Valley RAEN

All of us are well aware that the increasing rigor on the math sections of the new High School Equivalency Exam has caused lots of anxiety and stress for teachers, students, and administrators alike. In order to address this "new" math, some programs have taken steps to acclimate their teachers and students to new ways of thinking about math and how best to teach and learn it.

I was lucky enough to be invited to yesterday's Student Numeracy Adventures Day at Sullivan County BOCES where this progressive approach to the content was on full display. Students from all of the different adult education classes in Sullivan County were brought to the St. John's Street School in Monticello to spend the day immersed in all things math. The gymnasium was setup as a haven for students to come and try their skills at different math games, problem solving activities, and interactive manipulative-based questions. Students came from ESOL, ABE, HSE, and even continuing education classes to participate in the multi-session extravaganza which turned out to be a hit for everyone.

All of the different math activities were created and setup by ABE/HSE teacher Patricia Helmuth and under the direction of the Adult Education principal Linda Blanton. Patricia is well known in the Hudson Valley for her work in Numeracy and was recently published in an issue of Adult Numeracy Network's (ANN) newsletter, along with presenting on the topic at the annual COABE conference in Denver, CO.

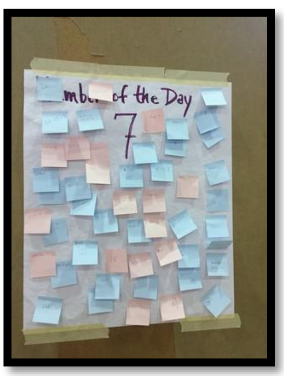
The setup and preparation for Student Numeracy Adventures Day took several weeks, many staff meetings, and lots of buy-in from teachers and student alike. Patricia came up with the floor plan and list of activities for the day. According to Patricia, "the idea for this Student Numeracy Day grew out of workshops that I presented at SC BOCES. We tossed around several ideas for a program wide event and this was suggested after the teachers had fun at our first 'problem solving' workshop. In putting the day together, I decided to develop activities that every student at every level would have access to, while at the same time provide opportunities and challenges for students at more advanced levels. We wanted students to be engaged and see that math could be a fun adventure."

Patricia went on to talk about the motivation for all of the different activities that were put together for the students noting, "All but a few of the activities I've tried out in my classroom. Some came from my Adult Numeracy Institute (ANI) PD, others came from CUNY (Teacher Leader Institutes in Albany), there were those that I developed myself, and then others that I've collected from colleagues. I wanted to use a variety of manipulatives that address numerous math topics at the conceptual level."

In discussing the layout and procession of the different math activities principal Linda Blanton noted that "Beginning the Student Numeracy Adventures Day with the number of the day activity (image 1), number line (image 2), and the product game (image 3) was key to boost student confidence. Students were then willing to attempt the more difficult activities." As for the variety in student activities Linda noted that the different activities, "allowed for all students to experience success and attempt additional math activities. It was great seeing students working with each other to complete tasks... Students truly enjoyed the day and their accomplishments!"

The day was a success, not only for the students, but for the teachers and support staff alike. Anyone and everyone in the building could see how a student-centered approach to math was driving what turned out to be an exciting and fun day. The questioning techniques Patricia had taught and practiced with her colleagues at Sullivan County BOCES led to open-ended discussions and collaboration amongst all participants. Even I, a self-admitted non-math person, got involved in the math spirit by working on problems and playing math games with some of the students who attended.

A true testament to the success of the day came when I spoke with a few young students who had just participated in the day's events. I asked them what they thought about the day. Both students told me they really enjoyed all of the activities and one of the boys finished with, "I really don't normally like math, but I do it because I am trying to get my high school diploma. But today was fun, much more fun than I thought it would be. It was really good."



This review of the *Student Numeracy Adventures Day* was originally posted at www.hudsonvalleyraen.org on May 15, 2015

INTERVIEW WITH LINDA BLANTON, PRINCIPAL
ADULT PROGRAM AT SULLIVAN COUNTY BOCES

Why did you decide to sponsor the math fair?

Personally I have struggled with math throughout my educational career, as far back as the sixth grade. My struggle and anxiety stemmed from a lack of confidence and the numbers not making any sense. I observed Patricia teaching a lesson at the Recovery Center to 12 students with a wide variety of abilities and confidence levels. Students were given a problem to solve but they chose how to approach the problem. This allowed them to decide on a strategy that fit their learning style, be it technology, manipulatives, and/or pictures. Patricia used a questioning technique that accounted for all of these differences and encouraged students to work through their doubts as well as justify the decisions they made in the process of solving the problem. The overall premise of the class was: it did not matter which strategy or method a student chose as long as the end result was correct.

I believe this approach to solving problems would have been advantageous to me during my education as I am an auditory and tactile learner. If I would have been allowed to choose a strategy or method that matched my learning style, I believe my math experiences would have been more positive.

I have also worked in several different capacities at SC BOCES as a Special Education teacher and Principal of Special Education and Alternative Education, where a majority of students shared the same frustrations as I did in regards to math and the requirement to pass the Algebra Regents. During my time at Adult Education, students from all corners of the globe with different experiences are preparing to take the TASC exam. Many of our students have not been in school for several years, experience language barriers, and struggle with the math required on the TASC exam.

What preparations do you think helped the math fair to be a success?

The preplanning was a contributing factor to the success of the event. A staff meeting was devoted to staff participating in math activities, where Patricia modeled the questioning strategies that enable participants to struggle through the process and solve the problem. I think it was interesting when staff shared their approaches, strategies, and rationale; how different each was! Before the event, the gym was set up with stations as a trial run and staff rotated to each station participating in each activity. Staff were then assigned a station to facilitate during the actual event.

What do you think worked out well during the math fair?

The event was held in the gymnasium which allowed plenty of room for the stations to be spread out. At the first station, the teacher was positive, inviting, and outgoing and every student was engaged and experienced success. This boosted their confidence levels and inspired them to attempt more difficult tasks.   I believe the arrangement of the stations (spread apart from each other, which limited distractions) had a positive impact on student success and willingness to attempt the subsequent activities at each station.

What would you say to another administrator who is thinking of sponsoring a math fair?

I would encourage other administrators to support this type of project/activity. Not only did it help to break down barriers, it gave all students a positive experience in math, encouraged problem solving, team work, and a sense of school community. Both breakfast, lunch, drinks and snacks were provided during the course of the day which promoted communication and discussion among all students.

PROFESSIONAL DEVELOPMENT FOR STAFF

Prior to our math fair, the staff at SC BOCES had their own math adventures. It started with: *A Shared Culture: Best Practices in Mathematics Learning and Instruction,* a set of workshops that featured Common Core instructional strategies that I learned at the NYSED/CUNY Mathematics Teacher Leader Training Institutes. Specifically we focused on:

- Common Core Standards for Mathematical Practice
- DOK Levels (Depth of Knowledge)
- Make sense of problems and persevere in solving them
- Open-ended and non-routine math problems
- Problem solving strategies
- TASC connections
- Facilitating productive struggle
- The art of questioning
- Examining student thinking
- Building meta-cognitive skills in our students

Learning how to change classroom practice so that math activities become student directed instead of being teacher centric takes some practice. In my case, I really needed to learn how to ask questions that effectively lead students to discover solutions to a problem rather than “show” students how to solve the problem This approach focuses authority on students as a problem solvers and helps students to develop meta-cognitive skills that will enable them to meet the demands of high-stakes exams and real-life problems at home and in the workplace with confidence.

I was excited about sharing these student directed activities with my colleagues and their response exceeded my expectations! We worked through two activities at our first workshop, the Zip-Zap-Zowie Problem and the Bicycle Shop Problem. Everyone enjoyed the activities so much that a suggestion was made that we should have a similar workshop for our students. At our next workshop we worked on the Painted Cube Problem and solidified our plan for a student math day.

It is essential for teachers to experience problem solving themselves, to have those satisfying aha moments, to share diverse problem strategies in a workshop setting, and to equip themselves with effective questioning resources if they are to be adequately prepared to help facilitate a math fair.

Drawing Out the 8 Common Core Math Practices with Questions* and additional teacher resources are in Appendix 1. The problems referred to are in Appendix 2. A PDF of the presentations: *A Shared Culture: Best Practices in Mathematics Learning and Instruction can be accessed at: CollectEdNY

OBJECTIVES & PLANNING

I met with our administrator, Linda Blanton, several times before the event. First, on the agenda was to decide upon our objectives for the math fair. We determined that we wanted the math fair to:

- Provide opportunity for teachers to practice asking effective questions when facilitating student driven open-ended math activities

- Demystify mathematics for students; show them that math can be fun

- Make math accessible for students at all ability levels

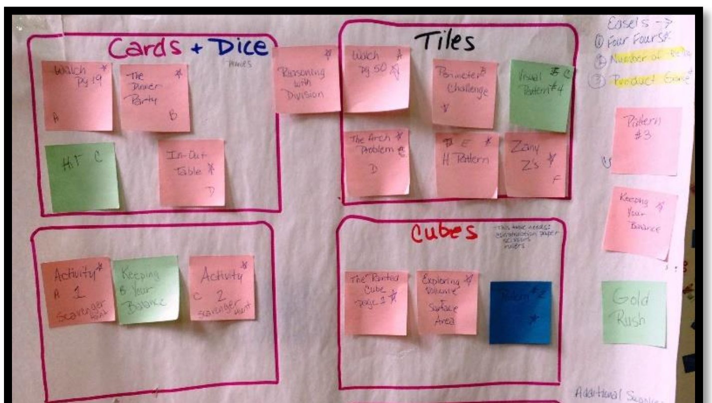
- Create community within our program, including our satellite locations

We elected to go for a full-day event that included, snacks, lunch, prizes for activities, and door prizes. Linda decided on granola bars and an assortment of additional snacks for mid-morning and mid-afternoon snacks, and she really liked the idea of pizza for lunch. She set about making all of that happen and I set about planning activities.

This proved to be the biggest challenge for me. I wanted to make sure that the activities had a low entry point so that all students, regardless of their levels, could find a way into the problems. At the same time, the activities would need to present a challenge for higher skilled students. I specifically looked for problems that lent themselves to using visual models and manipulatives as these activities help both lower and higher level students to develop conceptual understanding of math topics. To help myself narrow down the choices, I created the organizer that is pictured, and I found it extremely helpful.

Writing the activities that I was interested in using on sticky notes helped me to develop themed stations and to make decisions on which activities would best meet the objectives of the event. The six stations that emerged are as follows:

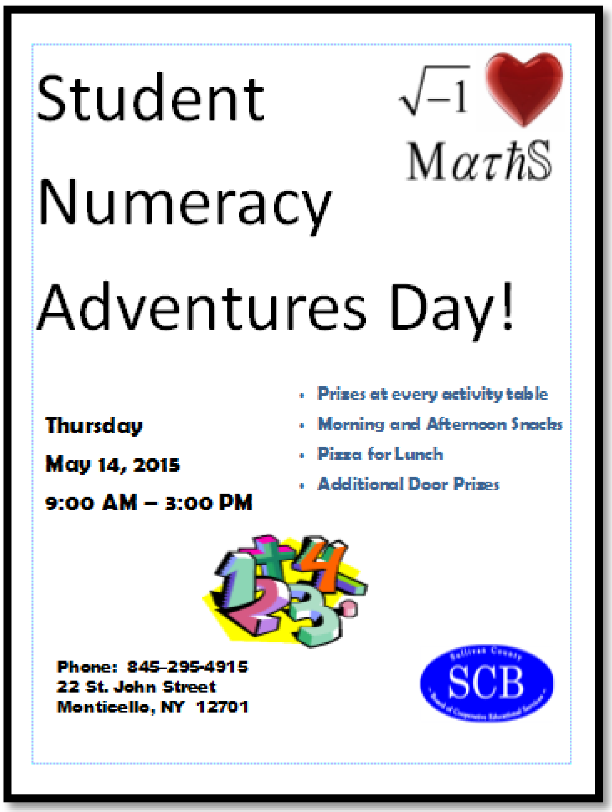
1. **Pattern Block Activities**
2. **Tile Activities**
3. **Snap Cube Activities**
4. **Cards & Dice Activities**
5. **Internet Station**
6. **Open-Ended Problems**



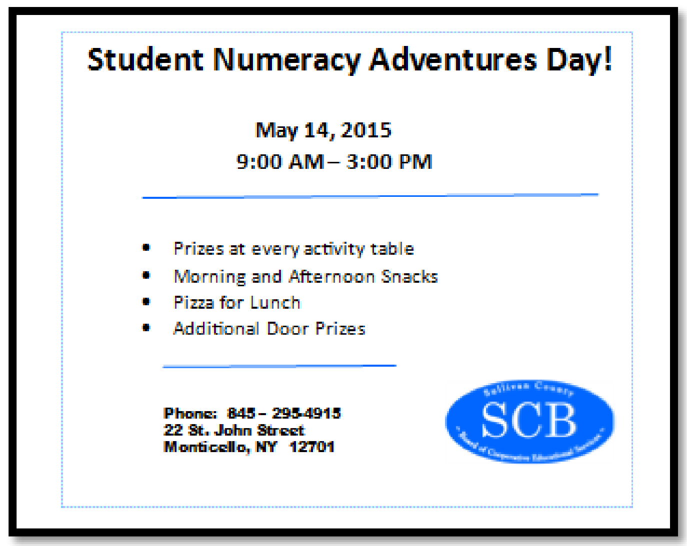
Many of the activities that we used for the Student Numeracy Adventures day are in **Appendix 2.** Links to additional activities are in **Appendix 3.**

PROMOTING THE EVENT

In addition to having a flyer for the event, which Chris Cimmino made up for us, I decided to create personal invitations to distribute to every student. I wanted to generate some positive anticipation for the math fair. I figured that if all the students knew that there would be pizza and prizes, they might be more inclined to want to attend, even though they might have a bit of math anxiety. All teachers were given invitations to distribute to their students. We made sure that every student got an invitation and we posted flyers everywhere!



The flyers, pictured on the left, were printed, in color on regular printer paper. The invitations below were printed out on card stock postcards.



All students were told that they would be attending the math fair during their regularly scheduled class(es). Teachers in our sattellite locations made arrangements for their students to come to our main building for the event. One teacher got a van and drove all her students to the math fair to make sure they had a chance to join in the fun!

The publisher template used for the student invitations is available online at: www.hudsonvalleyraen.org and CollectEdNY

MORE STAFF DEVELOPMENT

To prepare the staff for the student math fair we had a ***Teacher Numeracy Adventures Day***. The gym was set up with the stations that we planned on using for the student math fair and teachers were able to move from station to station and work through the various activities. Part of the objective of this staff development session was to get teachers thinking about how they could facilitate productive struggle by using effective questions to get students thinking during the actual *Student Numeracy Adventures Day*. Teachers were given a copy of *Drawing Out the 8 Common Core Math Practices with Questions* (Appendix 1) and asked to think about which stations they would be willing to man during the student math fair and which questions they might use to facilitate the activities at the station.

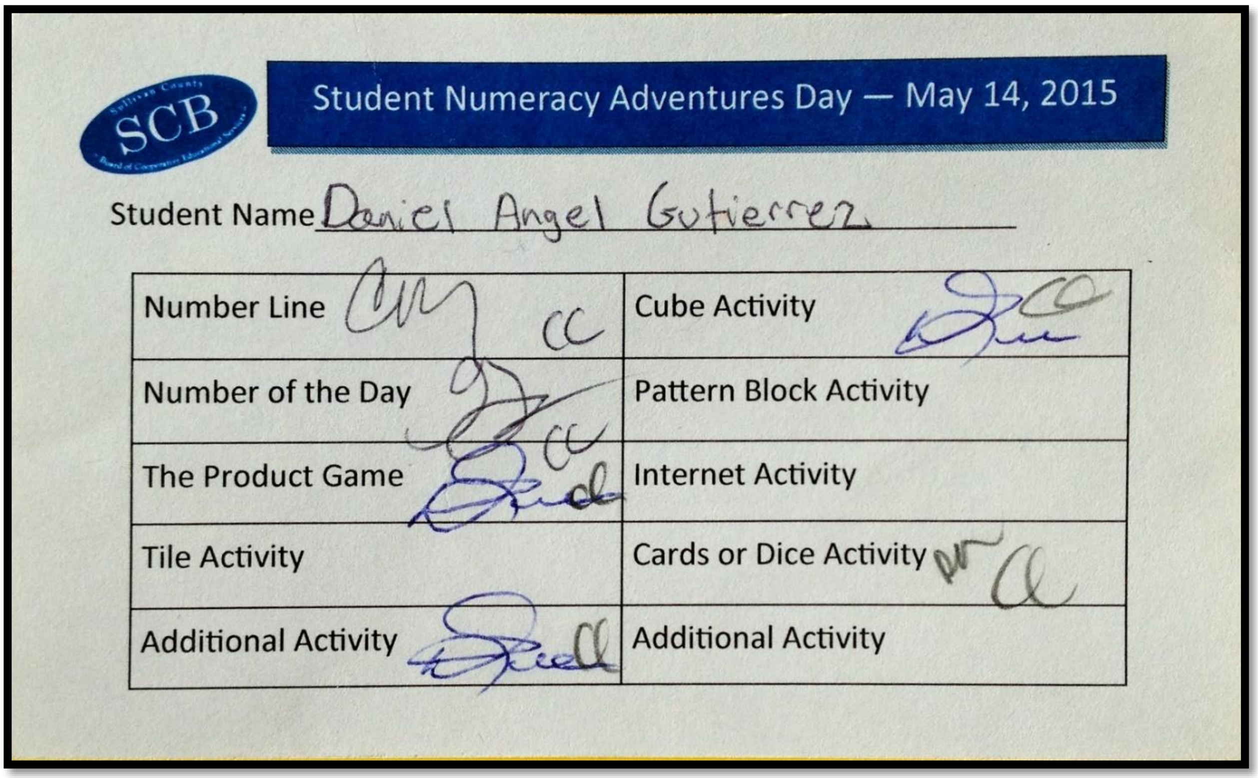
One added benefit that came of this teacher math day was that it made clear to me that a few of the activities I had chosen just didn't work well. The activity you see the teachers working on here involves putting together marshmallows with toothpicks and I thought it would be loads of fun for students, but it was just a bit too time consuming and complex.

We came together again, the day before the *Student Numeracy Adventures Day*, for a dry run, as you see pictured below. This was when teachers volunteered to man certain stations.

I would do this differently next time. The day before the event is not the best time to be confirming volunteers. I should have had a sign-up sheet that was passed around to teachers at the *Teacher Numeracy Adventures Day* pictured above to get the ball rolling. That way, I could have recruited more teachers as the day approached, if needed.

THE BUY IN

In order to make students feel welcome and get them involved right away in a few non-threatening math activities, they were greeted at the door with a sticky note, a number card, a bag to collect their prizes, and an index card. Then, they were guided to the first three activities listed on the left side of the index card. There was a teacher stationed at each of these three activities to facilitate.

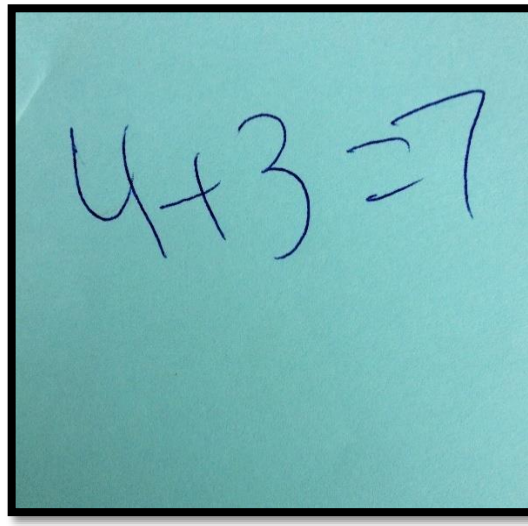


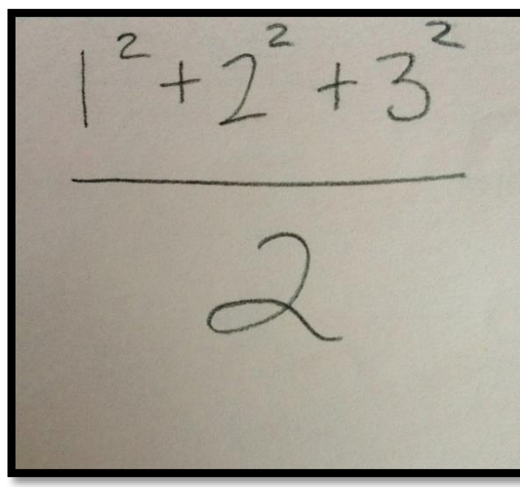
The card served a two-fold purpose. First, and probably most important to the students, they received a prize for every activity they finished. All they had to do was complete the activity, have the teacher facilitating the activity initial their card, and then they could collect their prize at the prize table. It also served to help the students to keep track of their progress throughout the event. A suggestion was made at our last staff meeting that it might be easier to have a hole puncher or a stamper to mark off each activity that a student completed on the student activity card. That might be easier if you have those items on hand.

The publisher template used for the student activity card available at:
www.hudsonvalleyraen.org and CollectEdNY

THE NUMBER OF THE DAY

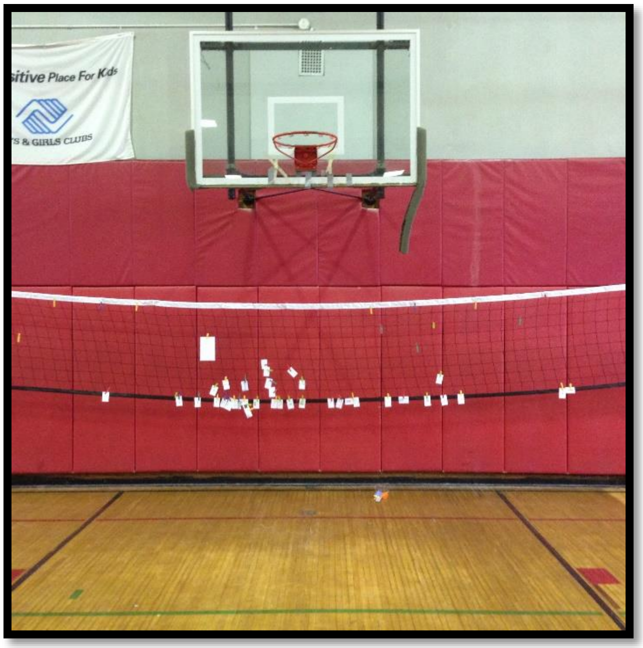
The Number of the Day Activity was the first activity we led students to as an ice breaker. The objective of this activity is for the students to create an equation that has the number of the day on one side of the equal sign, and their equation on the other side. By the end of the day, The Number of the Day had 100 sticky notes on it! Below are two examples that exemplify how students can create and "own" an equation, regardless of their skill level.





NUMBER LINE ACTIVITY

The Number Line Activity can be challenging for many students so it is necessary for the teacher who facilitates this activity to be ready with good questions that he or she can ask students to help to guide them in the direction of where their number would be placed on the number line. I had originally planned on setting up a clothesline to use for this activity but the volleyball net served the activity well. If you have one available it's fun to use, but a clothesline & clothespins, or a paper drawn number line & tape, will work just as well.



I made some changes to the number line activity due to some feedback from students and teachers. First, most of the number line cards had values between -1 and +1 (with lots of variations on how those negative and positive parts of a whole could be represented), so the cards ended up in a sort of web design between those two integers. Thus, I revised the number line cards so there would be more of an even spread of values between -3 and +3.

Also, in the original set of number line cards, the marker cards were the same size as the number cards we gave to students, but some students had difficulty locating where the marker numbers were, especially after the number line got really crowded. The larger marker cards should help out with this. You might even want to print out the marker cards in a different color so the marker numbers are clearly visible.

Note: There are a few cards that are open-ended in that they could correctly be placed in more than one place on the number line, depending on what the student interprets the picture to mean. Just make sure the student can correctly defend the placement on the number line!

The number line student cards and the marker cards for the number line are in **Appendix 2.**

THE PRODUCT GAME

"I don't know my multiplication facts." How often have you heard that from one of your students? The Product Game is a fun way for students to practice multiplication, collaborate with peers, and build up their problem solving skills as teams work together to get four in a row. Part of the strategy involves blocking the other team from getting there first. We used a magnetic, double-sided whiteboard for this activity, with a game board on both sides. That way, two games can be played at the same time if you have a lot of students who converge on The Product Game simultaneously.

We made two game boards out of plain easel size paper and taped one to each side of the magnetic whiteboard. Then, we used magnetic pattern blocks as game pieces on the board. This way you can use the same board over and over again.

For information on how to download directions and handouts for the Product Game, see Appendix 3.

PATTERN BLOCK ACTIVITIES

Pattern blocks provide opportunity to introduce students to mathematical concepts or deepen their existing understanding of them. Most students enter our programs with fraction angst. The ones that aren't filled with anxiety at the thought of tackling fractions are prone to misremember and misapply procedures typically used to solve fractions. Pattern blocks are a valuable tool for both groups of students, as in the first case it provides a visual model that the student can manipulate to gain understanding of what a fraction actually is and in the latter, students find themselves challenged when they are asked to demonstrate comprehension of fractional relationships when using a visual model. In both cases, pattern blocks can lay the groundwork that leads to a conceptual understanding of procedural knowledge of fractions. Aha! Now I understand why this procedure works!

Pattern blocks can also be used for algebraic thinking and an exploration of angle measurements. Samples of a number of pattern block activities are in Appendix 2. Links to additional pattern block activities, including angle measurement activities, are noted in Appendix 3.

TILE ACTIVITIES

There are so many wonderful activities that can be done with square tiles that I had some difficulty figuring out which ones I thought would work the best for the math fair. When students are physically able to manipulative visual models like the tiles, they guide their own learning through a process of trial, error, and discovery. Furthermore, it can help them to gain conceptual understanding of patterns and spatial structuring, a key bridge to understanding relationships within and between most mathematics. According to Mulligan & Mitchelmore [1], “Virtually all mathematics is based on pattern and structure.”

A number of tile activities are included in Appendix 2.

SNAP CUBE ACTIVITIES

As with the tile activities, snap cubes enhance student understanding of spatial structuring and math terms that might otherwise forever remain mysterious numbers on a page. For example, what does it mean to cube a number? What is volume? What is surface area? These concepts are sometimes represented with pictures in math resources, but these pictures are attempting to represent a three-dimensional object on a flat surface. With snap cubes, these mathematical concepts jump off the page and into the hands of our students, giving them a tool to solve problems and visualize math concepts.

A detailed lesson plan for [The Painted Cube] is available at The Hudson Valley RAEN website; however, for the math fair you should use only the first student page. The entire activity works well for a professional development workshop or as an extended multi-day activity in your classroom.

INTERNET ACTIVITIES

Students benefit from using both physical manipulatives and virtual manipulatives. According to a 2011 study [2], there are some advantages to using virtual manipulatives. They provide immediate feedback, sometimes giving hints to the student if they get the answer wrong, and they help students to grasp concepts more quickly because the student is able to connect a physical model to abstract math symbols. Plus, it's fun! See my review of The National Library of Virtual Manipulatives at CollectEdNY and my posts at World Educations's Tech Tips for Teachers Blog, Functions: Bridging from Concrete Understanding to Abstract Representations and Balance Mathematics Instruction by Balancing Shapes, for some tips on how to use virtual manipulatives.

Two activities that have been adapted from NCTM Illuminations and The National Library of Virtual Manipulatives are included in Appendix 2.

CARDS AND DICE

Cards and dice are inexpensive tools that give students some control over the outcome of a math activity. In *Cards in the Classroom: Mathematics and Methods*, Robert N. Baker asserts, “The use of cards as a focus of mathematical content enables techniques for the classroom that are particularly helpful among the math-anxious, adults, and non-traditional students.”[3] Students are challenged in solving an open-ended problem, yet grounded in something familiar, a simple deck of cards or pair of dice.

One of the card activities that we used for the math fair came from, *Acing Math (One Deck at a Time)*, a collection of over 50 math games that can be played with a deck of cards. See Appendix 2 for the cards and dice activities we used at the math fair, as well as a link to access *Acing Math.*

OPEN ENDED AND NON-ROUTINE PROBLEMS

At this station we had a few open ended and non-routine problems, meaning that there was more than one way to solve the problem and/or there was more than one solution to the problem. I did have some bingo chips at this table in case someone wanted to use them for a visual model but most students chose not to use them as you see pictured below.

The ability to find a way into and solve a type of problem that a student has never seen before is a skill that is being assessed on the TASC exam, so we serve our students well if we provide them with plenty of opportunities to experience productive struggle and to develop problem solving strategies that they can call their own.

In Appendix 1 you'll find a great resource, *Depth of Knowledge Level Sample Math Descriptors,* that describes the earmarks of a good open-ended DOK Level 3 problem. As Mark Trushkowsky would say, "It's a problem that makes you go hmmmmmm?"

Go to www.mathmemos.org to access a collection of rich math problems that have been peer reviewed. The website includes samples of student work and practical suggestions for using problem-solving activities in class. One of the problems posted there, the Zip Zap Zowie problem, is in Appendix 2.

INCENTIVES AND DOOR PRIZES

We wanted every student to walk away with rewards for participating in activities, so we offered a prize for each activity that the student completed. We had a variety of items at a table for students to choose from including: candy, water bottles, Frisbees, lanyards, rulers, flip-flops, piggy banks, pot holders, digital thermometers, magnet clips, and insulated can holders. Some of the items we already had on hand as giveaways for BOCES events and some we purchased specifically for the math fair. The prize table is pictured on the left and the picture on the right is when we were drawing names for door prizes.

At a recent staff meeting we discussed the possibility of handling the prizes a bit differently next time by assigning each prize a specific cost. A ruler, for example, might be had for participating in one activity, while a water bottle might cost 3 activities.

Also, the prize table at times was over busy with students converging to claim prizes all at the same time. Perhaps next time we will have small prizes that we keep at the first 3 mandatory activities.

We had drawings for door prizes twice during the day: once at the end of the morning session and once at the end of the afternoon session. All students placed their index cards in a container and we drew names for prizes. We had a few extra things we'd put aside for the drawings such as calculators and boxes of cookies.

Tip: If you're short on prizes, start collecting donations a few months ahead of time from students and teachers. Doing so will be sure to create positive anticipation from the students!

FINAL THOUGHTS

There are a few things that we plan to do differently at the next math fair that we have planned for this spring. For example, I'm going to create a short student survey this year and ask students to fill it out on their way out of the door. That will help to better inform decisions we make about future events. We're also preparing a special photo/video release form just for that day. While most programs have their students fill out a generic photo release form that can be used for program events, it's probably a good idea to have a special form for the math fair. I've included samples of both of those forms in **Appendix 1**.

While there were manipulatives at most of the stations that we set up, some students prefer to draw pictures or use traditional procedures to solve the problems. The key is to provide students with a variety of tools that they can try out if they want to. If you'd like to include manipulatives at your math fair but don't have them, templates for pattern blocks are readily available online. You can print them out on cardstock and can even laminate them if you have a laminator.

Most students will need support as they work through these activities. At our math fair this happened in several different ways. Some teachers manned one activity, while others floated about the gym, listening and stopping to work through activities with students as needed. Our ESOL students worked together in groups, for the most part, with their instructors staying close by and working right alongside them, giving them the extra support that they needed to understand the directions of the activities, which were all in English.

Most of the students who attended that math fair in the spring of 2015 have come and gone, but I went about asking the few I still had contact with for some of their reflections on it. I got comments such as, "Overall, it was great. I really liked the one table where I had to solve the problem about the legs of the chickens and goats. There was a guy who sat there with us and helped us to figure it out. That was good." I'm pretty sure he was talking about Eric Appleton, because another student of mine from the same class (who is now long gone), had commented on that same activity a year ago and he talked about how Eric helped them. Now, I know Eric. When my students said that Eric "helped" them, I'm sure he was busy asking them good questions so as to support their ability to solve the problem. Thus, the math would come from the student instead of the instructor. This was one of the goals of the math fair.

What I think is interesting about that comment from the former student, whose story you read about in the introduction, is that after the passing of a year, what stood out in his mind was the pleasure he felt at problem solving. He didn't even mention the pizza and prizes; although, I know that was part of the drawing power. A year later, though, it was the math adventures that had the staying power.

Happy Math Adventures to All!

Patricia Helmuth

Citations:
[1] ***Awareness of Pattern and Structure in Early Mathematical Development***, Mulligan & Mitchelmore 2009
http://files.eric.ed.gov/fulltext/EJ883867.pdf

[2] ***Virtual vs. Concrete Manipulatives in Mathematics Teacher Education: Is One Type More Effective Than the Other?***,Hunt, Nash, & Nipper 2011.
http://www.napomle.org/cimle/fall2011/hunt.pdf

[3] ***Cards in the Classroom: Mathematics and Methods***, Robert N, Baker 1999.
http://files.eric.ed.gov/fulltext/ED428786.pdf

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My colleagues at Sullivan County BOCES for their tireless contribution to make the Student Numeracy Day successful.

All of the students who dug in their heels and persevered in problem solving at the Student Numeracy Adventures Day.

Please share your math fair adventures with me at:

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Student Created List

APPENDIX 1

TEACHER RESOURCES
• Depth of Knowledge Level Sample Math Descriptors • The 4 Roles of Questions in a Mathematical Discussion • Creating & Categorizing - The 4 Roles of Questions in a Mathematical Discussion • Drawing Out the 8 Common Core Math Practices with Questions • Five Teaching Practices for Improving Discourse in Math Classes • Some Practical Advice for Teaching Problem Solving • Supporting & Extending Problems *Practicing with the Sum and Difference Problem* *NYSED/CUNY Mathematics Teacher Leader Training Institutes*
• Sample Photograph/Video Release Form • Sample Student Satisfaction Survey • The Number of the Day *Developed by Patricia Helmuth*

Example Math Descriptors for Depth of Knowledge Levels 1 to 4

D.O.K. Level 1: Recall & Reproduction	D.O.K. Level 2: Basic Skills & Concepts
• There is usually a right answer • Recall or recognize a fact, definition, term or property • Apply/commute a well-known algorithm or formula (i.e. sum, quotient, etc.) • Perform a specified or routine procedure • Solve a one-step word problem • Retrieve information from a graph or table • Make conversions between and among representations or numbers (fractions, decimals, percent) or within and between customary metric measures • Locate points on a coordinate grid • Determine the area or perimeter of rectangles or triangles given a drawing a labels. • Identify shapes and figures • Identify a pattern • <u>Teacher's Roles</u>: Tells, Shows, Demonstrates, Directs • <u>Student's Roles</u>: Memorizes, Restates, Absorbs, Remembers, Repeats	• There is usually a right answer • Classify shapes and figures • Interpret information from a simple graph • Solve a routine problem that require multiple steps/decision points or the application of multiple concepts • Provide justification for steps in a solution process • Use models or diagrams to represent and explain mathematical concepts • Make and explain estimates • Make basic inferences or logical predictions from data/observations • Organize or order data • Choose an appropriate graph type and organize and display data • Extend/continue a pattern • Retrieve information from a table, graph, or figure and use it to solve a problem requiring multiple steps • Specify and explain relationships between facts, terms, properties, or operations • <u>Teacher's Roles</u>: Shows, Evaluates, Questions, Observes, Organizes, Facilitates • <u>Student's Roles</u>: Solves Problems, Calculates, Illustrates, Compiles, Demonstrates Use of knowledge

D.O.K. Level 3: Strategic Thinking/Reasoning	D.O.K. Level 4: Extended Thinking
• May be more than one right answer and/or more than one way to get there • Use concepts to solve non-routine problems • Explain your reasoning *when more than one response is possible* • Having to plan a strategy and decide how to approach a math task *when more than one approach is possible* • Generalize a pattern • Write your own problem, given a situation • Find all the possible answers • Describe, compare, contrast different solution methods • Use evidence to develop logical arguments for a concept • Draw conclusions from observations/data, citing evidence • Interpreting information from a complex graph • Make and/or justify conjectures • Perform procedure with multiple steps and multiple decision points • Solve a multi-step problem and provide support with a mathematical explanation that justifies the answer • Interpret data from a complex graph • Verify the reasonableness of results • Teacher's Roles: Probes, Observes, Organizes, Guides, Evaluates, Frames, Questions • Student's Roles: Discusses, Questions, Debates, Examines, Judges, Justifies, Reasons, Decides, Tests, Compares	• Relate math concepts to other content areas • Relate math concepts to real-world applications *in new situations* • Conduct a project that specifies a problem, identifies solution paths, solves the problem and reports results • Conduct an investigation to solve a real-world problem with unpredictable outcomes • Design a mathematical model to inform and solve a practical or abstract situation • Apply understanding in a novel way, providing an argument/justification for the application • Teacher's Roles: Facilitates, Evaluates, Extends, Analyses • Student's Roles: Designs, Proposes, Formulates, Modifies, Creates, Plans

The 4 Roles of Questions in a Mathematical Discussion

Discuss the following questions with your partner(s) and decide which role (if any) the question could play in a math classroom. Also, put a star next to any question you'd like to try in your classroom and be prepared to talk about what you like about the question.

1. Can you draw a picture or diagram of the situation in this problem?
2. Raise your hand if you think the two ratios are equivalent. Raise your hand if you think they are not equivalent. Raise your hand if you are not sure. Ok. Now, turn to a partner who agrees with you and come up with some arguments to try to convince the students who are not sure.
3. Compare your method to her method. How are they similar? How are they different?
4. Can you show us how you did that?
5. What do you think about what she just said?
6. Did anyone solve it a different way?
7. Do you see a pattern?
8. Who tried something that didn't work? How did you figure out it
9. Do you agree or disagree?
10. What mathematical ideas did you have to use to solve this one?
11. Which of these methods makes the most sense to you?
12. This is a really interesting mistake. Does anyone see why I really like this mistake?
13. That is interesting. How could we prove that?
14. Have we ever worked on a problem like this before?
15. What did people do when they got stuck?
16. Everyone write down what you would do next.
17. What questions do you have?
18. It sounds like you have an idea and you have an idea, but you're not putting your ideas together to come up with a solution. What's your plan?
19. How do we know this answer is correct?
20. What kind of future problems could we solve using this method?
21. What do you want to remember about the way you solved this problem?
22. Will your method always work? How do you know it will always work?
23. That worked when we solved that other problem. Why isn't it working now? What's different?
24. Can you say what she just said in her own words?

The Art of Questioning: The 4 roles of questions in a math classroom			
In each category, write a few examples of questions you could ask students.			
Help students work together to make sense of mathematics	Help students rely on themselves to determine if something is mathematically correct	Help students learn to conjecture, invent, and solve problems	Help students connect mathematics, its ideas and its applications

The Art of Questioning			
The 4 roles of questions in a math classroom			
Help students work together to make sense of mathematics	Help students rely on themselves to determine if something is mathematically correct	Help students learn to conjecture, invent, and solve problems	Help students connect mathematics, its ideas and its applications

MP1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining the meaning of a problem and looking for entry points to its solution.

- ☑ They analyze givens, constraints, relationships, and goals.
- ☑ They make conjectures about the form and meaning of the solution
- ☑ They plan a solution pathway rather than simply jumping into a solution attempt.
- ☑ They consider similar problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution.
- ☑ They monitor and evaluate their progress. And change course if necessary.
- ☑ They check their answers to problems using a different method than the one they used to solve the problem.
- ☑ They continually ask, "Does this make sense?"
- ☑ They understand the approaches of others to solving complex problems
- ☑ They identify similarities and differences between different approaches.

Questions that can help students persevere and develop deeper self-awareness of their process, to be asked while they are working

- ☑ Can you explain the situation in your own words?
- ☑ How did people get started on this problem?
- ☑ Have we ever seen a problem like this before? How was the problem similar to this problem?
- ☑ Talk me through what you have done so far, step by step.
- ☑ What is the relationship between the quantities?
- ☑ How will you know if your strategy is working?

Questions that focus student thinking on process, to be asked during class discussions of different student solution methods

- ☑ How did people get started on this problem?
- ☑ How is <student's name> method similar to <student's name> method?
- ☑ How is <student's name> method different from <student's name> method?
- ☑ How do you think <student's name> would use their method to solve this problem?
- ☑ What did you do when you got stuck?
- ☑ How do you know that your answer is correct?
- ☑ Is there another way to solve this problem?
- ☑ Did anyone start with/try a strategy that didn't work?
- ☑ Which one of these strategies helped you see this problem more clearly?
- ☑ What do you appreciate about <student's name>'s strategy?
- ☑ Look over your classmates' work up on the board. What did each student do to make sense of and solve this problem? Can we pull out any general problem-solving strategies that might help us in the future?

MP2. Reason abstractly and quantitatively	
Mathematically proficient students make sense of the numbers (quantities) and relationships in problem situations ☑ They represent abstract situations symbolically - *decontextualize* ☑ The manipulate the representing symbols as if they have a life of their own, without attending to their referents ☑ They contextualize symbols, pausing to connect them to the situation in the problem ☑ They create a coherent representation of the problem They use the properties of the four operations flexibly	
Questions Teachers Can Ask to Draw Out and Develop this Mathematical Practice	☑ What do the numbers in this situation represent? ☑ What does this number represent? (referring to a number appearing a students' work) ☑ Can you make a drawing of the situation? ☑ What does it mean to multiply/divide/add/subtract? ☑ Can you represent the problem with symbols/ equations/ pictures/ sentences/ numbers?

MP3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and established results in constructing arguments.

- ☑ They make conjectures & build a logical progression of statements to explore the truth of their conjectures.
- ☑ They can analyze situations by breaking them into cases, and can recognize and use counterexamples.
- ☑ They justify their conclusions, communicate them to others, and respond to the arguments of others.
- ☑ They can compare the effectiveness of two arguments, and determine correct or flawed logic
- ☑ Listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Questions Teachers Can Ask to Draw Out and Develop this Mathematical Practice

**It is important that students are working on problems that lend themselves to discussion, arguments or critique*

- ☑ Summarize what <student's name> just said in your own words.
- ☑ How is <student's name>'s answer different from <student's name>'s answer? How are they similar?
- ☑ What questions do you have for <student's name> about their method? (As a follow-up: After a student has presented their work, use the work they put up to ask the rest of the class specific questions…"I see <student's name> did this. What were they thinking here?)
- ☑ What do you appreciate about <student's name>'s method?
- ☑ How can you prove that your answer is correct?
- ☑ Will that always be true?
- ☑ Raise your hand if you agree with Jane. (Count) Now, raise your hand if you agree with Daphne. (Count) Raise your hand if you're not sure. (Count). Ok, so everyone who is unsure is an undecided voter. Everyone else, your job is to convince them to agree with you.
- ☑ Can you come up with some examples that will prove your argument? Or disprove someone else's?
- ☑ Which explanation makes the most sense to you? What did <student's name> do well to make their ideas clear to you?

MP4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems from everyday life, society, and the workplace.

☑ They can simplify a complicated situation, realizing that they may need to revise later.
☑ They can identify important quantities in a practical situation and show their relationships.
☑ They can analyze those relationships mathematically to draw conclusions.
☑ They routinely interpret their mathematical results in the context of the situation
☑ They reflect on whether the result makes sense, possibly improving the model if it does not.

Questions Teachers Can Ask to Draw Out and Develop this Mathematical Practice	*It is important that students are working on problems that involve real-world situations ☑ Write a number sentence(s) to describe this situation ☑ How could we draw a picture/make a diagram/visually represent this situation? ☑ What do you already know about solving this problem? ☑ What information would we need to answer this problem? Where could we get that information? ☑ How can you tell if the results make sense? ☑ What factors of the situation did you choose to focus on? Explain your thinking. ☑What are the practical implications of your findings? Who might be able to use your findings? How might your findings be used by other people?

Example of a Problem Targeting this Mathematical Practice

Wikipedia reports that each day, 8% of all Americans eat at McDonald's. In 2012, there were about 310 million Americans and 12,800 McDonald's restaurants in the United States.
Do you believe the Wikipedia report to be true? Create a mathematical argument to justify your position.

A Word on Mathematical Modelling

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

(from The Math Assessment Project)

When we hear the word "modelling" in a classroom context, we often think about the teaching strategy where a teacher demonstrates a skill or an approach to a problem for students. When we talk about mathematical modelling, we are talking about something a little different.

To begin to think about mathematical modelling, let's look at two quotes by Henry Pollak

"When you use mathematics to understand a situation in the real world, and then perhaps use it to take action or even to predict the future, both the real-world situation and the ensuing mathematics are taken seriously."	"Mathematical modeling begins in the unedited real world, requires problem formulation before problem solving and once the problem is solved, moves back into the real world where the results are considered in their original context. Are the results practical, the answers reasonable, the consequences acceptable? If so, great! If not, take another look at the choices made at the beginning, and try again. This entire process is what's called mathematical modeling."

(quotes above are from EngageNY PowerPoint on Mathematical Modelling.)

Now, consider the following problem, also from Henry Pollak:

Your grandmother will be arriving at the airport at 6:00 pm. You live 20 miles from the airport. The speed limit is 40 miles per hour. When should you leave to get her?

In a traditional math classroom the answer to this problem would be 5:30, since driving 20 miles at a speed of 40 MPH, will get you to the airport in a half hour.

But if you left your house at 5:30, you would most certainly be late to pick-up your grandmother. What are some other things you might factor in to your calculations?

What about traffic, stop lights, parking, time to meet your grandmother at the baggage claim to help her with her luggage, etc? This is a very simple example, but it begins to get at what we mean by mathematical modelling.

MP5. Use appropriate tools strategically	
Mathematically proficient students consider the available tools when solving a mathematical problem. ☑ They make good decisions about the use of specific tools (calculator, concrete models, digital technology, paper/pencil, ruler, compass, protractor, etc.) ☑ They detect possible errors by strategically using estimation and other mathematical knowledge ☑ They use tools to visualize the results of assumptions, explore consequences and compare predictions with data ☑ They use technological tools to explore and deepen understanding of concepts ☑ They identify relevant external math resources and use them to pose or solve problems	
Questions Teachers Can Ask to Draw Out and Develop this Mathematical Practice	☑ Can you draw a picture to show your thinking? ☑ What would be the best tools for working on this problem? (Or offering students a selection of tools and asking them to choose one and then later to explain and reflect on their choice) ☑ What mathematical tool(s) could you use to visualize/represent this situation? ☑ How did it help us to use a _______?

MP6. Attend to precision.

Mathematically proficient students try to communicate precisely to others.

- ☑ They try to use clear definitions when discussing their reasoning with others
- ☑ They express the meaning of the symbols they choose, including using the equal sign consistently and appropriately.
- ☑ They are careful about specifying units of measure, and labeling quantities in a problem.
- ☑ They calculate accurately and efficiently.
- ☑ They express numerical answers with a degree of precision appropriate for the problem context.

Questions Teachers Can Ask to Draw Out and Develop this Mathematical Practice	☑ What does the word ____ mean? ☑ Explain what you did to solve this problem. ☑ How could you label your work to make it clearer? ☑ Is there a more efficient strategy? ☑ How could you organize your work to make it clearer? ☑ How do you know your answer is reasonable? ☑ How exact does your answer need to be? Explain your thinking. ☑ What symbols or mathematical notations are important in this problem? ☑ <Student's name> just explained their strategy to us. What was clear about their strategy? What questions do you have for <Student's name>?

MP7. Look for and make use of structure	
Mathematically proficient students look closely for patterns or structure. ☑ They recognize quantities can be represented in different ways ☑ They can shift back, look at the big picture and shift perspective ☑ They can see complicated quantities both as single objects or compositions of several objects and use operations to make sense of problems	
Questions Teachers Can Ask to Draw Out and Develop this Mathematical Practice	*Moving from general to specific ☑ How is ____ related to _____? ☑ Is there another way to look at this problem? ☑ What do you know about ____ that would be helpful in this situation? ☑ What patterns do you notice? How can we use that pattern? ☑ How do you know if something is a pattern? ☑ What problems have we done that are similar to this one? How are they similar? ☑ What mathematical concepts/strategies have we learned that helped you work on this problem?

MP8. Look for and express regularity in repeated reasoning	
Mathematically proficient students notice repeated calculations and look for general methods and shortcuts ☑ While working on a problem, mathematically proficient students maintain oversight of the process, while attending to the details. ☑ They continually evaluate the reasonableness of intermediate results ☑ They make generalizations based on findings	
Questions Teachers Can Ask to Draw Out and Develop this Mathematical Practice	☑ (Making generalizations) Can you come up with a rule that will help us solve the problem whatever the numbers are? ☑ Will the same strategy work in other situations? ☑ Now that you have the answer, go back and see if there are any patterns you notice. ☑ How can working on this problem help us solve another problem? ☑ Is there another way to solve this problem using less calculation?

Five Teaching Practices for Improving the Quality of Discourse in Mathematics Classrooms

1. Talk moves that engage students in discourse

- **Revoicing**: *"So let me say that back to you to make sure I understand what you are saying…"*
- **Asking students to restate someone else's reasoning**: *"Can you repeat what she just said in your own words?"*
- **Ask students to apply their reasoning to someone else's:** *"Do you agree or disagree with what _____ just said?"*
- **Prompt students for further participation:** *"Would someone like to add to that?"*
- **Wait time:** *Don't fear the crickets*

2. The art of questioning

<u>The 4 roles of questions in a math classroom[1]:</u>

- to help students work together to make sense of mathematics
- to help students rely on themselves to determine if something is mathematically correct
- to help students learn to conjecture, invent, and solve problems
- to help students connect mathematics, its ideas and its applications

3. Using student thinking to propel discussions

- Be an active listener
- Be strategic and choose ideas, methods, representations, misconceptions in a purposeful way that enhances the quality of the discussion

4. Setting up a supportive environment

- Be conscious of the physical and emotional environment
- Respond neutrally to errors, but seek out novel or common misconceptions and bring them into discussion

5. Orchestrating the discourse

The teacher's role in orchestrating the discourse[2] is to:

- Anticipate student responses to challenging mathematical tasks,
- Monitor students' work and engagement with the tasks;
- Select particular students to present their mathematical work;
- Sequence the student responses that will be displayed in specific order; and
- Connect different student' responses and connect the responses to key mathematical ideas.

Adapted from How to Get Students Talking! Generating Math Talk That Supports Math Learning by *Lisa Ann de Garcia*

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[1] *Professional Standards for Teaching Mathematics (*National Council of Teachers of Mathematics)
[2] Smith, M.S., E.K. Hughes, R.A. Engle & M.K. Stein *Orchestrating Discussions, (Mathematical Teaching in Middle School)*

<table>
<tr><th colspan="2">Some Practical Advice for Teaching Problem-Solving</th></tr>
<tr><td>Encourage Persistence

Allow students time to understand and engage with the problem

Discourage students from rushing in too quickly or from asking you to help</td><td>When students ask a question about one of the conditions that make the problem “problematic”, encourage them and reflect question back to them
Answer most questions with “Good question. What do you think?”
When students start to shut down, get them talking
When students are stuck, suggest a strategy – for example, “Can you draw a picture?” or “What could the answer be? Is there a way you can check that?”
What have you tried from our list of problem-solving strategies?</td></tr>
<tr><td>Focus Authority with Students

Offer strategic rather than technical hints

Avoid simplifying problems for students by breaking it down into steps</td><td>Don’t exert authority by saying what is right or wrong
Never take the pen/pencil out of a student’s hand
Sit in student’s chair when student goes up to demonstrate their thinking
Respond to most student explanations with “What do the rest of you think?”
Ask, “How do we know this answer is correct?
Model thinking and powerful methods. When students have done all they can, the teacher can demonstrate another powerful, elegant approach. If this is done at the beginning, however, students will simply imitate the method and not appreciate why it was needed. Whenever possible, teachers should draw from presented student work.</td></tr>
<tr><td>Help Student Reveal Their Thinking

Encourage explanation

Make students do the reasoning, and encourage them to explain to one another</td><td rowspan="2">Constantly ask “Can you show us how you did that?”
When a student presents their thinking and part of their reasoning is unclear, ask them to tell the class more about what they did there
When students present their thinking, give other students an opportunity to ask questions – if they don’t have any, ask at least one question Respond to most student explanations with “What do the rest of you think?”
After a student explains their thinking, ask the rest of class to explain a potentially confusing aspect of the student’s thinking
Ask, “How are these methods similar/different?”
Name one thing you like/appreciate about other students’ methods.</td></tr>
<tr><td>Help Students Understand the Ideas of Their Classmates

Encourage students to consider alternative methods and approaches
Encourage students to compare their own methods</td></tr>
</table>

Have Students Reflect on Their Own Process	• How did you get started? • What were the hardest parts of this problem? • What do you want to remember about the way we solved this problem? • Have we ever seen a problem like this before? How was this problem similar? Different? • Which method makes the most sense to you? • What did ______ do to help make her thinking and strategy clear to us? • What general problem-solving strategies can we add to our class list? • What did you learn from working on this problem?

Supporting & Extending Problems

Practicing with the Sum and Difference Problem

The left column is divided into some potential categories, to help guide you as you create support and extension questions for your students. In the right column there are examples of these questions, written for the sums and difference problem.

Questions We Can Ask to Help Support "Stuck" Students	
Kinds of Questions	Examples of Questions
Ask students to break down the problem / explain the situation	❖ What are we looking for? ❖ What do we know about those two numbers?
Ask students about important/relevant concepts/vocabulary	❖ What does "sum" mean? ❖ What does "difference" mean?
Ask students questions that model how to reach into their problem-solving toolbox	❖ When you have felt stuck on a problem, what problem-solving strategies have helped you get started? ❖ Can you draw a picture/visual representation of the situation? ❖ What *could* those two numbers be? How could you test if those two numbers work? ❖ Can you draw a number line that shows the two numbers, the sum and the difference?
Ask students reflect on their solution and feel confident that their answer is correct	❖ <If the numbers did not work> How do you know those numbers are not the answer? What could you try next? ❖ <Once they have found the correct numbers> How do you know those two numbers are correct?
Simplify the problem / Model simplifying the problem as a problem-solving strategy	❖ Can you find two numbers with a sum of 5 and a difference of 1? Can you find two numbers with a sum of 12 and a difference of 4? How did you figure it out?

Questions We Can Ask to Extend the Problem	
Kinds of Extensions	Examples of Questions
Ask students to create their own, similar problem	❖ Can you write a similar problem with different numbers?
Ask students to generalize their method	❖ Can you describe a procedure that would work to find any two numbers if you are given the sum and difference of those two numbers? Test your procedure with different numbers? ❖ Do you see any patterns in the relationship between the sums and the differences and the two numbers? ❖ What other patterns do you notice? ❖ Can you express your procedure as an equation?
Add (or change) one condition	❖ Can you create a similar problem with two numbers who have a difference that is larger than their sum? ❖ Can you create your own example where the difference is larger than the sum, and the sum is not zero? ❖ Find the two consecutive numbers that have a sum of 487.
Ask students to make and explore a hypothesis	❖ Is the sum of two numbers always larger than the difference of those same two numbers? ❖ Is it possible to have two *different* pairs of numbers that have the same sum and difference? Prove it.
Ask students to reflect on their process/solution method	❖ Now that you have an answer, can you organize (or re-write) your work in a way that would be clear to someone else? ❖ Now that you have an answer, can you find another solution method? ❖ Will your procedure always work? How do you know it will always work? Why does your procedure work?

Other Strategies for Extending Problems

- For open-ended problems with many possible solutions: Ask students to find other solutions. Ask students to find all possible solutions (and then ask them how they know they have found all possible solutions)

STUDENT NUMERACY ADVENTURES DAY
Photograph and Video Release Form

Student Name:____

Program:____

I understand that at the math fair today pictures and/or video recordings may be taken for the purpose of documenting and promoting the event. The photographs or video recordings may be made available to other educational programs or adult education teachers as a training resource, either online, at workshops, or in printed form. By signing this form, I agree to allow photographs or video recordings of myself to be used for the reasons listed in this statement.

My program may use my name _____

My program may not use my name _____

Signature__________________________________ Date_________

Student Numeracy Adventures Day Survey

1. Was the math fair better than you expected, not what you expected, or about what you expected?

    - Better than I expected
    - Not what I expected
    - About what I expected

2. Which activity that you completed did you like the most? Explain why it was your favorite.

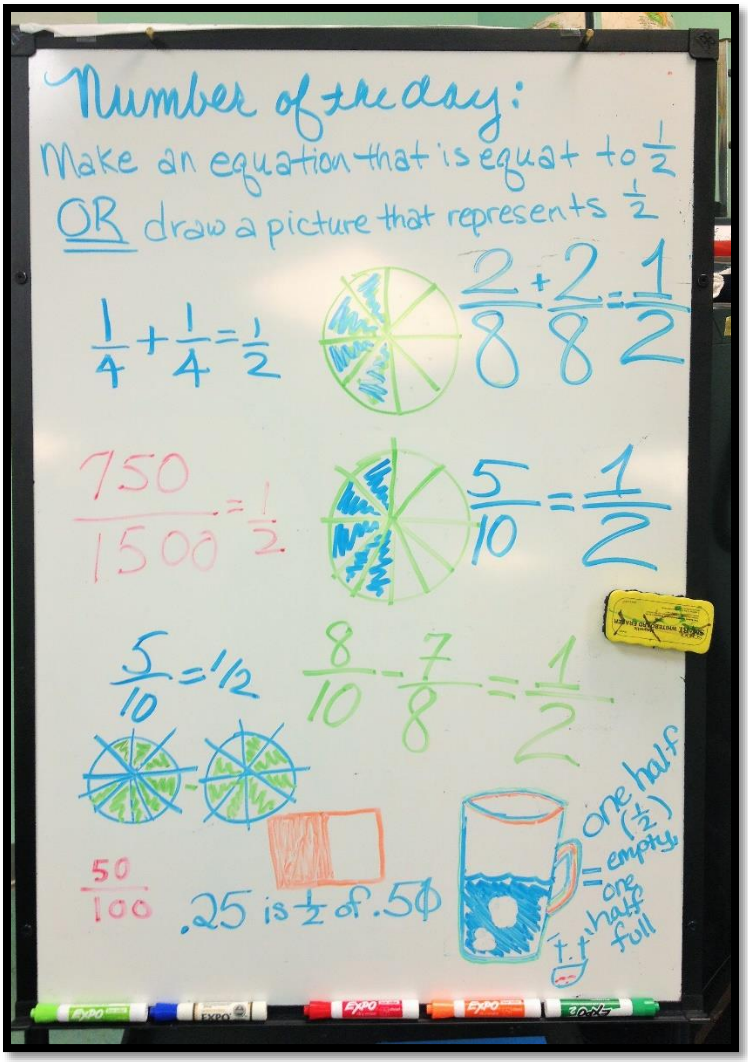
3. Which activity did you like the least? Explain why you didn't like it.

4. What was the best part of the day for you?

5. Any additional comments or suggestions you'd like to make?

The Number of the Day

I use the Number of the Day almost every day in my classes. Most times I choose the number of the day but sometimes I'll ask the students to choose the number. For the math fair, a simple whole number serves the event well; however, in your classroom you can get a little more complicated as shown in the picture below.



It's the Number of the Day with a twist. I wanted my students to develop a conceptual understanding of benchmark fractions so I had been asking them to make an equation that equals a particular fraction OR draw a picture that represents that fraction.

This serves several purposes. One, it allows students at all ability levels to participate in scary fractions. It also works well as a formative assessment. I can see what students know, what misconceptions they may have, and then take it from there. You'll notice, for example, in the photo that a student made a very common error by writing:

$$8/10 - 7/8 = 1/2$$

When I asked her to draw a picture to represent the equation, which you can see underneath it and to the left, she realized that the equation she wrote couldn't be true. Aha!

For more ideas on how to use the Number of the Day, access the 2015 summer issue of The Math Practitioner and read how Barbara Leonard uses it in her classroom.

http://www.nwlincs.org/mtlincs/opi/Math/ANN_Newsletter_10-1-15.pdf

APPENDIX 2

Math Fair Activities

Number Line Cards & Marker Cards

Developed by Connie Rivera and Patricia Helmuth

Pattern Block Activities

- Triangle Pattern
- Pattern Block Fractions
- Pattern Block Percents

Pattern Block Fractions & Percents adapted from EMPower
Developed by Patricia Helmuth

Sample Pattern Block Activities

- Pattern Blocks – Another Tool
- Adding and Subtracting with Pattern Blocks
- Pattern Block Division

Reprinted with permission from the Adult Numeracy Center at TERC

Card Activity

- Hit

Reprinted with permission from The Positive Engagement Project

Card Activity

- The Dinner Party

Developed by Larry Gotham

Dice Activities

- One-Step Rule Function Activity
- Creating a Two-Step Rule

Developed by Patricia Helmuth

Internet Activities • Internet Activity 1 – Balancing Shapes • Internet Activity 2 – Circle 0 *Adapted from NCTM Illuminations & The Virtual Library of Virtual Manipulatives*
Support for Internet Activity 1 • Keeping Your Balance *From Math Matters: Understanding the Math You Teach* *NYSED/CUNY Mathematics Teacher Leader Training Institutes*
Tile Activities • Exploring Area and Perimeter • How Many Squares? • The Pinwheel Pattern *Developed by Patricia Helmuth*
Tile Activity • The Arch Problem *CUNY HSE Curriculum Framework*
Tile Activity • Reasoning with Division Adapted from *The Math Practitioner*
Open-Ended Problem Solving • Zip Zap Zowie • The Handshake Problem • The Bicycle Shop • Sums and Differences • Chicken & Goats Problem *NYSED/CUNY Mathematics Teacher Leader Training Institutes*
Snap Cube Activity • The Painted Cube *Adapted from NYSED/CUNY Mathematics Teacher Leader Training Institutes*

Snap Cube Activity   Condo Challenge   *NYSED/CUNY Mathematics Teacher Leader Training Institutes*
Snap Cube Activity   The Rising Smokestack   *Developed by Patricia Helmuth* *Adapted from Visual Patterns.org and CollectEdNY*
Snap Cube Activity   The Aquarium Problem   *Contributed by Tyler Holzer* Adapted from *From Kabiri, M. S., and N. L. Smith "Turning Traditional Textbook Problems into Open-Ended Problems," Mathematics Teaching in the Middle School, Vol. 9, No. 3, 2003, 186-92.*

Number Line Marker Cards

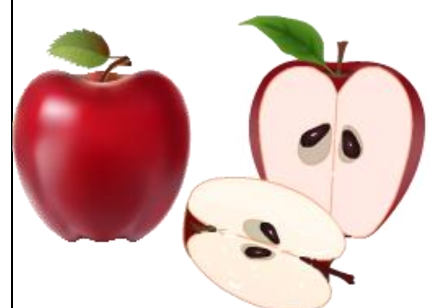
-3	-2
-1	0

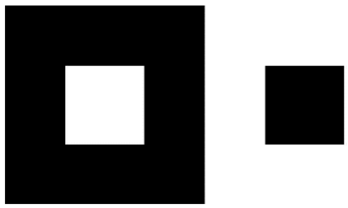
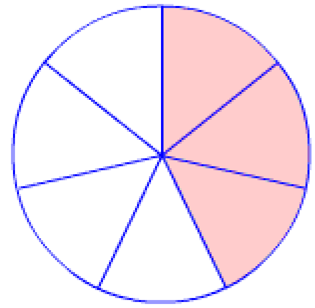
Number Line Marker Cards

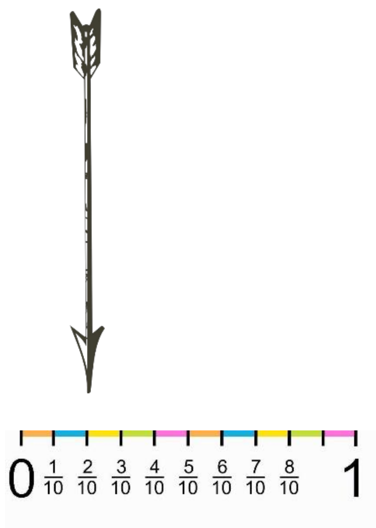
1	2
3	

account balance: \$4.25 spend: \$6.75 New Account Balance: ________________	account balance: \$1.75 spend: \$3.50 New Account Balance: ________________	account balance: \$0.50 deposit: \$1.75 New Account Balance: ________________
account balance: \$1.25 spend: \$3.25 New Account Balance: ________________	-2.9	-2¾

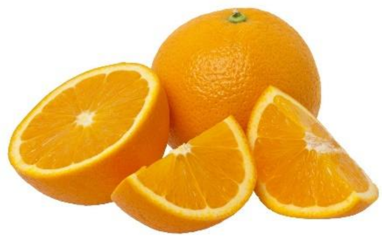
account balance: $4.25 spend: $7.05 New Account Balance: ________________	account balance: $1.75 spend: $3.70 New Account Balance: ________________	account balance: $0.22 deposit: $1.75 New Account Balance: ________________
account balance: $1.25 spend: $3.90 New Account Balance: ________________	-1.199	1 ¾

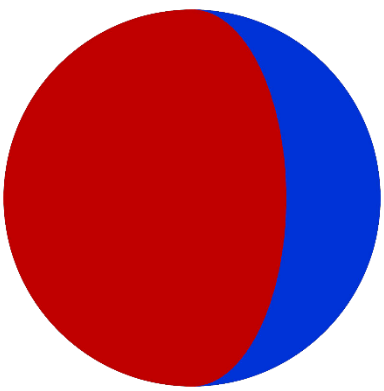
$-1\frac{1}{3}$	$\frac{2}{5}$	
$\frac{7}{8}$	$-2\frac{7}{8}$	

	-1.09	
1 ⅔	1 ⅚	-0.09

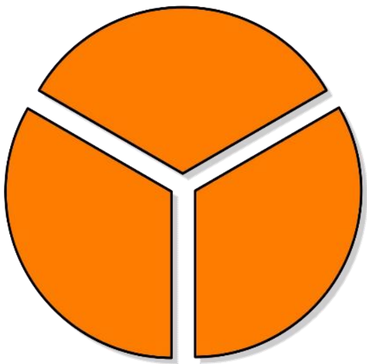
	-.492	$-\frac{7}{8}$
-0.9	0.33	$-1\frac{1}{6}$

		0.7
-1.6	⅛	2.909

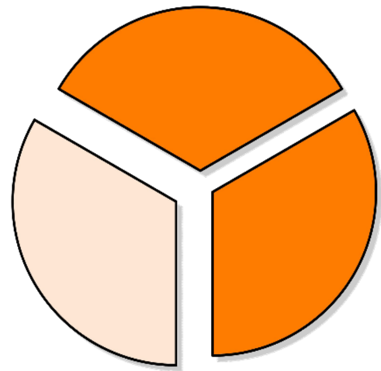
$2\frac{3}{4}$		$2\frac{1}{4}$
$1\frac{3}{8}$	2.05	

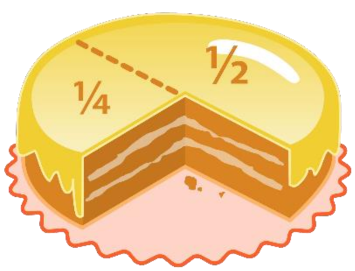
$-2\frac{1}{2}$	$-1\frac{3}{4}$	$-1\frac{1}{2}$
		$-\frac{1}{3}$

-1.5	1.1	-1.1
2.2	-2.2	1.4

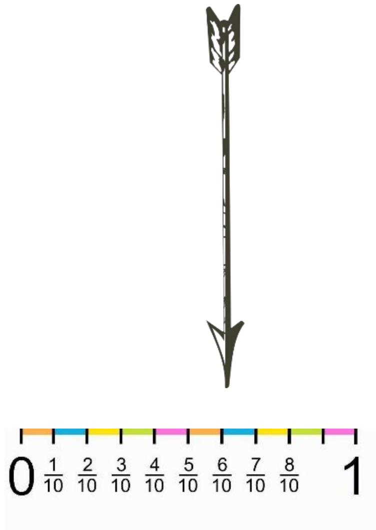


$\frac{4}{2}$



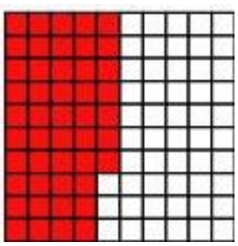


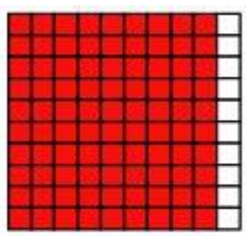
$\frac{27}{10}$



-.50

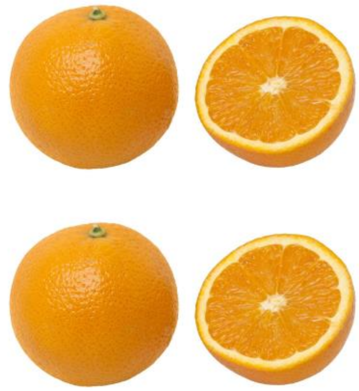
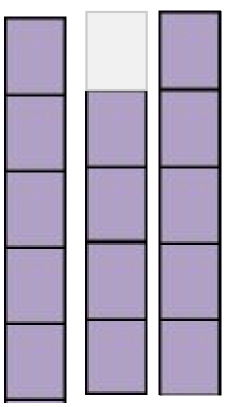
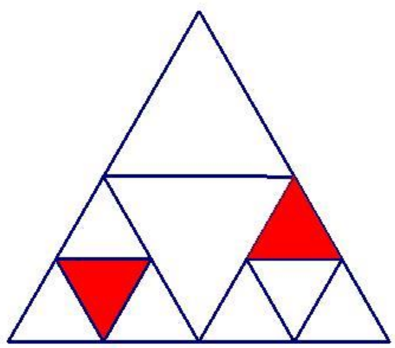
-.10

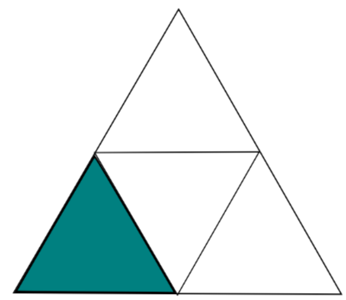


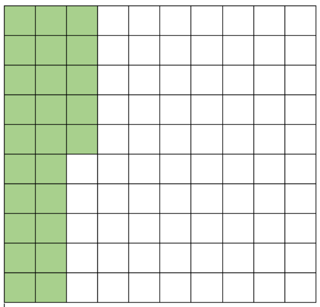
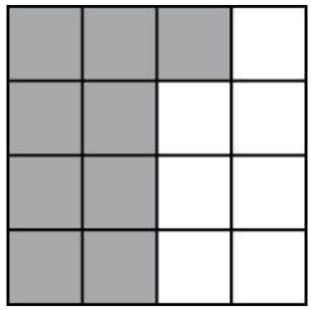


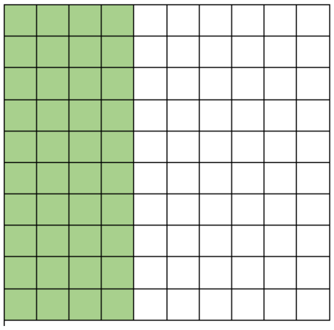
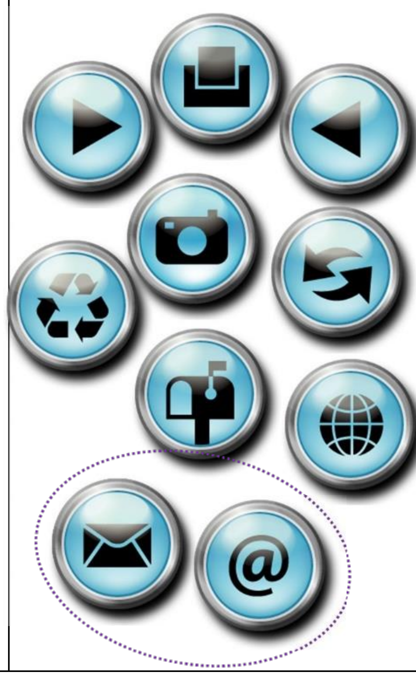
$1\frac{1}{6}$

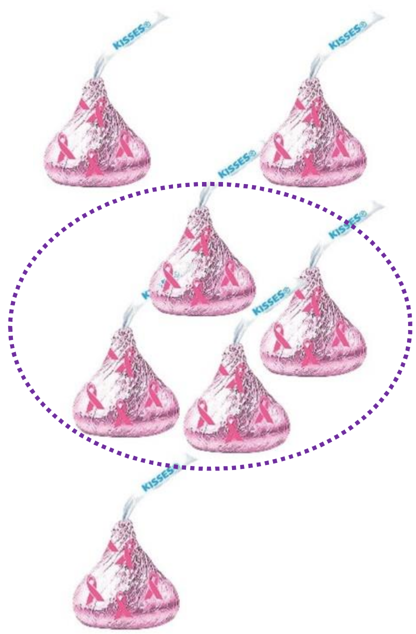
		$2\frac{2}{3}$
$\frac{10}{4}$	$-\frac{3}{4}$	$-\frac{1}{4}$

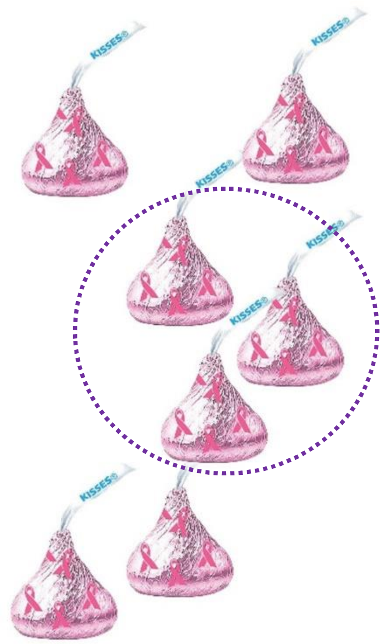
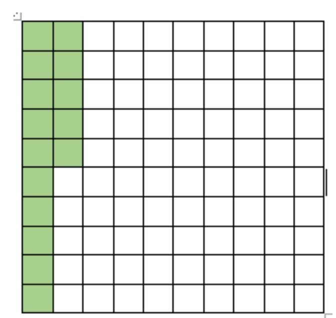
$-2\frac{3}{4}$		$-2\frac{1}{4}$
		

1.89	-1.437	
1.765	-.79	2.1998

		0.25
1.05	$\frac{1}{3}$	.333

	.4	40%
0.40		$\frac{1}{2}$

$\frac{4}{7}$	57%	0.57
	$\frac{3}{7}$	42.8%

.428		$\frac{3}{20}$
$\frac{15}{100}$	$\frac{1.5}{10}$	

2.31	1.5	-2.1

Triangles Pattern

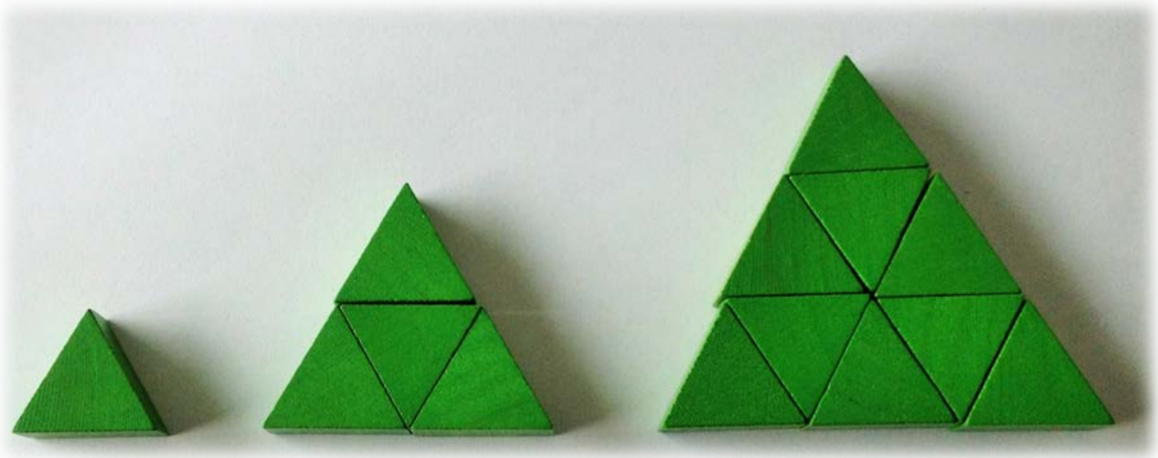


Figure 1 **Figure 2** **Figure 3**

1. What would the 5th figure look like? Sketch it below or use the triangles to make a model of the 5th figure.

2. What patterns do you see?

Triangles Pattern

3. In a few sentences, describe what the 10th figure would look like.

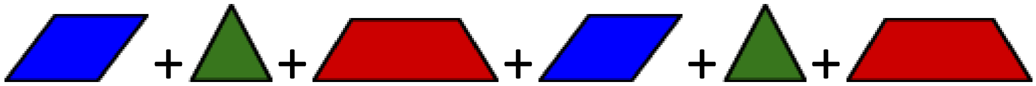
4. In a few sentences, describe how to figure out how many triangles there would be in any figure in this pattern.

5. Write the equation.

Adapted from:

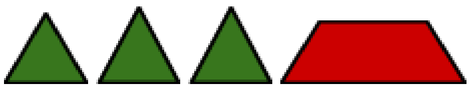
www.collectedny.org

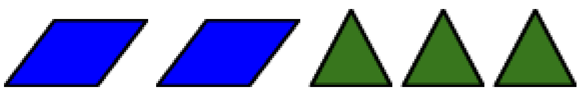
Pattern Block Fractions

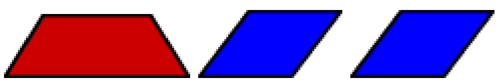


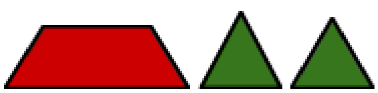
= ?

1. If the yellow hexagon = 1 whole, which combinations of shapes shows 2/3 + 1/2?

a. 

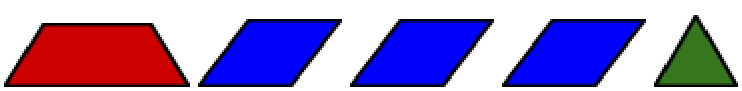
b. 

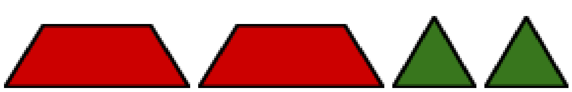
c. 

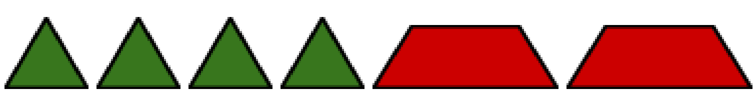
d. 

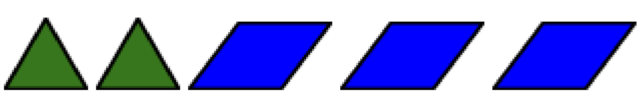
2. If the yellow hexagon = 1 whole, which combinations of shapes shows

1 ⅔?

a. 

b. 

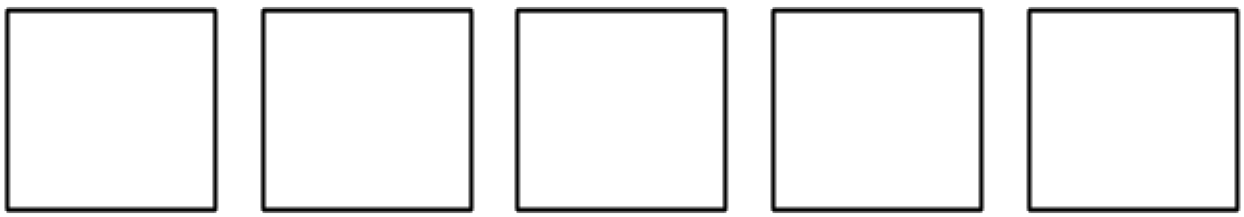
c. 

d. 

Pattern Block Percent

Name___________________________ Date____________

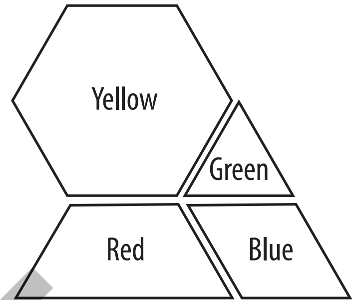
The printer ran out of ink when it was printing this page and only 20% of the boxes were printed. Please draw the boxes that are missing.



Explain how you figured out how many boxes to draw:

Activity 2: Pattern Blocks—Another Tool

You will need a pile of Pattern Blocks for this activity.



1. Counting the yellow hexagon as one, the whole, find the *fractional* value of

a. the green triangle __________

b. the red trapezoid __________

c. the blue parallelogram __________

2. Examine the Pattern Blocks to answer each of the following questions. Show the equivalence with a picture, a math equation, and words.

a. How many green triangles equal 1 yellow hexagon?

The picture

The math equation

The math in words

b. How many blue parallelograms equal 1 yellow hexagon?

The picture

The math equation

The math in words

c. How many red trapezoids equal 1 yellow hexagon?

The picture

The math equation
The math in words

d. How many green triangles equal 1 red trapezoid?

The picture

The math equation
The math in words

e. How many green triangles equal $2\frac{1}{2}$ yellow hexagons?

The picture

The math equation
The math in words

f. How many blue parallelograms equal 1 red trapezoid?

The picture	The math equation
	The math in words

g. How many blue parallelograms equal 2 red trapezoids?

The picture	The math equation
	The math in words

3. Show two more equivalent statements.

Activity 2: Adding and Subtracting with Pattern Blocks

Use Pattern Blocks to show the addition or subtraction and to arrive at the answer. Use one hexagon as a one whole.

1. $\frac{5}{6} - \frac{1}{3} =$

2. $1 - \frac{2}{3} =$

3. $\frac{2}{3} - \frac{1}{2} =$

4. $\frac{1}{2} - \frac{1}{6} =$

5. $\frac{1}{2} + \frac{1}{3} + \frac{1}{6} =$

6. $\frac{2}{3} + \frac{1}{6} =$

Make up 3 more!

7. $\quad - \quad =$

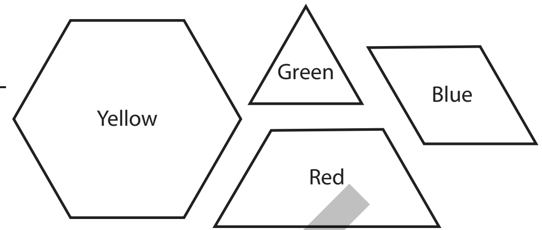
8. $\quad + \quad =$

9. $\quad - \quad =$

Activity 1: Pattern Block Division

Use Pattern Blocks to explore these questions.

For this activity the yellow hexagon will have a value of 1.
What is the value of



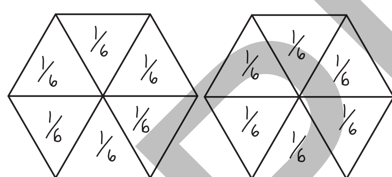
- The green triangle? ________
- The red trapezoid? ________
- The blue parallelogram? ________

An example has been done for you:

How many triangles are in 2 hexagons?

a. Answer: 12

b. A picture:



c. Math symbols: $2 \div \frac{1}{6} = 12$

d. Show another way to explain the answer.

$1 \div \frac{1}{6} = 6$, so $2 \times 6 = 12$

1. How many parallelograms are in a trapezoid?

a. Answer: ______

b. A picture:

c. Math symbols:

d. Show another way to explain the answer.

2. How many triangles are in two parallelograms?

   a. Answer: ______

   b. A picture:

   c. Math symbols:

   d. Show another way to explain the answer.

3. Now use the shapes to ask and answer your own "How many ___ in ___?" question. Show the answer with a picture and using math symbols.

   a. Answer: ______

   b. A picture:

   c. Math symbols:

   d. Show another way to explain the answer.

Hit (Grades 5 - 8)

Players: Groups of two or more

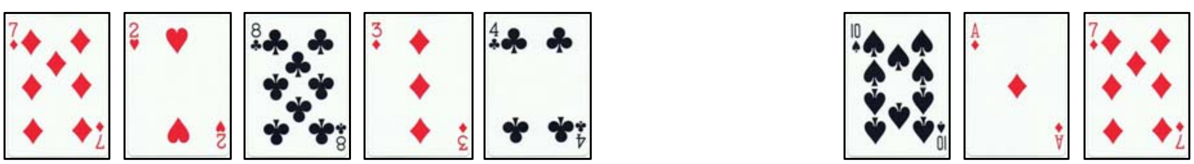
Materials: Deck of cards with face cards removed

Skill: Number recognition, addition, subtraction, multiplication, positive integers, negative integers, and mathematical reasoning

How to Play: Black cards are positive numbers; red cards are negative numbers. For each player, turn one card face down and one card face up. Everyone can see the face-up card, but only the player gets to look at his/her face-down card (until the end of the game, when all cards are revealed). The goal of the game is to get as close to zero as possible.

Player 1 Player 2

Each player adds his/her cards together in their head. Then he/she may ask for up to 5 "hits," or extra cards, that are dealt face up, for a maximum of 7 cards total.



Player 1: asked for 3 hits, total is 0
$-7 + -2 = -9 + 8 = -1 + -3 = -4 + 4 = 0$

Player 2: asked for 1 hit, total is 2
$10 + -1 = 9 + -7 = 2$

Player 1 is closest to zero, so they win that round.

When everyone is done asking for hits, all cards are turned face up. Whatever each player's cards add up to is his/her score, and whoever scores closest to zero when all of the cards are revealed wins that round and becomes the dealer for the next round.

The Dinner Party

The King and Queen are hosting a dinner party for 10 of their subjects. The Jack is out of town on a diplomatic mission. Your mission is to design a rectangular table to seat the King, Queen, and their guests. Your Queen would like to review your models and make her choice from all of the possible options.

Given a standard deck of playing cards, have each member of your design team choose one suit, remove the Jack, and with their cards construct a model given a scale of 1 inch=1 foot. (Round the measurement of the cards to the nearest ½ inch.)

Calculate and compare all possible areas, perimeters, and the available area of each guests' place setting.

Be prepared to demonstrate your results and give your Queen a professional recommendation as to which table would provide the best seating arrangement.

Contributed by Larry Gotham

Create Your Own In-Out Table – With One Dice

For this activity, you will create your own function table with a one-step rule.

In (x)	Out (y)
1	
2	
3	
4	
5	
6	

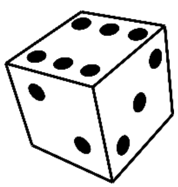
EXAMPLE OF A ONE-STEP RULE:

The rule of this table is

x + 3 = y

Fill in the missing numbers on this table.

In (x)	Out(y)
1	**4**
2	
3	**6**
4	**7**
5	
10	



ROLL THE DICE!

Now, create a rule with the dice that you rolled but don't tell anyone what it is. You can write it on the back of this worksheet or on a scrap piece of paper.

Fill in your table using your rule and then exchange papers with someone and see if you can guess their rule and if they can guess your rule!

Create Your Own In-Out Table – With Two Dice

For this activity, you will create your own function table with a two—step rule.

An example of a 2-step rule:

$5x + 4 = y$

Fill in the numbers that are missing on this table.

In (x)	Out(y)
1	9
2	14
3	
4	24
5	
6	34

Roll the dice!

In (x)	Out (y)
1	
2	
3	
4	
5	
6	

Now, ***you create a rule with the dice that you rolled*** but don't tell anyone what it is.

You can write it on the back of this worksheet or on a scrap piece of paper.

Fill in your table using your rule and then exchange papers with someone and see if you can guess their rule and if they can guess your rule!

Internet Activity 1 – Balancing Shapes

1. Open up Google Chrome Browser. Search for: **Pan Balance Shapes Illuminations.** It should come up as your first search option. Click on this link.

2. Click on **"Set 3"** on the top menu of the Pan Balance Activity.

3. Start by dragging one yellow shape down and dropping it on the right side of the pan balance. Then, drag one red shape down on the left side of the pan balance. Continue to build only yellow on one side and red on the other side until both sides are equal. What do you notice?

   Record your observations here:

   ____
   ____
   ____

4. Click on **"Reset Balance"**

5. This time put only blue shapes on one side and only yellow shapes on the other side until the scale is balanced equally. Click on **"Reset Balance."**

6. Look at the table on the right side of the pan balance. What can you infer about the relationship between the red and the blue? Which one do you think weighs more?

   ____

7. Test out your theory. Put only blue on one side and red on the other side until the scale is balanced. What did you discover?____

   ____

8. Click on **"Reset Balance"**

9. Now put only pink on one side and yellow on the other side until the scale is balanced.

10. Click on **"Guess Weights",** enter your guesses for the weights, and check your answers.

11. Now try out one of the other sets on the menu on the top of the pan balance and work through it the same way you worked through Set 2.

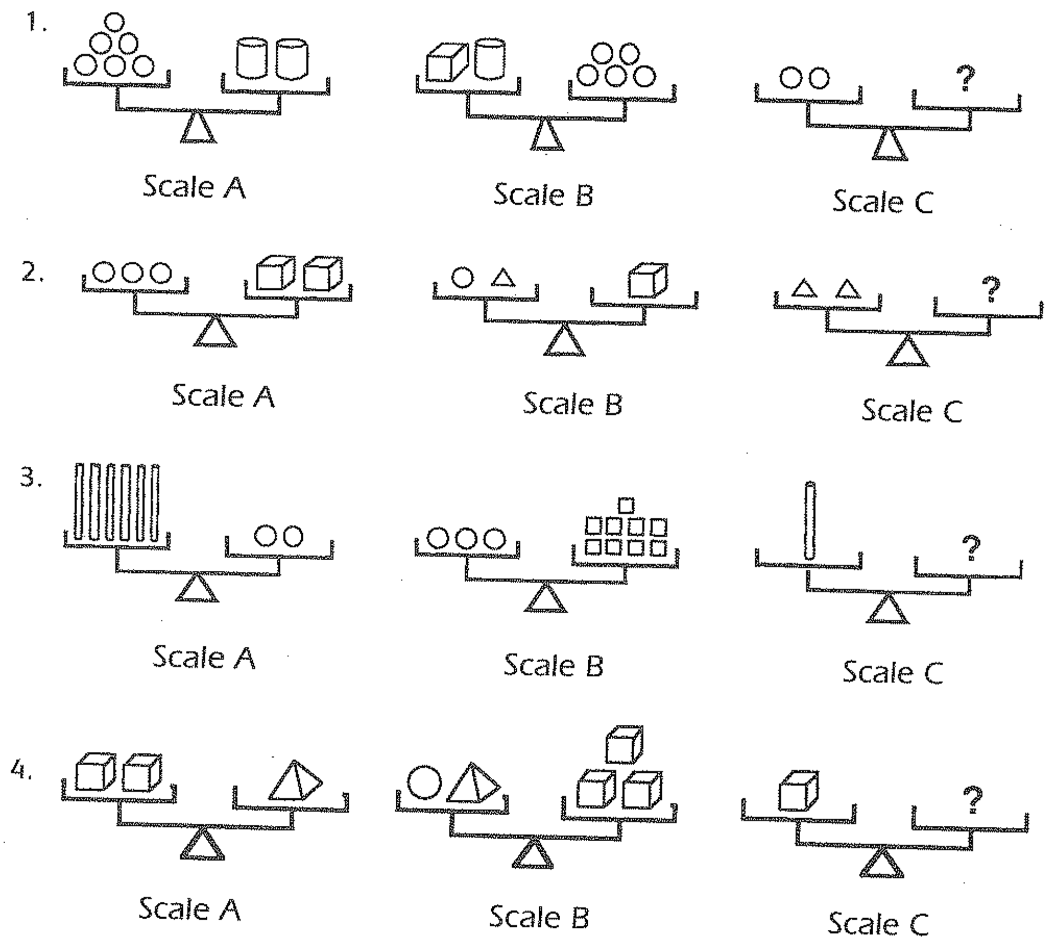
12. What was different about this set? Record your observations here:

    ____
    ____
    ____

Keeping Your Balance

Solve the four balance problems below. In each problem, use the information from the balanced scales A and B to figure out what is needed to balance scale C.

From



From Math Matters: Understanding the Math You Teach

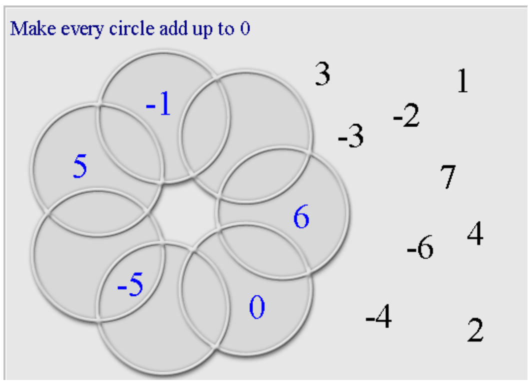
Internet Activity 2 – Circle 0

This is a really good partner activity but you can work it by yourself if you feel more comfortable doing so.

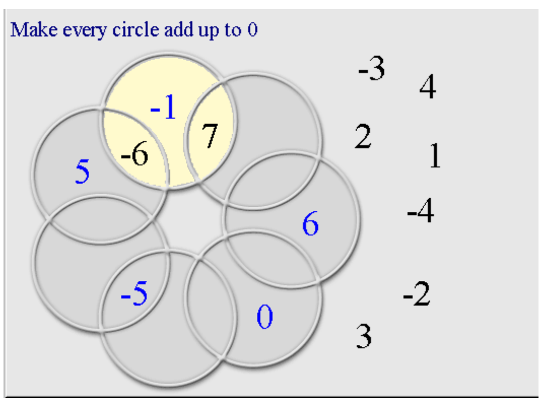
1. Open up Google Chrome Browser and Search for: **National Library of Virtual Manipulatives**. It should be the first search option – Click on it

2. Click on "**Number and Operations, Grades 6-8"**

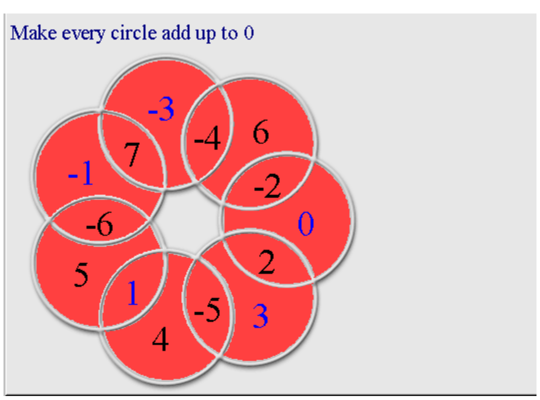
3. Scroll down to: "**Circle 0"** and click on that link.



You should see something like this (only maybe with different numbers).The object of this game is to get all the circles to add up to zero. You can't move the blue numbers but you can move around the black numbers as many times as you need to until the 3 numbers inside the circle add up to zero.



When the circle turns white, like the one pictured here, it means that all the numbers inside that circle add up to zero. You might have to move the numbers around several times until you get the right combination.



When all your circles add up to zero they will all turn red!

If you liked this game, try ***Circle 3, Circle 21,*** or ***Circle 99.***

Exploring Area and Perimeter

Count out 42 tiles and organize them into as many rectangles as possible. You should be able to make 4 different size rectangles with 42 squares. Record the length, width, area, and perimeter of each rectangle.

Area is the amount of square units within each rectangle.

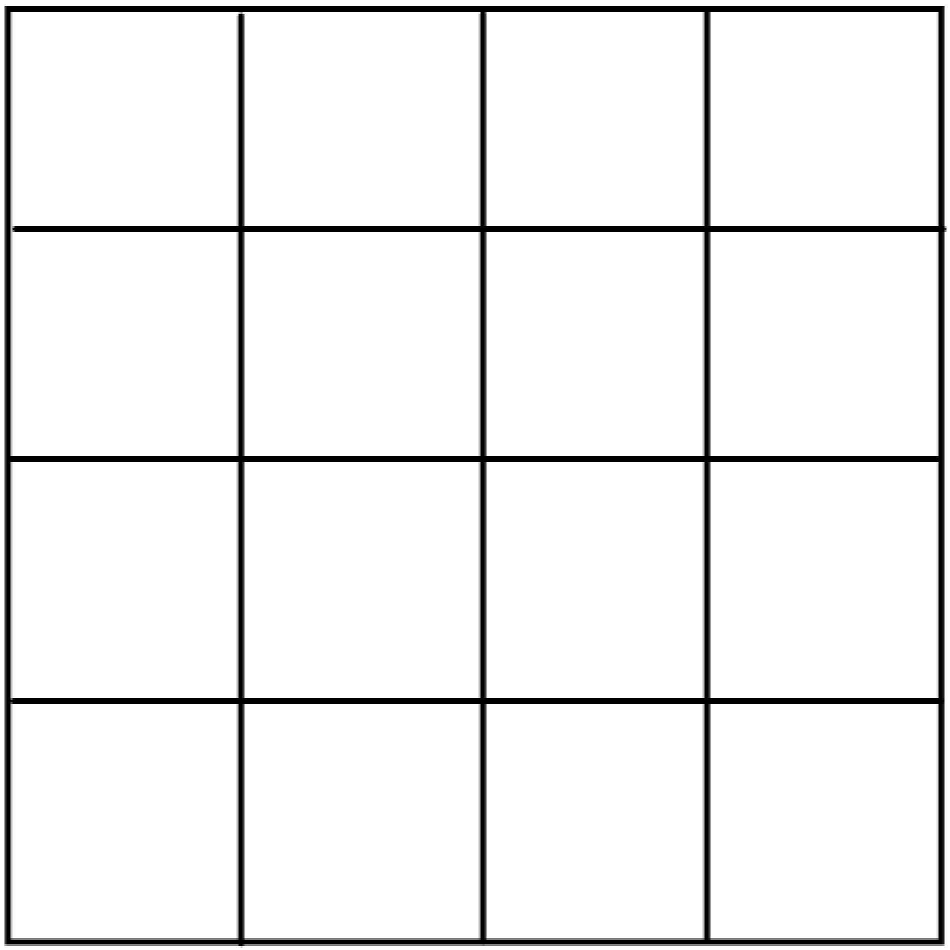
Perimeter is the distance around the outside of each rectangle.

LENGTH	WIDTH	AREA	PERIMETER

What do you notice about the area and perimeters of your four rectangles?

What do the length and width of each rectangle have in common with the number 42?

How Many Squares?



How many squares can you find in the above 4 x 4 square? You can use tiles to help you find the answer.

The Pinwheel Pattern

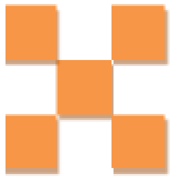


Figure 1

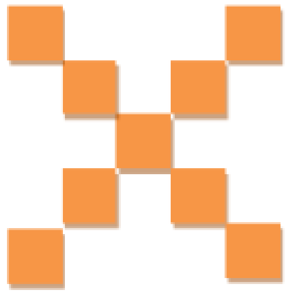


Figure 2

Figure 3

1. What would the 5th figure look like? Sketch it below or use the tiles to make a model of the 5th figure.

2. What patterns do you see?

The Pinwheel Pattern

3. In a few sentences, describe what the 10th figure would look like.

4. In a few sentences, describe how to figure out how many squares there would be in any figure in this pattern.

5. Write the equation.

Adapted from:

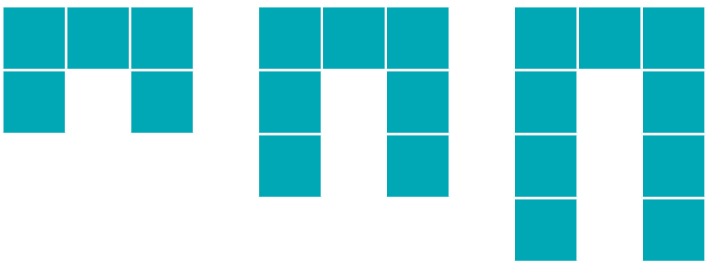
www.collectedny.org

www.visualpatterns.org - Visual Pattern #4

The Arch Problem, part 1

Look at the figures below. What do you notice?

Figure 1 Figure 2 Figure 3



Complete the following sentence:

As the figure number changes, ________________________________ also changes.

The Arch Problem, part 2

1. Sketch the next two figures.

Figure 1 Figure 2 Figure 3

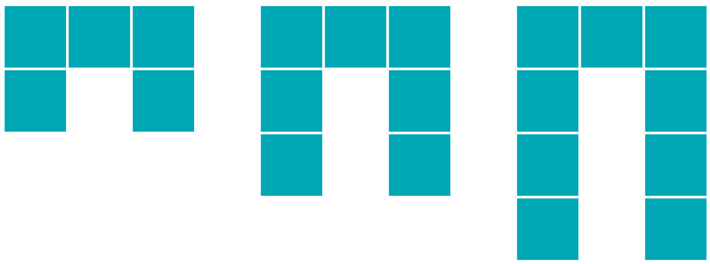


Figure 4:

Figure 5:

2. Complete the table to the right.

Figure Number	Number of Squares
1	
2	
3	
4	
5	

3. In a few sentences, describe what the tenth figure would look like.

4. Explain how you would figure out the number of squares in the 99th figure.

5. BONUS QUESTION: In a few sentences, describe how you would determine how many squares there are in any figure in the pattern.

The Arch Problem, part 3

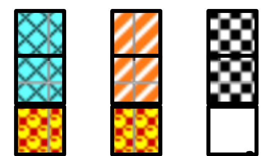
Which figure will have 175 squares in it?

BONUS QUESTION:

Which figure will have 44 squares in it?

Reasoning with Division of Fractions

Directions: First, think about what is being asked. Then think about what might be a reasonable answer. Finally, figure out the answer. You can draw a picture, use a number line, or use tiles to solve. Then, check your answer with a calculator.

Problem	Question Being Asked	Estimate	Strategy to Solve
Example: 3 ÷ 2/3	How many 2/3 are there in 3?	At least 3 since 2/3 is less than 1; probably 4 but less than 6 since 2/3 is more than 1/2	 I have 4 sets of 2/3 and 1/2 of another 2/3. 3 ÷ 2/3 = 4 1/2
4 ÷ 1/2			
2 ÷ 1/5			

Problem	Question Being Asked	Estimate	Strategy to Solve
6 ÷ 1/3			
4 ÷ 3/5			
4 ÷ 3/4			

One zip weighs as much as 3 zaps.

2 zaps weigh as much as 5 zowies.

3 zowies weigh as much as 2 swooshes.

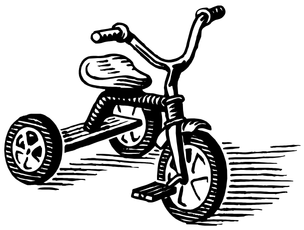
If one swoosh weighs 60 pounds, how many pounds does a zip weigh?

The Handshake Problem:
The Supreme Court of the United States

There are nine justices on the U.S. Supreme Court. Every year, the Supreme Court session begins with each judge shaking hands with every other judge.

Chief Justice Melville W. Fuller (1888-1910) started this custom, saying it shows "that the harmony of aims, if not views, is the court's guiding principle."

If each justice shakes hands exactly once with each of the other justices, how many handshakes will there be? Show how you got your answer.



The Bicycle Shop Problem

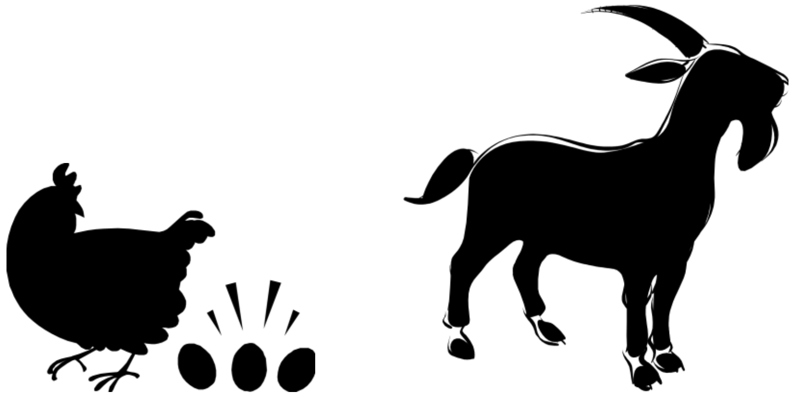
A bicycle shop has a total inventory of 36, some bicycles and some tricycles. Altogether, the bicycles and tricycles have a total of 80 wheels. How many of each type of bike are in the bicycle shop?

- The sum of two numbers is 65 and the difference of the same two numbers is 19. Can you figure out what the two numbers are?

- The sum of two numbers is 33 and the difference of the same two numbers is 11. Can you figure out what the two numbers are?

- The sum of two numbers is 98 and the difference of the same two numbers is 14. Can you figure out what the two numbers are?

- The sum of two numbers is 243 and the difference of the same two numbers is 69. Can you figure out what the two numbers are?



Farmer Montague raises chickens and goats. She is not sure how many she has of each animal, but she does know she has 22 animals all together. She also knows that all together, her animals have 56 legs. How many of each type of animal does Farmer Montague have?

The Painted Cube

Activity 1 – How many faces are painted yellow?

Using 1-centimeter white cubes, Sally builds a 3 x 3 x 3 cube. Sally paints all of the faces of the big cube with yellow paint. Then she breaks it back down into 1-centimeter cubes.

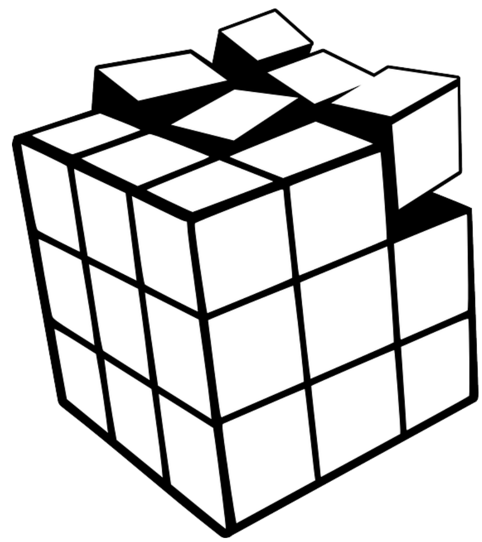
How many of the 1-centimeter cubes have no yellow faces at all?

Exactly one face painted yellow?

Exactly two faces painted yellow?

Exactly three faces painted yellow?

More than three faces painted yellow?



How do you know your conclusions are correct? Explain.

The Painted Cube

Activity 2 – What if the cube was bigger or smaller?

Fill in the table below with your findings from ***Activity 1*** for a cube that is 3 x 3 x 3, then explore cubes with different side lengths and complete the rest of the table.

Length of the Sides (x)	**Number of cubes with *0 faces* painted**	**Number of cubes with *1 face* painted**	**Number of cubes with *2 faces* painted**	**Number of cubes with *3 faces* painted**	**Number of cubes with *more than 3 faces* painted**	***Total* number of 1-cm cubes in larger cube**
2						
3						
4						
5						

What patterns do you notice after all the information is filled in?

The Painted Cube

Activity 3 – A closer look at 2 faces painted

1. How many cubes would have 2 faces painted if the cube was 10 x 10 x 10?

2. Can you create a rule that would predict how many 1 cm cubes would have 2 faces painted if the side length of the cube was any number (n)?

3. Use graph paper to graph the information you gathered for cubes that have 2 faces painted. Put the length of sides on the x-axis and the number of cubes with 2 faces painted on the y-axis.

The Painted Cube

Activity 4 – A closer look at 1 face painted

1. How may cubes would have 1 face painted if the cube was 10 x 10 x 10?

2. Can you create a rule that would predict how many 1 cm cubes would have 1 face painted if the side length of the cube was any number (n)?

3. Use graph paper to graph the information you gathered for cubes that have 1 face painted. Put the length of sides on the x-axis and the number of cubes with 1 face painted on the y-axis.

The Painted Cube

Activity 5 – Comparing Your Graphs

Compare the graphs that you made in ***Activity 3*** and ***Activity 4.***

1. How are they the same?

2. How are they different?

3. Which graph do you think represents a linear relationship? Explain your reasoning.

4. Which graph do you think represents a quadratic relationship? Explain your reasoning.

The Painted Cube – Teacher's Notes

About This Set of Activities

This version of *The Painted Cube* is segmented into several activities that increase in difficulty with the idea of making *The Painted Cube* accessible to all of our students, while providing extension activities for students that finish the activity quickly and are working at a more advanced level.

If you plan to do the entire set of activities with all your students, you might want to scaffold the activities over several days, doing one each day, and providing students with follow up activities to build on each new level of understanding. Each activity should be accompanied by lots of discussion, with students explaining how they came to their conclusions. It is important that students come to a concrete understanding of each of the conditions of Activity 1 before moving to the next activity and so on.

Allow plenty of time for productive struggle. Students may not understand what they are looking for at first, so students should be encouraged to deconstruct the problem and explain it in their own words before they jump into problem solving.

Building Background Knowledge

Before doing this set of activities with your students, you may want to try doing a similar one-dimensional pattern activity that leads to graphing, such as *The Patio Project* in *EMPower Seeking Patterns, Building Rules*. Then, students will be able to transition to *The Painted Cube* with some experience in looking for patterns in square and rectangle designs, thinking about rules that describe those patterns, and graphing linear equations.

Encourage Use of Visual Models

- Interlocking cube manipulatives
- Centimeter dot paper
- Graph paper
- Animated version of Painted Cube at: http://nrich.maths.org/2322

The Painted Cube – Teacher's Notes

Questions to ask to facilitate productive struggle

- Can you draw a picture of the situation?
- Can you make a model of the cube with the blocks?
- Can you use the centimeter dot paper to draw/make a cube?
- What do you already know about the cube?
- What do you see (when you look at the drawing or the cube)?
- What do you notice about the cubes that have 3 faces painted?
- What do you notice about the cubes that have 2 faces painted?
- Where would the cubes with no faces painted be?
- What questions do you have?
- How do you know this answer is correct?
- Can you show us how you arrived at your conclusion?

Questions to ask to extend the activity

Vocabulary

- How many **edges** does any size cube have?
- How many **faces** does any size cube have?
- How many **vertices** does any size cube have?

Comparing cubes with different side lengths:

- What stays the same when the cube side lengths increase?
- What changes when the cube side lengths increase?

Refer to your table from Activity 2 for the following:

- What would a graph that represents 0-faces painted look like?
- What would a graph that represents 3-faces painted look like?
- What would a graph that represents total number of cubes look like?
- How do you know?

The Painted Cube – Teacher's Notes

Activity 1 – How many faces are painted yellow?

Using 1-centimeter white cubes, Sally builds a 3 x 3 x 3 cube. Sally paints all of the faces of the big cube with yellow paint. Then she breaks it back down into 1-centimeter cubes.

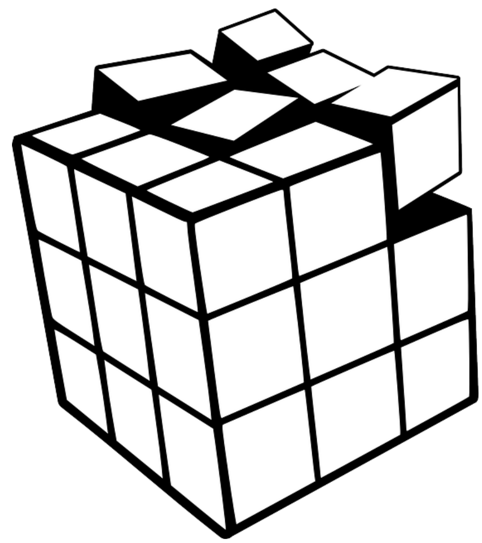
How many of the 1-centimeter cubes have no yellow faces at all? = **1**

Exactly one face painted yellow? = **6**

Exactly two faces painted yellow? = **12**

Exactly three faces painted yellow? = **8**

More than three faces painted yellow? = **0**



How do you know your conclusions are correct? Explain.

Explanations will vary.

Many students will have difficulty putting their thinking into words on paper. Ask them to show the class how they came up with their conclusions.

The Painted Cube – Teacher's Notes

Activity 2 – What if the cube was bigger or smaller?

Fill in the table below with your findings from ***Activity 1*** for a cube that is 3 x 3 x 3, then explore cubes with different side lengths and complete the rest of the table.

Length of the Sides (x)	**Number of cubes with *0 faces* painted**	**Number of cubes with *1 face* painted**	**Number of cubes with *2 faces* painted**	**Number of cubes with *3 faces* painted**	**Number of cubes with *more than 3 faces* painted**	***Total* number of 1-cm cubes in larger cube**
2	**0**	**0**	**0**	**8**	**0**	**8**
3	**1**	**6**	**12**	**8**	**0**	**27**
4	**8**	**24**	**24**	**8**	**0**	**64**
5	**27**	**54**	**36**	**8**	**0**	**125**

What patterns do you notice after all the information is filled in?

Possible responses:

The number of cubes with 2 faces painted increases by 12 every time the side length is increased by 1.

The number of cubes with 3 faces painted is always 8.

I don't see any patterns. (Respond to this with questions such as: How do you know if something is a pattern? What do you see happening in the column with 3 faces painted? How is that different from the column with 2 faces painted?)

The Painted Cube – Teacher's Notes

Extension Activity 1 – A closer look at 2 faces painted

1. How many cubes would have <u>2 faces painted</u> if the cube was 10 x 10 x 10?

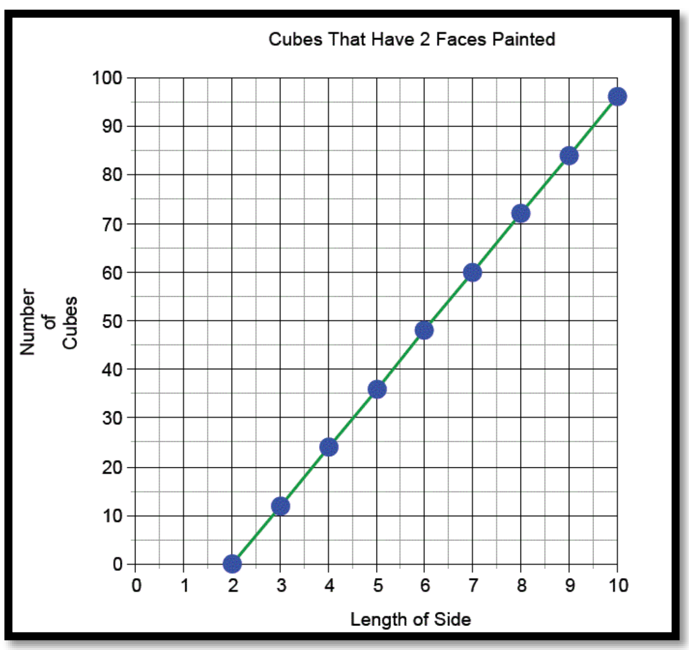
96

2. Can you create a rule that would predict how many 1 cm cubes would have <u>2 faces painted</u> if the side length of the cube was any number (n)?

Students should be encouraged here to write an explanation of the pattern or rule. Some students may be able to represent the rule as a linear equation or function.

12(n-2) or 12n – 24 or f(*n*) = 12*n* – 24

3. Use graph paper to graph the information you gathered for cubes that have <u>2 faces painted</u>. Put the length of side on the x-axis and the number of cubes with <u>2 faces painted</u> on the y-axis.



The Painted Cube – Teacher's Notes

Extension Activity 1 - A closer look at 1 face painted

1. How may cubes would have 1 face painted if the cube was 10 x 10 x 10?

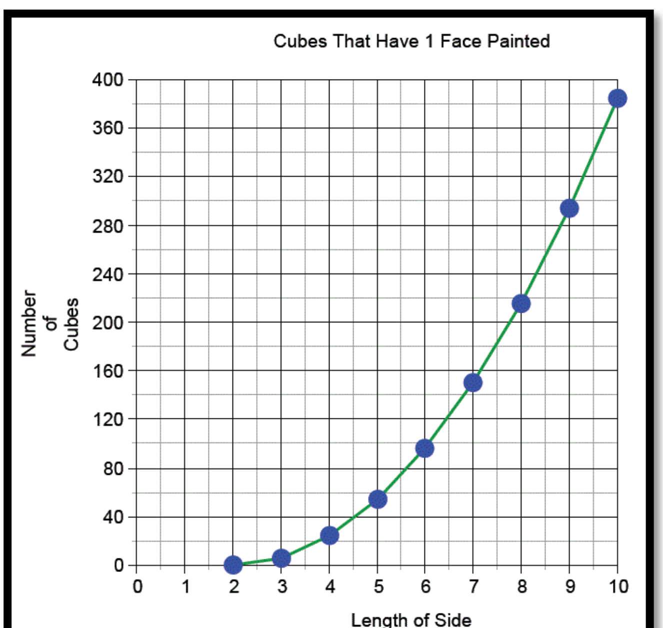
   384

2. Can you create a rule that would predict how many 1 cm cubes would have 1 face painted if the side length of the cube was any number (n)?

   Students should be encouraged here to write an explanation of the pattern or rule. Some students may be able to represent the rule as a quadratic equation or function.

   $$6(n-2)^2 \text{ or } 6(n-2)(n-2) \text{ or } f(n) = 6n^2 - 24n + 24$$

3. Use graph paper to graph the information you gathered for cubes that have 1 face painted. Put the length of side on the x-axis and the number of cubes with 1 face painted on the y-axis.



The Painted Cube – Teacher's Notes

Extension Activity 1 – Comparing Your Graphs

Compare the graphs that you made in *Activity 3* and *Activity 4.*

1. How are they the same?

Possible responses:

They both have the same edge length on the x-axis

2. How are they different?

Possible responses:

The graph with one-face painted is a curved line and the graph with two-faces painted is a straight line.

The numbers go higher up on the y-axis on the graph with one-face painted.

3. Which graph do you think represents a linear relationship? Explain your reasoning.

Cubes with 2 faces painted. Explanations will vary.

4. Which graph do you think represents a quadratic relationship? Explain your reasoning.

Cubes with 1 face painted. Explanations will vary.

The Painted Cube – Teacher's Notes

Extension Activity 2 – Group Work – Graphing

As an alternative to *Extension Activity 1* you may want to do the following extension activity after students have completed the table in *Activity 2*.

Break students up into four groups and assign each group a graph to represent one column of the table in *Activity 2.*

- **Group 1 – Graph cubes with 1 face**
  - Area – a quadratic relationship
  - **$6(n-2)^2$ or $6(n-2)(n-2)$ or $f(n) = 6n^2 - 24n + 24$**
- **Group 2 – Graph cubes with 2 faces**
  - The edges grow consistently – linear relationship.
  - **$12(n-2)$ or $12n-24$ or $f(n) = 12n-24$**
- **Group 3 – Graph cubes with 3 faces**
  - This is a type of linear relationship that is constant
  - **$n = 8$ or $(f)\ n = 8$**
- **Group 4 – Graph total number of cubes**
  - This grows in volume – a cubic relationship
  - **n^3 or $f(n) = n^3$**

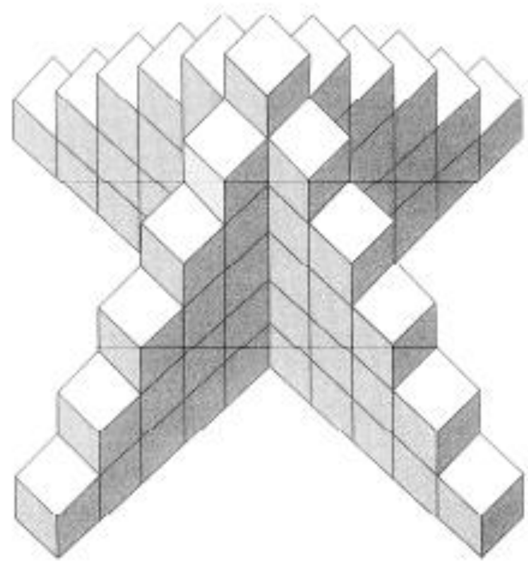
After all four groups have completed their graphs, lead a discussion to compare the graphs and explore the following questions:

- Can we create a rule that would describe what is happening in each graph? (Rule may be described by student as explicit or recursive.)
- Which graph represents a **linear** relationship?
- Which graph represents a **quadratic** relationship?
- Which graph represents a **cubic** relationship?
- Which graph represents a **constant** relationship?
- Can we represent the rules as an equation or function?

Resources: *NYSED Common Core/TASC Math Teacher Learning & Leadership Institute, Frogs, Fleas, and Painted Cubes, education.ti.com, http://nrich.maths.org/2322*

Condo Challenge

Building planners are designing a new condo building. They develop the model to the right, with each small cube representing a condominium.



1. How many cubes are needed to build the six-condo high model of Quadratic Condominiums shown to the right?

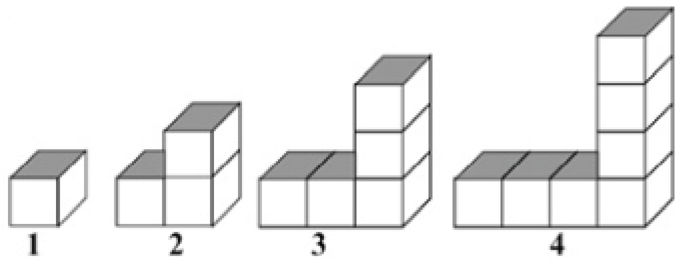
2. How many cubes would be needed to build a twelve-cube high model?

3. Explain how you got your answers for the first and second questions. Can you think of any other ways of solving this problem?

4. How many cubes would be needed to build a model of any height? Explain your thinking and make a rule.

5. How could your findings help the builders of Quadratic Condominiums?

The Rising Smokestack



1. What would the 6th figure look like? Sketch it below or use the snap cubes to make a model of the 6th figure.

2. What patterns do you see?

The Rising Smokestack

3. In a few sentences, describe what the 10th figure would look like.

4. In a few sentences, describe how to figure out how many cubes there would be in any figure in this pattern.

5. Write the equation.

Adapted from:

www.collectedny.org

www.visualpatterns.org - Visual Pattern #2

The Aquarium Problem

You have been asked to design an aquarium in the shape of a rectangular prism for the school visitor's lounge. Because of the type of fish being purchased, the pet store recommends that the aquarium should hold 36 cubic feet of water. Find as many different dimensions for the aquarium as possible. Then decide which aquarium you would recommend for the lounge and explain why you made that choice.

You can use the snap cubes to build models of the aquariums. Each snap cube represents 1 cubic feet of water.

Adapted from *From Kabiri, M. S., and N. L. Smith "Turning Traditional Textbook Problems into Open-Ended Problems," Mathematics Teaching in the Middle School, Vol. 9, No. 3, 2003, 186-92.*

APPENDIX 3

ANN – The Adult Numeracy Network
http://www.adultnumeracynetwork.org/

Balance Mathematics Instruction by Balancing Shapes
http://techtipsforteachers.weebly.com/blog/balance-mathematics-instruction-by-balancing-shapes

CollectEdNY
http://www.collectedny.org/category/subjects/math/

Deepen Conceptual Understanding in Math with Virtual Manipulatives (and it's fun!)
http://www.collectedny.org/2015/02/deepen-conceptual-understanding-in-math-with-virtual-manipulatives/

Functions: Bridging from Concrete Understanding to Abstract Representations
http://techtipsforteachers.weebly.com/blog/functions-bridging-from-concrete-understanding-to-abstract-representations

Mathematics Assessment Project
http://map.mathshell.org/

Math Memos
http://www.collectedny.org/mathmemos/

Pattern Block Lessons – The Math Learning Center
http://catalog.mathlearningcenter.org/files/pdfs/PBLCCSS35-0412w.pdf

The CUNY HSE Curriculum Framework – Problem Solving in Functions and Algebra
http://www.cuny.edu/academics/programs/notable/CATA/lit/hseframework/Section4CUNYHSEFrameworkMath.pdf

The Positive Engagement Project
http://www.pepnonprofit.org/mathematics.html

The Product Game – NCTM Illuminations
http://illuminations.nctm.org/Lesson.aspx?id=5729

Using Benchmark Fractions Plus – EMPower Sampler
http://empower.terc.edu/pdf/UB%20Sampler_March2014.pdf

Visual Patterns
http://www.visualpatterns.org/