

Rate of Change and Starting Amount

Lesson Plan

Rate of change is a fundamental concept when working with functions. Too often it is presented by writing y = mx + b on the board, with m—the rate of change—being defined as the slope. This lesson focuses students on function tables, looking for patterns and making observations. From there, we connect what students see to how the things they observe appear in function rules, and we introduce the vocabulary of "rate of change" and "starting amount," which can later be referred to as "slope" and "y-intercept." Before we talk about slope and y-intercept though, we want students to have a flexible understanding of how to identify rate of change and starting amount in tables, rules, and graphs, and to know how they can use these concepts in solving problems.

OBJECTIVES

- Students will discover the relationship between rate of change, starting amount, and function rules.
- Students will use rate of change and starting amount to determine function rules from tables.
- Students will determine rate of change, starting amount, and function rules for contextualized problems.

COMMON CORE STANDARDS OF MATHEMATICAL PRACTICE

MP1; MP2; MP4; MP6; MP7; MP8

HANDOUTS

- 1 What's My Rule?
- 2 Discovering Rules 1
- 3 Discovering Rules 2
- 4 The Power of the Rate of Change
- 5 The Many Faces of Functions (see Teacher Note below)
- 6 Counting Antibodies

- 7 Exploring Polygons
- 8 Weekend Getaway
- 9 Counting Cricket Chirps
- 10 Lightning and Thunder
- 11 Temperature Scales
- 12 Measuring Babble

KEY VOCABULARY

rate of change: the constant change in the output when the inputs are consecutive

starting amount: the value of the output when the input is zero

ACTIVITY 1 Launch

1 This opening activity is crucial to the lesson and cannot be rushed or skipped. You will need at least 45–60 minutes to do the whole thing. The point of this activity is to create the "headache" for which the rate of change discovery will provide the "aspirin." Like they did with the Function Machine activity in the first unit, students are going to look at abstract function tables and figure out the function rule that fits. The big difference is that most of the rules here have two operations. Though students struggle with this activity, it is productive and they tend to persevere, particularly because it has the elements of a puzzle that they feel motivated to solve.

2 Some of your students may be able to get straight to work on these. Others may have a hard time getting started and/or get frustrated easily. You might want to draw the first function table (top-left corner) from the What's My Rule? handout on the board and discuss it as a class. Here's an example of how you might frame your questions:

Teacher: As I go from the input to the output, how are the

numbers changing?

Students: They are getting bigger.

Teacher: What operations usually make things bigger?

Students: Addition and multiplication.

Teacher: Ok, let's try one of those. How could you use addition or

multiplication to get from 1 to 5?

Students: Add 4.

UNIT 3: RATE OF CHANGE AND STARTING AMOUNT

Teacher: Good. Is that the function rule? How do you know?

Students: 1 + 4 is 5, but 2 + 4 is not 7.

Teacher: Okay. Let's try something else. But first, let's keep track

of all the rules we try.

(Write "+4" on the side of the table and cross it out.)

What else can we do to get from 1 to 5?

Students: Multiply by 5.

Teacher: Good. Is that the rule for this function? And how do you

know?

Students: No, because 2 times 5 is not 7. It doesn't work for the

other inputs and outputs.

Teacher: Good. Add that to the list of rules we've tried so far. So

it looks like we are going to need a two-step rule. How can we use multiplication and addition together to get

from 1 to 5?

Students: ?

Teacher: When you multiplied 2 by 5, was the output too big or

too small?

Students: Too big.

Teacher: So what is another number we could multiply by?

Students: 4?

Teacher: Excellent! Let's try it. If you put the 1 in and multiply

by 4, what do you need to do to get the output?

- 3 Walk around the room as students are looking for the rules that fit each function table. Encourage them to keep track of their guesses in the space on the side of the function tables or on a separate piece of paper. After they come up with a rule or two, ask them if keeping track of their guesses has been helpful.
- 4 If any students happen to notice the constant change in the outputs when the inputs are consecutive, congratulate them and say that is an important observation that the class will come back to later. But don't make too much of it at this point.
- It will probably take your class about 45 minutes to complete the sheet. If you have a few students who finish early, make sure the rest of the class still has time to struggle through the handout. This handout is a good equalizer in some ways, so it is unlikely that you will have anyone who finishes too fast. For those who do, I like to ask them to spend a few minutes to play the Function Game from the Introducing Functions lesson in Unit 1. After working hard, students enjoy the thought of trying to stump their teacher.
- Once everyone is done, write the function tables on the board or on newsprint. If you only have time to write some of them, focus on the ones that have two-step rules. Now ask different students come up and add the function rules they found. Make sure that the whole class agrees on the correct rule for each table, and then ask for volunteers to explain why each rule is correct.

7 As a final piece, ask students to reflect on the process. Ask them, How did it feel working on finding these rules? and What strategies were helpful to you?

ACTIVITY 2 Rate of Change Discovery

- Tell students they are going to spend some time looking at functions rules and tables to see if they notice any patterns that might help them find rules in the future. Distribute the handout Discovering Rules 1 and have students use the function rules to complete the tables. Once they are done, ask them, "What do you notice about these two functions? How are they similar? How are they different?" Some possible student responses:
 - *The outputs for both alternate between odd and even numbers.*
 - They both have \times 3 in the rule.
 - *The outputs for both functions go up by 3.*
 - *The inputs for both functions go up by 1.* (See if there is a student who knows the math word for this. If not you can introduce the word *consecutive*.)
 - The outputs on the right are 4 larger than the ones on the left.
- We want to draw their attention to the difference in the outputs. Once students have observed this difference, we can introduce the formal name. Ask students to write the following definition on their handout, and tell them that this is a crucial concept in functions:

The rate of change is the constant change in the outputs when the inputs are consecutive.

For both of these functions the rate of change is 3. That means the outputs grow by 3 when the inputs are consecutive.

- 3 Ask your students if the rate of change of 3 is visible anywhere else, aside from the constant change in the outputs. We want them to notice that the rate of change appears in two places: the table and the rule.
- 4 Have students look back at the function tables from the Activity 1 Launch and ask them to identify the rate of change for each function. Make sure they can identify the rate of change in both the rule and the table. Almost all of the tables have consecutive inputs,

but two do not: " \times 2 + 4" and " \times 3 + 4." For both of those, if students don't raise it themselves, ask why only the first 4 outputs show the rate of change of 2 and 3, respectively. We want students to notice that when the inputs are not consecutive, the rate of change is not visible in the same way as when the inputs are consecutive.

You can talk about this more here, though students will have an opportunity to explore this idea further during Activity 4. If you want to pursue it, you might look at the function table from the launch for the function "x 2 + 4". The first four inputs are consecutive, so you can easily observe the rate of change of 2. But then the inputs jump from 4 to 9. And as the inputs make that jump, the outputs go from 12 to 22. You can have students figure out the outputs for the missing inputs of 5, 6, 7, and 8.

ACTIVITY 3 Starting Amount Discovery

- 1 Give out Discovering Rules 2 and give students time to complete the tables. Ask them to identify the rate of change for each function, and make sure they can see it in both the table and in each rule.
- Now, ask students to make observations about the similarities and differences between the two functions in the same way you did for the rate of change discovery. The one observation you want to focus in on is the fact that both functions share one solution: (0, 4). They may also notice that both functions have a "+ 4" in the rule. You can ask them how the output could be the same for both of them if they have different function rules (and different rates of change). If the input is 0, it will knock out the first part of the function, since anything times 0 is 0. That leaves you with the + 4.
- 3 Next, tell students that the shared solution is our second important function concept of the day. Ask students to write the following definition on the DISCOVERING RULES 2 handout:

The starting amount is the value of the output when the input is zero.

Again, look back at the function tables from the and find the starting amount for each function. They should be able to find it in the rule for all of them. For each table that doesn't have an input of 0, ask students what would come out if you put a 0 into that function rule.

ACTIVITY 4 The Power of Rate of Change

- 1 Give students an opportunity to recognize the implications of these two concepts. Have them work on The Power of Rate of Change handout.
- The two functions tables on the top of the page have consecutive inputs while the two on the bottom do not. You should not mention this to students. In this activity, we want students to feel the speed and efficiency with which their new knowledge will allow them to come up with function rules. We also want them to make an important mistake. The function table on the bottom left has the same outputs as the function table above it. Many students will write the same function rule for both because they don't notice the inputs on the bottom one are not consecutive. This is an important mistake and you should not take away their opportunity to make it and reflect on it. Most students will struggle with the function table on the bottom right. Most will work on it as they would have worked on all tables before they know about rate of change and starting amount—guessing and checking rules. That is a fine strategy while they are working. It is not necessary for everyone to have gotten that last function rule before debriefing.
- 3 Have students come up, share their rule and explain how they got it for the first two functions. Make sure to ask the class how we know each rule works—does it work for all of the given inputs and outputs?
- 4 Ask the class how knowing about rate of change and starting amount impacts their abilities to come up with two-step function rules.
- Write the function table from the bottom left up on the board. Ask if anyone found a rule for it. Then ask if anyone came up with a different rule. The most common answer is likely to be "× 4 + 6". Some students may have come up with the correct answer— "× 2 + 6". If you get both, write them both on the board without revealing which one is correct. Ask them which one fits all of the inputs and outputs in the table and they will see it is "× 2 + 6". Ask them about why they think the "× 4 + 6"—which will be the most common answer—doesn't work. We want them to recognize that the inputs being consecutive is an important part of the rate of change. To help students see the rate of change of 2, you can add the "missing" numbers to make the inputs consecutive.

The function table on the bottom right is tricky because the inputs are not in size order, and they are not consecutive. Most students will assume this means the rate of change cannot be determined. We want them to notice that there are two pairs of inputs that are consecutive, if they rearrange the order of the inputs—the 8 and 9 and the 4 and 5. From there, they can identify the rate of change of 4.

7 From this point on, you should always ask students to identify the starting amount and rate of change for every function they look at.

TEACHER NOTE: INTRODUCING FORMAL FUNCTION NOTATION

We have purposely held back on introducing formal notation of functions before now. We believe it is important for students to have a strong foundation in the concepts of the functions—the function machine model, the three views of a function, rate of change and starting amount—before they move into some of the more abstract notation. That said, at some point we do need to introduce our students (at least those at the HSE level) to the notation they are likely to encounter during the exam.

If you are working with lower-level students, or if you think you want to keep your students working on the foundational concepts, you will have to adapt the handouts for the rest of this unit, which all employ more formal notation.

We are including an activity titled, "The Many Faces of Function Rules" to help you and your students make the transition and be comfortable with different ways of writing function rules. If you feel your students are ready, this is a good moment in their function study to do so.

As they work through this activity, a good way to keep students centered is to ask them, for each different version of the rule, what is different and what is the same. The principles of inputs and outputs are the same. The rate of change is the same. The starting amount is the same. If students get thrown by the new notation, have them focus on translating it back to a form they are comfortable with.

The first change is to write the function rule as an equation–including the words "In" and "Out" in the rule. The next change is writing the table horizontally. The third change is to substitute x for the In and y for the Out. The next major change is on the last page: p=10c+120. We want students to realize that x and y are the most commonly used variables to represent, but that other variables can be used as well. We also want them to see that regardless of what variables are used, we can tell which are the inputs and which are the outputs by where they appear in the rule and the table. The final change might be the strangest looking, but it is one that students are likely to see on their HSE exam.

f(x) = 6x + 5 can be read as "f of x is 6x + 5," or "the function of x is equal to 6x + 5." Also, you can ask for an output in the following way: "For the function f(x) = 6x + 5, what is f(21)?" which means, "If you put a 21 into this function, what comes out?"

ACTIVITY 5

Looking at Functions in Real-World Contexts

To use their understanding of rates of change and starting amounts to determine rules. Additionally, they all allow students to consider the meaning of rate of change and starting amounts in real-life contexts. They also give students an opportunity to both work with function rules that describe real-world situations and which are written in formal notation.

Below are brief descriptions of each activity. Teachers should use the descriptions to decide which activities you want your students to do and how you might have students work on them. You can certainly have your students work on all of them, but that would likely take several classes.

Counting Antibodies

This is the most comprehensive activity, with the widest variety of different questions. It deals with a health care situation in which a scientist is testing two medicines to determine their effect on the number of antibodies in the blood sample of a patient. Students have to use a function rule to complete a table, come up with an output for a given input not in the table, find an input for a given output, find the rate of change and starting amount for two different functions, create a function rule from a function table, come up with a solution that does not fit a function rule, graph two functions, interpret a point of intersection and make a choice based on their work, all within the context of a contextualized healthcare situation.

Exploring Polygons

This activity incorporates some geometry into our function exploration. You should make sure all of your students understand the column that reads "Sum of the interior angles". You might have them use the pictures on the top to mark the interior angles. You might even have them calculate the measure of each interior angle, given that the polygons are regular (and therefore have equal angles).

This activity is more open than *Counting Antibodies* and allows for a few different solution methods. Whichever methods students use, make sure to raise using the rate of change to determine the rule as one option.

Weekend Getaway

This activity gives students a chance to work with a rate of change that is a decimal. They will have to complete a table, come up with a rule, plug an input into the rule, find an input for a given output, and interpret both the rate of change and the starting amount in the context of the situation—which is about the cost of renting a car.

Counting Cricket Chirps

There is a relationship between the number of times a cricket chirps per minute and the temperature. This activity involves a function that describes that relationship. It gives students another opportunity to work with inputs that are not consecutive, to discuss the inputs and outputs in the context of the situation, to find the rate of change, and to consider the domain of the function. The domain of a function is the set of inputs for which the function is defined. You can put any number into the cricket function, but any number less than 40° will result in a negative number of chirps per minute—fun to think about, but not actually possible.

Lightning and Thunder

This is another function with connections to science. This time students are looking for a function rule that describes the relationship between the distance between you and a lightning strike and the amount of time it takes for the sound of the thunder to reach you. This one also involves a rate of change that is less than one—expressed as 1/5 or 0.2. Like *Exploring Polygons*, this activity is open and the questions can be answered without determining a function rule. If no one uses that strategy, I would pursue it after they have presented all of their methods.

Temperature Scales

This activity is also has science connections—it involves a chart with information about temperature equivalencies between Celsius, Fahrenheit, and Kelvin. It is another opportunity for students to work with inputs that are not consecutive. Depending on which conversion you are making, it will also involve a fraction as the rate of change. This activity also allows students to write their own questions. You can have them share them with the whole class, or write them on the board. Then you can have the students chose a question or two they want to work on. It is a nice way for students at all different levels to be engaged in the same activity. As with *Exploring Polygons* and *Lightning & Thunder*, there is no explicit question asking students to determine a function rule for converting Celsius to Fahrenheit. You can raise this question to the class, use it as a bonus question, or have it as an extension for students who finish early.

Measuring Babble

This activity is different in that it is the only one with a negative rate of change. It deals with a politician who wants to make her opponent look bad in front of the press. The function describes the rate at which an audience leaves a speech over time. Given the starting amount, students need to figure out what time to invite the press to come hear the speech so that no one is left.

What's My Rule?

Fill in the missing values. All rules use whole numbers.

Rule:	
In	Out
1	5
2	7
3	9
4	11
5	13
6	15

Rule:		
In	Out	Solution
		(0, 0)
		(1, 7)
		(2, 14)
		(3, 21)
		(4, 28)
		(5, 35)

Rule:		
In	Out	Solution
1	9	
2	15	
3	21	
4	27	
5	33	

Ruie.	
In	Out
0	2
1	7
2	12
3	17
4	22

Rule:		
In	Out	Solution
		(1, 3)
2	6	
		(3, 9)
4	12	
		(5, 15)
6	18	

Rule:	
In	Out
0	1
1	7
2	13
3	19
4	25
5	31

Bulo

Rule:	
In	Out
1	6
2	8
3	10
4	12
9	

Rule:		
In	Out	Solution
		(0, 4)
		(1, 7)
		(2, 10)
		(3, 13)
100		

Discovering Rules 1

Fill in the missing values.

Function: × 3 + 1		
In	Out	
1		
2		
3		
4		
5		
6		

Function: × 3 + 5		
In	Out	
1		
2		
3		
4		
5		
6		

Discovering Rules 2

Fill in the missing values.

Function: × 5 + 4		
In	Out	
0		
1		
2		
3		
4		
5		

Function: × 2 + 4		
In	Out	
0		
1		
2		
3		
4		
5		

The Power of Rate of Change

Function:					
In	Out				
0	6				
1	10				
2	14				
3	18				
4	22				
5	26				

Function:				
In	Out			
0	7			
1	10			
2	13			
3	16			
4	19			
5	22			

Function:				
In	Out			
0	6			
2	10			
4	14			
6	18			
8	22			
10	26			

Function:				
In	Out			
1	13			
11	53			
8	41			
5	29			
9	45			

The Many Faces of Function Rules and Tables

Textbooks, teachers, and professors may differ in their visual display of function rules and tables. Below, you will find several different function rule and table formats. Even though some will look very different from the rules and tables you have seen up until now, the input-output principles are the same.

ĺ)
Ī		

Rule: Out = In × 6 + 2				
In Out				



In	1	2	5	8	20	
Out	11	19	43			99

3 This function rule is the same as the following:

Out =
$$\ln \times 4 + 1$$

Notice that x indicates the input, and y indicates the output. When a number is written immediately in front of x, this indicates multiplication of the number and x. For example, 4x means "4 times x".

Function: $y = 4x + 1$			
x	У		
0			
2			
6			
	49		
	61		
	121		

4 This function rule is the same as the following:

Function: y = 5x - 2				
X	У			
1				
2				
6				
	48			

5 Re-write this function rule in In/Out format:

Out =

Function: $y = 5x + 1$			
х у			
0			
1			
2			
3			
	56		
	101		

6 Re-write this function rule in In/Out format:

X	0	1	2	7	11		
y						60	140

7 Re-write this function rule in In/Out format:

Out =

Function: $y = 25x + 10$			
х у			
4			
6			
9			
	17.50		
	10		

Re-write this function rule in In/Out format:

Out =

Function: y=		
X	У	
1	8	
2	11	
3	14	
4	17	
6		
	38	

9 Re-write this function rule in In/Out format:

Function: y =			
х	У		
1	10		
2	17		
3	24		
4	31		
6			
	59		

Re-write this function rule in In/Out format:

Out =

Function: $\rho = 10c + 120$			
с р			
0			
1			
2			
10			
.5			
	270		
	620		

Re-write this function rule in x/y format:

y =

Rewrite this function rule in In/Out format:

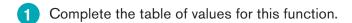
Function: $f(x) = 6x + 5$			
Х	f(x)		
0			
1			
2			
10			
.5			

Counting Antibodies

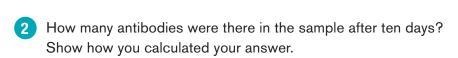
A scientist tested a medicine in order to determine how effective it is in producing antibodies. The following is a function that represents the number of antibodies in a sample of blood from a patient:

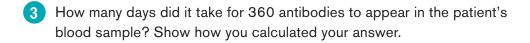
$$a = 20d + 100$$

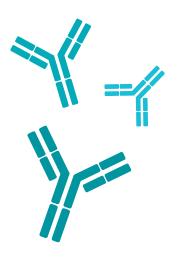
In this function, d represents the number of days that passed in the experiment, and a represents the number of antibodies in a sample of the patient's blood.



Data for Medicine A			
Days Passed	Antibodies in the Sample of Blood		
d	а		
0			
1			
2			
3			
4			
5			







Antibodies are y-shaped molecules of the immune system.

4 How many antibodies were there at the beginning of the experiment? Show how you calculated your answer.

5 What is the rate of change for this function? How can it be interpreted using the context of the problem? In other words, can you describe the rate of change in terms of "days" and the "number of antibodies"?

6 Identify an ordered pair that would not be a solution to this function. Explain why it is not a function solution.

The scientist tested a different medicine, and recorded the following data after following a patient for several days.

Data for Medicine B				
Days Passed	Antibodies in the Sample of Blood			
0	60			
1	90			
2	120			
3	150			
4	180			
5	210			
6	240			

7 If the growth of antibodies continues at the rate shown in the table, predict the number of antibodies in the blood sample after 12 days. Show how you calculated your answer.

8 How many days would be needed to produce 600 antibodies in the patient's blood sample at this same rate? Show how you calculated your answer.

9 Find a function rule that describes the data, where *d* represents the number of days that passed in the experiment, and *a* represents the number of antibodies in a sample of blood from a patient.

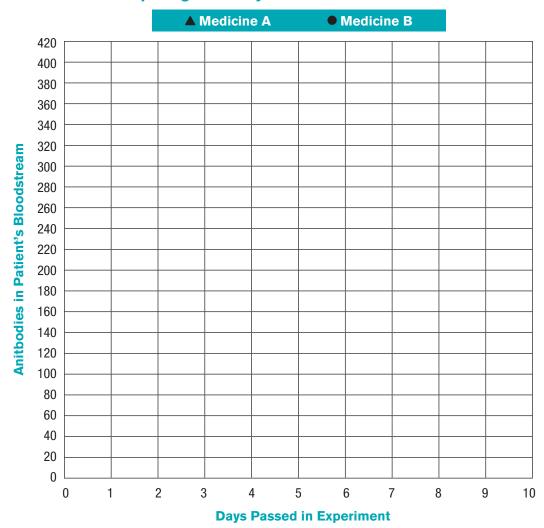
What is the starting amount for this function? How can it be interpreted using the context of the problem? In other words, can you describe the starting amount in terms of "days" and the "number of antibodies"?

What is the *rate of change* for this function? How can it be interpreted using the context of the problem?

If you were ill and you had both medicines to choose from, which would you choose? Why?

Graph the data for both medicines. Use pencil. Use a triangular symbol for each point for Medicine A. Use a circular point for Medicine B.

Comparing Antibody Production of Two Medicines



14	What is the significance of the place on the graph where the two lines intersect?

On the graph, use arrows to indicate the point that represents the starting amount for each function.

Exploring Polygons

A regular polygon is a polygon that has equal angles. Some regular polygons are shown below. Name them based on the number of sides they have.









The table to the right has information on the sum of the interior angles for different polygons. What do you notice? Are there any patterns you see?

1 What will be the sum of the interior angles of a polygon that has 24 sides? Be prepared to show how your group determined your answer.

Number of Sides in the Polygon	Sum of the Interior Angles
3	180°
4	360°
5	540°
6	720°
7	900°
8	1080°

2 How many sides will a polygon have that has an interior angle sum of 2700°? Be prepared to show how your group determined your answer.

Weekend Getaway

You need to rent a car for the weekend. You locate the following advertisement for a local rental agency.

Using the information from the advertisement, complete the table at the right.

BROOKLYN'S BEST CAR RENTAL

Special Weekend Price!

One mid-sized car rental
for a flat rate of \$96
plus \$0.12 per mile

- 1 Determine the function that shows the costs of renting a mid-sized car from *Brooklyn's Best Car Rentals*, where the inputs (x) represent the number of miles driven, and the outputs (y) represent the weekend rental charge in dollars.
- What would be the cost in dollars, not including taxes or insurance, of renting a car and driving it a total of 200 miles over the weekend? You may use a calculator if you wish, but show here what calculations you made to determine your answer.

X	У
Miles Driven	Total Rental Charge (\$)
0	
1	
2	
3	
4	
5	

- 3 After you rent the car for the weekend, *Brooklyn's Best Car Rentals* presents you with a bill for \$110.88, not including taxes or insurance. How many miles are they claiming you drove? Show how you determined your answer.
- 4 What is the *rate of change* for this function? How can it be interpreted using the problem context?
- 5 What is the *starting amount* for this function? How can it be interpreted using the problem context?

Counting Cricket Chirps

The speed at which crickets chirp is based on the temperature. The following function is a pretty good measure of the number of chirps per minute depending on the temperature:

$$c = 4t - 150$$

where *t* represents the temperature in degrees Fahrenheit and *c* represents the number of cricket chirps per minute. Complete the data table for this function below.

t	40	50	60	70	80	90	100
С			90				

When the value of *t* is 60, the value of *c* is 90. Explain what this means using the problem context.

What is the *rate of change* for this function? (Be careful!)
How can it be interpreted using the problem context?

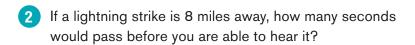
3 What is the *starting amount* for this function? Does it make sense given the context of this problem? Why or why not?

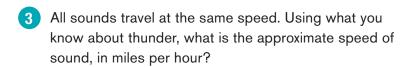
Lightning and Thunder

Thunder is the sound produced by lightning. When you see a lightning bolt, you can figure out how far away the it is by counting the seconds before you hear the thunder. The farther away the storm, the more time there will be between seeing the lightning and hearing the thunder.

A person standing 1/5 (or .2) of a mile away will hear the thunder approximately 1 second after the lightning strikes.

1 If there was 30 seconds between the time you saw the lightning and when you heard the thunder, how far away was the lightning strike?







Lightning travels at 90,000 miles/sec!

Time (in seconds)	Distance (in miles)

Temperature Scales

There are three main ways used to measure temperature-Celsius, Fahrenheit, and Kelvin.

Celsius (C°)	Fahrenheit (F°)	Kelvin (K°)	
0°	32°	273°	Freezing point of water / Melting point of ice
10°	50°	283°	
20°	68°	293°	Average room temperature
30°	86°	303°	
40°	104°	313°	
50°	122°	323°	
60°	140°	333°	
70°	158°	343°	
80°	176°	353°	
90°	194°	363°	
100°	212°	373°	Boiling point of water (at sea level)

1 If it is 25° Celsius, what is the temperature in Fahrenheit?

2 If it is 28° Celsius, what is the temperature in Fahrenheit?

3 Write three questions based on the table above.

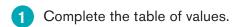
Measuring Babble

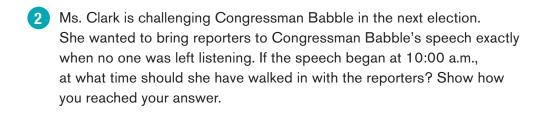
Congressman Babble recently gave a speech outlining his new policy proposals. A researcher recorded the number of people listening by counting how many both stayed in their seats and remained awake.

The following function was created to describe the number of listeners:

l = 600 - 15m

where *m* represents the number of minutes that Congressman Babble was speaking, and *l* represents the number of listeners.







Congressman Babble moments before his speech.

m	1
0	
5	
10	
15	

- What is the *starting amount* for this function? How can it be interpreted using the problem context? In other words, can you describe the starting amount in terms of "number of minutes passed in the speech" and the "number of listeners"?
- 4 What is the *rate of change* for this function? How can it be interpreted using the problem context?
- 5 Is (22, 270) a solution to this function? Explain why or why not.

66 Understanding takes place in students' minds as they connect new information with previously developed ideas, and teaching through problem-solving is a powerful way to promote this kind of thinking. Teachers can help guide their students, but understanding occurs as a by-product of solving problems and reflecting on the thinking that went into those problem solutions.

-Diana Lambdin