UNIT 1

Introducing Functions

Lesson Plan

OBJECTIVES

- ✔ Students will understand that for each input there can be only one output.
- ✔ Students will use a one-operation rule to complete an In/Out function table.
- ✔ Students will use a two-operation rule to complete an In/Out function table.
- ✔ Students will identify a one-operation rule that fits a given table of In/Out values.
- ✔ Students will identify a two-operation rule that fits a given table of In/Out values.
- ✔ Students will understand that a function rule must fit all the In/Out values in the table for that function.
- ✔ Students will create their own function rules and use it to fill out an In/Out function table.

ACTIVITY 1 Launch: Math in Love

NOTE TO TEACHER

This first activity gives students experience in working from a rule with inputs to create a table of data. Students will use *Maxine's Rules for Love* to figure out appropriate ages to date. Their own ages will be used as inputs in order to determine the outputs of acceptable ages of dating partners. At the beginning, these skills (and vocabulary) are approached in an implicit way, but they will be made explicit by the end of the lesson.

MATERIALS: ***Maxine's Rules for Love*** **handout**

STEPS:

1. Give students a few minutes to discuss the following question in pairs:

   In your experience, does age matter when people are dating?

2. Give students a few minutes to share their opinions with the class. Ask if anyone has (or had) a rule about the youngest or oldest person they are (or were) willing to date.

We begin our lesson with a launch to engage student interest and to draw out some work that can serve as a reference to build off of as we explore a content area. Students tend to see mathematics as disjointed set of procedures to be memorized. We see that as an obstacle to their learning and so we strive to connect mathematical ideas to work they have already done.

In some cases that means connecting to work with previous content. In other cases that means having them do some work and then adding structure and vocabulary. This lesson models the latter.

3. Tell them your friend Maxine has a very specific rule when it comes to the age of people she would be willing to date.

4. Give students the handout and have them work in pairs to answer the questions. Walk around for a few minutes and address any questions.

5. On the board, draw two tables.

The first should look like this:

Name	Age	Youngest person you should date

When lesson planning, it can be incredibly helpful to actually map out what you want your board to look like in advance. If you have a big enough board, it's nice to be able to move from left to right and leave the board work up through to the end of class. It can help you make connections between activities and help students see how activities fit together and build off each other.

The second should look like this:

Name	Age	Oldest person you should date

NOTE TO TEACHER: Make sure to leave some space above the Age columns and the Youngest/Oldest Person columns. Later on in the lesson you'll be adding the words "In" and "Out" above each, respectively.

As students finish the questions on the handout, ask them if they are willing to go up and add their numbers to the charts. If some students don't want to share their age, that is ok, but make sure you look over their work.

6. Tell students that *Maxine's Rules for Love* is an example of something called a "function." I like to compare functions to car factories. What goes into car factories? Glass, metal, tires, rubber, seats, carburetors, etc. Something happens to all those parts and materials in the factory. And then something comes out—cars. Functions work the same way, but with numbers. You have one number that goes in, something happens to it, and another number comes out.

7. Add the word "In" on top of the column for students' ages. Add the word "Out" on top of the column for the ages of the youngest/oldest person they should date. For *Maxine's Rules for Love*: One number goes in (your age), you follow Maxine's Rule, and then another number comes out (the age of the youngest or oldest person you should date). You are likely to have two students who are the same age. If that happens, they should both talk and come up with the

same youngest/oldest ages. Make sure to draw students' attention to this. If it doesn't, ask what they would expect to happen if two students of the same age used Maxine's rule. You want them to realize something along the lines of, *If the same number goes in, and we do the same thing to it the same number will come out.* Once everyone feels good about the rationale, write the following sentence of the board and have them copy it into their notes: *When we are working with a function, each input can only have one output.* This is a rule that should be referenced in future classes.

8. Ask students what they think about Maxine's Rule. Do they think it's reasonable? When does it make sense? When does it not make sense?

9. Tell students we are now going to explore the idea of functions in a game called, "My Teacher is a Computer."

ACTIVITY 2 My Teacher is a Computer

MATERIALS: Board/Newsprint and marker

STEPS:

1. The way the game works is as follows: students give you a number, you do something to that number in your head, and then you tell students what number comes out. They are going to keep giving you numbers to put into your computer and try to guess what is going on inside the computer, i.e. what you are doing to the numbers in your head. It is very important to make clear that students should not call out the rule. When they think they have figured out the rule, they should raise their hand and say so without revealing what they think the rule is.

2. Tell students that you are going to help them keep track of their guesses in a table on the board or newsprint. Write **In** over the top of the left column and explain that is where the numbers they give you will be recorded. When write **Out** over the right column and explain that is where you'll record the numbers that come out of your "computer."

3. Next, give students a limit (maximum and minimum) for their guesses. This step isn't absolutely necessary, but might be a good idea to keep things simple at first. I usually ask students to keep their numbers between 0 and 20. Another option is to use the range to introduce some algebraic notation. You might write $0 \leq In \leq 20$ and see if anyone knows what it means. If someone does, great, but chances are, very few of your students will be able to say, "The input has to be greater than or equal to 0 and less than or equal to 20."

As with all important points you want to emphasize in class, don't assume that because one student understands that everyone else understands too. Ask other students what they think of what was said. Ask them to restate it in their own words. Ask them if it will always be true.

That's fine. Just write it on the board and have them copy both the notation and the statement explaining what it means in their notes. Whatever you decide, don't spend more than a few minutes on this step.

4 Repeat the request that students not call out the rule once they think they have figured it out. Use "add 9" as your first rule. Obviously, don't tell students your rule. Instead, ask a student to give you a number. Put that number in the "computer" and record the number as an entry in *In* column. I like to give my students a visual and actually act out the number going in by gesturing as if I am actually putting the number into my head with my right hand (so students will see it going in on the left). For the calculation part I use both hands to mimic washing my hair. Finally I gesture with my left hand of a number coming out of my head. Then enter the number that comes out to the *Out* column of the table, next to the corresponding In number. Now, ask the class to give you different numbers to put in the computer.

5 Once a student says they have figured out the rule, remind them not to say it out loud. Instead of giving the rule, the student will get an opportunity to show if their rule matches the computer. Give the student a number to put in (still recording it in the table). Ask her, *If you put this number in, what number would come out?* If they are correct, tell them so and add the output in the table. If they give you a number that is not what should come out, tell them so and record the correct output in the chart—*If you put a __ in, you will actually get a ___ out.* Either way, keep going until more students have figured out the rule, handling their guesses in the same way.

6 Once you have a good number of students who have correctly identified the rule (by giving the correct output), ask one of them to say what it is that is happening inside your "computer" and write it above the table, next to the word, **Rule**. Once the rule is established, tell the class you want to go back and check to make sure that rule works with all of the inputs and outputs. Give them an opportunity to check. Once they are sure the rule works for all of the inputs and outputs, ask a few students to talk you through their process. Encourage them to use the language of functions: *When you put a 14 in and subtract 4, you get a 10 out.*

6

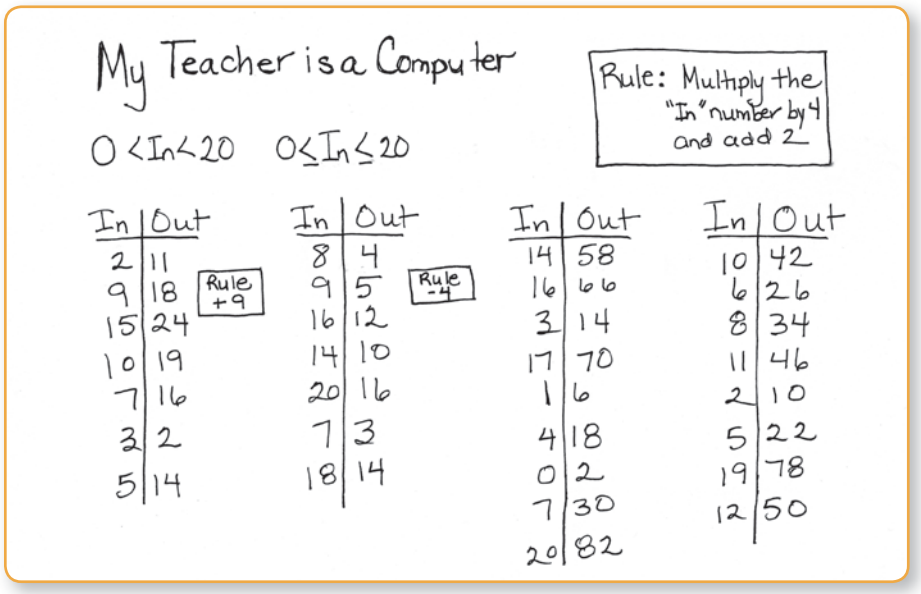
This language helps students internalize a structure of thinking that will help them develop flexibility as they have to adapt to different kinds of function questions.

7 I recommend playing this game at least three times on the first day. (The game can be used at different points in a curriculum on functions, since you can use more complex rules as students become comfortable analyzing input/output charts.) I like to use rules that get more and more complicated. Try a few single step rules, and

use different operations. You might try *add 9,* then *subtract 4* and finally *multiply by 3.*

8. Finally, play a round with a two-step rule. Try something simple, but not too easy—like *multiply by 4, plus 2.* Follow the directions for the game in steps 1 to 6, but I would recommend telling students in the beginning that the rule they are looking for is a two-step rule. It is not necessary, but if you don't, it makes it more challenging and at this point we still want to prioritize students getting a feel for the flow of the input to the rule to the output. Keep in mind that a two-step rule will likely take longer for students to figure out than a single step rule.

   After it is all said and done, your board might look something like this:



9. Ask, *Were any numbers you put in especially helpful for figuring out the rule? How so?* You can point them back to all of the tables and rules written on the board. There is no need to push to get too much out of students with this question, at this point. What they say will depend on the In numbers that students suggested. But even if their answer is "No," it is a good question to ask to raise the idea that there might be a strategy in choosing the numbers we put in. Ask them to keep the question in mind. They might say things like "putting a zero in was helpful" or "Choosing consecutive numbers helped." Though it is too early to make this point now, these are useful connections to concepts like starting amount (y-intercept) and rate of change (slope) in later classes.

ACTIVITY 3 One Rule to Bind Them All

MATERIALS:

- ***One Rule to Bind Them*** **handout, cut into individual strips, one for each group**
- **Several sheets of newsprint and several markers (one for each student group)**

STEPS:

1. Break students into groups of 3 to 4 students. Give each group a piece of newsprint and marker. If newsprint is not available, consider breaking them into smaller groups and have them do the activity on regular paper.

2. Explain that for the next activity they are going to be working with functions, but in a different way. You are going to give each group one input and one output—which is to say, *"If this number goes in, something happens to it and that number comes out."* Ask them to write and label their *In* and their *Out*. Their job is to come up with as many different ways as possible to explain what is the something that might be happening.

3. The One Rule to Bind Them handout has some sample Input/Output pairs you can use. Cut out each pair and give one to each group. Remember to give only one input and its corresponding output to each group.

   There is only one function rule that works with all of the inputs and outputs: **"times 2, then plus 2."**

4. Walk around as they are coming up with all the different ways to get from their input number to their output number. If any groups struggle, give them at least five minutes without any intervention from you (unless they are struggling with understanding the instructions). Here's a potential series of questions you can ask to help them get unstuck. Ask only enough for them to start working.

   a. *Is the number getting bigger or smaller?*

   b. *What operations could you try to make the numbers bigger?*

   c. *Is there a way to get from the input to the output in one step?*

   d. *Are there any other ways to do it in one step?*

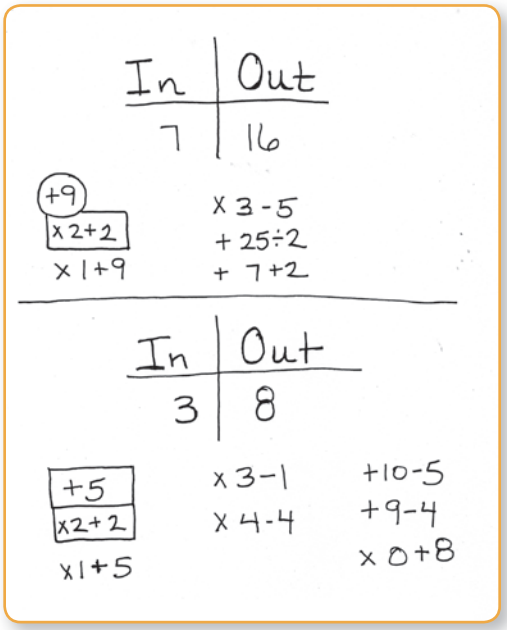
   e. *How could we get from the number that went IN to the number that came OUT doing two things?*

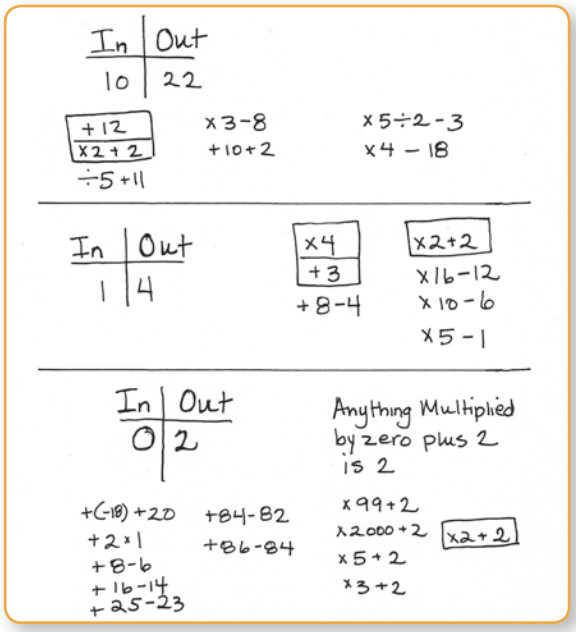
   f. *Pick a number to multiply the IN by. What did you get? How could you get from that number to the OUT?*

5 You want each group to have several ways to get from their input to their output. As you walk around, you might encourage some friendly competition and say things like, *Hmmm… this group found 8 different ways to go from their input to their output. And they're still looking for more!*

6 You also need every group to have "times 2, then plus 2" in their list of rules. Once that happens, have everyone tape their newsprint next to each other on the wall. Ask each group to share all the different ways they came up with to get from the IN to the OUT. After every group has shared, ask the class what they notice. At least one person will notice is that there is one way that is true for all of the INs and OUTs: "times 2, then plus 2." Ask the class what they think about that.

Here's an example of what it might all look like:





7 Draw a table with all of the inputs and outputs they've been working with.

Point to one of the rules for an input of 1 and an output of 4 other than "times 2, then plus 2." For example, "times 4." Ask the class if you should write that rule in the space above the table. Whatever students say, yes or no, ask them to defend their answer and ask other students what they think. You want students to arrive to the conclusion that you can't put "times 4" in as the function rule because it only works for one of the input/output pairs. A function rule must fit all of the input/output pairs.

In	Out
1	4
3	8
5	12
7	16
10	22

For a final question, ask the class how many input/output pairs they would need to come up with a rule for a function.

Can you do it with one pair? Can you do it with two?

ACTIVITY 4 Function Machines

MATERIALS:

- ***Function Machines 1*** **handout;** ***Function Machines 2*** **handout**
- **Blank** ***Function Machines*** **(optional)**

STEPS:

1. Tell students that they've been looking at a few different kinds of function skills.

   For *Maxine's Rule*, they were given a rule and put numbers in to find the numbers that come out. You might go back to the tables on the board for that activity and ask students to add the rules for each (divide by 2, then add 7; subtract 7 then times 2).

   For *My Teacher is a Computer* and *One Rule to Bind Them*, they had the inputs and the outputs and they had to come up with the rule.

2. Explain they are now going to have an opportunity to practice those skills using something called a "function machine." Explain that a function machine can be a helpful way to think of functions, similar to a car factory and to *My Teacher is a Computer*.

   Draw the following picture and table on the board:

In	Out
3	6
5	10
7	14
9	18
11	22
13	26

   Explain that function machines are another way to represent and visualize functions. It is very similar to the other models we've

2

When we know there is a common mistake that students might make—such as not checking a rule against several input/output pairs—it can be really nice for teachers to "make" the mistake themselves. It raises the issue but it puts the responsibility to correct it onto your students. It is always better if they are the ones who say, "Add 3 only works for the first input and output. A function rule has to work for all of the inputs and outputs."

looked at. A number from the left column of the table goes in (at the top of the machine), something happens to that number in the middle of the machine and then a number comes out of the bottom of the machine.

Ask students what the rule is. You'll get a range of answers, and if they are correct, they will be equivalent—"times 2," "double," "the in plus itself." Add them all to the box and give students a chance to decide if they are different rules or if they are all the same. If a student does not suggest it, ask if the rule could be "add 3." When students say no, ask them why not. Even if they say the rule can not be "add 3," push back a little with, "But if you put a 3 in and add 3, you will get a 6 to come out."

3 Give out FUNCTION MACHINE 1 handouts and tell students that each machine will give them different pieces of information about each function—their job is to use that information to figure out what's missing. Some of the machines will give inputs and outputs but the rule will be missing. Some machines will give a rule and students will have to find the inputs and outputs. The bottom line is, whatever is missing, they should try to find it.

Walk around and look at student work. Unless they ask you a question, try to spend at least five minutes just watching what they are doing. Be on the lookout for challenges, struggles, interesting strategies and common mistakes. These will give shape to the whole-class discussion.

One thing that you will definitely want to talk about in the whole-class discussion is what students do when they are given a rule and an output and asked to find the input. For example, look at the machine on the top right on page 1 of the FUNCTION MACHINES 1 handout. Students first have to figure out that the rule is "subtract 4." Once they do, they put the 20 in and get a 16 out. What many students do next is put the 50 in (even though it is in the *Out* column) and write 46 in the *In* column. This is a great and important mistake for students to make and you should not address it beforehand or intercede if you see a student making the mistake.

Rather than just correcting them, you want to model a process they could use in the future to (a) catch a similar mistake and (b) correct themselves. Instead of stepping in, walk around and get a sense of which students are making the mistake. After a little while, start asking students to talk you through one of the machines. I'm going to give you a sense of how the conversation might go for the function machine on the top right, but you can use it as a guide to discuss any of the machines where the output is given and students

3

You may have some students who use the inverse operation and work the rule backwards to go from the output to the input. You may be tempted to teach your students how to do that yourself. I would strongly encourage you to hold off at this point in your class' work with functions. If a student happens to be able to do it on their own and is consistent and understands why it works, great. But for many students it can be confusing and our goal for this first lesson is to get students comfortable with one function rule and its relationship with its function table.

have to determine the input. Rather than asking them about the 46 right away, have them talk you through the whole machine. Ask how they know the rule is " subtract 4." Encourage them to use the language of the function machine: *If you put a 6 in and subtract 4, a two will come out. If you put an 8 in and subtract 4, a 4 will come out….*

Once they get to the 46 listen to what they say. Some students may catch the mistake in their own throats when they say, "If you put a 46 in and subtract 4… wait." Some students may switch columns and say, "If you put a 50 in, and subtract 4 a 46 will come out."

If that happens, use the visual of the machine and columns to ask whether one of the earlier outputs is an input or an output.

You want to help students see that when they are given a rule and an output the question changes and becomes, *What number has to go in, to follow that rule and give us this output?* Another question to help students to see this to ask, *What is the difference between the 20 and the 50?"*

Once they understand they are looking for the number that goes in, encourage them to use guess and check as an effective strategy. The questions *What could it be?* or *What if it was __?* are good questions for students as general problem-solving strategies and they can be really helpful here. Students try a number, use the rule and see if they get the given output. If not, are they too high or too low? Should the next number they try be larger or smaller? How can we use the other inputs and outputs to make our guesses?

4

Function Machines 2 introduces rules that involve decimal multiplication. It can help students think of the decimals in terms of money and the multiplication in terms of groups. Students may not remember the rules for multiplying decimals. Another approach is to think of 3 x 1.50 as "three groups of 1.50." Seen this way, many student will be able to do the calculations in their heads

4 Give out Function Machines 2 handout. Function machines are a great way to introduce/review/explore different content areas. Once students have an understanding of how they work, you can use them to focus on topics you are working on in class. For example, you will see Function Machines 2 has multiplication with decimals, percent change and offers an opportunity to talk about square roots and exponents.

5 In addition to the two function machine handouts, you will find two pages of blank function machines. Use these blank machines to create your own rules over the course of a semester—it is a good way to continue student exposure to the core concepts of functions as well as incorporate other mathematical topics.

6 For students who want to practice function machines on their own, I recommend the following Internet resource:

http://www.mathplayground.com/functionmachine.html

ACTIVITY 5 The Function Game

MATERIALS: ***Function Game*** **handouts**

STEPS:

1. Tell students that they have been doing a great job working on all the different functions you've thrown at them and now it is their turn. They are going to have an opportunity to create their own function rules and tables. Ask them to write a one-operation rule and a two-operation rule on a piece of paper. Tell them you are going to try to figure out their rule and that they can try to make it as difficult as they like.
2. Give out the FUNCTION GAME handouts. Ask students to use each of the rules they came up with to fill out each of the In/Out function tables. Make sure they do not write the rule on the handout! It will be good practice for them to actually do the calculations themselves, so for this activity I would encourage teachers to ask students not to use calculators.
3. Have students write their names on the handouts and collect them. I always try to work on them, figure out all the rules, and give it back to students by the next class
4. *The Function Game* is a great activity that works as an in-class assignment or as a homefun assignment. It serves as a good assessment that provides some insight into what students are retaining from today's function activities.

3

I like to pick out a few favorites before the next class and ask those students if they would be willing to have the class guess their rule using the structure of "My Teacher is a Computer" but changing it to "My Classmate is a Computer."

NOTE TO TEACHER

As you are working through your students' rules, look for ones that don't work. Spend some time and analyze why they don't work and ask yourself whether it is a good mistake for other students to consider. For example, I once had a student whose table looked like this *(see right)*.

In	Out
5	5
5	10
5	15
5	20
5	25
5	30

I started off our next class having the students who had written some of my favorite rules play the *My Classmate is a Computer* game with the class. Then I said I wanted to share *My Favorite No*, which I explained as a really interesting mistake that I thought everyone could learn from. I wrote the table on the board and asked everyone to spend a minute just thinking about what this mistake could teach us. After a few minutes, students shared some of their ideas. One thing I definitely wanted to see if anyone would say is that this does not fit with what we learned

about functions because you have different outputs for the same input. Someone else said something that the class liked when our conversation shifted to where the mistake came from and how we could all be careful to avoid it in the future. She said, "I think the rule is 'times 5' and they meant '1 times 5', '2 times 5', etc. but maybe in their head they were saying '5 times 1', '5 times 2', '5 times 3', etc., and so they got confused with the 5 and the *In* because they were doing the 5 first."

Check-Out/Exit Ticket

- **Ask students to look at the board to remember all the activities they've done (*Maxine's Rules for Love*, *My Teacher is a Computer, One Rule to Bind Them* and the function machines.) Tell them that they are going to write for a few minutes about what they want to remember about today's lesson.**
- **On a separate piece of paper—you'll be collecting it—have students answer the following two questions:**
  - What do you know about functions?
  - What is one question you have about functions?

  They will say different things and all are okay. Give them a couple of minutes to think about the question and share their answers, either in pairs or in a whole-class reflection. You may hear things like:
  - *"There are lots of different ways to get from an input to an output but there is only one that works for all of them."*
  - *"What I understand is the rule has to be used with each in number to equal the out number to make it a pattern."*
  - *"If you are looking for different ways to get from an input to an output, it is easy to start with the one-operation possibilities first."*
  - *"When you are trying to figure out a rule, don't cross out any of your guesses."*
- **Collect all of the responses as an assessment to make sure you hear all voices and get a sense of what impressions students are leaving the room with in terms of functions. Look for any common misconceptions and misconceptions of individual students. Look for some well-phrased reflections that you can bring back and share with the class.**

Maxine's Rules for Love

Maxine has a mathematical rule that she uses when it comes to love. Maxine says that, in her opinion, when it comes to the age difference between two people in a romantic relationship, the younger person should never be younger than half the older person's age plus seven more years.

1. Following Maxine's rule, would a couple made up of a 44 year old and a 27 year old work? Explain your answer.

2. According to Maxine's rule, what is the age of the youngest person you should date?

Maxine has a slightly different rule for figuring out the age of the oldest person she is willing to date. She says to find out the oldest person you should date, take away 7 years from your current age and then double that number.

3. According to Maxine's rule, what is the age of the oldest person you should date?

One Rule to Bind Them

Teacher Instructions:

Cut out each In/Out pair. Give one to each group.

In	Out
7	16

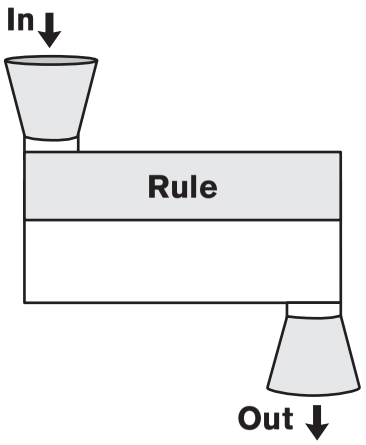
In	Out
3	8

In	Out
1	4

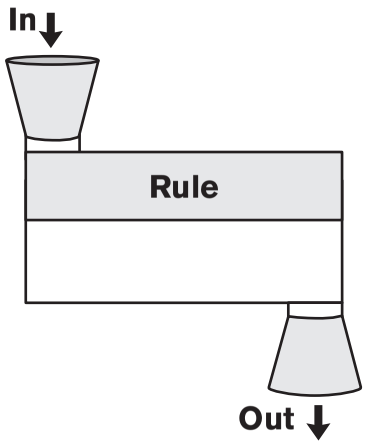
In	Out
10	22

In	Out
5	12

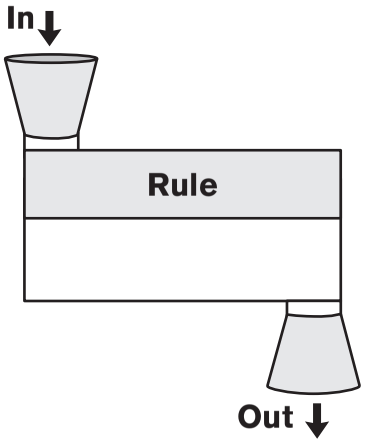
Function Machines 1



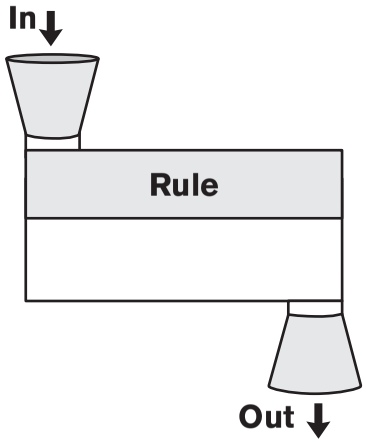
In	Out
2	4
3	6
5	10
10	20
20	40
100	200



In	Out
6	2
8	4
10	6
14	10
20	
	50

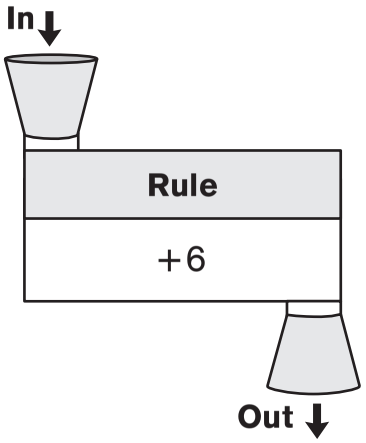


In	Out
1	6
15	20
4	9
30	
22	
	58

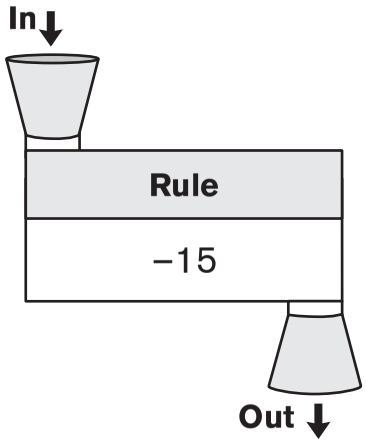


In	Out
10	5
14	7
24	12
	6
	20
	300

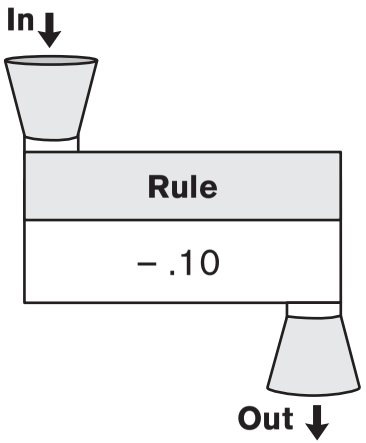
Function Machines 1



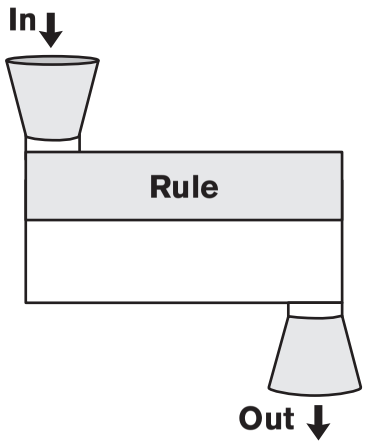
In	Out
2	8
3	
5	
10	
	20
	100



In	Out
60	45
80	
15	
115	
	115
	26

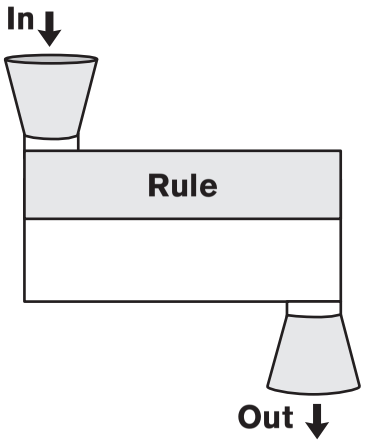


In	Out
.50	
.65	
1	
1.2	
	5
	6.25

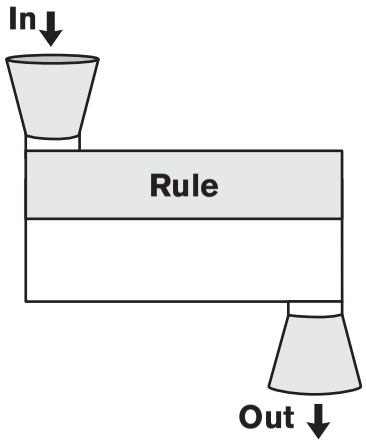


In	Out
6	8.25
1.50	3.75
5	
10.75	
	21.25
	30

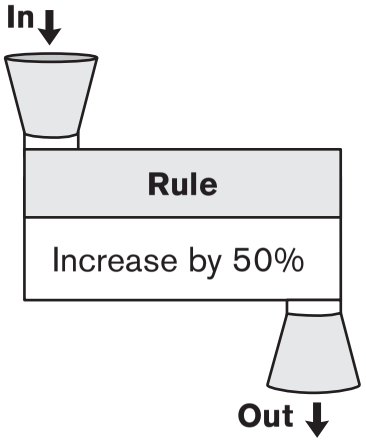
Function Machines 2



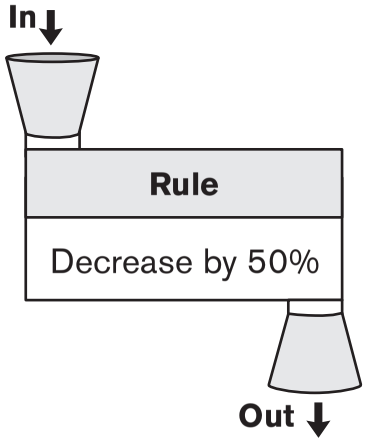
In	Out
16	8
10	2
20	
	100
	0
	95



In	Out
10	40
2	8
6	
	60
	36
	320

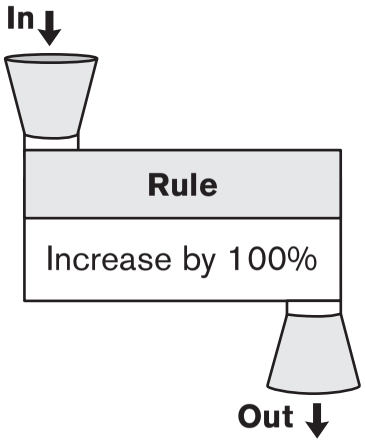


In	Out
10	
16	
20	
	6
	75
	36

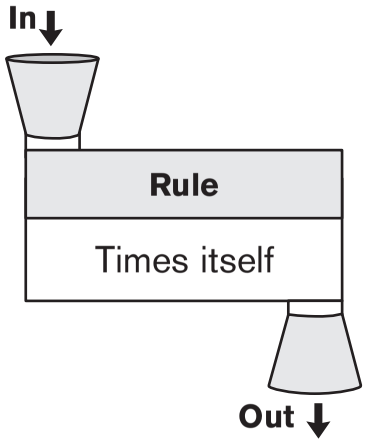


In	Out
4	
12	
16	
	24
	50
	100

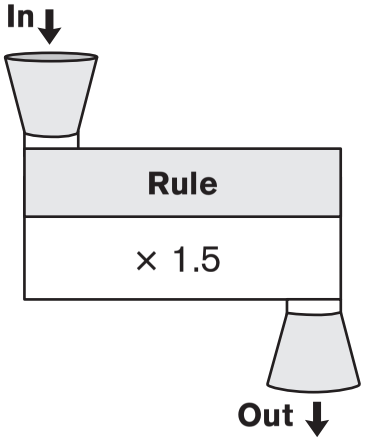
Function Machines 2



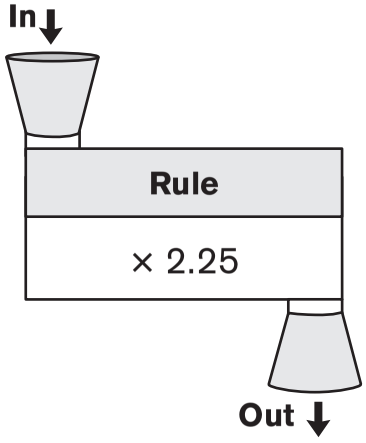
In	Out
1	
4	
12	
	15
	2.6
	170



In	Out
1	
2	
5	
	36
	64
	289



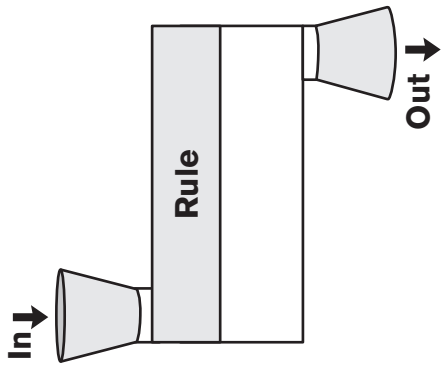
In	Out
1	
2	
3	
10	
	7.50
	16.50



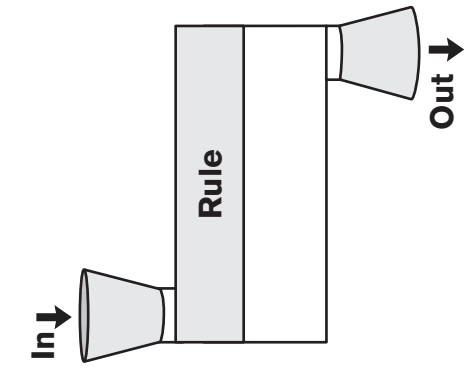
In	Out
1	
2	
3	
10	
40	
	18

More Function Machines

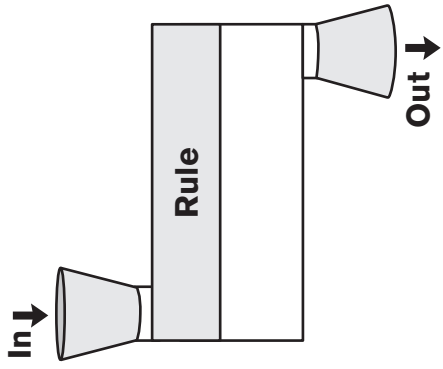
In	Out



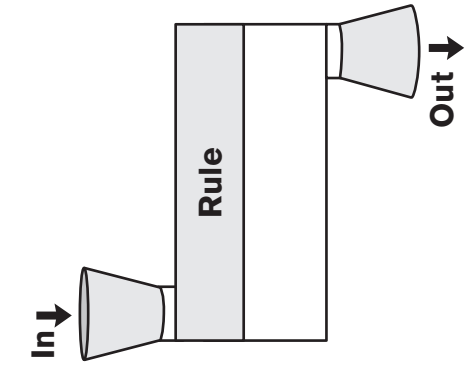
In	Out



In	Out



In	Out



More Function Machines

Fill in missing values and/or rules.

Rule:	
In	**Out**

Rule:	
In	**Out**

Rule:	
In	**Out**

Rule:	
In	**Out**

The Function Game

Now it is your turn. You are going to create your own function rules!

Write a one-operation rule and a two-operation rule on a separate piece of paper. I am going to figure out your rule and you can try to make it as difficult as you like. Use each rule to fill in the Inputs and Outputs for one of the tables below. Do not write your rule on this handout!

Rule:	
In	**Out**

Rule:	
In	**Out**

One of our greatest strengths—our practice of starting from where students are and having a serious respect for their learning processes.