Adding and Subtracting Integers with 2-Color Counters

Model Lesson Using Manipulatives

Objective: The student will be able to add and subtract integers using 2-color counters and using pencil sketches of color counters.

Before the Lesson: Students can compare positive and negative numbers using $\leq, \geq, <, >$, and =. Students have some understanding of signed numbers in applied contexts. For example, they understand that $-10°$ is ten degrees colder $0°$, or that -15 could express a debt of \$15.

Materials: Have about 16 counters for each student. You need a dry erase board or butcher paper to demonstrate so that solid circles represent red counters and hollow circles represent yellow counters. A blackboard will be confusing. Magnetic boards with magnetic color counters are best. Have copies each of the opening activity, in-class practice, and homework handouts. Leave enough time at the end of class to collect the counters.

Opening Activity:

Quantitative Comparisons of Rational Numbers

For each of the following problems, use the given material, if any, to determine whether:

Column A is greater than column B (answer A)
Column B is greater than column A (answer B)
Column A and B are equal (answer C)
It cannot be determined (answer D)

	Column A	Column B	Answer
1.	2	-5	
2.	-3	-6	
3.	.25	$\frac{1}{4}$	
4.	0	$-.2$	
5.	10	x	
6.	*y*	-6	
7.	*a*	*b*	
8.	*p*	*-p*	
9.	$\frac{1}{4}$	$\frac{1}{5}$	
10.	$-\frac{2}{3}$	$-\frac{3}{4}$	

Lesson Development:

1. By the end of today's class, you will be able to add and subtract integers using 2-color counters and with sketches of 2-color counters. This is a topic that gives students all kinds of trouble, but if you focus on today's lesson, you will get it. Better yet, you should be able to teach your kids or friends how to add and subtract integers using this method.

2. We are going to start our lesson with an important concept called a ***zero pair***. Take out a clean sheet of paper for some notes.

3. If someone has $6 but they have a $6 debt to pay, what will they end up with? (zero) Remember that we sometimes use positive numbers to represent money you have, and negative numbers for debts.

$$6 + (-6) = 0$$

4. Similarly, if someone owes $45 on their credit card, and you give them $45 to pay it off, they will end up with $0.

$$-45 + 45 = 0$$

5. Who can finish this sentence for me? "The sum of a number and its ___*(opposite)*___ is zero."

6. These examples are more complex versions of the zero pair idea. Be sure to write this down in your notes.

   A *zero pair is the sum of one and negative one, which is zero*. $1 + (-1) = 0$

7. I want you to turn your sheet of paper over to the back and put everything else under your desk. We are going to call this blank piece of paper your 'mat'.

8. I am going to pass out 16 counters to each of you. Please be sure to not lose any. (Give the students a minute to explore with their counters. They aren't kids, but they will still want to organize them in various ways, and this exploration is going to happen if you like it or not. Waiting a minute ensures that you will have their full attention when you continue.)

9. Which of the counters is the closest to the color of gold? (yellow) Remember that because gold is valuable and that might help you remember that the yellow counters are worth +1. Who knows the expression we use to describe somebody in debt that involves the color red? (to be in the red) That might help you remember that the red counters are each worth –1.

10. I am going to show you a zero pair on my mat. Here's what one looks like. (In this lesson plan, dark gray represents red and light gray represents yellow. On your dry erase board or

butcher paper, use a solid circle for red and open circle for yellow. Explain this to your students.)

11. Can anyone tell me why this is a zero pair? (You have +1 and –1 together, whose sum is zero). Who can show me how to make more zero pairs? (two reds and two yellows, etc.)

12. Now that you understand zero pairs on our integer mat, lets talk about adding integers.

13. Let's suppose that you need to find the following sum: $3+(-5)=$

14. Start by putting 3 on the mat. Should I use three reds or three yellows? (yellows) When you **add** using an integer mat, do you think that you put counters **on** the mat, or do you think that you take counters **off** the mat? (put them on) What should I put on the mat? (five reds for –5).

15. Do you see any zero pairs on the mat? How many are there? (there are three zero pairs because there are three reds matched up with three yellows). What are the zero pairs worth? (zero).

16. Simply remove the zero pairs, because adding zero does not change our answer. What do you have left? (two reds) What is the value of two reds? (negative 2). What is the sum of three and negative five? (negative two)

17. Now I'd like everyone to use their mats to solve the following problem: $7+(-2)=?$
(Circulate and help the students having difficulty. Let a student demonstrate his/her solution on the board, on an overhead, or using the magnetic counters.)

18. Let's look at the following sum: $-1+(-5)$. Who can tell me how to proceed? (start with one red, then add 5 more reds. There are no zero pairs, so we are done.) When there are no zero pairs, you simply read your mat. In this case, $-1+(-5)=-6$.

19. (Have the students complete problems 1&2 from the handout on adding signed numbers. Review the sums before continuing.)

20. You do not even have to have the mat and counters in front of you to solve integer sums. You can simply make a little sketch of the counters to do the very same thing. Use solid

circles for reds and hollow circles for yellows. Turn back to your notes and solve the following problem by making a sketch. Cross out any zero pairs.

$$4+(-5)=$$

21. Once you have done this several times, you can start to do the math in your head. For example, let's consider the following problem:

$$100+(-4)=$$

Would I have 100 reds or yellows? (yellows)

What would I add to the mat? (four reds)

How many zero pairs would I have? (four)

When I remove them, what is left? (96 yellows)

What is the sum of 100 and –4? (positive 96)

22. You should also be able to tell me the sign of a sum immediately after you look at a problem. For example:

$$40+(-28)$$

Do I have more yellows or reds? (yellows) How do you know this?

When I remove zero pairs, will I have yellows or reds left over? (yellows)

Will this sum be positive or negative? (positive)

23. Subtraction is a bit more difficult. Before I do, though, look at the following equations and tell me if they are true or false:

$$4+0=4$$

$$4+0+0=4$$

$$4+0+0+0=4$$

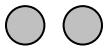
When I add one or more zeros to a number, does it change the number? (no) Good.

24. Please turn back to your mats. Consider the problem $-4-(-2)=$
25. What color are my initial 4 counters? (red) When you subtract using an integer mat, do you think that you put counters **on** the mat, or do you think that you take counters **off** the mat? (take them off—"take away" is an expression we sometimes use for subtraction.)

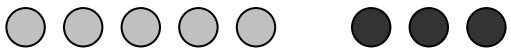
26. Once you take off two counters, do you have any zero pairs to remove? (no) So, that's it. You're done. What's your result? $(-4-(-2)=-2)$

27. Here is a tougher example. Watch closely. Consider the problem $2-5=$

28. What color are my initial 2 counters? (yellow) Am I subtracting positive 5 or negative 5? (positive) Are there enough yellows for me to remove five from the mat? (no)



29. We have to get 5 positive counters on the mat *without changing the number we started with*. The only way to do that is to bring on zeros, or zero pairs. After bringing on 3 zero pairs we have a total of 5 positives.



30. Now, can we remove 5 positive counters? (yes) Do it, and what is left? (negative 3) In other words, $2-5=-3$.

31. I'd like everyone to use their mat to solve the following problem: $-1-(-4)=?$ (Circulate and help the students having difficulty. Let a student demonstrate his/her solution on the board. Do the following examples before moving back to the handout $-1-4$, $2-6$. Do at least one example using pencil and paper.)

32. (Have the students complete problem 3 from the handout on adding and subtracting signed numbers. Have students review how they solved each before continuing on.)

Closing:

1. What is a zero pair?

2. If there are an equal number of red and yellow counters on the mat, what will be the sign of the number?

3. If there are more yellows than reds, what result are you going to have?

4. Who can explain to me in words how you would use counters to solve the following problem: $-8+5=?$

5. If you need to remove counters from the mat because you are subtracting, and you don't have enough on the mat, what do you need to do? Be specific.

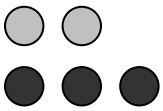
6. Please complete the homework handout before our next class. Instead of using the counters, try to draw sketches or use a mental picture of the counters to solve the problems. Use open circles for positives, and solid circles as negatives. Cross out zero pairs.

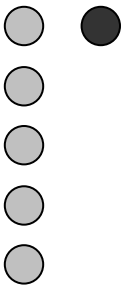
7. I am going to come around to collect your counters. You should have 16 counters.

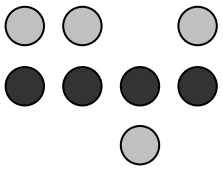
Adding and Subtracting Signed Numbers

1. What number is represented by each model? Dark gray circles represent red counters, and light circles represent yellow counters.

a. 

b. 

c. 

d. 

2. Use your counters to model and find the sum.

a. $4+(-2)$ b. $-4+5$ c. $-3+(-2)$ d. $-5+3$

e. $7+(-4)$ f. $-1+(-5)$ g. $-1+(-2)+5$ h. $4+(-3)+1$

3. Use your counters, or a sketch of your counters to model and find the difference.

a. $-4-(-1)$ b. $-4-2$ c. $3-5$ d. $-5-1$

e. $1-7$ f. $-2-(-6)$ g. $-5-(-4)$ h. $8-4$

Adding and Subtracting Signed Numbers

Extra Practice

1. Find the following sums using 2-color counters, a sketch of counters, or using mental math.

   a. $5+(-4)$ b. $-3+(-6)$ c. $5+(-14)$ d. $6+(-3)$

   e. $-5+(-4)$ f. $-3+8$ g. $-6+50$ h.* $6.2+(-3)$

2. Find the following differences using 2-color counters, a sketch of counters, or using mental math.

   a. $4-10$ b. $5-(-4)$ c. $-6-(-4)$ d. $-6-3$

   e. $1-7$ f. $-1-(-4)$ g. $-2-50$ h.* $6.2-(-3)$

3. For each of the following exercises, model the problem using signed numbers, then solve it.

   a. You are working with chemicals that are at $-20°F$. During a chemical reaction, the temperature rises $8°F$. After the reaction, what temperature are your chemicals?

b. The temperature was at $14°F$ before falling $26°F$ overnight. What was the low temperature reached?

4. Can the sum of two negative numbers ever be positive? Defend your answer with a written explanation that includes sketches of our counters to clarify your written answer.