PROBLEM SOLVING

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

Comparing Fuel Consumptions: *Buying Cars*

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley

Comparing Fuel Consumption: *Buying Cars*

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to solve a real-world problem that involves rates of change. In particular, it will help you assess how well students are able to create, compare, and evaluate different representations of functions.

COMMON CORE STATE STANDARDS

This lesson relates to **all** *Mathematical Practices* in the *Common Core State Standards for Mathematics*, with a particular emphasis on:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.

This lesson gives students the opportunity to apply their knowledge of the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

8.F: Define, evaluate, and compare functions.
Use functions to model relationships between quantities.

8.EE: Understand the connections between proportional relationships, lines, and linear equations.

INTRODUCTION

- Before the lesson students tackle the problem individually. You then review their work and write questions to help students improve their solutions.
- At the beginning of the lesson students respond to your questions. Students are then grouped into pairs and work collaboratively to produce better solutions to the same task.
- There is a whole-class discussion to compare and evaluate different approaches.
- A second collaborative activity follows where students work in small groups to evaluate and comment on sample solutions, followed by a second whole-class discussion about the work.
- Finally, students review and evaluate their work on the problem.

MATERIALS REQUIRED

- Each student will need a copy of the assessment task: *Buying Cars*, two or three sheets of paper, and a copy of the review questionnaire *How Did You Work?*
- Each small group of students will need a large sheet of paper for making a poster, felt tip pens, and copies of the *Sample Responses to Discuss*.
- You will need a supply of graph paper and rulers. There is a projector resource to support whole-class discussions.

TIME NEEDED

Approximately 20 minutes before the lesson, a 90-minute lesson (or two shorter lessons) and 10 minutes in a follow-up lesson. These timings are approximate.

BEFORE THE LESSON

Assessment task: *Buying Cars* (20 minutes)

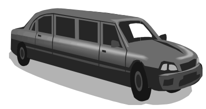
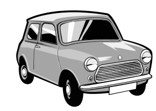
Have students complete this task, in class or for homework, a few days before the formative assessment lesson. This will give you an opportunity to assess the work and to find out the kinds of difficulties students have with it. You should then be able to target your help more effectively in the subsequent lesson.

Give each student a copy of the assessment task: *Buying Cars* and a sheet of paper to work on. Introduce the task, and help the class to understand the problem and its context.

> *Most people today are concerned with saving energy. One way of doing this is to buy a more fuel-efficient car. Today's task is all about the money that may be saved by doing this.*
> *Read the task carefully.*

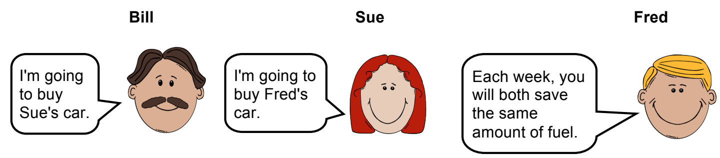
Buying Cars

Bill, Sue and Fred each own a car.
Every week, all three drive the same distance.

Bill's car	Sue's Car	Fred's car
		
Average fuel consumption: 20 miles per gallon	Average fuel consumption: 30 miles per gallon	Average fuel consumption: 40 miles per gallon

1. Which car is the cheapest to run?
   Explain your answer clearly.

2. They are trying to work out how much fuel they will each save if they change cars.



   Fred is **incorrect**.
   Use mathematics to show why his statement is wrong.
   State any assumptions you make and explain your reasoning clearly.

3. Fred wants to buy a new car.
   Each week, he wants to save the same amount of fuel as Bill.

   What should be the average fuel consumption of Fred's new car?
   Explain your reasoning clearly.

> *It mentions 'average fuel consumption'. Does anyone know what this means? [The average amount of miles a car goes on each gallon of fuel - miles per gallon].*
>
> *Why do we say 'average fuel consumption' not just 'fuel consumption'?*
> *[The fuel consumption depends on the speed of the car, whether the air conditioning is on etc.]*
>
> *Can anyone name a type of car that is fuel-efficient?*
> *What is its average fuel consumption?*
> *Does anyone know of a car that is a 'gas guzzler'?*

Have graph paper and rulers available for those who request them.

> *Read through the questions and try to answer them as carefully as you can. I have graph paper and rulers here if you feel you need them, but you may not.*

It is important that, as far as possible, students are allowed to answer the questions without assistance. If students are struggling to get started then ask questions that help them understand what is required, but make sure you do not do the task for them.

Students who sit together often produce similar answers, then, when they come to compare their work, they have little to discuss. For this reason, we suggest that when students do the task individually you ask them to move to different seats. At the beginning of the formative assessment lesson allow them to return to their usual seats. Experience has shown that this produces more profitable discussions.

You may be interested to read the article *The MPG Illusion*, a fascinating report on the misperception in judging fuel efficiency when it is expressed as miles per gallon.

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding and their different problem solving approaches.

We suggest that you do not score students' work. The research shows that this will be counterproductive, as it will encourage students to compare their scores and distract their attention from what they can do to improve their mathematics.

Instead, help students to further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given in the *Common issues* table on the next page. These have been drawn from common difficulties observed in trials of this unit.

We suggest you make a list of your own questions, based on your students' work. We recommend you either:

- Write one or two questions on each student's work, or
- Give each student a printed version of your list of questions and highlight the questions for each individual student.

If you do not have time to do this, you could select a few questions that will be of help to the majority of students and write these on the board when you return the work to the students at the start of the lesson.

Common issues	Suggested questions and prompts
Compares rates rather than fuel usage For example: The student claims Bill and Sue will be able to drive 10 more miles per gallon, so they save the same money. Or: The student incorrectly assumes Fred will need to buy a car that has an average fuel consumption of 50 mpg.	• What do you know? What do you need to find out? How can you use what you know to do this? • How many gallons of fuel does Bill/Sue use each week? How can you use this information?
Gets confused by the context For example: The student confuses cost and fuel consumption. Or: The student states the speed or air-conditioning in the car will change the fuel consumption and concludes that the problem cannot be solved.	• What is the question asking you to figure out? Will the cost of gas affect the fuel consumption? • What does 'average fuel consumption' mean?
Assumes there is insufficient data For example: The student thinks no comparisons can be made because the distance driven each week is not given.	• Can you make an assumption about the distances traveled each week? • How much fuel would Bill and Sue save if they each traveled 120 miles each week?
Uses an inefficient method For example: The student figures out the fuel consumption for the three cars at several different mileages. Or: The student uses guess and check to figure out the fuel consumption of Fred's new car.	• Would the answer to Question 1 change if Bill, Sue and Fred drove 60 miles a week instead of 120 miles? Explain your answer. • Can you think of a more efficient method? • Can you use the math in Question 1 to help you answer Question 2?
Calculates the fuel consumption for each car for a specific distance (e.g. 120 miles) but remains unable to use the information to answer the questions	• How does the information you have figured out help you answer the question? • How many gallons of fuel did Bill use when driving his old car/his new car?
Plots a graph of the fuel consumption but is unable to use it to answer the questions	• How does the graph you have drawn help you answer the question?
Creates an incorrect equation to represent the savings in fuel consumption	• Check the equation is correct.
Has technical difficulty For example: The student incorrectly manipulates fractions. Or: The student incorrectly plots a graph. Or: The student incorrectly manipulates an equation.	• Check your work. Does it make sense? Can you spot any mistakes? • What does the slope of your graph represent? Is it correct?

SUGGESTED LESSON OUTLINE:

Individual work (10 minutes)

Have graph paper and rulers available for those who request them. Return the assessment task to the students. Give each student a sheet of paper. Begin the lesson by briefly reintroducing the problem. If you did not add questions to individual pieces of work, write your list of questions on the board. Students select questions appropriate to their own work and spend a few minutes answering them.

Recall what we were looking at in a previous lesson. What was the task about?

Today we are going to work together to try to improve your initial attempts at this task.

I have had a look at your work and I have some questions I would like you to think about.

On your own, carefully read through the questions I have written. I would like you to use the questions to help you to think about ways of improving your own work.

Use your sheet of paper to make a note of anything you think will help to improve your work.

Collaborative small-group work (25 minutes)

Organize the class into small groups of two or three students. Give each group a large piece of paper and a felt-tipped pen. On request, give students graph paper and a ruler.

Deciding on a Strategy

Use Slide P-1 of the projector resource to explain the activity.

Planning a Joint Method

1. Take turns to explain your method and how your work could be improved.
2. Listen carefully to each other.
   Ask questions if you don't understand.
3. When everyone in the group has explained their method, plan a joint method that is better than your separate ideas.
4. Make sure that everyone in the group can explain the reasons for your chosen method.
5. Write a brief outline of your method on one side of your large sheet of paper.

At this point students should have used only one side of their large sheet of paper to plan what they will do.

Implementing the Strategy

Students are now to turn their large sheet of paper over and write their joint solution clearly in the form of a poster.

While students work in small groups you have two tasks: to note different student approaches to the task and to support student problem solving.

Note different student approaches to the task

Listen and watch students carefully. Note different approaches to the task and what assumptions students make. Do students work systematically? How do they organize their work? Are they concerned about the context, such as the speed of a car or the cost of fuel? What do students do if they get stuck? Do they check their answers? In particular, note any common mistakes. You can then use this information as a focus for a whole-class discussion.

Support student problem solving

Try not to make suggestions that move students towards a particular approach to the task. Instead, ask questions that help students clarify their thinking. In particular, focus on the strategies rather than the solution. Encourage students to justify their work. If they get really stuck, suggest they assume that Bill, Sue, and Fred all travel 120 miles each week.

You may want to use the questions in the *Common issues* table to support your own questioning.

What do you know? What do you need to find out?

How can you use what you know to do this?

How many gallons of fuel does Bill /Sue use each week?

Can you make an assumption about the distances traveled each week?

How much fuel would Bill and Sue save with their new cars if they each traveled 120 miles each week? How can you use this information?

Do you think fuel used by a car is proportional to the distance travelled? How do you know?

If the whole class is struggling on the same issue, you could write one or two relevant questions on the board and hold a brief whole-class discussion.

If a group successfully completes the task, ask them:

Suppose Sue did save the same amount of fuel as Bill, how does this change the distance Sue traveled each week?
Explain your reasoning.

Whole-class discussion (10 minutes)

The purpose of this activity is to evaluate and compare different approaches.

I want us to share all the different ways you've interpreted the task.

What was the most challenging part of this task? Why?

What did you do to get 'unstuck'?

What did you learn when working together on the problem?

You may have noticed some interesting ways of working or some incorrect methods, if so, you may want to focus the discussion on these. Equally, if you have noticed different groups using similar strategies, you may want to compare solutions.

Has anyone used a similar/different method to Jake's? Please explain.

Which method do you prefer? Why?

Extending the lesson over two days

If you are taking two days to complete this lesson unit then you may want to end the first lesson here. At the start of the second day, briefly remind students of the task they have been working on before moving on to the collaborative analysis of sample responses.

Collaborative analysis of *Sample Responses to Discuss* (30 minutes)

Distribute copies of the *Sample Responses to Discuss* to each group of students. This task gives students an opportunity to evaluate a variety of possible approaches to the task, without providing a complete solution strategy.

There may not be time for all groups to look at all three sample responses. If time is short, it may be helpful for everyone to look just at Madison's and Ethan's work.

In your groups you are now going to look at some student work on the task. Notice in which ways this work is similar to yours and in which ways it is different.

There are some questions for you to answer as you look at the work. You may want to add notes to the work to make it easier to follow.

Slide P-2 of the projector resource, *Evaluating Sample Student Responses*, describes how students should work together:

Evaluating Sample Student Responses

1. Take turns to work through a student's solution.
   Write your answers on your mini-whiteboards.
2. Explain your answer to the rest of the group.
3. Listen carefully to explanations.
   Ask questions if you don't understand.
4. When everyone is satisfied with the explanations, write the answers below the student's solution or on a separate piece of paper.
   Make sure the student who writes the answers is not the student who explained them.

Encourage students to focus on evaluating the math contained in the student work, not on its superficial appearance.

During the small group work, support the students as in the first collaborative activity. Also, check to see which of the explanations students find more difficult to understand. Note similarities and differences between the sample approaches and those the students took in the collaborative group work.

Madison has figured out the fuel consumption for Bill, Sue, and Fred for several different distances. She has organized her work into a table and figured out the savings for Bill and Sue at each distance. The distance of 10 miles resulted in fuel consumptions of less than one gallon. Madison did not include these results in her table. She carefully chose multiples of 20, 30, and 40 for her distances.

I will assume the 3 people travel 10 miles

Bill: 20mpg 10/20 = ½ gallon 1/6 g saving
Sue: 30mpg 10/30 = 1/3 gallon 1/12 g saving
Fred: 40mpg ~~40/3~~ 10/40 = ¼ gallon

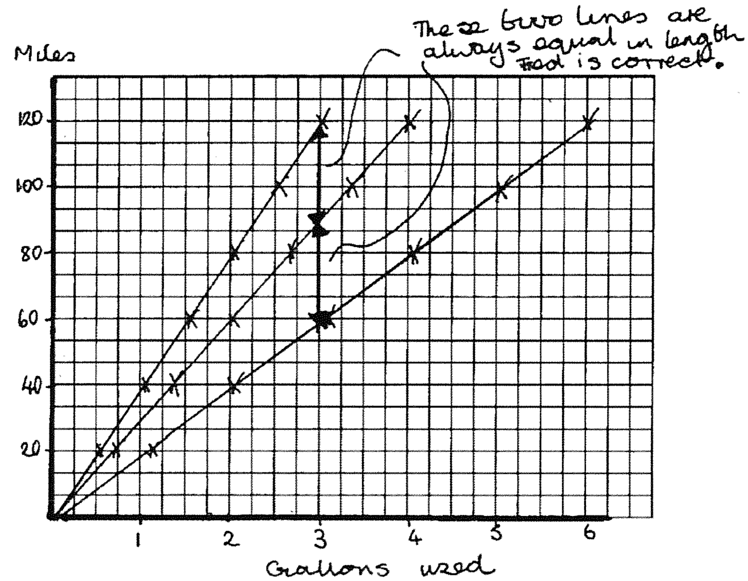
Now I will assume the 3 people travel 120 miles
Bill: 120/20 = 6 gallons 2g saved
Sue: 120/30 = 4 gallons 1g saved
Fred: 120/40 = 3 gallons

	120 miles	240 miles	360 miles	480 miles
Bill	6	12	18	24
Sue	4 ↲ 2	8 ↲ 4	12 ↲ 6	16 ↲ 8
Fred	3 ↲ 1	6 ↲ 2	9 ↲ 3	12 ↲ 4

How has Madison selected the distances traveled?

Madison could conclude that Bill saves twice as much as Sue; however, to show that this is the case for every possible weekly mileage, an algebraic solution is needed.

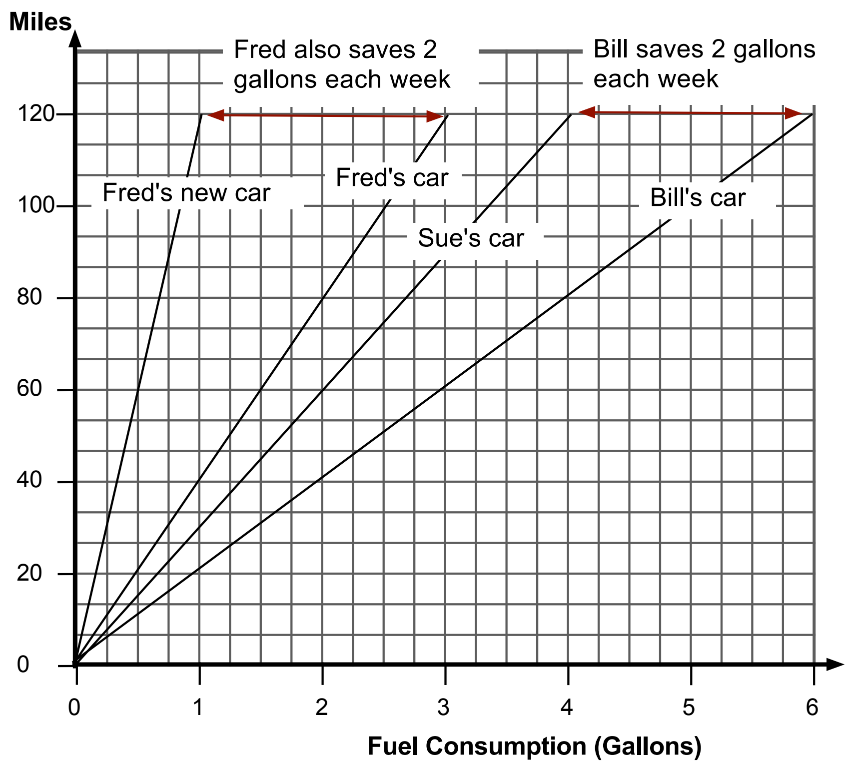
Ethan has correctly drawn the graph, however, his lines are not labeled and it is unclear how he positioned the lines. Rather than measuring vertical distances he should have measured the horizontal distances between points on each line. These horizontal distances are not equal. This shows Bill's and Sue's savings on fuel consumption are not equal.



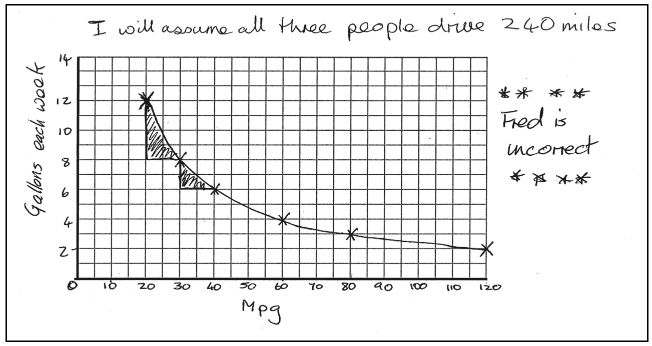
What does the slope of each line represent? [mpg of car]

What do the vertical lines Ethan has drawn represent? Does this help you answer the questions on the sheet? How?

To use the graph to figure out the mpg of Fred's new car, students should add a line to the graph ensuring that the two horizontal distances are equal. The graph shows Fred will need to buy a car that does 120 mpg. This is a very unrealistic fuel consumption for most cars, although a few hybrid cars that boast this mpg rating are just beginning to be built!



Ava's method is quite sophisticated. She has figured out the amount of fuel consumed in 240 miles for each car. She has also figured out the fuel consumption for cars with a mpg of 60, 80, and 120.

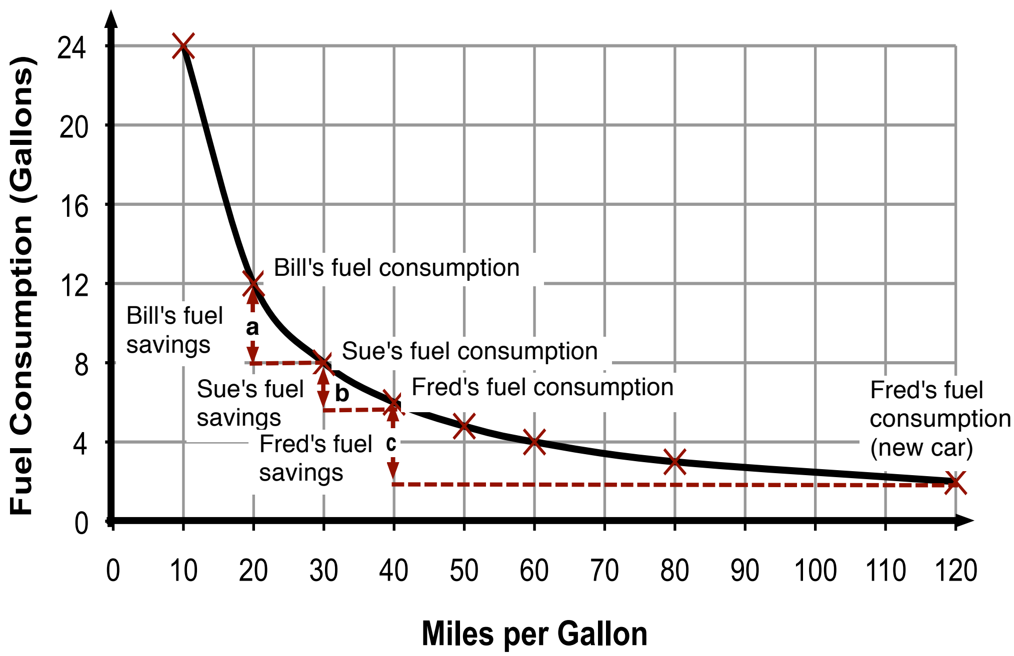


Ava could have labeled the curve with the names of the three people next to the mpgs of their car.

Ava has not explained her conclusion.

Ava has drawn vertical and horizontal lines.
What do they represent?

To use the graph to figure out the fuel consumption of Fred's new car (120mpg), students should draw the horizontal dashed line shown below:



Whole-class discussion: comparing different approaches (15 minutes)

Hold a whole-class discussion to consider the different approaches used in the sample work. This activity provides students with the opportunity to share with each other their understandings of the three solutions, the mathematical connections between the solutions and their criteria for evaluating each solution.

You may first want to check their understanding of the solutions.

What can you tell me about the savings Bill and Sue will make? Please explain your thinking.

Connections between responses:

What do these two responses have in common?

How are the different measures (distance, MPG, gallons, savings) represented in each response?

Comparing the strengths and weaknesses of each approach:

Which student has adopted the most appropriate method? Why?

Does anyone agree/disagree with this answer?

What information can you deduce from this response, but not this one?
Is this extra information useful?

Which response clearly demonstrates the benefits to the environment of removing the most fuel inefficient cars from our roads? Please explain.

Suppose the solution was to be understood by a student in another class/a math teacher/Fred. Would one method be preferable? Why?

If students have not already stated the following criteria when comparing solutions-methods, encourage them to consider :

- Which is the most efficient method? Why?
- Which method is easiest to understand? Why?
- Which method gives you the most information? Why?
- Which method would be the easiest to do? Why?

If there is time encourage students to recognize the decisions they made themselves when attempting to solve the task:

Who used a method similar to one of the student's solutions?

How is your solution similar/different from the Sample student work?
Why did you decide to use that method?

Did anyone use a different method to the three we have been discussing? Please explain.

How does it compare to the sample pieces of work?

You may also want students to reflect on their own learning:

What have you found difficult/easy about the task?
What have you learnt today?

Draw students' attention to the surprisingly large increase in the mpg of Fred's new car:

Fred wants to save the same amount of fuel as Bill.
Bill sells a car that does 20 mpg and buys a car that does 30 mpg.
Fred's old car does 40 mpg, what is the mpg of Fred's new car?
[120 mpg, quite a difference in fuel consumption! Cars with this mpg are very rare but hybrid cars that claim this fuel efficiency are now being built].

When thinking about benefits to the environment, why is it important to remove the most fuel inefficient cars from our roads?

Follow-up lesson: individual review of work (10 minutes)

Give out the sheet *How Did You Work?* and ask students to complete this questionnaire. The questionnaire should help students review their progress.

If you have time you may also want to ask your students to read through their original solution and using what they have learned, attempt the task again. In this case, give each student a blank copy of the assessment *Buying Cars*.

Some teachers give this task as homework.

SOLUTIONS

Assessment task: *Buying Cars*

There are many ways students could answer these questions. Below are four possible ones.

Solution A: Numerical

This solution assumes that each person drives 120 miles per week.

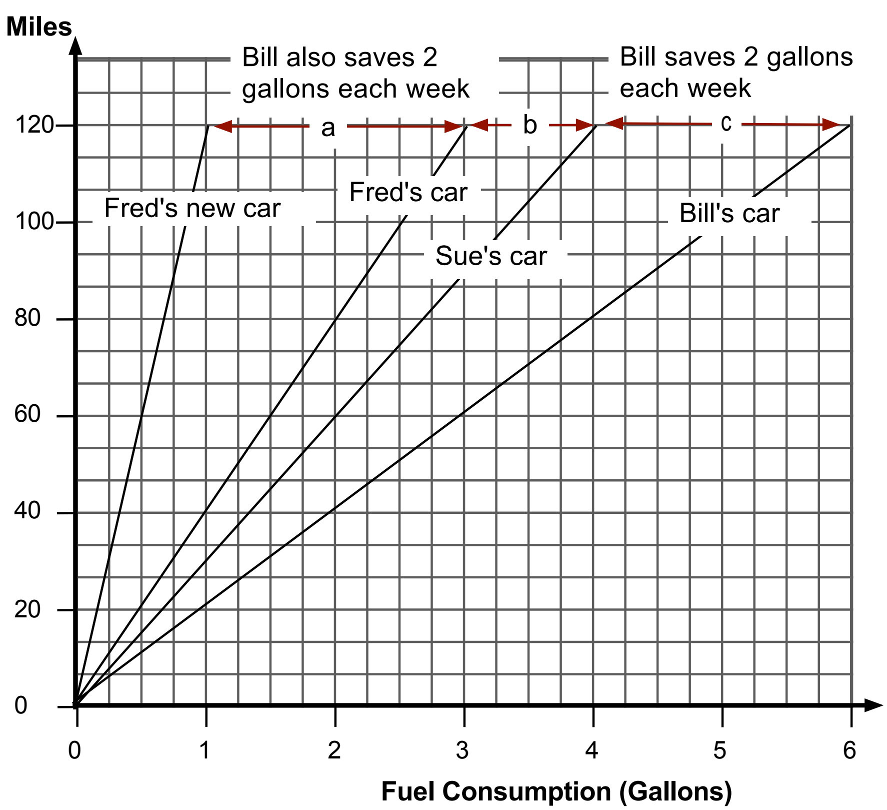
1. Bill's car uses 120 ÷ 20 = 6 gallons each week.
   Sue's car uses 120 ÷ 30 = 4 gallons each week.
   Fred's car uses 120 ÷ 40 = 3 gallons each week.
   Bill will save 6 - 4 = 2 gallons.
   Sue will save 4 - 3 = 1 gallon.
   Bill's and Sue's savings on gas will be different.

2. Fred's new car must save Fred 2 gallons for each 120 miles driven.
   $3 - x = 2$ where x represents the number of gallons consumed in 120 miles, therefore $x = 1$ gallon.
   Fred will need to buy a car that uses 1 gallon for each 120 miles he drives.

Solution B: Graphical

The lengths *b* and *c* are different, indicating Bill's and Sue's saving on fuel will be different.

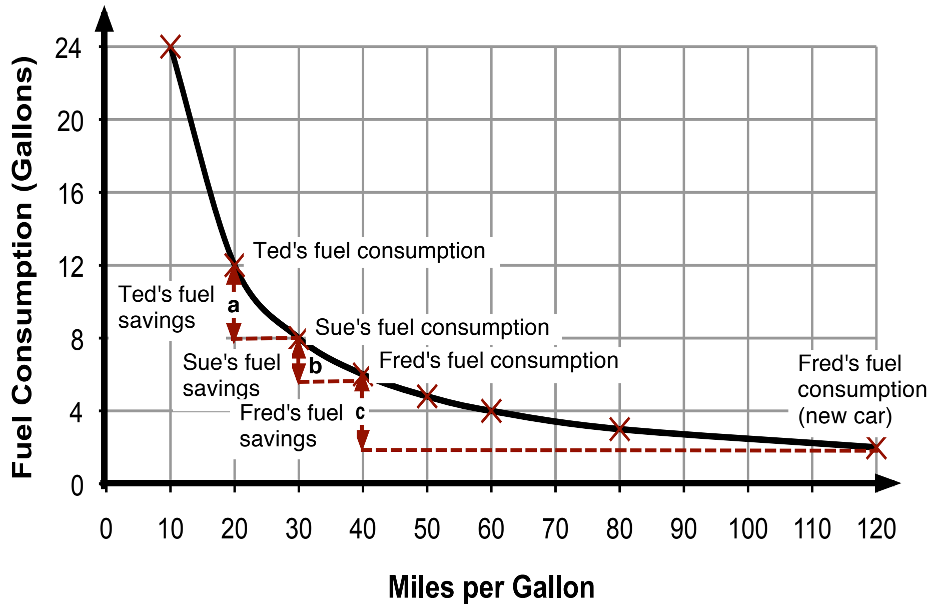
For Fred to save the same amount in fuel as Bill, length *a* must be equal to length *c*. The fuel consumption of Fred's new car must be 120 mpg.



Solution C: Graphical

The lengths *a* and *b* are different, indicating Bill's and Sue's saving on fuel will be different.

For Fred to save the same amount in fuel as Bill, length *a* must be equal to length *c*. The fuel consumption of Fred's new car must be 120 mpg.



Solution D: Algebraic

1. Suppose the three people each drive a distance of *d* miles each week.

   The fuel used by each is as follows:

   $$\text{Ted}: \frac{d}{20} \qquad \text{Sue}: \frac{d}{30} \qquad \text{Fred}: \frac{d}{40}$$

   If Ted buys Sue's car, he saves $\frac{d}{20} - \frac{d}{30} = \frac{d}{60}$ gallons per week.

   If Sue buys Fred's car, she saves $\frac{d}{30} - \frac{d}{40} = \frac{d}{120}$ gallons per week.

   So Sue only saves one half the money that Ted saves.

2. Fred wants to save $\frac{d}{60}$ gallons per week.

   If his new car does *x* miles per gallon, then subtracting the new fuel consumption from the old fuel consumption gives:

   $$\frac{d}{40} - \frac{d}{x} = \frac{d}{60} \quad \text{gallons per week}$$

   $$\Rightarrow \frac{d}{40} - \frac{d}{60} = \frac{d}{x} \quad \text{gallons per week}$$

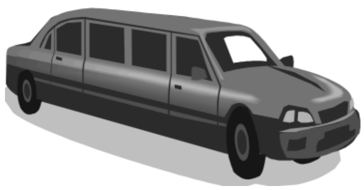
   $$\Rightarrow \frac{d}{120} = \frac{d}{x} \quad \text{gallons per week}$$

   $$\Rightarrow x = 120 \quad \text{miles per gallon}$$

   So his new car must do 120 miles per gallon.

Buying Cars

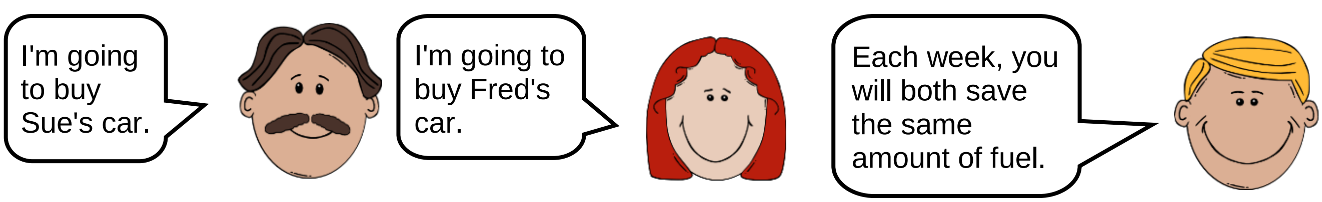
Bill, Sue and Fred each own a car.
Every week, all three drive the same distance.

Bill's car	**Sue's Car**	**Fred's car**
		
Average fuel consumption: 20 miles per gallon	Average fuel consumption: 30 miles per gallon	Average fuel consumption: 40 miles per gallon

1. Which car is the cheapest to run?
   Explain your answer clearly.

2. They are trying to work out how much fuel they will each save if they change cars.

Bill **Sue** **Fred**



Fred is **incorrect**.
Use mathematics to show why his statement is wrong.
State any assumptions you make and explain your reasoning clearly.

3. Fred wants to buy a new car.
   Each week, he wants to save the same amount of fuel as Bill.

   What should be the average fuel consumption of Fred's new car?
   Explain your reasoning clearly.

Sample Response to Discuss: Madison

I will assume the 3 people travel 10 miles

Bill : 20mpg 10/20 = ½ gallon
Sue: 30mpg 10/30 = ⅓ gallon ⅙ g saving
Fred: 40mpg ~~40/3~~ 10/40 = ¼ gallon $\frac{1}{12}$ g saving

Now I will assume the 3 people travel 120 miles
Bill : 120/20 = 6 gallons
Sue: 120/30 = 4 gallons 2g saved
Fred : 120/40 = 3 gallons 1g saved

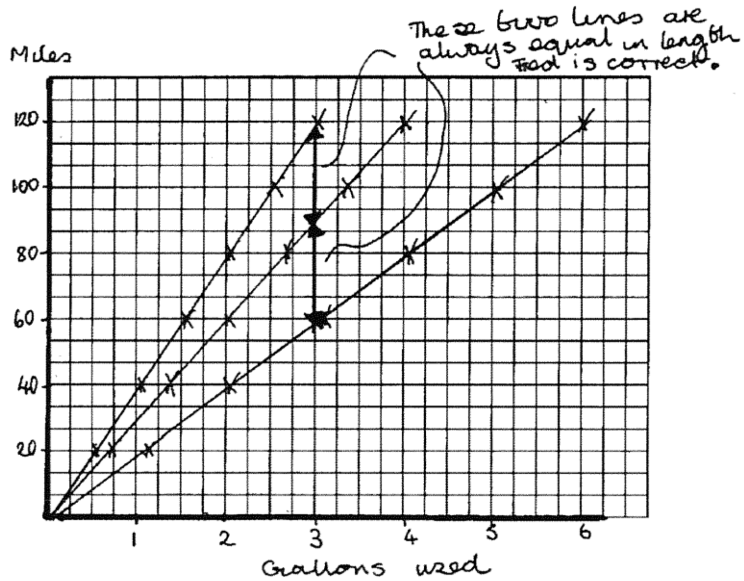
	120 miles	240 miles	360 miles	480 miles
Bill	6	12	18	24
	2	4	6	8
Sue	4	8	12	16
	1	2	3	4
Fred	3	6	9	12

Add another column of figures to Madison's table.

What conclusions can Madison make?

Use Madison's method to figure out the average fuel consumption of Fred's new car.
Explain your reasoning clearly.

Sample Response to Discuss: Ethan

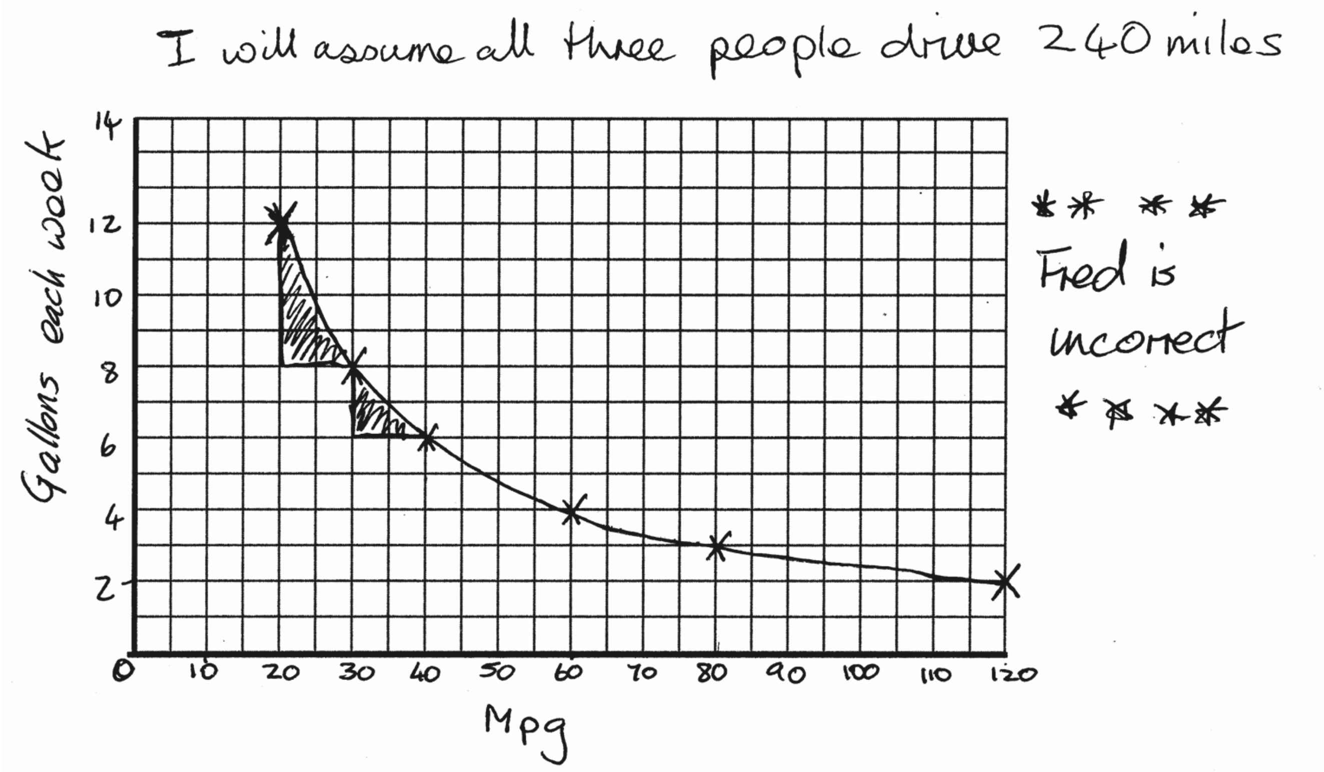


How has Ethan decided where to draw the lines on his graph?
What do these lines represent?
It may help to add text to the graph.

How can Ethan improve his method? Mention any mistakes Ethan has made.

Use the graph to figure out the average fuel consumption of Fred's new car.
Explain your reasoning clearly.

Sample Response to Discuss: Ava



How has Ava decided where to draw the line?
It may help to add text to the graph.

Explain how the graph shows Fred is incorrect?

Use the graph to figure out the average fuel consumption of Fred's new car.
Explain your reasoning clearly.

How did you work?

Tick the boxes, circle an option and complete the sentences that apply to your work.

1. Our group work was better than my own work **Yes/ No**

   This is because ....

   ....

2. Our solution is similar to one of the sample responses ☐ **OR** Our solution is different from **all** of the sample responses ☐

   Our solution is similar to .... *(add name of the student)*

   I prefer **our solution** / **the student's solution** (*circle*)

   This is because ....

   Our solution is different from all of the sample responses because ....

3. Compare the sample responses. What are the advantages and disadvantages of each approach?

	Advantages	Disadvantages
Madison:		
Eva:		
Ava:		

4. What difficulties do you think someone new to the task would face?

   ....

Planning a Joint Method

1. Take turns to explain your method and how your work could be improved.

2. Listen carefully to each other.
   Ask questions if you don’t understand.

3. When everyone in the group has explained their method, plan a joint method that is better than your separate ideas.

4. Make sure that everyone in the group can explain the reasons for your chosen method.

5. Write a brief outline of your method on one side of your large sheet of paper.

Evaluating Sample Student Responses

1. Take turns to work through a student's solution.
   Write your answers on your mini-whiteboards.

2. Explain your answer to the rest of the group.

3. Listen carefully to explanations.
   Ask questions if you don't understand.

4. When everyone is satisfied with the explanations, write the answers below the student's solution or on a separate piece of paper.
   Make sure the student who writes the answers is not the student who explained them.

Sample Response to Discuss: Madison

I will assume the 3 people travel 10 miles

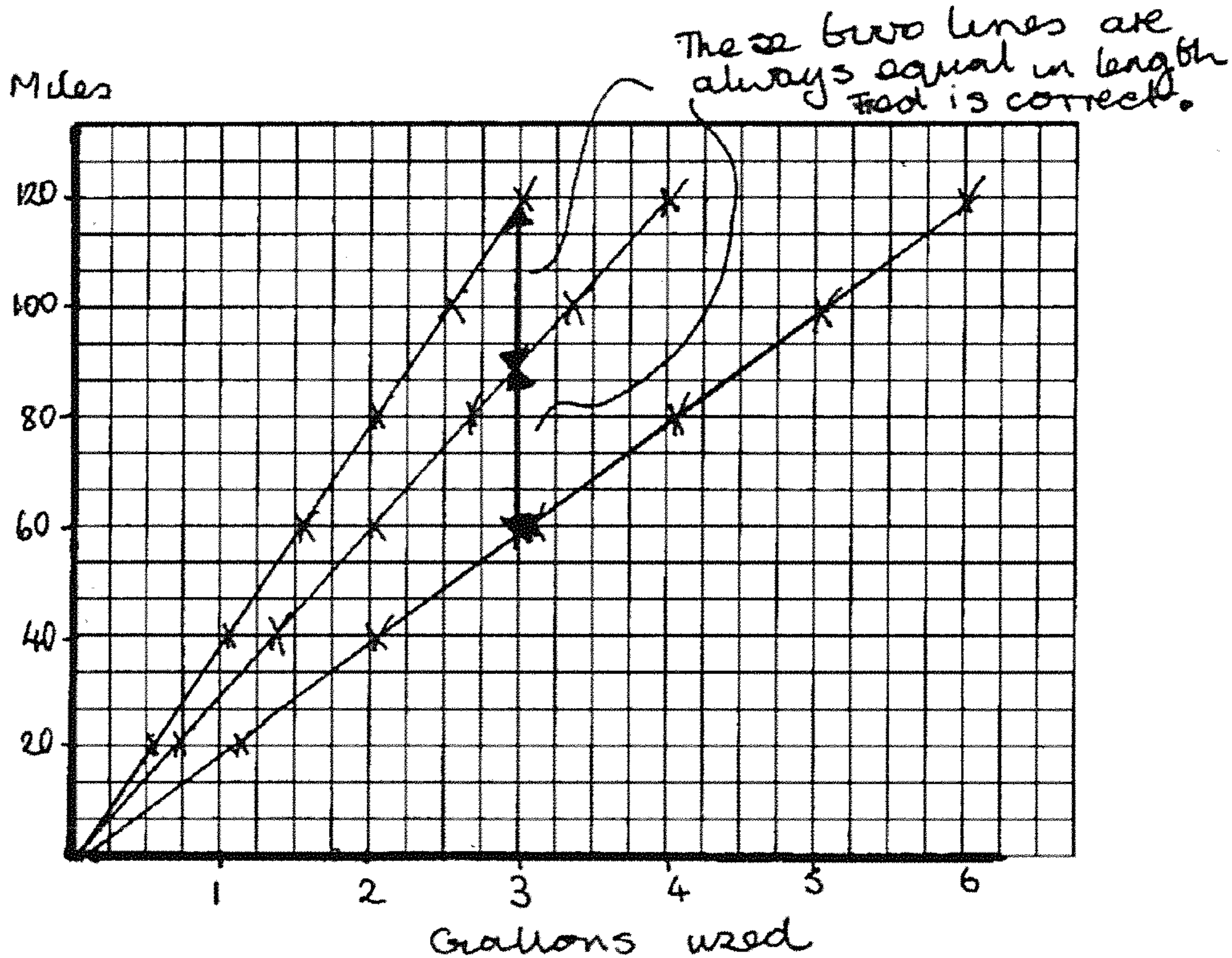
Bill : 20mpg $10/20$ = ½ gallon ↓ 1/6 g saving
Sue: 30mpg $10/30$ = ⅓ gallon ↓ 1/12 g saving
Fred: 40mpg ~~40/3~~ $10/40$ = ¼ gallon

Now I will assume the 3 people travel 120 miles

Bill : $120/20$ = 6 gallons ↓ 2g saved
Sue: $120/30$ = 4 gallons ↓ 1g saved
Fred : $120/40$ = 3 gallons

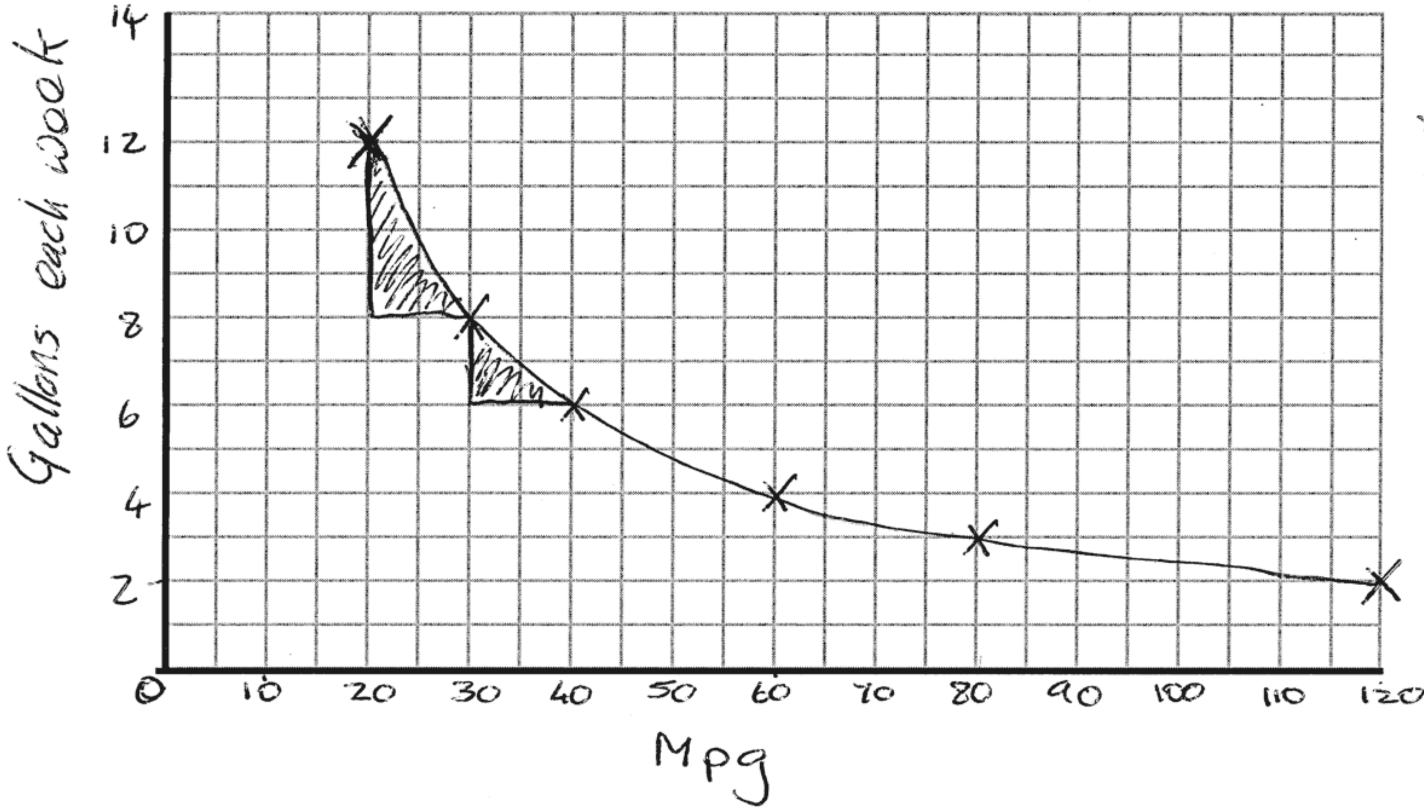
	120 miles	240 miles	360 miles	480 miles
Bill	6 ↓ 2	12 ↓ 4	18 ↓ 6	24 ↓ 8
Sue	4 ↓ 1	8 ↓ 2	12 ↓ 3	16 ↓ 4
Fred	3	6	9	12

Sample Response to Discuss: Ethan



Sample Response to Discuss: Ava

I will assume all three people drive 240 miles



** **
Fred is incorrect
****

Mathematics Assessment Project

Classroom Challenges

These materials were designed and developed by the
Shell Center Team at the Center for Research in Mathematical Education
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The full collection of Mathematics Assessment Project materials is available from

http://map.mathshell.org