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INTRODUCING ALGEBRA

from "A Collection of Math Lessons: from Grades 6 through 8" Marilyn Burns and Cathy Humphreys

Students often get their first glimpse of algebra from older sisters or brothers. The math papers they see are filled with more letters than numbers and seem very mysterious. Students are generally impressed and curious; some are a bit fearful.

For some students, algebra classes unlock the mystery of this new subject. These students learn to solve equations easily. They like the orderliness of procedures, enjoy solving word problems, and feel satisfied when they get to the final $x =$.

Other students have different reactions. All teachers who have taught algebra have encountered students who report, "I don't get it." "This is dumb" is another common reaction. Students who do not take pleasure in the orderly procedures ask, "What's it good for anyway?"

For many students, important elements are missing from algebra instruction. They don't learn how algebra fits with the rest of the mathematics they've been studying. They focus on the various procedures and problems to be solved and pay attention to the rules and rituals of algebra while ignoring its place in the world of mathematics.

This chapter explains how algebra was introduced to a class of eighth graders. The five-day investigation integrated algebra with geometry and arithmetic. Each student was given a 10-by-10 grid of squared-centimeter paper and asked to figure the number of squares in its border. After describing and comparing their different methods, the students solved the same problem for grids of other sizes. Finally, they generalized their methods of calculation into algebraic formulas. By introducing algebra as an extension of arithmetic and geometry, the students were helped to see algebra as connected to their previous math learning.

Day 1

"What do you know about algebra?" I asked the class.

I was interested in learning what these eighth graders knew or had heard about algebra. My question was met with general silence. Finally, Jesse raised his hand.

"It has to do with using letters to stand for numbers," he said.

"Yes," I responded, "letters are often used for numbers in algebra. Does anyone have a different idea?"

Zack raised his hand. "You have to solve for unknowns," he said.

No other students had ideas to offer.

I then wrote on the chalkboard:

Algebra is a generalization of arithmetic.

"What is arithmetic?" I asked.

Lots of hands went up to answer this question. "It has to do with numbers." "It's addition and subtraction and like that." "It's multiplication and division, too." "You do things to numbers and get answers." "You do it with whole numbers and fractions and decimals."

I gave all the students who raised their hands the chance to contribute. Then I asked another question, "What is a generalization?"

Fewer hands went up. "It's the opposite of *specific*," Chris said.

"It's like those statements we write about graphs," Krystal said.

"They're conclusions," Jeremy added.

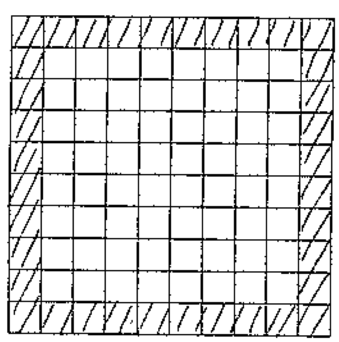
"In order to help you learn about algebra," I then said to the class, "I'm going to start from what you already know. I'm going to give you a problem you can solve using arithmetic. Then I'll introduce you to how to use algebra to generalize your solutions. We won't get to the algebra part today; instead, we'll focus just on the problem."

In general, students learn from connecting new experiences to what they know. I want students to see algebra in relation to what they've already learned, not as a topic separate from the mathematics they've been studying.

I had cut a ten-by-ten grid from squared-centimeter paper for each student. I distributed the grids at this point and asked the students to find out how many squares were on the grid. It was easy for them to figure there were 100 squares.

I had colored in a border on my grid, one row of squares around the outside. "Now I'd like you to find out how many squares there are in the border of your grid," I said, "the row all around the outer edge."

To watch a video of Cathy doing the first part of the Border Problem, search for "Border Problem Part 1" on youtube



There were murmurs as the students worked. "I know, it's 40." "It's got to be 40." "I'll count to check." "Hey, it's not 40." "Oops, I'd better count again." "I got 36." "Yeah, I got 36, too." "You could have fooled me."

When all the students had convinced themselves that there were 36 squares in the border, I asked for volunteers to explain how they figured it.

I called on Jason. "I did 10 times 4," he said, "and then I subtracted 4 and got 36."

"Why do you think your method makes sense?" I asked.

"Because you can't count the corners twice," Jason answered, "so I subtracted them at the end."

I recorded Jason's method on the board numerically.

$$\begin{array}{r} 10 \\ \times\ 4 \\ \hline 40 \\ -\ 4 \\ \hline 36 \end{array}$$

"Did anyone do it a different way?" I asked.

Robert had a suggestion. "I counted the sides and added," he said. "The top has 10, then the next side has 9 because you already counted the corner, and the next side has 9, and the last side has 8 because you counted both corners."

Robert and Steven figure out how many squares there are in the border of the grid.

I recorded Robert's method on the board.

$$\begin{array}{r} 10 \\ 9 \\ 9 \\ +8 \\ \hline 36 \end{array}$$

"Any other way?" I asked.

Tami reported next. "I just multiplied 9 times 4," she said.

"Why does that make sense?" I asked.

"Because I didn't want to count the corners twice," Tami answered. "I just said that each side gets one corner, so they each have 9 squares, and I multiplied that by 4."

I numerically recorded Tami's method on the board. Then I asked, "Any other ideas?"

I called on Zack. "There are 64 squares in the middle," he said, "because that's 8 times 8. I subtracted 64 from 100 and got 36."

I added Zack's method to the board. "Another way?" I asked.

Chris had an idea. "I added 10 and 10 for the top and bottom," he said. "Then there are only 8 left on each side, so add 8 and 8, and you get 36."

I recorded Chris's idea on the board. "Another idea?" I asked again. There were no more volunteers.

"I have another method," I said. "I'll write it on the board numerically and then see if someone can explain what I was thinking." I wrote:

$$\begin{array}{r} 8 \\ \times 4 \\ \hline 32 \\ +4 \\ \hline 36 \end{array}$$

Several hands shot up. I called on Juliette. "You took out the corners first and then added them back in at the end," she said.

"Where does the 8 come from?" I asked.

Steven answered. "That's how much is on each edge without the corners," he said.

I now had six methods recorded on the chalkboard:

1	2	3	4	5	6
$\begin{array}{r} 10 \\ \times 4 \\ \hline 40 \\ -4 \\ \hline 36 \end{array}$	$\begin{array}{r} 10 \\ 9 \\ 9 \\ +8 \\ \hline 36 \end{array}$	$\begin{array}{r} 9 \\ \times 4 \\ \hline 36 \end{array}$	$\begin{array}{r} 100 \\ -64 \\ \hline 36 \end{array}$	$\begin{array}{r} 10 \\ 10 \\ 8 \\ +8 \\ \hline 36 \end{array}$	$\begin{array}{r} 8 \\ \times 4 \\ \hline 32 \\ +4 \\ \hline 36 \end{array}$

I then gave the students a writing assignment to do. "Choose one of the methods for solving the border problem," I said. "Describe it in your notebook so that someone who wasn't in class would have a way to find the answer. Be sure to explain why the method works."

Having the students write helps them find out what they know and what they don't know. Also, their writing gives me insights into their understanding. I knew that writing was difficult for many of the students in the class. They had not had much previous experience with writing about their thinking processes in their math classes.

When I read through their explanations that night, I found most to be very poor, both in the thoughts they expressed and the grammar they used. The following are samples from about half the students.

Michael wrote: *You can figure it out by taking 9 from each side and multiplying it by four.*

From Sinead: *All you had to do was add the top and bottom which is 10 + 10 = 20 and then add the two sides 8 + 8 = 16 and then add them together 20 + 16 = 36.*

Quinn wrote: *take the bottom and the top and add them. then add the sides and theres your answer.*

$$\begin{array}{r} 10 \\ 10 \\ 8 \\ +8 \\ \hline 36 \end{array}$$

Aneitra wrote: *I think Jason had a good method because there are four sides and one of the sides has ten. You have to tell how many squares are around the border. So you multiply 10 × 4 and come up with 40. Then I subtract 4 because I have to take out a corner and come up with 36.*

From Steven: *The easiest way to find out the number of squares in the border is 9 × 4 = 36.*

Hasani wrote: *You can figure this out by multiplying 9 and 4 because it is easer then doing anything else.*

From Zack: *You can figure it out by 8 × 8 inside subtract 64 from 100 = 36.*

Jason wrote: *You add the squares on the border then subtract the 4 corners that you don't use.*

Jesse wrote: *You can figure this out by the Tami 9 on each side method. Each side has ten sqares but you can't count the corners twice. So you just multiply 9 × 4.*

From Rachel: *Take away the border and find how many squares are left 8 × 8 = 64 Then subtract 64 from 100 and you get the answer*

The border Problem
The border has 36 squares.
You can figure this out by multiplying 9 and 4 because it is easer the doing any thing else.

Algebra
Doublestuff
10
10
8
+8
36
take the bottom and the top and add them. then add the sides and theres your answer.

Students explained how they figured how many squares were in the border.

Jeremy wrote: *You can find this out by taking the number off the top which is 10. Then you add two sides which are 9 each and then add the bottom which is 8.*

$$\begin{array}{r} 8 \\ 9 \\ 9 \\ +10 \\ \hline 36 \end{array}$$

Juliette wrote: *You can figure this out by adding the 2 vertical sides and then adding the 2 middles which add up too 8 so 10 + 10 = 20 and 8 + 8 = 16 so 16 + 20 = 36*

I decided to focus the next lesson on helping the students improve their ability to explain their reasoning in writing.

Day 2

To prepare for this lesson, I wrote five of the explanations the students had written on large sheets of newsprint. I planned to use these examples to talk about what parts of the explanations were clear and what parts needed more information. Also, I planned to edit each of the explanations to model for the students how to make improvements.

I posted the chart of the six methods I had recorded the day before. Then I posted one of their explanations. I chose one of the more complete explanations to review first:

> You can find this out by taking the number off the top which is 10. Then you add two sides which are 9 each and then add the bottom which is 8.

$$\begin{array}{r} 8 \\ 9 \\ 9 \\ +10 \\ \hline 36 \end{array}$$

"Which method does this explain?" I asked.

Several hands went up. "It's like number 2," Juliette said, "the one that Robert said."

"That's right," I answered, "and the explanation is fairly clear. There is some missing information, however. The explanation doesn't tell where the 9 and 8 come from. Let's start with the 9. Why are the two sides 9 each?"

"I can explain," Robert said. "It's because you already used the corner."

"That's just the kind of information that should be included," I responded, and inserted "because you can't count the corner again" into the explanation.

"How can you explain the 8?" I then asked.

I called on Michael. "You already counted both corners for the last row," he said, "so you can't count them."

I added "without the two corners" after the 8. The explanation now read:

You can find this out by taking the number off the top which is 10. Then you add two sides which are 9 each because you can't count the corner again and then add the bottom which is 8 (without the two corners).

$$\begin{array}{r} 8 \\ 9 \\ 9 \\ +10 \\ \hline 36 \end{array}$$

I added one more comment. "Including the numerical representation is a good idea," I said, "because it adds to the explanation."

I then went on to a second explanation, also a fairly complete one:

> I think Jason had a good method because there are four sides and one of the sides has ten. You have to tell how many squares are around the border. So you multiply 10×4 and come up with 40. Then I subtract 4 because I have to take out a corner and come up with 36.

"I didn't write that," Jason called out.

"No, Aneitra did," I answered. "She was describing the method you explained."

"It seems clear to me," Zack said.

"I think so, too," I said, "except that I have two changes to recommend." I changed "one of the sides" to "each of the sides" and "a corner" to "all four corners." Though these were small changes, I wanted them to know that I expected careful reading and attention to all details.

I posted another:

> You can figure this out by multiplying 9 and 4 because it is easier than doing anything else.

"There were more explanations of this method than any of the others," I said. "Though it was a popular choice, none of the explanations had enough information. What's missing is an explanation of why it makes sense to multiply 9 and 4."

"I can explain," Tami said. She came to the board and drew just the border of squares with 10 on a side.

"See," she continued, "you count 9 on each side by stopping before the last one so you don't count the corners twice." Tami illustrated this by counting and marking the squares she had drawn.

"Your explanation makes sense," I said. "How can I describe what you did in writing?"

Tami was stumped. Andy raised his hand. "You take a corner off each side," he said, "and that leaves 9."

"Then what?" I said.

"You do 9 times 4," Tami added.

I rewrote the explanation on the chart:

> Take one corner off each side. That leaves 9 on each side. Then multiply 9 times 4 to find out how many squares in the border.

"This isn't the only possible way to explain Tami's method," I said. "There isn't one right way, but you have to be sure that what you write has all the information needed to make sense."

I then posted a fourth explanation:

> Take away the border and find how many squares are left $8 \times 8 = 64$ Then subtract 64 from 100 and you get the answer

I went through a similar discussion, talking with the class about explaining why multiplying 8 times 8 made sense and where the number 100 came from. Also, I reminded them to include a period at the end of each sentence.

The last explanation I posted read:

> take the bottom and the top and add them. then add the sides and theres your answer.
>
> $$\begin{array}{r} 10 \\ 10 \\ 8 \\ +8 \\ \hline 36 \end{array}$$

First I corrected the grammar, adding capital letters at the beginning of each sentence and an apostrophe in *theres*.

"Having the numbers included helps," I said, "but I think the explanation would be clear if you explained why you added the two 8s."

"They're the sides," Tri said.

"Why does 8 make sense for the sides?" I asked.

"Because you can't count the corners again," Aneitra answered.

I edited the explanation to read:

> Take the bottom and the top and add them. Then add the sides which are 2 less than the top because you can't count the corners again, and there's your answer.

"Now," I said to the students, "I'm going to post one more. This time, each of you is to write in your notebook an improved version of what I've posted." I posted an explanation that was another version of a method we had already discussed:

> You add the squares on the border then subtract the 4 corners that you don't use.

The students' explanations showed improvement.

Krystal wrote: *There are ten squares on each side. You can't count any square twice. So subtract four corner squares. Now there are 36 squares.* Krystal included a diagram to illustrate her explanation.

Steven wrote: *OK you have 10 squares on each side and there are 4 sides so then you multiply 10×4 which is 40 but you have added the corners twice so you have to take out the 4 corners which makes it 36 squares. $10 \times 4 - 4 = 36$.*

From Juliette: *You add the top which is 10 and then the next side which is ten (counting the corner you've already counted). Add the bottom which is ten and then the next side up which is ten. And then you subtract the 4 corners that you counted twice each.* Juliette also included a numerical recording.

Zack wrote: *There are 10 squares on each side. Multiply 10×4, then subtract the corners (4), because you counted each twice.*

From Tami: *You count the 10 on each side and you get 40. Then subtract the 4 corners because you counted all corners twice and then you come up with the answer 36.*

Subtract-the-Corners

There are ten squares on each side. You can't count any square twice. So subtract four corner squares. Now there are 36 squares.

You add the top which is 10 and then the next side which is ten (counting the corner you've already counted) ~~count the co~~ Add the bottom which is ten and then the next side up which is ten. And then you subtract the 4 corners that you counted twice ~~ea~~ each

$$\begin{array}{r} 10 \\ +10 \\ +10 \\ 10 \\ \hline 40 \\ -\ 4 \\ \hline 36. \end{array}$$

The students' revised explanations showed improvement.

Hasani wrote: *You add the number on the top and then multiply it by 4 and then you subtract the corners that you used all ready.*

Andy wrote: *Assuming that the sides don't overlap on the corners, you multiply the 10 squares on a side by four for the four sides. Then you subtract the overlapping corners.*

The student who had the hardest time was Tri, a Vietnamese boy, who has lived in the United States for only three years and has difficulty with English. Tri wrote: *We count each side of borders are 10, we subtract 4 because we count twice on the corner.* Practice with writing is necessary for Tri to learn to express his ideas in more conventional English.

When I noticed that most of the students had written an explanation, I interrupted them to give additional directions. "I'm going to give each pair of you a worksheet I've prepared. When you complete your explanation," I said, "compare what you've written with your partner. Agree on one way to write this explanation and record this for number 1 on the worksheet. Then work together to write improved versions of the others on the worksheet."

I had included five explanations on the worksheet, all taken from what the students had written in their notebooks the day before. The students worked for the remaining twenty minutes rewriting the explanations on the worksheet.

When I read the worksheets that night, only one pair of students had completed all five explanations. I was faced with the natural difficulty that some students work more quickly than others. I wanted to give the others time to finish their work while keeping them all moving toward describing the methods algebraically. I decided to give an additional problem to extend their work into figuring out the borders of squares of different sizes.

Day 3

I began the class by drawing a 5-by-5 grid on the chalkboard. "How many around the border of this grid?" I asked.

I waited, giving the students a chance to think about the problem. When more than half the hands were raised, I called on Steven. "There are 16," he said.

"How did you figure that out?" I asked.

"I did it by Tami's method," he said. "I multiplied 4 times 4."

"Who did it a different way?" I asked. In this way, students again had the opportunity to describe the different methods for figuring the border.

"When you finish rewriting the explanations," I said, "then you are to find the number of squares in the border of grids of different sizes. Record your answers on a chart like this one." I drew a chart for them on the chalkboard, filling in the answers they already knew.

Number of squares on edge	Number of squares on border
3	
4	
5	16
6	
7	
8	
9	
10	36

"Underneath your chart," I said, "write what you notice about the pattern in the borders as the grids increase in size."

"Can we draw the squares to figure them out?" Andy asked.

"Yes," I answered, "making a drawing often helps in solving problems."

"Can we do this first before we finish the writing?" Joe asked.

"No," I answered, "you need to complete the other work and have me check it before you begin this."

There were no other questions, and the students got to work. Near the end of the period, all but two had finished what I had assigned. I asked those two to finish their work as homework. I called the class back to attention and told them that tomorrow we would be talking more about how algebra can be related to the border problem.

"Oooh, that's going to be hard," Sinead said.

"How many of you think algebra is going to be hard to learn?" I asked. Sinead and Joe raised their hands.

"I don't think it's going to be hard," Robert said, "but I don't think it's going to be easy." There was general agreement with Robert's feelings from the class.

"We'll see tomorrow," I said. I felt they were anxious but also interested and curious.

Day 4

I began the fourth day's lesson by discussing the charts on which the students had recorded the number of squares in the borders of grids of different sizes. They all had noticed that the number of squares in the borders increased by 4 as the number of squares on the edges increased by 1.

Number of squares on edge	Number of squares on border
3	8
4	12
5	16
6	20
7	24
8	28
9	32
10	36

"Why do you think this is so?" I asked.

"Because squares have four sides," Krystal said. Several of the others nodded in agreement.

"No, because they have four corners," Steven said. More nodded or murmured their agreement with Steven's thought.

"I think I know why," Zack said. Zack had come to be respected by the others for his understanding of math, and he had the attention of the class. "It's kind of like what Krystal said," he continued. "When the edge of the squares increases by 1, the border increases by 4 because 1 square is added to each side."

"That's it," Steven said.

"I don't get it," Rachel said.

"Look," Steven said, "every time a square is bigger by 1, you have to make each side bigger by 1, so that's like adding 4."

"Does anyone have another way to explain that to Rachel?" I asked.

"I can," Jesse said. "Adding 1 to each side is adding 4 altogether, and those 4 are on the border, so the border gets bigger by 4."

"Is there another way to say that?" I probed. There were no other volunteers. Rachel and the others seemed satisfied.

I was pleased that Zack offered his idea. I much prefer it when ideas such as these come from the students instead of from me. The class always seems to be more curious about and receptive to a class member's ideas than to mine. My ideas are taken more as pronouncements, laden with the authority of my position. (If Zack hadn't offered his idea, however, I would have offered an explanation. Then I would have asked for others to explain my idea in their own words.)

At this point, I moved ahead in my lesson plan. "Suppose I told you that I had a grid in my pocket," I said, "and that I wanted to figure out how many squares were in the border. Who could explain how to do it?"

About a third of the students raised their hands. I called on Halbert. "Go slowly, Halbert," I said, "because I'm going to write down what you would do as you explain it."

"First count the squares on an edge," he said. I stopped him so I could write this on the board. Then I asked him to continue.

"Multiply that number times 4," he continued, "and then subtract 4." I wrote that on the board as well.

Steven explains to Rachel why the number of squares in the border increases by four as the number of squares on the edges increases by one.

"Anything else?" I asked.

"No," Halbert said, "that tells you the answer." I wrote that as well. I had now written:

> First count the squares on an edge. Multiply that number by 4 and then subtract 4. That tells how many squares are in the border.

"Do you agree that this method works for any size grid?" I asked. The students indicated their agreement.

"That makes it a method that is generalized," I said. "It works for all grids, not just the ones we've been exploring. Algebra is a way of describing a generalized method. Let me show you how I could translate Halbert's method to algebra."

"That wasn't Halbert's method," Jason blurted out. "That was my idea." Jason's need for attention was characteristic of him.

"Yes," I acknowledged, "Halbert described the method you first reported in a more generalized way." That seemed to satisfy Jason for the moment.

"Because I don't know yet how many squares are on the edge," I continued, "I'm going to use a letter instead of a number. I'll use *e* to represent the number of squares on the edge. What shall I use to represent the number of squares in the border?"

"*B*," several students called out together.

"So I'll write Halbert's generalization like this," I said, and wrote on the board:

$$e \times 4 - 4 = b$$

"This formula says that if I multiply the number of squares on the edge, e, by 4 and then subtract 4, I'll know how many squares are in the border. That's an algebraic way to express the method."

There were positive reactions to this. "That's cool." "I get it." "That's not so hard." "Yeah, that's OK."

Andy raised his hand. "I learned that you could write $4e$ instead of e times 4," he said. I wrote that on the board and explained to the class that this was OK, that when a number and letter were written together like this, it was understood that it meant to multiply.

"Could you write $e4$ instead?" Juliette asked.

"No," I answered, "the number is written first. Also, though it's not necessary, you could use parentheses as punctuation when you use the times sign." I wrote the first formula again with parentheses. Now there were three formulas on the board:

$$e \times 4 - 4 = b$$
$$4e - 4 = b$$
$$(e \times 4) - 4 = b$$

"Each of these three formulas," I said, "describes how to apply Jason's original method to a grid of any size. This formula doesn't describe the other methods. Tami, for example, didn't multiply the edge by 4 in her method. She removed one corner from each edge first before multiplying."

"How do you write that in algebra?" Hasani asked.

"I know," Jesse said. "You have to use e minus 1 instead of e." I wrote "$e - 1$" on the board.

"Can you explain why that makes sense to you?" I asked.

"Because you take 1 away from each edge, and that's e minus 1," Jesse said.

"I get it," Hasani said.

1 $4e-4=b$
2 $e+e-1+e-1+e-2=b$
3 $4(e-1)=b$
4 $e\,e(e-2)\times 2=b$
5 $2e+e-2\times 2=b$
6 $e-2\times 4+4=b$

Border problem in Algebra

1. $(e \times 4)-4=b$
2. $e+e-1+e-1+e-2=b$
3. $(e-1)\times 4=b$
4. $(e\times e)-(e-2)^2=b$
5. $e\times 2+e-2+e-2=b$
6. $e-2+e-2+e-2+e-2+4=b$

Working in pairs, students wrote formulas to describe each of the methods.

I continued with directions. "Working with your partner, try to write formulas for the other five methods," I said. "Though you'll work together, each of you should record in your own notebook. If you get stuck, try writing an explanation in words first and then translate it to a formula."

The students were willing to try this assignment. Their discussions were animated. Most found some way to write a formula for each method. The method that gave most of the students difficulty was Zack's method of removing the middle and leaving just the border.

I collected their notebooks at the end of class to see what they had accomplished. When I looked at their work, I found a great variety in what they had written. Some formulas were correct and others were incorrect.

As with any lesson, I now had a decision to make about what to do. It made no sense to belabor the work for those who understood. And it did not make sense to belabor the work for those who did not yet understand. My goal for this instruction was to give the students a beginning experience with algebra that I could build on over time. Though I would have liked each student to be able to write formulas easily and correctly, for the methods we had been studying, this wasn't the case. (It rarely is in a class.)

What I decided to do was to give the formulas some more attention for part of the next class, then have them write about their individual experiences with the lesson.

Day 5

I began class by writing on the board five different formulas the students had written for method number 5—adding the top and bottom edges and then the sides, each 2 less than the top and bottom.

$$e + e + e - 2 + e - 2 = b$$
$$e \times 2 + e - 2 + e - 2 = b$$
$$2e + (e - 2) \times 2 = b$$
$$e + e + (e - 2) + (e - 2) = b$$
$$e \times 2 + e - 2 \times 2 = b$$

"What's the same and what's different about each of these algebraic formulas?" I asked.

"They all explain Chris's method," Michael said.

"Some use times and some don't," Steven said.

"Some have parentheses and some don't," Andy said.

"What do the parentheses do?" I asked.

"They make it clearer," Juliette said.

"It looks neater," Jesse added.

"More than that," I said, "parentheses are sometimes necessary." I pointed out to the students that in the last formula I had written on the board, it wasn't clear that I was to multiply the quantity of e minus 2 by 2. I added parentheses. Then I asked them what else they noticed about the formulas.

"They all work," Zack said.

"With the parentheses added to the last formula, I agree with you," I said. "However, all the formulas you wrote did not work. I'm going to give you a chance to take another look at what you've done, this time with other students so you can get different points of view. I'm going to organize you into groups of three instead of partners so you have more formulas to compare."

I did this and had the groups work for about ten minutes. During this time, I talked with the groups that asked for help. There was much sharing of ideas and much erasing in notebooks.

When I interrupted them, I asked them to write a lesson log about this experience. Writing lesson logs is an idea I learned about in a workshop on writing in math and science I attended at the Lawrence Hall of Science in Berkeley, Calif. The workshop was given by teachers involved with the Bay Area Writing Project.

In my adaptation of the idea, I have students describe what has happened during the lesson and what they have learned so that it would be possible for someone who is not present to have a sense of what has occurred. I ask that they include as many details and specific examples as possible. I also require that their logs be about one page in length. The form I ask them to use is this:

Date of lesson:
General title: (Describing the lesson)
Description: (What went on)
Math content: (What you learned about)

This was the students' third experience writing a lesson log, and their approach to writing was more purposeful than in either of their earlier attempts. I circulated as students wrote. When someone asked me if what he or she had written was OK or enough, I had a stock answer. "That's a good beginning," I would say. Then I would add, "Include more examples," or "Explain more about what you learned," or "Write more about the mathematics."

As with all their writing, I gained insights into their thinking. Their comments about algebra revealed their perceptions and understanding.

Juliette wrote: *We learned about the beginning of algerbra and how to take a normal problem and change it to an algerbra equation.*

Chris wrote: *I learned about basic Algebra. How to make equastions and how to seperate algebraic promblems. I at first had a hard time figuring out what e & b ment but I figured out what they ment at the end. For example, e ment the question and b ment the answer.*

Hasani wrote: *Algebra is not easy but its not as hard as people say it is if you put your mind in.*

From Krystal: *I think algebra is finding new ways to write things and solve things, shortening things and extending things. I also think it helps you use your mind in difficult situations.*

From Jesse: *Algebra is a way of generalizing mathematics. You substitute letters for numbers. When we did it we used E for Edge and B for Border. This activity was just right because we had small groups.*

Michael wrote: *We learned about different ways to solve problems and there is no best way although one may fit you better than another. Some of us learned something about algebra.*

Jason wrote: *Algebra is the way to write how to figure out the problems. There are many ways to write algebra. Algebra is a way of adding, subtracting, dividing and multiplying letters to mean something with numbers.*

From Alesia: *I think algebra is letters that discribe numbers.*

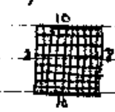
Halbert wrote: *I think algebra is another way to find the answer to a math problem.*

MATH LESSON LOG

Date. Oct. 13, 14, 17, 18, 19

The Border problem

Description: First Mrs. Burns had us try to figure out all the different ways to find the border of a square that has 10 on each side. First thing that popped into everybody's head was 40 but you have to eliminate the corners which we already counted. The way I found the easiest was to count the 2 horizontal sides which are ten (with corners) and then count the two vertical sides which is 8 which you multiply by 2 and add to the 2 tens which add up to 20.



The next day we worked on improving the way that Mrs. Burns had written problems on a sheet of paper so they were easy to understand. Next we wrote all 6 of the ways to figure the border out into ~~algebraic~~ equations for algerbra. Zach's was the hardest to figure out. His was to multiply the two edges (10x10, exe) and then that gives you the area of the whole square and then he took the area and subtracted just the middle which was everything but the border which turned out the equation was exe - e-2 = b. This unit helped us understand equations better. It helped us not just do the problem, but understand how we did the problem.

Math Content: We learned about the beginning ~~and~~ of algerbra and how to take a normal problem and change it to an algerbra equation.

Logs help students reflect on their learning experiences and also help teachers assess students' understandings and misconceptions.

Math Lesson Log

October, 14, 17, 18, 19

ALGEBRA =

THE BORDER PROBLEM

Description: We try to find out The border of a square by counting the edge, add the four edges, and then subtract the four corners. Here is an example. We cut it down, shortened it into a different form.

$e + e + e + e - 4 = b$

$(e \times 4) - 4 = b$

$4e - 4 = b$

Math Content: I think algebra is finding new ways to write things and solve things, Shortening things and extending things. I also think it helps you use your mind in difficult situations.

Math Lesson Log.

Date: October 13, 14, 17, 18, 19.

Title: Algebra: The Border Problem.

Description: We have been working on the border problem for the past five days. We had a square and we tried to figure out the area of the square, when we found it it was 36. Then we had to figure out how someone got that.

Math Content: I learned about basic Algebra. How to make equations and how to operate algebra problems. I at first had a hard time figuring out what e and b meant but I figured out what they meant at the end. For example e meant the question and b meant the answer.

$(e-1)+(e-1)+(e-1)+(e-1)=B$

This equations means, edge-1 + edge-1 plus edge-1 plus edge-1, then B would be the final answer.

Final Thoughts

Looking at the students' notebooks later, I noticed that although there was improvement in their formulas, there were still errors. Clearly, not all the students had "mastered" generalizing arithmetic procedures to algebraic representations. This did not concern me.

I've come to understand that partially grasped ideas and periods of confusion are natural to the learning process. I've come to understand that students' mathematical knowledge is developed, elaborated, deepened, and made more complete over time. I've come to understand that I cannot expect all students to get the same thing out of the same experience. (The *Mathematics Model Curriculum Guide, Kindergarten Through Grade Eight* from the California State Department of Education [1987] includes these notions as guiding principles for teaching for understanding.)

I feel that students benefited from the lesson in different ways. Zack, Jesse, Juliette, Krystal, and others gained insights and facility with algebraic notation. Sinead, Quinn, and Rachel learned that there was more than one way to solve a problem. Halbert and Joe learned more about the benefits of working collaboratively with others. Tri gained experience with expressing his ideas about math in English. Hasani learned he could do something he thought was hard.

And I learned more about each of my students, which will help me better meet their needs as the year progresses.